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SOLUTION OF THE SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS BY COMBINED LAPLACE TRANSFORM–ADOMIAN DECOMPOSITION METHOD

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Abstract- In this paper, combined Laplace transform–Adomian decomposition method is presented to solve differential equations systems. Theoretical considerations are being discussed. Some examples are presented to show the ability of the method for linear and non-linear systems of differential equations. The results obtained are in good agreement with the exact solution and Runge-Kutta method.

Key Words- Differential equations systems; Laplace transform; Adomian decomposition method

1. INTRODUCTION

A system of ordinary differential equations of the first order can be considered as:

$$\begin{cases} y_1' = f_1(x, y_1, \ldots, y_n) \\ y_2' = f_2(x, y_1, \ldots, y_n) \\ \vdots \\ y_n' = f_n(x, y_1, \ldots, y_n) \end{cases} \tag{1}$$

where each equation represents the first derivative of each unkown functions as a mapping depending on the independent variable x, and n unknown functions $f_1, f_2, \cdots, f_n$ and the initial conditions $y_1(0), y_2(0), \ldots, y_n(0)$ are prescribed.

The main purpose of this paper is to extend the application of combined Laplace transform–Adomian decomposition method [1,2,3,4] to obtain an approximate solution of differential equations systems . The paper is organised as follows: In Section 2, how to use of combined laplace transform–adomian decomposition method is presented. In Section3, combined laplace transform–adomian decomposition method is demonstrated by applying it on three problems and conclusion is given at the last section.

2. THE USE OF COMBINED LAPLACE TRANSFORM–ADOMIAN DECOMPOSITION METHOD

We can present the system (1), by using the i_{th} equation as:

$$y_i' = g_i\left(x, y_1, \ldots, y_n\right) + F_i\left(x, y_1, \ldots, y_n\right), i = 1, 2, \ldots, n \tag{2}$$

Here $g_i\left(x, y_1, \ldots, y_n\right)$ and $F_i\left(x, y_1, \ldots, y_n\right)$ are linear and nonlinear parts of $f_i\left(x, y_1, \ldots, y_n\right)$ respectively. To solve the system of ordinary differential equations of the first order (1) by using the combined Laplace transform–Adomian decomposition method, we recall that the Laplace transform of the derivative of y_i' are defined by

$$L\left\{y_i'\right\} = s.L\left\{y_i\right\} - y_i\left(0\right), i = 1, 2, \ldots, n .$$

Applying the Laplace transform to both sides of (2) gives

$$s.L\left\{y_i\right\} - y_i\left(0\right) = L\left\{g_i\left(x, y_1, \ldots, y_n\right)\right\} + L\left\{F_i\left(x, y_1, \ldots, y_n\right)\right\}, i = 1, 2, \ldots, n \tag{3}$$

This can be reduced to

$$L\left\{y_i\right\} = \frac{y_i\left(0\right)}{s} + \frac{1}{s} L\left\{g_i\left(x, y_1, \ldots, y_n\right)\right\} + \frac{1}{s} L\left\{F_i\left(x, y_1, \ldots, y_n\right)\right\}, i = 1, 2, \ldots, n \tag{4}$$

The Adomian decomposition method and the Adomian polynomials can be used to handle (4) and to address the nonlinear term $F_i\left(x, y_1, \ldots, y_n\right)$. Solutions are represented as infinite series in this method, such that

$$y_i = \sum_{k=0}^{\infty} y_{ik}, i = 1, 2, \ldots, n \tag{5}$$

where the components y_{in} are to be recursively computed. However, the nonlinear term $F_i\left(x, y_1, \ldots, y_n\right)$ at the right side of equation (5) will be represented by an infinite series of the Adomian polynomials A_{ik} in the form

$$F_i\left(x, y_1, \ldots, y_n\right) = \sum_{k=0}^{\infty} A_{ik}\left(x\right), i = 1, 2, \ldots, n \ . \tag{6}$$

Where $A_{ki}, n \geq 0$ are defined by

$$A_{ik} = \frac{1}{k!} \frac{d^k}{d\lambda^k} \left[F_i\left(x, \sum_{j=0}^{k} \lambda^j y_{1j}, \sum_{j=0}^{k} \lambda^j y_{2j}, \ldots, \sum_{j=0}^{n} \lambda^j y_{nj} \right) \right]_{\lambda=0}, k = 0, 1, 2, \ldots; i = 1, 2, \ldots, n \tag{7}$$

where the so-called Adomian polynomials A_{ik} can be evaluated for all forms of nonlinearity. In other words, assuming that the nonlinear functions is $F_i\left(x, y_1, \ldots, y_n\right)$, therefore the Adomian polynomials are given by

$$
\begin{aligned}
&A_{i0}=F_i\left(y_{i0}\right),\\
&A_{i1}=y_{i1}F_i'\left(y_{i0}\right),\\
&A_{i2}=y_{i2}F_i'\left(y_{i0}\right)+\frac{1}{2!}y_{i1}{}^{2}F_i''\left(y_{i0}\right),\\
&A_{i3}=y_{i3}F_i'\left(y_{i0}\right)+y_{i1}y_{i2}F_i''\left(y_{i0}\right)+\frac{1}{3!}y_{i1}{}^{3}F_i'''\left(y_{i0}\right),\\
&A_{i4}=y_{i4}F_i'\left(y_{i0}\right)+\left(\frac{1}{2!}y_{i2}{}^{2}+y_{i1}y_{i3}\right)F_i''\left(y_{i0}\right)+\frac{1}{2!}y_{i1}{}^{2}y_{i2}F_i'''\left(y_{i0}\right)+\frac{1}{4!}y_{i1}{}^{4}F_i^{(iv)}\left(y_{i0}\right).
\end{aligned}
\tag{8}
$$

Substituting (5) and (6) into (4) leads to

$$
L\left\{\sum_{k=0}^{\infty}y_{ik}\right\}=\frac{y_i(0)}{s}+\frac{1}{s}L\left\{g_i\left(x,\sum_{k=0}^{\infty}y_{1k},\ldots,\sum_{k=0}^{\infty}y_{nk}\right)\right\}+\frac{1}{s}L\left\{\sum_{k=0}^{\infty}A_{ik}(x)\right\},i=1,2,\ldots,n \tag{9}
$$

Matching both sides of (9) yields the following iterative algorithm.

$$
L\{y_{i0}\}=\frac{y_i(0)}{s},L\{y_{ik+1}\}=\frac{1}{s}L\{g_i(x,y_{1k},\ldots,y_{nk})\}+\frac{1}{s}L\{A_{ik}(x)\},i=1,2,\ldots,n \tag{10}
$$

Applying the inverse Laplace transform to the first part of (10) gives y_{i0}, that will define A_{i0}. Using A_{i0} will enable us to evaluate y_{i1}. The determination of y_{i0} and y_{i1} leads to the determination of A_{i1} that will allows us to determine y_{i2} ,and so on. This successively will lead to the complete determination of the components of $y_{ik}, k\geq 0$ upon using the second part of (10). The series solution follows immediately after using equation (5). The combined Laplace transform–Adomian decomposition method to solve systems of differential equations of the first and second order are illustrated by studying the following examples.

3. NUMERICAL EXAMPLES

Three examples are presented in this part. The first and second examples are considered to illustrate the method for linear and non-linear ordinary differential equations systems of order one while in third example a differential equations system of order two is solved.

Example 1. In this example we solve the following non-linear system of differential equations, with initial values $y_1(0)=1,\ y_2(0)=1,\ y_3(0)=0$ [5]. Exact solutions are $y_1=e^{2x}$, $y_2(x)=e^{x}$ and $y_3(x)=x.e^{x}$.

$$\begin{aligned} y_1' &= 2y_2^{\,2} \\ y_2' &= e^{-x} y_1 \\ y_3' &= y_2 + y_3 \end{aligned} \tag{11}$$

Applying the Laplace transformation we get

$$\begin{aligned} L\{y_1\} &= \frac{1}{s} + \frac{2}{s} L\{y_2^{\,2}\} \\ L\{y_2\} &= \frac{1}{s} + \frac{1}{s} L\{e^{-x} y_1\} \\ L\{y_3\} &= \frac{1}{s} L\{y_2\} + \frac{1}{s} L\{y_3\} \end{aligned} \tag{12}$$

Substituting $y_1 = \sum_{k=0}^{\infty} y_{1k}$, $y_2 = \sum_{k=0}^{\infty} y_{2k}$, $y_3 = \sum_{k=0}^{\infty} y_{3k}$ and $F_1 = y_2^{\,2} = \sum_{k=1}^{\infty} A_{1k}$ into (12) leads to

$$\begin{aligned} L\left\{\sum_{k=0}^{\infty} y_{1k}\right\} &= \frac{1}{s} + \frac{2}{s} L\left\{\sum_{k=1}^{\infty} A_{1k}\right\} \\ L\left\{\sum_{k=0}^{\infty} y_{2k}\right\} &= \frac{1}{s} + \frac{1}{s} L\left\{e^{-x}\sum_{k=0}^{\infty} y_{1k}\right\} \\ L\left\{\sum_{k=0}^{\infty} y_{3k}\right\} &= \frac{1}{s} L\left\{\sum_{k=0}^{\infty} y_{2k}\right\} + \frac{1}{s} L\left\{\sum_{k=0}^{\infty} y_{3k}\right\} \end{aligned} \tag{13}$$

Where A_{1k} are Adomian polynomials defined by

$$A_{1k} = \frac{1}{k!}\frac{d^k}{d\lambda^k}\left[F_1\left(x, \sum_{j=0}^{k} \lambda^j y_{1j}, \sum_{j=0}^{k} \lambda^j y_{2j}, \sum_{j=0}^{n} \lambda^j y_{3j}\right)\right]_{\lambda=0}, k = 0,1,2,\ldots$$

Table 1

The absolute error involved in the combined Laplace transform–Adomian decomposition method along with the exact solution for Example 1

x_i	$y_1(x_i)$	$err(y_1(x_i))$	$y_2(x_i)$	$err(y_2(x_i))$	$y_3(x_i)$	$err(y_3(x_i))$
0	1.	0.	1.	0.	0.	0.
0.1	1.2214	$9.32904\ \mathrm{e}^{-7}$	1.10517	$1.97308e^{-7}$	0.100167	$1.03504e^{-2}$
0.2	1.49177	$5.58506e^{-5}$	1.22139	$1.12364e^{-5}$	0.201334	$4.29468e^{-2}$
0.3	1.82152	$5.96726e^{-4}$	1.34974	$1.14138e^{-4}$	0.304497	$1.00461e^{-1}$
0.4	2.22239	$3.15363e^{-3}$	1.49125	$5.73158e^{-4}$	0.410631	$1.86099e^{-1}$
0.5	2.70693	$1.13473e^{-2}$	1.64676	$1.95841e^{-3}$	0.520667	$3.03694e^{-1}$
0.6	3.28807	$3.20509e^{-2}$	1.81687	$5.24959e^{-3}$	0.635471	$4.57800e^{-1}$
0.7	3.97853	$7.66715e^{-2}$	2.00184	$1.19098e^{-2}$	0.755826	$6.53801e^{-1}$
0.8	4.79049	$1.62542e^{-1}$	2.20161	$2.39291e^{-2}$	0.882413	$8.98019e^{-1}$
0.9	5.7352	$3.14451e^{-1}$	2.41576	$4.38416e^{-2}$	1.01581	1.19783
1	6.82272	$5.66335e^{-1}$	2.64356	$7.47225e^{-2}$	1.15649	1.5618

We obtain the following procedure by using the combined Laplace transform–Adomian decomposition method.

$$L\{y_{10}\}=\frac{1}{s},L\{y_{1k+1}\}=\frac{2}{s}L\{A_k\}$$
$$L\{y_{20}\}=\frac{1}{s},L\{y_{2k+1}\}=\frac{1}{s}L\{e^{-x}y_{1k}\} \tag{14}$$
$$L\{y_{30}\}=0,L\{y_{3k+1}\}=\frac{1}{s}L\{y_{2k}\}+\frac{1}{s}L\{y_{3k}\}$$

Approximations to the solutions with five terms are as follows:

$$y_1=-\frac{1214}{9}-\frac{28}{9}e^{-3x}+15e^{-2x}+124e^{-x}+\frac{176}{3}x-16e^{-2x}x+104e^{-x}x-4e^{-2x}x^2$$
$$y_2=\frac{104}{9}+\frac{13}{9}e^{-3x}-40e^{-2x}+28e^{-x}+\frac{4}{3}e^{-3x}x-16e^{-2x}x-32e^{-x}x$$

$$y_3=-\frac{781}{18}-\frac{1}{9}e^{-3x}+\frac{17}{2}e^{-2x}+35e^{-x}+\frac{80}{3}x+2e^{-2x}x+24e^{-x}x-6x^2+\frac{5}{6}x^3-\frac{1}{12}x^4+\frac{1}{120}x^5$$

Table 1 shows the results of from the solution of Example 1 and illustrates the absolute errors between exact solution and combined Laplace transform–Adomian decomposition method, respectively. We achieved a good approximation with combined Laplace transform–Adomian decomposition method with only six iterations.

Example 2. Now we consider stiff system of differential equations [7, 8, 9].

$$\frac{dy_1}{dx}=-\kappa_1 y_1(x)+\kappa_2 y_2(x)y_3(x),$$
$$\frac{dy_2}{dx}=\kappa_3 y_1(x)+\kappa_4 y_2(x)y_3(x)-\kappa_5 y_2{}^2(x), \tag{15}$$
$$\frac{dy_3}{dx}=\kappa_6 y_2{}^2(x),$$

Where $\kappa_1=0.04$, $\kappa_2=0.01$, $\kappa_3=400$, $\kappa_4=100$, $\kappa_5=3000$ and $\kappa_6=30$. The initial conditions are given by $y_1(0)=1$, $y_2(0)=0$, $y_3(0)=0$.

Taking the Laplace transformation of Eqs. (15), we get

$$L\{y_1\}=\frac{1}{s}-\frac{\kappa_1}{s}L\{y_1\}+\frac{\kappa_2}{s}L\{y_2y_3\},$$
$$L\{y_2\}=\frac{\kappa_3}{s}L\{y_1\}+\frac{\kappa_4}{s}L\{y_2y_3\}-\frac{\kappa_5}{s}L\{y_2{}^2\}, \tag{16}$$
$$L\{y_3\}=\frac{\kappa_6}{s}L\{y_2{}^2\},$$

Substituting $y_1=\sum_{k=0}^{\infty} y_{1k}$, $y_2=\sum_{k=0}^{\infty} y_{2k}$, $y_3=\sum_{k=0}^{\infty} y_{3k}$ and $F_1=y_2y_3=\sum_{k=1}^{\infty} A_{1k}$
$F_2=y_2{}^2=\sum_{k=1}^{\infty} A_{2k}$ in (16) leads to

$$\begin{aligned} &L\left\{\sum_{k=0}^{\infty} y_{1k}\right\}=\frac{1}{s}-\frac{\kappa_1}{s}L\left\{\sum_{k=0}^{\infty} y_{1k}\right\}+\frac{\kappa_2}{s}L\left\{\sum_{k=1}^{\infty} A_{1k}\right\},\\ &L\left\{\sum_{k=0}^{\infty} y_{2k}\right\}=\frac{\kappa_3}{s}L\left\{\sum_{k=0}^{\infty} y_{1k}\right\}+\frac{\kappa_4}{s}L\left\{\sum_{k=1}^{\infty} A_{1k}\right\}-\frac{\kappa_5}{s}L\left\{\sum_{k=1}^{\infty} A_{2k}\right\},\\ &L\left\{\sum_{k=0}^{\infty} y_{3k}\right\}=\frac{\kappa_6}{s}L\left\{\sum_{k=1}^{\infty} A_{2k}\right\}, \end{aligned} \tag{17}$$

Where A_{1k} and A_{2k} are Adomian polynomials defined by

$$A_{1k}=\frac{1}{k!}\frac{d^k}{d\lambda^k}\left[F_1\left(x,\sum_{j=0}^{k}\lambda^j y_{1j},\sum_{j=0}^{k}\lambda^j y_{2j},\sum_{j=0}^{n}\lambda^j y_{3j}\right)\right]_{\lambda=0},k=0,1,2,\ldots \tag{18}$$

$$A_{2k}=\frac{1}{k!}\frac{d^k}{d\lambda^k}\left[F_2\left(x,\sum_{j=0}^{k}\lambda^j y_{1j},\sum_{j=0}^{k}\lambda^j y_{2j},\sum_{j=0}^{n}\lambda^j y_{3j}\right)\right]_{\lambda=0},k=0,1,2,\ldots \tag{19}$$

We obtain the following procedure by using the combined Laplace transform–Adomian decomposition method.

$$\begin{aligned} &L\{y_{10}\}=\frac{1}{s},L\{y_{1k+1}\}=-\frac{\kappa_1}{s}L\{y_{1k}\}+\frac{\kappa_2}{s}L\{A_{1k}\},\\ &L\{y_{20}\}=0,L\{y_{2k+1}\}=\frac{\kappa_3}{s}L\{y_{1k}\}+\frac{\kappa_4}{s}L\{A_{1k}\}-\frac{\kappa_5}{s}L\{A_{2k}\},\\ &L\{y_{30}\}=0,L\{y_{3k+1}\}=\frac{\kappa_6}{s}L\{A_{2k}\}, \end{aligned} \tag{20}$$

Approximations to the solutions with five terms are as follows:

$$\begin{aligned} y1(x)=&1-x\kappa_1+\frac{1}{2}x^2\kappa_1^2-\frac{1}{6}x^3\kappa_1^3+\frac{1}{24}x^4\kappa_1^4-\frac{1}{120}x^5\kappa_1^5+\frac{1}{720}x^6\kappa_1^6\\ &+\frac{1}{15}x^5\kappa_2\kappa_3^3\kappa_6-\frac{29}{360}x^6\kappa_1\kappa_2\kappa_3^3\kappa_6 \end{aligned}$$

$$\begin{aligned} y2(x)=&x\kappa_3-\frac{1}{2}x^2\kappa_1\kappa_3+\frac{1}{6}x^3\kappa_1^2\kappa_3-\frac{1}{24}x^4\kappa_1^3\kappa_3+\frac{1}{120}x^5\kappa_1^4\kappa_3\\ &-\frac{1}{720}x^6\kappa_1^5\kappa_3-\frac{1}{3}x^3\kappa_3^2\kappa_5+\frac{1}{4}x^4\kappa_1\kappa_3^2\kappa_5-\frac{7}{60}x^5\kappa_1^2\kappa_3^2\kappa_5+\frac{1}{24}x^6\kappa_1^3\kappa_3^2\kappa_5\\ &+\frac{2}{15}x^5\kappa_3^3\kappa_5^2-\frac{5}{36}x^6\kappa_1\kappa_3^3\kappa_5^2+\frac{1}{90}x^6\kappa_2\kappa_3^4\kappa_6+\frac{1}{15}x^5\kappa_3^3\kappa_4\kappa_6-\frac{5}{72}x^6\kappa_1\kappa_3^3\kappa_4\kappa_6 \end{aligned}$$

$$y3(x)=\frac{1}{3}x^3\kappa_3^2\kappa_6-\frac{1}{4}x^4\kappa_1\kappa_3^2\kappa_6+\frac{7}{60}x^5\kappa_1^2\kappa_3^2\kappa_6-\frac{1}{24}x^6\kappa_1^3\kappa_3^2\kappa_6-\frac{2}{15}x^5\kappa_3^3\kappa_5\kappa_6+\frac{5}{36}x^6\kappa_1\kappa_3^3\kappa_5\kappa_6$$

Table 2 shows the results of the solution of Example 2 and illustrates the absolute errors between solution obtained by using Runge-Kutta fourth order method and solution obtained by using combined Laplace transform–Adomian decomposition method. We achieved a good approximation by using presented method with only six iterations.

Table 2.
The absolute error involved in the combined Laplace transform–Adomian decomposition method along with the result obtained by the Runge-Kutta fourth order method for Example 2

x_i	$\lvert y_1(x_i)-RKM\rvert$	$\lvert y_2(x_i)-RKM\rvert$	$\lvert y_3(x_i)-RKM\rvert$
0.0000	0.	$3.666064475e^{-21}$	$4.449668544e^{-30}$
0.0002	$7.105427358e^{-15}$	$4.499723839e^{-7}$	$4.498063952e^{-9}$
0.0004	$6.883382753e^{-15}$	0.00005671875213	$5.670656463e^{-7}$
0.0006	$6.994405055e^{-15}$	0.0008889447241	$8.887566672e^{-6}$
0.0008	$6.772360450e^{-15}$	0.005967764147	0.00005966518137
0.0010	$6.994405055e^{-15}$	0.02510443631	0.0002509926943

Example 3. Now we consider the following system of differential equations [6].

$$\begin{aligned}\frac{d^2y_1}{dx^2}+y_2&=1\\\frac{d^2y_2}{dx^2}+y_1&=0\end{aligned}\tag{21}$$

With the initial conditions $y_1(0)=y_2(0)=y_1'(0)=y_2'(0)=0$. Exact solutions are

$$y_1(x)=\frac{e^x}{4}+\frac{e^{-x}}{4}-\frac{\cos x}{2},\ y_2(x)=1-\frac{e^x}{4}-\frac{e^{-x}}{4}-\frac{\cos x}{2}.$$

Taking the Laplace transformation of Eqs. (21), we get

$$\begin{aligned}L\{y_1\}&=\frac{1}{s^3}-\frac{1}{s^2}L\{y_2\}\\L\{y_2\}&=-\frac{1}{s^2}L\{y_1\}\end{aligned}\tag{22}$$

Substituting $y_1=\sum_{k=0}^{\infty}y_{1k}$ and $y_2=\sum_{k=0}^{\infty}y_{2k}$ in (22) leads to

$$\begin{aligned}L\left\{\sum_{k=0}^{\infty}y_{1k}\right\}&=\frac{1}{s^3}-\frac{1}{s^2}L\left\{\sum_{k=0}^{\infty}y_{2k}\right\}\\L\left\{\sum_{k=0}^{\infty}y_{2k}\right\}&=-\frac{1}{s^2}L\left\{\sum_{k=0}^{\infty}y_{1k}\right\}\end{aligned}\tag{23}$$

We obtain the following procedure by using the combined Laplace transform–Adomian decomposition method.

$$\begin{aligned} &L\{y_{10}\}=\frac{1}{s^3}, L\{y_{1k+1}\}=-\frac{1}{s^2}L\{y_{2k}\} \\ &L\{y_{20}\}=0, L\{y_{2k+1}\}=-\frac{1}{s^2}L\{y_{1k}\} \end{aligned} \tag{24}$$

Table 3
The absolute error involved in the combined Laplace transform–Adomian decomposition method along with the exact solution for Example 3

x_i	$y_1(x_i)$	$err(y_1(x_i))$	$y_2(x_i)$	$err(y_2(x_i))$
0	0.	0.	0.	0.
0.1	0.005	1.11022×10^{-16}	-4.16667×10^{-6}	5.55112×10^{-17}
0.2	0.0200001	0.	-0.0000666667	1.11022×10^{-16}
0.3	0.045001	0.	-0.000337502	0.
0.4	0.0800057	0.	-0.00106668	0.
0.5	0.125022	0.	-0.00260426	5.55112×10^{-17}
0.6	0.180065	0.	-0.00540042	0.
0.7	0.245163	0.	-0.0100056	0.
0.8	0.320364	0.	-0.0170708	5.55112×10^{-17}
0.9	0.405738	1.11022×10^{-16}	-0.0273482	5.55112×10^{-17}
1	0.501389	0.	-0.0416915	0.

We obtain the following solutions

$$y_1(t)=\frac{t^2}{2}+\frac{t^6}{720}+\frac{t^{10}}{3628800}+\frac{t^{14}}{87178291200}+\frac{t^{18}}{6402373705728000}$$
$$+\frac{t^{22}}{1124000727777607680000}$$

$$y_2(t)=-\frac{t^4}{24}-\frac{t^8}{40320}-\frac{t^{12}}{479001600}-\frac{t^{16}}{20922789888000}$$
$$-\frac{t^{20}}{2432902008176640000}$$

Table 3 shows the results of the solution of Example 3 and illustrates the absolute errors between the exact solution and the solution of presented method. We achieved a good approximation with the presented method with only six iterations.

4. CONCLUSION

Combined Laplace transform–Adomian decomposition method has been applied to linear and non-linear systems of ordinary differential equations. Numerical examples have been presented to show that the approach is promising and the research is worth continue in this direction. All the calculations are performed easily. The calculated results are quite reliable. Since every ordinary differential equations of order n can be written as a linear algebraic equation by using Laplace Transform, this method is very useful and reliable for any order ordinary differential equation systems. Therefore, this method can be applied to many complicated linear and non-linear ODEs.

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