Mathematical and Computational Applications, Vol. 18, No. 3, pp. 548-557, 2013

ON THE THERMAL BUCKLING BEHAVIOR OF LAMINATED HYBRID COMPOSITE PLATES DUE TO SQUARE/CIRCULAR CUT-OUTS

Ahmet Erkliğ[1,]*, Eyüp Yeter[1]

[1]Department of Mechanical Engineering, University of Gaziantep, 27310, Gaziantep, Turkey. erklig@gantep.edu.tr

Abstract- Cut-outs like circular, rectangular, elliptical, triangular are generally used in composite structures due to access ports for mechanical and electrical systems, damage inspection, to serve as doors and windows, and sometimes to reduce the overall weight of the structure. In this paper the effects of cut-outs on the thermal buckling behavior of hybrid composite plates in cross-ply and angle-ply laminate are presented. The effects of eccentric cut-out size in different plate aspect ratios and boundary conditions on the thermal buckling behavior of the cross-ply and angle-ply laminated hybrid composite plates are also investigated. Finite element analysis is also performed to calculate thermal buckling temperatures for Kevlar/Epoxy, Boron/Epoxy and E-glass/Epoxy. Several outcomes and behavioral characteristics are discussed. These outcomes include the effects of cut-out size, shape, plate aspect ratio and boundary conditions.

Key Words- Thermal buckling, Hybrid composite plates, Cut-out, Finite element

1. INTRODUCTION

Fiber reinforced composites structures are used in aerospace, marine and automotive applications due to their light weight and directional properties. During the operation life of vehicles, high temperature throughout their structure is to be experienced. As a result of this environmental condition, thermal buckling can occur without the application of mechanical loads. Sometimes thermal stability of composite laminates is one of the factors governing their design.

There are many publications about thermal buckling of composite plates. Murphy and Ferreira [1] presented the results of a thermal buckling analysis of a clamped rectangular plates based on energy considerations. Shariyat [2] worked on the thermal buckling analysis of rectangular composite multilayered plates under uniform temperature rise by using layer-wise plate theory. Kabir *et al*. [3] presented an analytical solution to thermal buckling response of moderately thick symmetric angle-ply laminated, rectangular plate which is clamped from all the edges. Li *et al*. [4] investigated the axisymmetric vibrations of a statically buckled polar orthotropic circular plate due to uniform temperature rise. Laura and Rossit [5] worked on thermal bending of thin, anisotropic, clamped elliptic plates and their study deals with the exact analytical solution of thermal bending. Kalyan and Bhaskar [6] studied on the buckling of rectangular orthotropic plates subjected to non-uniform compressive loads using Galerkin method. Lee [7] derived governing buckling equations from the variational principle and a finite element method to analyze thermal buckling of laminated composites by using a layer-wise theory. Lee and Lee [8] investigated the behaviors of thermally post-buckled anisotropic plates. The finite element model is used based on the first-order shear deformable plate theory and von Karman strain-displacement relation to account for large deflection. Prabhu and Dhanaraj [9] researched the thermal buckling of symmetric cross-ply, symmetric angle-ply laminated composite plates using the finite element method based on the Reissner-Mindlin first order shear deformation theory. Huang and Tauchert [10] investigated the buckling behavior of moderately thick symmetric angle-ply laminates having clamped edges, subjected to a uniform temperature rise. Nath and Shukla [11] investigated the buckling and post-buckling analysis of the moderately thick angle ply laminated composite rectangular plates subjected to combined in-plane mechanical load and temperature gradient across the thickness. Shiau *et al*. [12] studied thermal buckling behavior of composite laminated plates by making the use of finite element method. The thermal buckling mode shapes of cross-ply and angle-ply laminates with various E1 /E2 ratios, aspect ratios, fiber angle, stacking sequence and boundary condition were studied in detail. Jones [13] worked on to derive simple solutions to the most fundamental thermal buckling problems for uniformly heated unidirectional and symmetric cross-ply laminated fiber-reinforced composite rectangular plates that are restrained in-plane at their edges in a single direction on two of the four edges, but are free to rotate on all edges. Barton [14] presented an approximate closed-form solution to compute the thermal buckling response of a symmetric angle-ply laminates that are clamped in one edge and free along the other edge. Results are compared with Rayleigh-Ritz method solutions.

Aydogdu [15] researched the thermal buckling analysis of rectangular cross-ply laminated beams subjected to different sets of boundary conditions by applying the Ritz method. Yapici [16] studied on the thermal buckling analysis of symmetric and antisymmetric angle-ply laminated hybrid composite plates with an inclined crack subjected to a uniform temperature rise. Avci *et al*. [17] performed the thermal buckling analysis of symmetric and antisymmetric cross-ply laminated hybrid composite plates with an inclined crack subjected to a uniform temperature rise. Avci *et al*. [18] extended their work on the thermal buckling analysis of symmetric and antisymmetric laminated composite plates with clamped and simply supported edges, and containing a hole. Then, Avci *et al*. [19] studied on the thermal buckling analysis of symmetric and antisymmetric cross-ply laminated hybrid composite plates with a hole subjected to a uniform temperature rise for different boundary conditions. Sahin [20] worked on thermal buckling analysis of symmetric and antisymmetric laminated hybrid composite plates with a hole subjected to a uniform temperature rise for different boundary conditions. Akbulut and Sayman [21] used finite element method to investigate buckling behavior of laminated composite plates with central square openings for various boundary conditions and stacking sequences. Erklig and Yeter [22] studied on the effects of different cutouts on the mechanical buckling behavior of plates made of polymer matrix composites. Circular, rectangular, square, elliptical, triangular were used in experimental and finite element analysis.

In this paper the effects of rectangular and circular cut-outs on the thermal buckling behavior of hybrid composite plates in cross-ply and angle-ply laminates are investigated based on the first order shear deformation theory. This study also contains the effect of eccentric cut-out size in different plate aspect ratios and boundary conditions on the thermal buckling behavior of the cross-ply and angle-ply laminated hybrid composite plates. The finite element method is used to calculate critical thermal buckling temperatures for Kevlar/Epoxy, Boron/Epoxy and E-glass/Epoxy.

2. FINITE ELEMENT FORMULATION

The first order shear deformation theory, used in the analysis, assumes a linear variation of in-plane displacement fields, u and v through the depth of the plate. The transverse displacement w(x,y) is assumed to be constant throughout the thickness of the plate. The displacement field of a rectangular shear deformable plate can be expressed as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\{w_{,x} + \gamma_x(x,y)\} \\ v(x,y,z) &= v_0(x,y) + z\{w_{,y} + \gamma_y(x,y)\} \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

From the large displacement theory, the strain-displacement relations can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ v_{,x} + u_{,y} \end{Bmatrix} + \begin{Bmatrix} w_{,x}^2/2 \\ w_{,y}^2/2 \\ w_{,x} w_{,y} \end{Bmatrix} - z \begin{Bmatrix} w_{,xx} + \gamma_{x,x} \\ w_{,yy} + \gamma_{y,y} \\ 2w_{,xy} + \gamma_{y,x} + \gamma_{x,y} \end{Bmatrix} = \{\varepsilon^0\} + z\{\kappa\} \tag{2a}$$

and

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} -\gamma_x \\ -\gamma_y \end{Bmatrix} \tag{2b}$$

where $(\ )_{,x}$ and $(\ )_{,y}$ represent partial differentiation with respect to x and y. The relationship between stress resultants and the strain terms for the laminated plate may be written as

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{Bmatrix} \gamma^0 \\ \kappa \end{Bmatrix} - \begin{Bmatrix} N^T \\ M^T \end{Bmatrix} \tag{3}$$

and the shear resultants may be written as

$$\begin{Bmatrix} R_x \\ R_y \end{Bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{54} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} dz$$

where stretching, stretching-bending and bending stiffnesses are defined as

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [Q_{ij}]^k (1, z, z^2) dz$$

where $[Q_{ij}]^k$ represents transformed plane stress reduced stiffness matrix of the kth lamina, which is a function of ply-angles $(\theta_1, \theta_2, \theta_3 \cdots \theta_n)$ and $h_k - h_{k-1}$ is the thickness of the lamina (Figure1).

The thermal load vector is given by the expression

$$\{N^T, M^T\} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [Q_{ij}].\{\alpha\}.T(z).(1,z)dz$$

Following the procedure given in Reference [23] (equating the first variation of total potential energy to zero) the governing equation of the problem may be written as

$$\{[K_s] + \lambda[K_g]\}\{\delta\} = \{0\}$$

where $[K_s]$ and $[K_g]$ are the linear and geometric stiffness matrices respectively. The lowest eigenvalue (λ) gives the buckling temperature T_c.

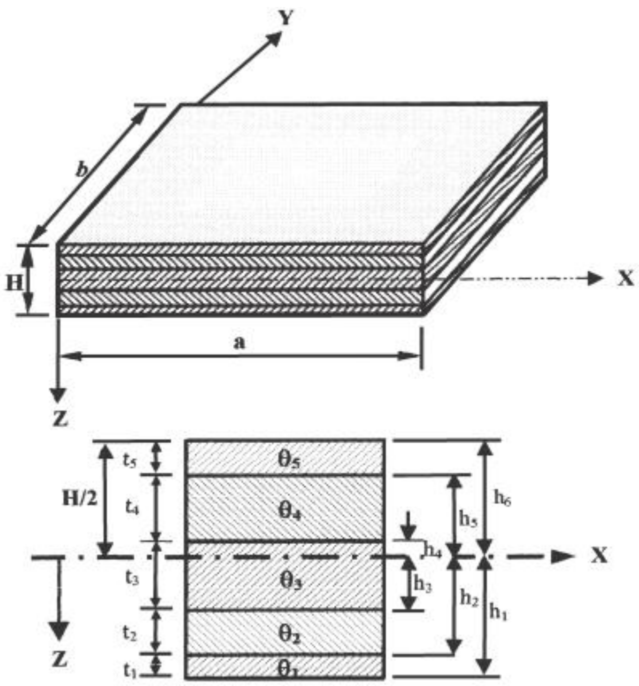

Figure 1. Geometry of laminated composite plate

3. FINITE ELEMENT SIMULATION

Equivalent buckling analysis is performed by using finite element analysis program. Finite element analysis of square cut-out composite lamina is performed by using via ANSYS 11.0 with an first order shear deformation element (SHELL91).

Critical thermal buckling values were found out by computational method. To test the correctness of the finite element model, simply supported $(\pm 45_3)_T$ laminated rectangular plates with a/h=100 and a/h=80 ratios having the following material properties are investigated:

$$\frac{E_2}{E_1} = 0.081, \quad \frac{G_{12}}{E_1} = \frac{G_{13}}{E_1} = 0.031, \frac{G_{12}}{E_1} = 0.0304, \; \nu_{12} = 0.21$$

$$\alpha_1 = -0.21 \times 10^{-6} ({}^0C^{-1}), \qquad \alpha_2 = 16 \times 10^{-6} ({}^0C^{-1}).$$

The finite element model thermal buckling results are compared in table 1 with the first-order shear deformation theory results of reference [2,24] and the higher-order shear deformation theory results of reference [25]. As it can be seen in table 1, finite element model results are nearly same the reference results.

Table 1. Buckling temperature variations with the (a/h) ratios (T_{cr} (^{0}C))

a/h	FEA	Shariyat [2]	Chen and Liu [24]	Shen [25]
80	189.1	191.6	191	191.9
100	121.5	122.8	122.7	123

The geometric model of the laminated plate with parametric dimensions is shown in Figure 2. Laminated hybrid composite plate material properties are given in table 2. The critical thermal buckling values are evaluated for cross-ply and angle-ply layers bonded symmetrically with different boundary conditions. The square hole edge is taken as free. The stacking sequences of hybrid composite plates are listed in Table 3. The letters B, K and G represent Boron/Epoxy, Kevlar/Epoxy and E-glass/Epoxy composites, respectively. Each layer has 0.15 mm thickness and rectangular plate and square hole is selected (a/b=1 and c/d=1). c/b ratio represents the ratio of the square hole size to length of one side of composite plate and h/b ratio represents the ratio of total thickness to length of one side of composite plate.

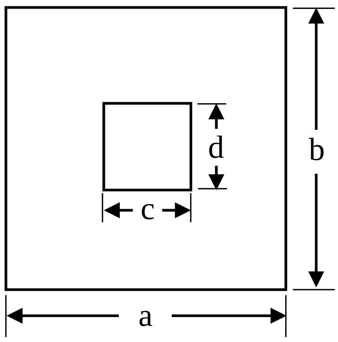

Figure 2. Dimensions of rectangular laminated hybrid composite plate with square cut-out.

Table 2. Material properties

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	υ_{12}	α_1 ($^0C^{-1}$)	α_2 ($^0C^{-1}$)
Boron/Epoxy	204	18.5	5.6	0.23	$0.61x10^{-5}$	$3.03x10^{-5}$
Kevlar/Epoxy	76	5.5	2.3	0.34	$-0.4x10^{-5}$	$7.6x10^{-5}$
E-glass/Epoxy	15	6	3	0.30	$0.7x10^{-5}$	$2.3x10^{-5}$

Table 3 Stacking sequence

Type 1	$(0_G/90_B/0_K/90_G)_S$
Type 2	$(15_G/-15_B/15_K/-15_G)_S$
Type 3	$(30_G/-30_B/30_K/-30_G)_S$
Type 4	$(45_G/-45_B/45_K/-45_G)_S$
Type 5	$(60_G/-60_B/60_K/-60_G)_S$

4. EFFECT OF CUT-OUT SIZE AND SHAPE

In this section the effect of eccentric square cut-out size is taken into consideration. The plate dimension is 120 x 120 mm. The plate normal is aligned in z direction and plate area is located on xy plane. The four different boundary conditions are applied on each four edges. The plates are meshed with quadratic composite shell elements as illustrated in Figure 3.

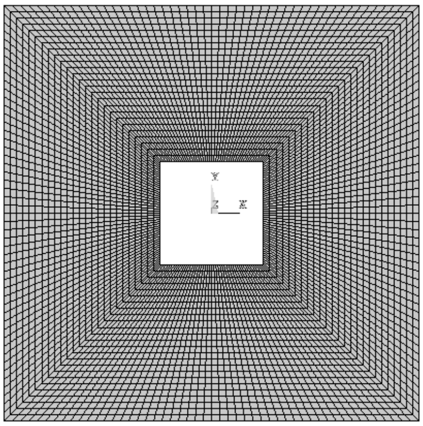

Figure 3. Typical mesh for a rectangular laminated plate with a square cut-out.

Figure 4 gives the buckling temperatures for cross-ply and angle-ply laminated hybrid composite plates with the cut-out width to the laminate width ratio (d/b) varying from 0.0 to 0.5. The cut-out width and length ratio (c/d) is taken as 1. Buckling temperature is increased by changing the cut-out size from 0.0 to 0.5 up to 55%. The buckling temperatures don't have any considerable change for the cut-out size from 0.0 to 0.25. The buckling temperature of perfect plate is initially decreased after the cut-out opened. Type 4 ($(45_G/-45_B/45_K/-45_G)_S$) gives the higher buckling temperature results.

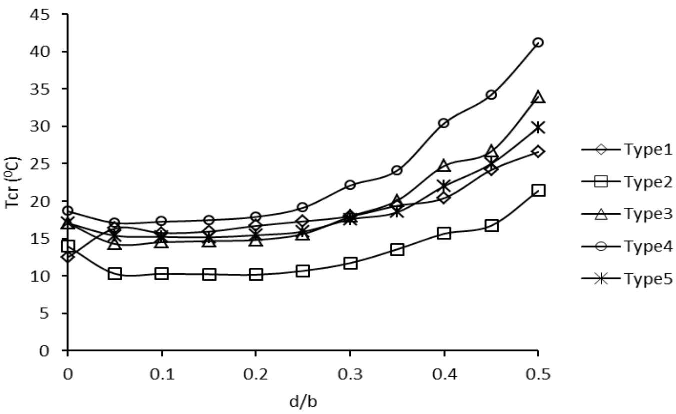

Figure 4. Variation of buckling temperatures with square cut-out dimensions

Because of design requirements and philosophy different cut-out shape may be used. The effect of circular cut-out is also taken into account. It is assumed that cut-out to be located at the center of line of the rectangular plates. The boundary conditions and stacking sequences are taken as constant.

Thermal buckling temperatures for cross-ply and angle-ply laminated hybrid composite plates with the cut-out diameter to the laminate width ratio (d/b) varying from 0.0 to 0.5 are displayed in Figure 5. It is seen that larger cut-out area causes the higher buckling temperature. Type 1 gives the better buckling temperature for the circular cut-out. Type 2 and type 3 plates give the worst result compare to the perfect plate. Sqaure and circular cut-outs are compared in Figure 6 for material type 4. As can be seen in the figure square cut-out has greater thermal buckling load against circular cut-outs for higher d/b ratios.

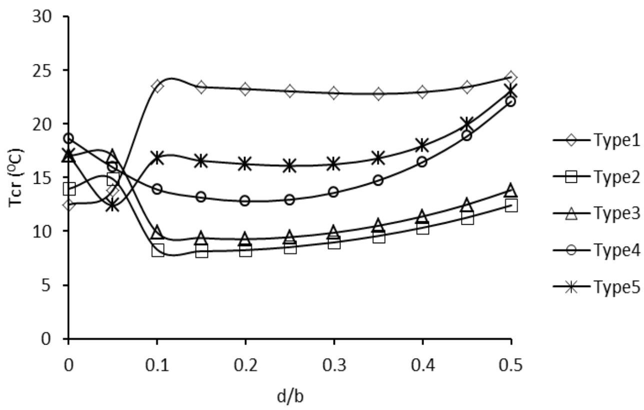

Figure 5. Variation of buckling temperatures with circular cut-out dimensions

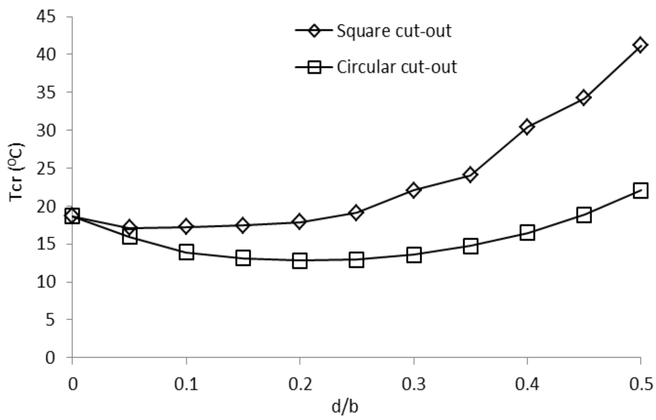

Figure 6. Variation of buckling temperatures for material type 4

5. EFFECT OF PLATE ASPECT RATIO

This section deals with the buckling behavior of perforated cross-ply and angle ply laminated plates in different plate aspect ratios. In this study the plate aspect ratios is selected to have integer value i.e. a/b = 1, 2, 3. The widths of these plates are equal to 120 mm, and all of the cut-outs are positioned in the center of the plates. The results of buckling temperature in different cut-out size are shown in Figure 6. As mentioned before the buckling temperature of the rectangular plate decreases with cut-out dimensions.

The buckling temperature change of plate with aspect ratios of 2 and 3 for d/b from 0 to 0.5 is not affected by cut-out size. Results show that the buckling temperature for the aspect ratio of 1 increases with increasing the cut-out size but aspect ratio 2 and 3 is not much affected by cut-out size.

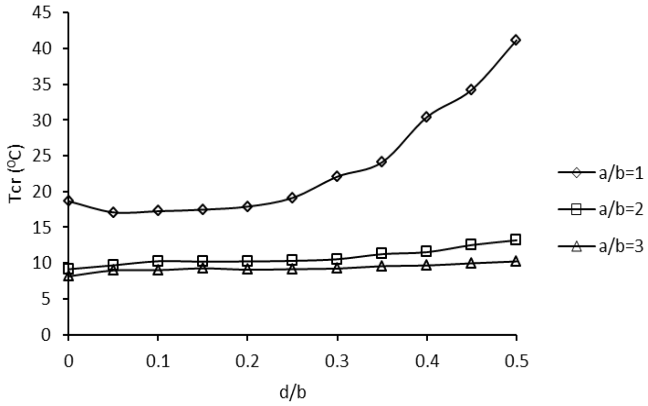

Figure 6 Variation of buckling temperature with circular cut-out dimensions in different plate aspect ratios.

6. EFFECT OF BOUNDARY CONDITION

The boundary condition has a significant effect on buckling temperatures. In this study the cross-ply laminated hybrid composite plates is evaluated at four different boundary conditions; as four edges simply supported (SSSS), two edges simply supported and two edges are free (SFSF), four edges clamped (CCCC), two edges clamped and two edges are free (CFCF). The size of plate is 120 mm. Figure 7 shows the results of buckling temperatures in different cut-out size and different boundary conditions. Because of the rigidity of clamped boundary condition buckling temperature is higher than simply supported boundary conditions.

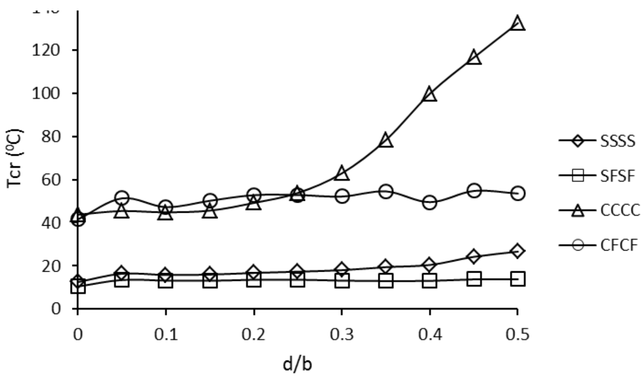

Figure 7. Variation of buckling temperatures with square cut-out dimensions in different boundary conditions.

8. CONCLUSION

This study considers the buckling response of laminated rectangular perforated hybrid composite plates under temperature loading with different boundary conditions.

The laminated composite plates have varying d/b ratio, aspect ratio, cut out shape and ply orientation. From the present study, the following conclusions can be made:

- The buckling temperature of rectangular plates containing square cut-out increases by the increment of cut-out dimension.
- It is seen that the different fiber orientation angles affected the critical buckling temperature. The plate with square cut-out and with $(45_G/-45_B/45_K/-45_G)_S$ layup has the highest buckling temperature and the plate with $(15_G/-15_B/15_K/-15_G)_S$ layup has the lowest buckling temperature.
- Imperforated plate buckling temperature is greater than small cut-outs. After cut-out ratio (d/b) 0.1 buckling temperature is increasing.
- Plate with square cut-out buckling temperature is higher than plate with circular cut-out. The type 4 gives the highest buckling temperature in a 0.5 cut-out ratio.
- By selecting integer value for plate aspect ratio the buckling temperature is increased by selecting the higher value for aspect ratio.
- The buckling temperatures of perforated hybrid composite plates are highly influenced by its boundary conditions. The buckling temperature for the plate with fully clamped boundary condition is higher than the buckling temperature for the plate with simply supported boundary conditions.

9. REFERENCES

1. K. D. Murphy, and D. Ferreira, Thermal buckling of rectangular plates, *International Journal of Solids and Structures* **38**, 3979-3994, 2001.
2. M. Shariyat, Thermal buckling analysis of rectangular composite plates with temperature dependent properties based on a layerwise theory, *Thin-Walled Structures* **45**, 439-452, 2007.
3. H. R. H. Kabir, M. A. M. Hamad, J. Al-Duaij, and M. J. John, Thermal buckling response of all-edge clamped rectangular with symmetric angle-ply lamination, *Composite Structures* **79**, 148-155, 2007.
4. S. R. Li, R. C. Batra, and L. Sheng, Vibration of thermally post-buckled orthotropic circular plates, *Journal of Thermal Stresses* **30**, 43-57, 2007.
5. P. A. A. Laura, and C.A. Rossit, Thermal bending of thin, anisotropic, clamped elliptic plates, *Ocean Engineering* **29**, 485-488, 1999.
6. J. B. Kalyan, and K. Bhaskar, An analytical parametric study on buckling of non-uniformly compressed orthotropic rectangular plates, *Composite Structures* **82**, 10-18, 2006.
7. J. Lee, Thermally induced buckling of laminated composites by a layer wise theory, *Computers and Structures,* **65(6),** 917-922, 1997.
8. D. M. Lee, and I. Lee: Vibration behaviors of thermally post-buckled anisotropic plates using first order shear deformable plate theory, *Computers and Structures* **63**, 371-378, 1997.
9. M. R. Prabhu, and R. Dhanaraj: Thermal buckling of laminated composite plates, *Computers and Structures* **53**, 1193-1204, 1994.

10. N. N. Huang, and T. R. Tauchert, Thermal buckling of clamped symmetrical laminated plates, *Thin-Walled Structures* **13**, 259-273, 1992.
11. Y. Nath, and K. K. Shukla, Postbuckling of angle ply laminated plates under thermal loading, *Communications in Nonlinear Science and Numerical Simulation* **6**, 1-16, 2001.
12. L. C. Shiau, S. Y. Kuo, and C. Y. Chen, Thermal buckling behavior of composite laminated plates, *Composite Structures* **92**, 508-514, 2010.
13. R. M. Jones, Thermal buckling of uniformly heated unidirectional and symmetric cross-ply laminated fiber-reinforced composite uniaxial in-plane restrained simply supported rectangular plates, *Composites: Part A* **36**, 1355–1367, 2005.
14. O. Barton, Approximate method for buckling of symmetric composite laminates under thermal loading, *Journal of Thermoplastic Composite Materials* **22**, 305-320, 2009.
15. M. Aydogdu, Thermal buckling analysis of cross-ply laminated composite beams with general boundary conditions, *Composites Science and Technology* **67**, 1096-1104, 2007.
16. A. Yapici, Thermal buckling behavior of hybrid-composite angle-ply laminated plates with an inclined crack, *Mechanics of Composite Materials* **41**, 131-138, 2005
17. A. Avci, O. S. Sahin, and N. Ataberk, Thermal buckling behavior of cross- ply hybrid composite laminates with inclined crack, *Composites Science and Technology* **66**, 2965-2970, 2006.
18. A. Avci, S.Kaya, and B. Daghan, Thermal buckling of rectangular laminated plates with a hole, *Journal of Reinforced Plastics and Composites* **24**, 259-272, 2005.
19. A. Avci, O. S. Sahin and M. Uyaner, Thermal buckling of hybrid laminated composite plates with a hole, *Composite Structures* **68**, 247-254, 2005.
20. O. S. Sahin, Thermal buckling of hybrid angle-ply laminated composite plates with a hole, *Composite Science and Technology* **65**, 1780-1790, 2005.
21. H. Akbulut, and O. Sayman, An investigation on buckling of laminated plates with a central square hole, *Journal of Reinforced Plastics and Composites* **20**, 1112-1124, 2001.
22. O. C. Zienkiewicz, *The Finite Element Method*, Third edition, McGraw Hill Publishing Company, England, 1971.
23. A.Erkliğ, and E. Yeter, The effects of cutouts on buckling behavior of composite plates, *Science and Engineering of Composite Materials* **19**(3), 323–330, 2012
24. W. C. Chen, W. H. Liu, Thermal buckling of antisymmetric angle-ply laminated plates-an analytical Levy-type solution, *Journal of Thermal Stress* **16**, 401–19, 1993.
25. H. S. Shen, Thermal postbuckling behavior of imperfect shear deformable laminated plates with temperature-dependent properties, *Computer Methods in Applied Mechanics and Engineering* **190**, 5377–90, 2001.