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ANALYSIS OF A MODIFIED LOGISTIC MODEL FOR DESCRIBING THE GROWTH OF DURABLE CUSTOMER GOODS IN CHINA

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Abstract- The ability of a modified logistic model for forecasting the growth of durable consumer goods in China was investigated. The fitting of the modified logistic model to the historical data uses a pattern search technique to establish the optimum parameter values. Two data sets on the quantity of air conditioner owned per 100 urban households at year-end and color TV set owned per 100 rural households at year-end were analyzed in this work. Additionally, the logistic model was applied to the same data. Both two models were compared using their goodness of fit to the historical data. The comparison has revealed that the modified logistic model is more appropriate for describing the growth of durable consumer goods in China.

Key Words- Logistic Model, Durable Consumer Goods, Pattern Search Method, Forecasting, Fit

1. INTRODUCTION

To create a simplified representation of reality and make a forecast about future developments, a forecasting model has been usually used [1, 2]. With the development of Chinese economy, the quantity of durable consumer goods owned by Chinese residents is increasing rapidly. Several growth models have been proposed for describing the growth of durable consumer goods in China [3-9]. Future growth forecasts are required for short and long term market planning activities of durable consumer goods. While it is hard to conclude which model predicts the growth of durable consumer goods most accurately, the logistic model and its modified forms have been proved to be effective for forecasting many social and technological patterns [10-14]. The main aim of this work is to investigate the effectiveness of the logistic model and one of its modified forms for describing the growth of air conditioners and color TVs owned by Chinese residents. It is hoped that this work will help Chinese businesses that supply air conditioners and color TVs.

2. MODIFIED LOGISTIC MODEL

The logistic growth model is attractive in the evolutionary S-shaped processes [10]. There have been a number of applications of the logistic growth curves to the biological, technological and economic fields [10-14]. The differential form of the logistic growth equation, which was originally developed by a Belgian mathematician Verhulst in 1838 [15], is given below:

$$\frac{dy_t}{dt} = ry_t\left[1 - \frac{y_t}{L}\right] \tag{1}$$

where y_t is the value of interest at time t, r is the intrinsic growth rate, L is the maximum value of y_t. Notice that the term $(1-y_t/L)$ is close to 1 when $y_t \ll L$ and approaches zero as $y_t \rightarrow L$. Thus, the growth rate begins exponentially but then decreases to zero as the y_t approaches the limit L, producing a sigmoidal growth trajectory [16].

Equation (1) has solution

$$y_t = \frac{L}{1 + \left(\frac{L}{y_0} - 1\right)e^{-rt}} \tag{2}$$

where y_0 is the value at time t=0.

Based on the logistic model described with Equation (1), Blumberg [17] proposed a modified logistic model:

$$\frac{dy_t}{dt} = ry_t^{\,a}\left[1 - \frac{y_t}{L}\right]^b \tag{3}$$

where $a \geq 0$, $b \geq 0$. Equation (1) is just a special case of this more general version of the logistic growth model where a=b=1. Equation (3) indicates growth is positively related to two factors: $y_t^{\,a}$ and $\left[1 - \frac{y_t}{L}\right]^b$.

Unlike the original logistic model, the modified logistic model, Equation (3), with $a \neq 1$, $a>0$, $b \neq 1$ and $b>0$, cannot be integrated analytically. But it can be numerically solved. For this purpose, either general purposed mathematical software or a computer program developed in any programming language should be used. In this work, the 'ode45' function in MATLAB® which employs the common fourth-order Runge-Kutta method (often referred to as the classical Runge-Kutta method or simply as the Runge-Kutta method [18]) for integration has been used to numerically solve Equation (3). Substitution of the obtained y_t data into Equation (3) gives the growth rate as a function of t.

Figure 1 shows several modified logistic curves for various values of r with y_0=1, L=100, a=1.25 and b=1.5. From Figure 1, it can be observed that the larger the r the faster in time the curve reaches the maximum value L.

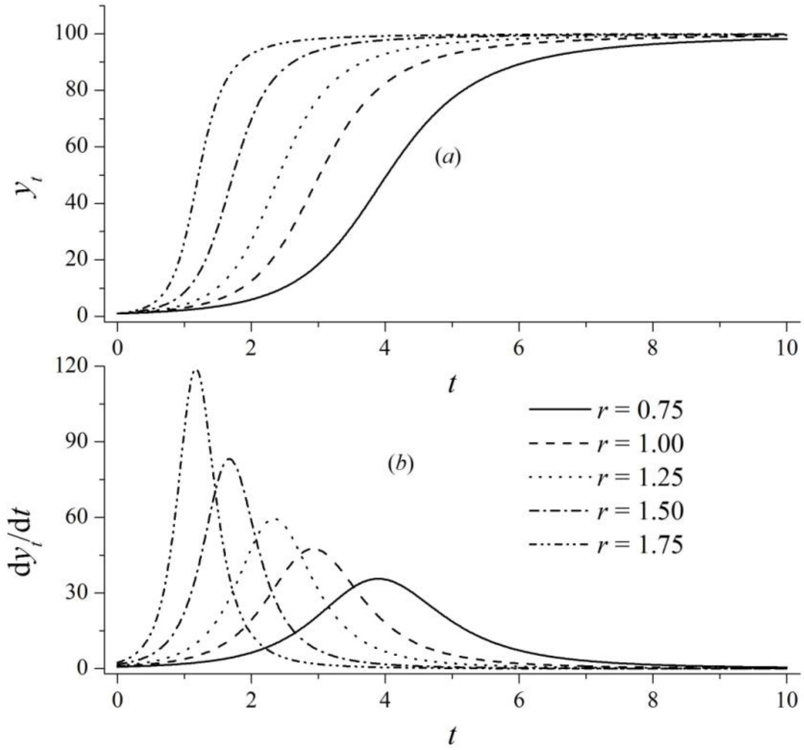

Figure 1. The influences of r on the numerical results of the modified logistic model (y_0=1, L=100, a=1.25 and b=1.5) (a) plots of y_t vs. t (b) plots of dy_t/dt vs. t

3. MODEL FIT

3.1. Parameter Estimation

For fitting the data to the logistic model and its modified form expressed in Equation (3), the estimation of the model parameter values is needed.

For fitting the data to the logistic model, the parameters L and r can be established by means of nonlinear regression, which is easy to perform with the help of some data analysis software (e. g. Origin® and DataFit®). In this work, DataFit® has been used for performing the regression analysis. Detailed information about the software can be found in the literature [19, 20].

In general, Equation (3) can't be analytically solved. The parameters, L, r, a and b, in Equation (3) can't be estimated by using direct regression analysis. In this work, the method of least squares, which is one of the most popular estimation techniques [21-24], has been used. In the implementation of this method, the following objective function is usually used:

$$\text{Minimize} \quad OF(L,r,a,b) = \sum_{i=1}^{n_d} \left(y_{t_i,cal} - y_{t_i,act} \right)^2 \tag{4}$$

where $y_{t_i,act}$ is the actual value, $y_{t_i,cal}$ is the value obtained by numerically solving Equation (3), the subscript i denotes the ordinal number of the datum point i, n_d is the number of datum points.

It is obvious that those optimization methods with making use of derivative information of the objective function can't be used to solve the optimization problem described by Equation (4). Therefore, one derivative-free optimization method should be used. In this work, a pattern search method, which is one of direct search methods for nonlinear optimization [25], is employed to find the minimum of the objective function. The pattern search method is a derivative-free method and superior to other direct search methods such as the Powell and Simplex methods in both robustness and number of function evaluations [26]. One of the problems that appear when using the pattern search method is to choose the starting points for the model parameters to estimate. In this work, I select the obtained optimum values of L and r, and a=b=1 as the starting points for the estimation of the parameters of the modified logistic model.

3.2. Statistical Measures

The mean squared error (MSE) has used to measure the goodness of the fit of each of the fitted model to the historical data. MSE is defined as:

$$\mathrm{MSE} = \frac{1}{n_d}\sum_{i=1}^{n_d}\left(y_{t_i,cal} - y_{t_i,act}\right)^2 \tag{5}$$

The larger the lower MSE, the better is the goodness of fit.

4. RESULTS AND DISCUSSION

4.1. Historical Data

The data used in this work consist of the historical data on the quantity of air conditioner owned per 100 urban households at year-end and color TV set owned per 100 rural household at year-end. Both two data sets are obtained from China Statistical Yearbook published by the National Bureau of Statistics of China. The growth of two kinds of durable consumer goods in China from 1985 to 2008 is shown in Figure 2. There is an increase in trend for both data sets.

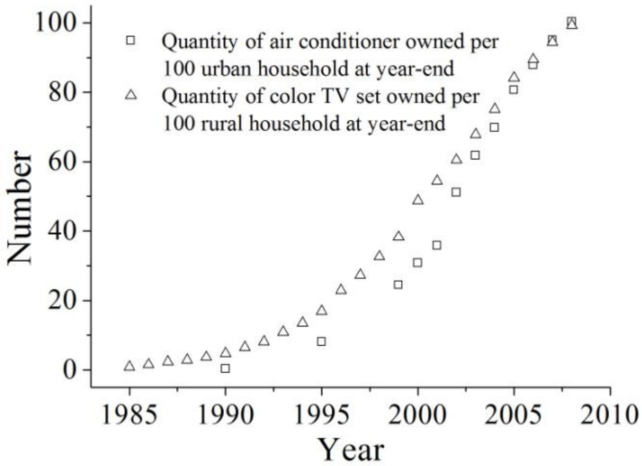

Figure 2. The annual quantity of durable consumer goods owned by Chinese residents

4.2. Application to the Historical Data

The logistic model and its modification have been used to fit both data sets. Table 1 shows the model parameter values and the corresponding values of R^2 and MSE.

Table 1. Results of the logistic models

Data set	Model	Model parameters				Statistical parameters
		L	*r*	*a*	*b*	MSE
Color TV set	Logistic model	107.4483	0.3084	---	---	3.0455
	Modified logistic model	126.3252	0.3784	0.8869	1.0070	0.5913
Air conditioner	Logistic model	103.8257	0.4701	---	---	7.4723
	Modified logistic model	110.2084	0.4434	0.9087	1.1483	1.9857

The comparison between the actual values and the values obtained from the logistic model and its modification for the growth of color TV set owned per 100 rural households at year-end are shown in Figures 4 and 5, respectively.

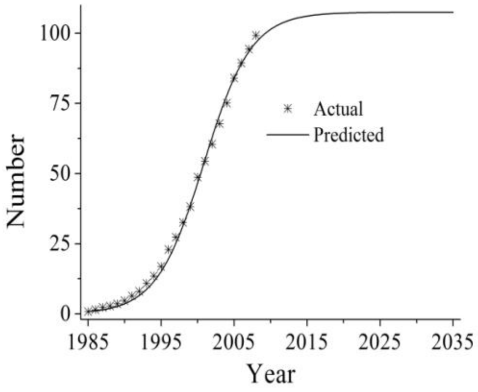

Figure 4. Comparison of the actual values and the values predicted by the logistic model for the growth of color TV set owned per 100 rural households at year-end

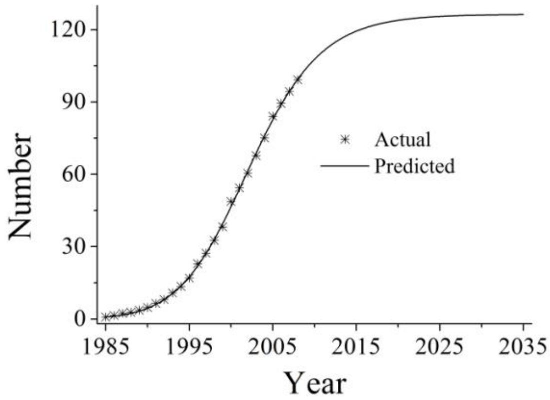

Figure 5. Comparison of the actual values and the values predicted by the modified logistic model for the growth of color TV set owned per 100 rural households at year-end

Figure 6 shows the comparison between the actual values and the values calculated from the logistic model for the growth of air conditioner owned per 100 urban households at year-end. Figure 7 shows the comparison between the actual values and the values calculated from the modified model for the growth of air conditioner owned per 100 urban households at year-end.

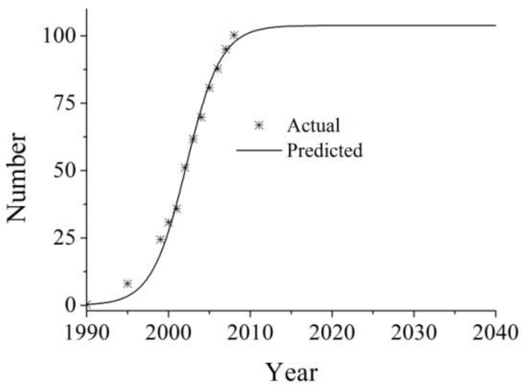

Figure 6. Comparison of the actual values and the values predicted by the logistic model for the growth of air conditioner owned per 100 urban households at year-end

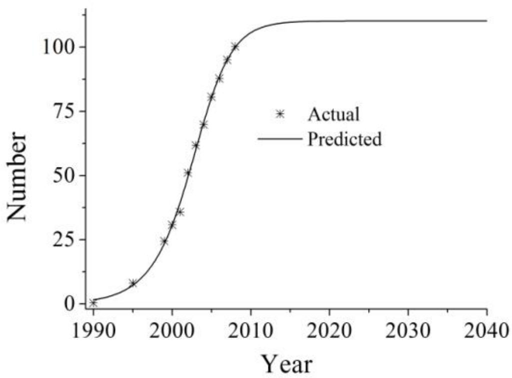

Figure 7. Comparison of the actual values and the values predicted by the modified logistic model for the growth of air conditioner owned per 100 urban households at year-end

The above results have shown that the modified logistic model fits the growth of color TV set and air conditioner owned by Chinese residents perfectly well. From the comparison between the logistic model and its modification, the modified logistic model is better than its original form for describing the growth of color TV set owned per 100 rural households at year-end and air conditioner owned per 100 urban households at year-end.

5. CONCLUSIONS

In this work, the logistic model and one of its modifications have been applied to describe the growth of durable consumer goods owned by Chinese residents. The study results have shown that the modified logistic model is better in predicting the growth of durable consumer goods in China than the logistic model. The good model fit has indicated that the modified logistic model is more suitable for predicting the growth of durable consumer goods in China.

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