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SOME I-CONVERGENT SEQUENCE SPACES OF FUZZY NUMBERS DEFINED BY INFINITE MATRIX

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Abstract- In this paper we introduce and study some new sequence spaces of fuzzy numbers defined by *I*-convergence using the sequences of Orlicz functions, infinite matrix. We study some basic topological and algebraic properties of these spaces. Also we investigate the relations related to these spaces.

Key Words- Ideal, I -convergent, infinite matrix, Orlicz function, fuzzy number.

1. INTRODUCTION

The theory of sequence of fuzzy numbers was first introduced by Matloka [10]. Matloka introduced bounded and convergent sequence of fuzzy numbers and studied some of their properties and showed that every convergent sequence of fuzzy numbers is bounded. Nanda [12] studied the sequence of fuzzy numbers and showed that the set of all convergent sequence of fuzzy numbers forms a complete metric space. J. S. Kwon [8] introduced the definition of strongly *p*-Cesaro summability of sequence of fuzzy numbers. Savas [17] introduced and discussed double convergent sequence of fuzzy numbers and showed that the set of all double convergent sequence of fuzzy numbers is complete. Savas [24] studied some equivalent alternative conditions for a sequence of fuzzy numbers to be statistically Cauchy and he continued to study the statistical convergence in [22, 25]. Recently, Mursaleen and Basarir [11] introduced and studied some new sequence space of fuzzy numbers generated by non-negative regular matrix. Also Savas and Mursaleen [21] defined statistically convergent and statistically Cauchy for double sequence of fuzzy numbers.

Different classes of sequence of fuzzy real numbers have been discussed by Nuray and Savas [13], Altinok *et al.* [1], Hazarika and Savas [2], Kumar and Kumar [7], Savas ([22], [23]), Savas and Mursaleen [21], Joong-Sung [8] and many others.
The notion of *I*-convergence initially introduced by Kostyrko *et al.* [6]. More investigations in this direction and more applications of ideals can be found in [18, 19, 20, 30, 31, 32] where many important references can be found.

Let X be a non-empty set, then a family of sets $I \subset 2^X$ (the class of all subsets of X) is called an ideal if and only if for each $A, B \in I$, we have $A \cup B \in I$ and for each $A \in I$ and each $B \subset A$, we have $B \in I$. A non-empty family of sets $F \subset 2^X$ is a *filter* on X if and only if $\varnothing \notin F$, for each $A, B \in F$, we have $A \cap B \in F$ and each $A \in F$ and each $A \subset B$, we have $B \in F$. An ideal I is called *non-trivial* ideal if $I \neq \varnothing$ I and $X \notin I$ .Clearly

$I \subset 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X - A : A \in I\}$ is a filter on X. A non-trivial ideal $I \subset 2^X$ is called *admissible* if and only if $\{\{x\} : x \in X\} \subset I$. A non-trivial ideal I is maximal if there cannot exists any non-trivial ideal $J \neq I$ containing I as a subset. Further details on ideals of 2^X can be found in Kostyrko *et al.* [22].

Recall in [5] that an Orlicz function M is continuous, convex, nondecreasing function such that $M(0)=0$ and $M(x)>0$ for $x>0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function is replaced by $M(x+y)=M(x)+M(y)$ then this function is called the modulus function and characterized by Ruckle [16]. An Orlicz function M is said to satisfy Δ_2-condition for all values of u, if there exists $K>0$ such that $M(2u) \leq KM(u)$, $u \geq 0$.

Lindenstrauss and Tzafriri [9] studied some Orlicz type sequence spaces defined as follows:

$$\ell_M = \left\{ (x_k) \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho} \right) < \infty, \textit{ for some } \rho > 0 \right\}.$$

The space ℓ_M with the norm

$$\|x\| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho} \right) \leq 1 \right\}.$$

becomes a Banach space which is called an Orlicz sequence space. The space ℓ_M is closely related to the space ℓ_p which is an Orlicz sequence space with $M(t)= |t|^p$, for $1 \leq p < \infty$.

The following well-known inequality will be used throughout the article. Let $p=(p_k)$be any sequence of positive real numbers with $0 \leq p_k \leq \sup_k p_k = H$, $D=\max\{1, 2^{H-1}\}$then

$$|a_k + b_k|^{p_k} \leq D\left(|a_k|^{p_k} + |b_k|^{p_k} \right)$$

for all $k \in N$ and $a_k, b_k \in C$. Also $|a_k|^{p_k} \leq \max\{1, |a|^H\}$ for all $a \in C$.

In the later stage different classes of Orlicz sequence spaces were introduced and studied by Parashar and Choudhary [14], Savas ([26]-[29]) and many others.

Throughout the article w^F denote the class of all fuzzy real-valued sequence space. Also N and R denote the set of positive integers and set of real numbers respectively.

In this paper, we study some new sequence spaces of fuzzy numbers defined by using I-convergence, the sequence of Orlicz functions and an infinite matrix. We establish inclusion relations between the sequence spaces $w^{I(F)}[A, M, p]$, $w^{I(F)}[A, M, p]_0$, $w^F[A, M, p]_\infty$ and $w^{I(F)}[A, M, p]_\infty$ where $p=(p_k)$ denote the sequence of positive real numbers for all $n \in N$ and $M=(M_k)$ be a sequence of Orlicz functions.

2. DEFINITIONS AND NOTATIONS

Before continuing with this paper we present a few definitions and preliminaries.

Given any interval A, we shall denote its end points by $\underline{A}, \overline{A}$ and by D set of all closed bounded intervals on real line R i.e., $D = \{A \subset R : A = [\underline{A}, \overline{A}]\}$. For A, $B \in D$ we define $A \leq B$ if and only if $\underline{A} \leq \underline{B}$ and $\overline{A} \leq \overline{B}$ and

$d(A,B) = \max\left\{ |\underline{A} - \underline{B}|, |\overline{A} - \overline{B}| \right\}$.

It is easy to see that d defines a Hausdorff metric on D and (D,d) is a complete metric space. Also $\leq$ partial order on D.

A fuzzy number is a function X from R to [0, 1] which satisfying the following conditions (i) X is normal, i.e. there exists an $x_0 \in R$ such that $X(x_0)=1$; (ii) X is fuzzy convex, *i.e*. for any $x,y \in R$ and $\lambda \in [0, 1]$, $X(\lambda x+(1-\lambda)y) \geq \min\{X(x), Y(y)\}$; (iii) X is upper semi continuous; (iv) The closure of the set $\{x \in R : X(x)>0\}$, denoted by X^0 is compact.

The properties (i) to (iv) imply that for each $\alpha \in [0,1]$, the α-level set $X^\alpha = \{x \in R: X(x) > \alpha\} = \left[\underline{X}^\alpha, \overline{X}^\alpha\right]$ is a non empty compact convex subset of R. Let $L(R)$ denotes the set of all fuzzy numbers. Define a map $\overline{d} : L(R) \times L(R) \to R$ by $\overline{d}(x,y) = \sup_{\alpha \in [0,1]} d(X^\alpha, Y^\alpha)$. Puri and Ralescu [15] proved that $\left(L(R), \overline{d}\right)$ is a complete metric space.

For X, $Y \in L(R)$, we define $X \leq Y$ if and only if $\underline{X}^\alpha \leq \underline{Y}^\alpha$ and $\overline{X}^\alpha, \overline{Y}^\alpha$ for each $\alpha \in [0, 1]$, we say that $X<Y$ if $X \leq Y$ and there exist $\alpha_0 \in [0, 1]$ such that $\underline{X}^{\alpha_0} \leq \underline{Y}^{\alpha_0}$ or $\overline{X}^{\alpha_0}, \overline{Y}^{\alpha_0}$ .The fuzzy number X and Y are said to be incomparable if neither $X \leq Y$ nor $Y \leq X$.

For any $X,Y,Z \in L(R)$, the linear structure of $L(R)$ induced addition $X+Y$ and scalar multiplication $\lambda X, \lambda \in R$, in terms of α-level sets, by $[X+Y]^\alpha = [X]^\alpha + [Y]^\alpha$ and $[\lambda X]^\alpha = \lambda [X]^\alpha$ for each $\alpha \in [0,1]$.

Proposition 1.1. If $\overline{d}$ is a translation invariant metric on $L(R)$ then

(i) $\overline{d}(X+Z,0) \leq \overline{d}(X,0) + \overline{d}(Y,0)$,

(ii) $\overline{d}(\lambda X,0) \leq |\lambda| \overline{d}(X, 0)$, $|\lambda|>1$.

A sequence $X=(X_k)$ of fuzzy numbers is said to converge to a fuzzy number X_0 if for every $\varepsilon>0$, there exists a positive integer n_0 such that $\overline{d}(X_k,X_0) < \varepsilon$ for all $n \geq n_0$.A sequence $X=(X_k)$ of fuzzy numbers is said to be bounded if the set $\{X_k : k \in N\}$ of fuzzy numbers is bounded.

A sequence $X=(X_k)$ of fuzzy numbers is said to be *I-convergent* to a fuzzy numberX_0 if for each $\varepsilon > 0$ such that

$$A=\{k \in N : \overline{d}(X_k,X_0) \geq \varepsilon\} \in I.$$

The fuzzy number X_0 is called *I-limit* of the sequence (X_k) of fuzzy numbers and we write I–lim $X_k = X_0$.

A sequence $X=(X_k)$ of fuzzy numbers is said to be *I-bounded* if there exists $M > 0$such that

$\{k \in N : \overline{d}(X_k,0) > M\} \in I$.

Let E_F be denote the sequence space of fuzzy numbers.

A sequence space E_F is said to be *solid* (*or normal*) if $(Y_k)\in E_F$ whenever $(X_k)\in E_F$ and $|Y_k|\leq|X_k|$ for all $k\in N$.

A sequence space E_F is said to be monotone if E_F contains the canonical pre-images of all its step spaces.

Example 2.1. If we take $I=I_f=\{A\subseteq N : A$ is a finite subset$\}$. Then I_f is a nontrivial admissible ideal of N and the corresponding convergence coincide with the usual convergence.

Example 2.2. If we take $I=I_\delta=\{A\subseteq N : \delta(A)=0\}$ where $\delta(A)$ denote the asymptotic density of the set A. Then I_δ is a non-trivial admissible ideal of N and the corresponding convergence coincide with the statistical convergence.

Lemma 2.1. *A* sequence space E_F is normal implies E_F is monotone. (For the crisp set case, one may refer to Kamthan and Gupta (see, [4]).

3. SOME NEW SEQUENCE SPACES OF FUZZY NUMBERS

In this section, using the sequence of Orlicz functions, an infinite matrix and I-convergence; we introduced the following new sequence spaces and examine some properties of the resulting sequence spaces. Let I be an admissible ideal of N and let $p=(p_k)$ bea sequence of positive real numbers for all $k\in N$, and $A=(a_{nk})$ an infinite matrix. Let $M=(M_k)$ be a sequence Orlicz functions and $X=(X_k)$ be a sequence of fuzzy numbers, we define the following new sequence spaces:

$$w^{I(F)}[A,M,p]=\left\{(X_k)\in w^F : \forall\varepsilon>0, \left\{n\in N : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k,X_0)}{\rho}\right)\right]^{p_k}\geq\varepsilon\right\}\in I,\ \textit{for some}\ \rho>0\ \textit{and}\ X_0\in L(R)\right\},$$

$$w^{I(F)}[A,M,p]_0=\left\{(X_k)\in w^F : \forall\varepsilon>0, \left\{n\in N : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k,\bar{0})}{\rho}\right)\right]^{p_k}\geq\varepsilon\right\}\in I,\ \textit{for some}\ \rho>0\right\},$$

$$w^{F}[A,M,p]_\infty=\left\{(X_k)\in w^F : \sup_n \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k,\bar{0})}{\rho}\right)\right]^{p_k}<\infty,\ \textit{for some}\ \rho>0\right\},$$

and

$$w^{I(F)}[A,M,p]_\infty=\left\{(X_k)\in w^F : \exists K>0\ s.t., \left\{n\in N : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k,X_0)}{\rho}\right)\right]^{p_k}\geq K\right\}\in I,\ \textit{for some}\ \rho>0\right\},$$

Let us consider a few special cases of the above sets.

(i) If $M_k(x)=x$ for all $k\in N$, then the above classes of sequences are denoted by $w^{I(F)}[A, p]$, $w^{I(F)}[A, p]_0$, $w^F[A, p]_\infty$, and $w^{I(F)}[A, p]_\infty$, respectively.

(iii) If $p=(p_k)=(1,1,1,...)$, then we denote the above spaces by $w^{I(F)}[A, M]$, $w^{I(F)}[A, M]_0$,

$w^F[A, M]_\infty$, and $w^{I(F)}[A, M]_\infty$.

(iv) If we take A=(C,1),i.e., the Cesaro matrix, then the above classes of sequences are denoted by $w^{I(F)}[w,M,p]$, $w^{I(F)}[w,M,p]_0$, $w^F[w,M,p]_\infty$, and $w^{I(F)}[\mathrm{w},M,p]_\infty$, respectively.

v) If we take $A=(a_{nk})$ is a de la Valeepoussin mean, i.e.,

$$a_{nk}=\begin{cases}\dfrac{1}{\lambda_n} & ; \quad if\ k\in I_n=[n-\lambda_n+1,n]\\ 0 & ; \quad otherwise\end{cases}$$

where (λ_n) is a non-decreasing sequence of positive numbers tending to ∞ and $\lambda_{n+1}\leq\lambda_n+1$, $\lambda_1= 1$, then the above classes of sequences are denoted by $w_\lambda^{I(F)}[M,p]$, $w_\lambda^{I(F)}[M,p]_0$, $w_\lambda^F[M,p]_\infty$, and $w_\lambda^{I(F)}[M,p]_\infty$, respectively.

(vi) By a lacunary $\theta=(k_r)$; r=0,1,2, ... where k_0=0, we shall mean an increasing sequence of non-negative integers with k_r - k_{r-1} as $r\to\infty$. The intervals determined byθ will be denoted by $I_r=(k_{r-1},k_r]$ and $h_r=k_r - k_{r-1}$. As a final illustration let

$$a_{nk}=\begin{cases}\dfrac{1}{h_r} & ; \quad if\ k_{r-1}<k<k_r,\\ 0 & ; \quad otherwise\end{cases}$$

Then we denote the above classes of sequences by $w_\theta^{I(F)}[\mathrm{A},M,p]$, $w_\theta^{I(F)}[M,p]_0$, $w_\theta^F[M,p]_\infty$, and $w_\theta^{I(F)}[M,p]_\infty$, respectively.

(vii) If $I=I_f$, then we obtain

$$w^F[A,M,p]=\left\{(X_k)\in w^F:\lim_{n\to\infty}\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(X_k,X_0)}{\rho}\right)\right]^{p_k}=0,\ for\ some\ \rho>0\ and\ X_0\in L(R)\right\},$$

$$w^F[A,M,p]_0=\left\{(X_k)\in w^F:\lim_{n\to\infty}\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(X_k,X_0)}{\rho}\right)\right]^{p_k}=0,\ for\ some\ \rho>0\right\},$$

$$w^F[A,M,p]_\infty=\left\{(X_k)\in w^F:\sup_n\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(X_k,\bar{0})}{\rho}\right)\right]^{p_k}<\infty,\ for\ some\ \rho>0\right\},$$

(viii) If $I=I_\delta$ is an admissible ideal of N, then we have

$$w^{I(F)}[A,M,\Delta^m,p]=\left\{(X_k)\in w^F:\forall\varepsilon>0,\left\{n\in N:\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(X_k,X_0)}{\rho}\right)\right]^{p_k}\geq\varepsilon\right\}\in I_\delta,\ for\ some\ \rho>0\ and\ X_0\in L(R)\right\},$$

$$w^{I(F)}[A,M,\Delta^m,p]_0=\left\{(X_k)\in w^F:\forall\varepsilon>0,\left\{n\in N:\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}(X_k,\bar{0})}{\rho}\right)\right]^{p_k}\geq\varepsilon\right\}\in I_\delta,\ for\ some\ \rho>0\right\},$$

and

$$w^{I(F)}[A,M,\Delta^m,p]_\infty=\left\{(X_k)\in w^F : \exists K>0 \ s.t. \left\{n\in N : \sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}\left(X_k,\bar{0}\right)}{\rho}\right)\right]^{p_k}\geq K\right\}\in I_\delta, \textit{for some } \rho>0\right\},$$

If $X=(X_k)\in w^F[A,M,p]$ then we say that $X=(X_k)$ is strongly (p)- Cesaro convergent with respect to the sequence of Orlicz functions M.

4. MAIN RESULTS

In this section, we examine the basic topological and algebraic properties of the new sequence spaces and obtain the inclusion relation related to these spaces.

Theorem 4.1. Let (p_k) be a bounded sequence. Then the sequence spaces $w^{I(F)}[A,M,p]$, $w^{I(F)}[A,M,p]_0$, $w^{I(F)}[A,M,p]_\infty$ are linear spaces.

Proof. We shall prove the result for the space $w_\theta^{I(F)}[\mathrm{A},M,p]_0$ only and the others can be proved in similar way.

Let $X=(X_k)$ and $Y=(Y_k)$ be two elements in $w_\theta^{I(F)}[\mathrm{A},M,p]_0$. Then there exist $\rho_1>0$ and $\rho_2>0$ such that

$$A_{\frac{\varepsilon}{2}}=\left\{r\in N : \sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}\left(X_k,\bar{0}\right)}{\rho_1}\right)\right]^{p_k}\geq\frac{\varepsilon}{2}\right\}\in I$$

and

$$B_{\frac{\varepsilon}{2}}=\left\{r\in N : \sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}\left(Y_k,\bar{0}\right)}{\rho_2}\right)\right]^{p_k}\geq\frac{\varepsilon}{2}\right\}\in I$$

Let α, ß be two scalars. By the continuity of the function $\mathrm{M}=(M_k)$ the following inequality holds:

$$\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}\left(\alpha X_k+\beta Y_k,\bar{0}\right)}{|\alpha|\rho_1+|\beta|\rho_2}\right)\right]^{p_k}$$

$$\leq D\sum_{k=1}^{\infty}a_{nk}\left[\frac{|\alpha|}{|\alpha|\rho_1+|\beta|\rho_2}M_k\left(\frac{\bar{d}\left(X_k,\bar{0}\right)}{\rho_1}\right)\right]^{p_k}+\leq D\sum_{k=1}^{\infty}a_{nk}\left[\frac{|\beta|}{|\alpha|\rho_1+|\beta|\rho_2}M_k\left(\frac{\bar{d}\left(Y_k,\bar{0}\right)}{\rho_2}\right)\right]^{p_k}$$

$$\leq DK\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}\left(X_k,\bar{0}\right)}{\rho_1}\right)\right]^{p_k}+DK\sum_{k=1}^{\infty}a_{nk}\left[M_k\left(\frac{\bar{d}\left(Y_k,\bar{0}\right)}{\rho_2}\right)\right]^{p_k}$$

where $K=\max\left\{1,\left(\frac{|\alpha|}{|\alpha|\rho_1+|\beta|\rho_2}\right)^H,\left(\frac{|\alpha|}{|\beta|\rho_1+|\beta|\rho_2}\right)^H\right\}$

From the above relation we obtain the following:

$$\left\{ n \in N : \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}\left(\alpha X_k + \beta Y_k, \bar{0}\right)}{|\alpha| \rho_1 + |\beta| \rho_2} \right) \right]^{p_k} \geq \varepsilon \right\} \subseteq$$

$$\left\{ n \in N : DK \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}\left(X_k, \bar{0}\right)}{\rho_1} \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \right\}$$

$$\cup \left\{ n \in N : DK \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}\left(Y_k, \bar{0}\right)}{\rho_2} \right) \right]^{p_k} \geq \frac{\varepsilon}{2} \right\} \in I$$

This completes the proof.

Theorem 4.2. $w^{I(F)}[A,M,p]$, $w^{I(F)}[A,M,p]_0$, and $w^{I(F)}[A,M,p]_\infty$ are linear are linear topological space spaces with the paranorm g defined by

$$g(X) = inf \left\{ \rho^{\frac{p_n}{H}} : \left(\sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}\left(X_k, \bar{0}\right)}{\rho} \right) \right]^{p_k} \right)^{1/H} \leq 1, \text{ for some } \rho > 0, n = 1,2,3,... \right\},$$

where $H=\max\{1, \sup_k p_k\}$.

Proof. This can be easily verified by using standard techniques and so is omitted.

Theorem 4.3. (a) Let $0<\inf p_k \leq p_k \leq 1$. Then

$w^{I(F)}[A,M,p] \subseteq w^{I(F)}[A,M]$, $w^{I(F)}[A,M,p]_0 \subseteq w^{I(F)}[A,M]_0$

(b) Let $1 \leq p_k \leq \sup p_k < \infty$. Then

$w^{I(F)}[A,M] \subseteq w^{I(F)}[A,M,p]$, $w^{I(F)}[A,M]_0 \subseteq w^{I(F)}[A,M,p]_0$

Proof. (a) Let $X=(X_k)$ be an element in $w^{I(F)}[A,M,p]$. Since $0<\inf p_k \leq p_k \leq 1$we have

$$\sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}\left(X_k, X_0\right)}{\rho} \right) \right] \leq \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}\left(X_k, X_0\right)}{\rho} \right) \right]^{p_k}$$

Therefore

$$\left\{ n \in N : \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}\left(X_k, X_0\right)}{\rho} \right) \right] \geq \varepsilon \right\}$$

$$\subseteq \left\{ n \in N : \sum_{k=1}^{\infty} a_{nk} \left[M_k \left(\frac{\bar{d}\left(X_k, X_0\right)}{\rho} \right) \right]^{p_k} \geq \varepsilon \right\}$$

The other part can be proved in similar way.

(b) Let $X=(X_k)$ be an element in $w^{I(F)}[A,M,p]$. Since $1 \leq p_k \leq \sup p_k < \infty$. Then for each $0<\varepsilon<1$ there exists a positive integer n_0 such that

$$\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k, X_0)}{\rho}\right)\right] \leq \varepsilon < 1 \quad for\ all\ n \geq n_0$$

This implies that

$$\sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k, X_0)}{\rho}\right)\right]^{p_k} \leq \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k, X_0)}{\rho}\right)\right]$$

Therefore we have

$$\left\{n \in N : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k, X_0)}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\} \subseteq$$

$$\subseteq \left\{n \in N : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k, X_0)}{\rho}\right)\right] \geq \varepsilon\right\} \in I$$

The other part can be proved in similar way.

Proposition 4.4. The sequence spaces $w^{I(F)}[A,M,p]_0$. and$w^{I(F)}[A,M,p]_\infty$ are normal as well as monotone.

Proof. We give the proof of the theorem for $w^{I(F)}$ $[A,M,p]_0$ only. Let X = $(X_k) \in w^{I(F)}[A,M,p]_0$ and $Y = (Y_k)$ be such that $\bar{d}(Y_k, \bar{0}) \leq \bar{d}(X_k, \bar{0})$ for all $k \in N$. Then forgiven $\varepsilon > 0$ we have

$$B = \left\{n \in N : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(X_k, \bar{0})}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\} \in I.$$

Again the set

$$B_1 = \left\{n \in N : \sum_{k=1}^{\infty} a_{nk}\left[M_k\left(\frac{\bar{d}(Y_k, \bar{0})}{\rho}\right)\right]^{p_k} \geq \varepsilon\right\} \subseteq B.$$

Hence $B_1 \in I$ and so $Y=(Y_k) \in w^{I(F)}[A,M,p]_0$. Thus the space $w^{I(F)}[A,M,p]_0$ is normal. Also from the Lemma 2.1, it follows that $w^{I(F)}[A,M,p]_0$ is monotone.

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