Mathematical and Computational Applications, Vol. 18, No. 3, pp. 330-339, 2013

A FULL FACTORIAL DESIGN BASED DESIRABILITY FUNCTION APPROACH FOR OPTIMIZATION OF PROPERTIES OF C 40/50 CONCRETE CLASS

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Abstract- In this study a full factorial design (FFD) based desirability function approach (DFA) was used to the modeling of determined quality criteria of C 40/50 (C50). A FFD based DFA was also applied to determine optimal mixture proportions of C50. The mixture proportion modeled by using FFD was determined as the function of variables such as water to binder materials ratio, coarse aggregate (II) to total aggregate ratio, the percentage of superplasticizer content and fly ash amount. The properties of C50 were identified as that the slump flow and 28th day compressive strength. The model results were tested with experimental runs. The results showed that the determined regression meta-models were useful for prediction of properties of C50 with the mixture parameters. The results also showed that the FFD based DFA are effective in solving the mixture proportions optimization problem.

Key Words- Desirability Function Approach, Full Factorial Design, C 40/50 Concrete Class, Optimization

1. INTRODUCTION

The optimization of a Ready Mixed Concrete (RMC) mixture for determination of the desired quality is an important issue in the field of material and design engineering [1, 2]. In literature several optimization and modelling methods have been proposed on investigating the optimal mixture proportions for many concrete types. For investigating the effects of parameters on concrete, design of experiment is commonly used in literature. Some of those, Sonebi [3] modeled mix proportion parameters of underwater composite cement grouts using a factorial design. Muthukumar et al. [4] optimized the mechanical properties of polymer concrete and recommended the mix-designed based on design of experiment. Özbay *et al.* [5] investigated the mix proportions of high strength self compacting concrete by using Taguchi method. Correia *et al.* [6] assessed of the recycling potential of fresh concrete waste using a factorial design of experiments. Correia *et al*. [7] practiced a Factorial design used to model the compressive strength of mortars containing recycled rubber. Santilli *et al.* [8] applied a factorial design study to determine the significant parameters of fresh concrete lateral pressure and initial rate of pressure decay. Alqadi *et al.* [9] developed a self compacting concrete using contrast constant factorial design.

Multiple response problems include three stages: data gathering, modeling and optimization [10]. In optimization phase; FFD is widely practiced with DFA. Some examples of these applications can be given as followings. Paterakis *et al*. [11] evaluated and optimized some pellets characteristics using a 3^3 factorial design and desirability function. Mukherjee and Ray [12] applied an optimal process design of two-stage multiple responses grinding processes using desirability functions and metaheuristic technique. Gottipati and Mishra [13] optimized the process of adsorption of Cr (VI) on activated carbons prepared from plant precursors by a two-level full factorial design.

Within the scope of this study, it is desired to obtain optimal mixture proportions of C50 in Turkey. First of all, the criteria of determining the quality of the concrete, factors and levels that affect these performance criteria were identified to obtain optimal mixture proportions. A 2^4 full factorial design with two replicates was used for the optimization of dual responses such as slump flow and 28^{th} day compressive strength. An analysis of variance (ANOVA) test was used to find out the significance and percentage contribution of each parameter [14]. The mathematical model of quality criteria has been developed using regression analysis as a function of the water to binder materials ratio, coarse aggregate (II) to total aggregate ratio, the percentage of super plasticizer content and fly ash amount. DFA approach was applied to determine optimal mixture proportions of C50.

2. MATERIALS AND METHODS

2.1. Materials

The cement used in this research for the normal weight concrete is a CEM I 42.5 R has a specific gravity of 3.15 and weighs 350 kg. Fly ash used in this research with a specific gravity of 2.46 weighs 80 kg. Chemical composition of the binder materials is given in Table 1. Crushed and which has particle size smaller than 4 mm (I) was used as the fine aggregate. The fine aggregate ratio was fixed at 50 % in all experiments. Aggregate number (II) with a size between 4mm to 11.2 mm and aggregate number (III) with a size between 11.2 mm to 22.4 mm were used as coarse aggregate in the concrete mixtures. The fine and coarse aggregates have specific gravities of 2.75 and 2.77 and mean water absorptions of 1.5% and 0.9 %, respectively. Superplasticizer content is defined as the ratio of superplasticizer amount of 100 kg cement.

Table 1. Chemical composition of cement and fly ash

Chemical analysis	CEM I 42.5 R (%)	FLY ASH (%)
CaO	66.25	4.76
SiO_2	21.79	56.21
Al_2O_3	5.98	23.1
Fe_2O_3	2.51	6.51
SO_3	1.54	0.73
MgO	1.15	2.11
K_2O	0.61	2.53
Na_2O	0.15	0.27
Cl	0.0071	0.0018
Loss of ignition	3.71	2.24

A polycarboxylic type superplasticizer (SP) was used in all concrete mixtures and physical properties of SP are given in Table 2.

Table 2. Properties of the SP at 20^{0}C

Properties	Superplasticizer
Chemical description	Polycarboxylic type polymer
Color	Brown
Specific gravity (kg/L)	1.08 - 1.14
Chlorin content % (EN 480-10)	< 0.1

2.2. Proposed multi-response optimization and modeling framework

Optimization of the mixture parameters is aimed to increase the performance and productivity of ready-mixed concrete plant. There are 6 flow steps in performance optimization of ready-mixed concrete plant. This flow diagram is given in Figure 1. Also, FFD used in this study provides possibility of creating a model. Models which may be created give us the ability to predict responses for mixture parameters and two quality characteristics. Modeling and optimization of mixture proportions plays an important role on providing competitive advantage and customer satisfaction.

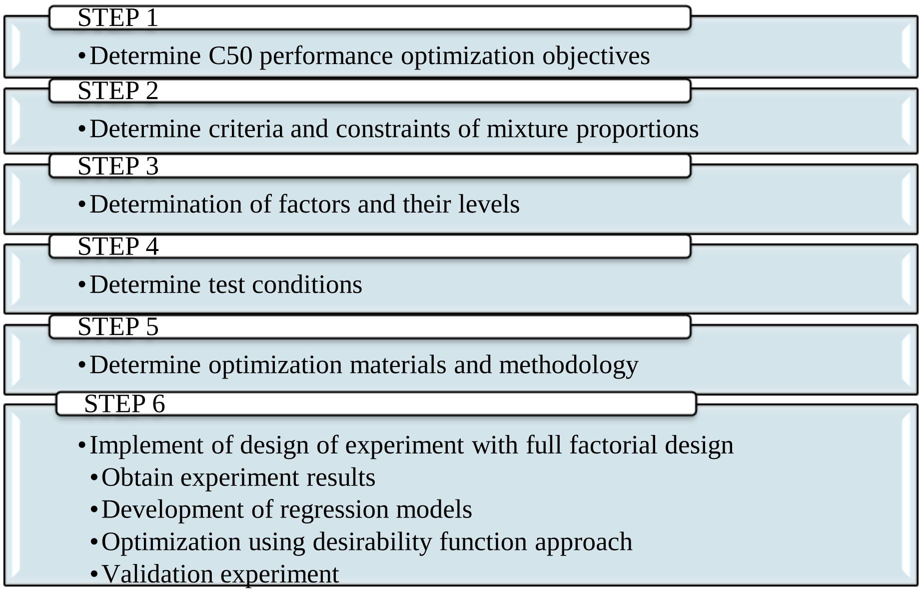

Figure 1. Proposed performance optimization framework

2.3. Determining criteria and constraints of optimization and modeling

Two quality characteristics were determined for C50 in the Ready-Mixed concrete plant (Table 3). The viscosity of produced concrete was evaluated through the slump flow test according to TS EN 12350/2 (Turkish standard) [15]. Slump flow range can be 10–220 mm for C50. Slump flow test is a kind of simple and fast implementation experiment both in laboratory and field [16]. The two performance criteria were identified respectively as the slump flow and 28th day compressive strength. Higher slump flow value gives higher workability of concrete. Therefore, the first criterion having effects on concrete quality is the slump flow value which should be maximized. For each concrete mixes, the compressive strength was determined on three 15 cm cubes for 28 days according to TS EN 12390/3 [17]. Each compressive strength experiment was an average of three 150 mm cube specimens. This criterion provides information about concrete durability [5, 18].

Table 3. Quality characteristic and their weights

Quality Characteristic	Symbol	Description	Type of concrete test	Target values	Weights
1	Y_1	Slump flow (cm)	Fresh concrete test	Larger is better	1
2	Y_2	Compressive strength (N/mm^2) 28 days	Hardened concrete test	Larger is better	1

The mixtures were prepared approximately in 4 min using a rotating planetary mixer. The total aggregate mixture weight is 1901 kg/m^3. The measured concrete temperature varies between 10C^0 and 16C^0.

2.4. Determination of factors and their levels

Four factors that each has two control levels affects these performance criteria are identified. Water to binder materials ratio, coarse aggregate (II) to total aggregate ratio, the percentage of superplasticizer content and fly ash amount were identified as two level factors. These factors are symbolized X_1, X_2, X_3 and X_4 respectively (Table 4).

Table 4. Levels of factors that affect quality characteristic

Factors	Description	Bounds -1(coded values)	 1(coded values)
		First bound	Second bound
X_1	Water to binder materials ratio	0.48	0,52
X_2	Coarse aggregate (II) to total aggregate ratio	0.28	0.32
X_3	Superplasticizer content (kg/m^3)	1.00	1.20
X_4	Fly ash amount (kg/m^3)	60	80

2.5. Full factorial design and desirability function

Experimental design is a statistical methodology used to analyze the effect of several factors simultaneously. It makes changes to the independent factors (input) to determine their effect on the dependent response (output). It not only determines the significant factors that affect the response, but also how these factors affect the response [19, 20].

The polynomial regression model which can be considered for the three input factors is given in Eq. (1) [19]. In addition to the main effects of the three factors, interactions among the factors were also included in the regression, as shown in Eq. (1) [20]:

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i} \sum_{j} \beta_{ij} x_i x_j + \beta_{123} x_1 x_2 x_3 + e, \quad i < j \tag{1}$$

Where in eq.1 Y is the predicted response, β_0 defines the offset term, β_i is the linear effect of factor i, β_{ij} is the two-factor interaction effect between factors i and j and β_{123} represents the three-factor interaction effect.

The desirability function approach transforms an estimated response (e.g., the *i*th estimated response $\hat{y}_i$) into a scale-free value, called a desirability (denoted as d_i for $\hat{y}_i$). It is a value between 0 and 1, and increases as the corresponding response value becomes more desirable. The overall desirability D, another value between 0 and 1, is defined by combining the individual desirability values (i.e., d_i's). Then, the optimal setting is determined by maximizing D [21]. In optimization study, different desirability functions are used depending on the selected criteria (maximum, minimum, target value assignment or have a certain range) for each response [22]. The desirability function for a larger-the-better (LB) - type response is defined as [23].

$$d_i\left(\hat{y}_i\right) = \begin{cases} 0 & \hat{y}_i(x) < L_i \\ \left[\dfrac{\hat{y}_i(x) - L_i}{T_i - L_i}\right]^s & L_i \le \hat{y}_i(x) < T_i \\ 1 & \hat{y}_i(x) > T_i \end{cases} \tag{2}$$

where $d_i(\hat{y}_i(x))$ is the desirability function of $\hat{y}_i(x)$, L_i is the lower bounds on the response, T_i is the desired target of the i^{th} response, where $L_i \le T_i$ and s_i is the parameters that determine the shape of $d_i(\hat{y}_i(x))$: if $s_i = 1$, the shape is linear; if $s_i > 1$, convex; and if $0 < s_i < 1$, concave. Derringer (1994) proposed a weighted geometric mean as a strategy to aggregate the individual d_i's:

$$D = \left(d_1^{\nu_1} \times d_2^{\nu_1} \times d_3^{\nu_1} \Lambda \times d_n^{\nu_1}\right)^{1/\sum \nu_i} \tag{3}$$

where ν_i is the relative weight of the *i*th response [3, 24].

3. RESULTS

In Table 5, columns 2–5 represent the four control factors and their levels. In this study, a full factorial design (2^4) was used to implement the experiments and results are given in Table 5.

Table 5. Findings were obtained in all experiments

Exp. No	FFD (2^4) uncoded variables				Y_1 mm	Y_1 mm	Y_2 (N/mm^2)	Y_2 (N/mm^2)
	X_1	X_2	X_3	X_4	Replicate 1	Replicate 2	Replicate 1	Replicate 2
1	0.48	0.28	1.0	60	90	100	54.3	54.1
2	0.52	0.28	1.0	60	170	160	52.2	52.0
3	0.48	0.32	1.0	60	70	80	54.0	53.8
4	0.52	0.32	1.0	60	150	150	53.2	53.0
5	0.48	0.28	1.2	60	110	110	53.0	53.0
6	0.52	0.28	1.2	60	200	180	51.0	51.3
7	0.48	0.32	1.2	60	100	100	51.7	51.4
8	0.52	0.32	1.2	60	180	170	51.1	51.5
9	0.48	0.28	1.0	80	100	110	54.5	54.3
10	0.52	0.28	1.0	80	190	190	52.5	52.5
11	0.48	0.32	1.0	80	80	90	54.3	54.6
12	0.52	0.32	1.0	80	170	170	53.8	53.8
13	0.48	0.28	1.2	80	130	130	53.4	53.2
14	0.52	0.28	1.2	80	240	220	51.4	51.2
15	0.48	0.32	1.2	80	120	120	52.0	52.1
16	0.52	0.32	1.2	80	230	210	51.5	51.1

3.1. Development of the regression meta-models

Relations among factors and regression equalities were determined by polynomial regression analysis and degree of accuracy by determination coefficient (R^2). MINITAB® Statistical Program Package (version 15.1.1) was used for the data analysis. Regression models obtained with MINITAB® were given in eq. (4) and eq. (5). Estimated effects and coefficients for all criteria were given in Table 6 and 7.

$$Y_1 = 144.375 + 41.875 * X_1 - 7.5 * X_2 + 15 * X_3 + 11.875 * X_4 + 4.375 * X_1 * X_4 + 3.75 * X_3 * X_4 \quad (4)$$

$$Y_2 = 52.7 - 0.64 * X_1 - 0.84 * X_3 + 0.18 * X_4 + 0.34 * X_1 * X_2 - 0.29 * X_2 * X_3 - 0.12 * X_1 * X_3 * X_4 \quad (5)$$

The experimental results are analyzed by ANOVA (ANalysis Of VAriance) procedures. The ANOVA table gives a summary of the main effects and interactions (Tables 6 and 7). MINITAB® 15 displays both the sequential sums of squares (Seq SS) and adjusted sums of squares (AdjSS) [19]. Table 6 and 7 shows the p-values associated with each individual model term. The ‘Term’ column in Table 6 presents the main effects and all interactions. The second and third column displays the main effects and coefficients of the terms. The fifth and sixth columns display the t-ratios and p-values. The rows of all significant factors are shown in bold in Table 6 and 7 ($p<0.05$). The p-values lead to following conclusions: (1) Two-way interaction X_1*X_4 ($p=0.005$) and X_3*X_4 ($p=0.012$) for slump flow and X_1*X_2 ($p=0.000$) and X_2*X_3 ($p=0.000$) for 28th day compressive strength is statistically significant, others are not. (2) Main factors X_1, X_3, X_4 ($p=0.000$) are statistically significant for two criteria; X_3, is only statistically significant for slump flow [20].

Table 6. Estimated effects and coefficients for modeling slump flow in coded values

Term	Effect	Coefficient	SE Coef	t	P value
Constant		**144.375**	**1.326**	**108.89**	**0.000**
X_1	**83.750**	**41.875**	**1.326**	**31.58**	**0.000**
X_2	**-15.00**	**-7.500**	**1.326**	**-5.66**	**0.000**
X_3	**30,00**	**15.000**	**1.326**	**11.31**	**0.000**
X_4	**23.750**	**11.875**	**1.326**	**8.96**	**0.000**
X_1*X_2	-0.000	-0.000	1.326	-0.00	1.000
X_1*X_3	5.000	2.500	1.326	1.89	0.078
X_1*X_4	**8.750**	**4.375**	**1.326**	**3.30**	**0.005**
X_2 *X_3	3.750	1.875	1.326	1.41	0.176
X_2 *X_4	0.000	0.000	1.326	0.00	1.000
X_3 *X_4	**7.500**	**3.750**	**1.326**	**2.83**	**0.012**
X_1 *X_2 *X_3	-1.250	-0.625	1.326	-0.47	0.644
X_1 *X_2 *X_4	-0.000	-0.000	1.326	-0.00	1.000
X_1 *X_3 *X_4	2.500	1.250	1.326	0.94	0.360
X_2 *X_3 *X_4	1.250	0.625	1.326	0.47	0.644
X_1 *X_2 *X_3*X_4	1.250	0.625	1.326	0.47	0.644

Table 7. Estimated effects and coefficients for modeling 28th day compressive strength in coded values

Term	Effect	Coefficient	SE Coef	t	P value
Constant		**52.7125**	**0.02932**	**1798.13**	**0.000**
X_1	**-1.2875**	**-0.6437**	**0.02932**	**-21.96**	**0.000**
X_2	-0.0625	-0.0313	0.02932	-1.07	0.302
X_3	**-1.6875**	**-0.8437**	**0.02932**	**-28.78**	**0.000**
X_4	**0.3500**	**0.1750**	**0.02932**	**5.97**	**0.000**
X_1*X_2	**0.675**	**0.3375**	**0.02932**	**11.51**	**0.000**
X_1*X_3	0.0750	0.0375	0.02932	1.28	0.219
X_1*X_4	-0.0375	-0.0188	0.02932	-0.64	0.531
X_2 *X_3	**-0.5750**	**-0.2875**	**0.02932**	**-9.81**	**0.000**
X_2 *X_4	0.0875	0.0437	0.02932	1.49	0.155
X_3 *X_4	-0.1125	-0.0562	0.02932	-1.92	0.073
X_1 *X_2 *X_3	0.0375	0.0188	0.02932	0.64	0.531
X_1 *X_2 *X_4	-0.0500	-0.0250	0.02932	-0.85	0.406
X_1 *X_3 *X_4	**-0.1250**	**-0.0625**	**0.02932**	**-2.13**	**0.049**
X_2 *X_3 *X_4	-0.0750	-0.0375	0.02932	-1.28	0.219
X_1 *X_2 *X_3*X_4	-0.0375	-0.0187	0.02932	-0.64	0.531

3.2. Validation of meta-models

The real data of responses versus the predicted responses are given in Figure 2 as the observed and predicted values respectively. The model explains that the experimental range was studied sufficiently. The fitted regression equation shows a good fit to the model [25].

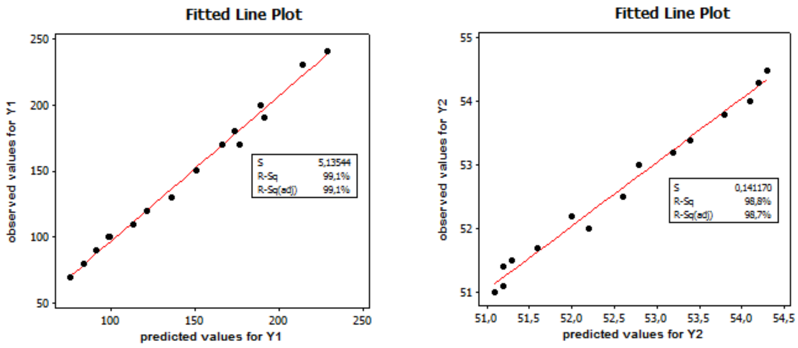

Figure 2. Observed values versus predicted values for criteria

3.3. Optimization using desirability function approach

Individual desirability values, the overall desirability D and predicted value were calculated by MINITAB and given in Figure 3. The factors obtained at the maximum points of Y_1 and Y_2 (target: 150 mm and 53 N/mm^2, respectively) were calculated as X_1=0.5090, X_2=0.3123, X_3=1.0012 and X_4=80 which are known as estimated condition (Figure 3).

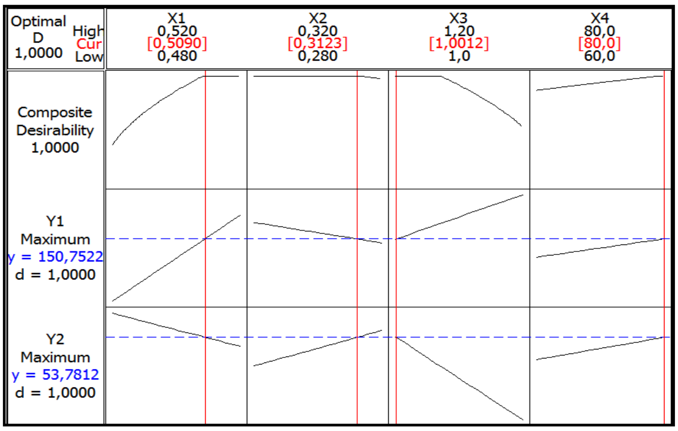

Figure 3. Optimization plot

3.4. Verification of optimal parameters

Validation experiments were applied at the estimated condition. The results illustrate that the experimental results are close to the estimated results (Table 8). The accuracy of the results can be taken as a proof for the validity of the estimated models.

Table 8. Test results for the verification of the results of FFD based DFA

Number	Responses	Predicted values	Verification experiment
1	Y_1	150.7522	150
2	Y_2	53.7812	54.0

4. CONCLUDING REMARKS

In this study, the optimization and the modeling of mixture proportions of C50 were performed by using a FFD based DFA. RMC consists of many conflicting factors; it is critical to use a systematic methodology for determining optimal mixes and modeling quality characteristics of C50 under a set of parameters that make up the concrete. For this reason, a FFD based DFA was used in this study to investigate ranking of the conflicting factor levels and best possible mix proportions of C50. The results showed that the proposed methodology is effective in solving the mixture proportions optimization problem. Also, it can be seen from the results of produced concrete samples, that they satisfied the expected properties of C50. Therefore, this study provides a tool for improvement of system performance for the organizations that have RMC. Outcomes were shared with the Concrete Manufacturer Company and more studies have been initiated to increase the system performance.

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