Mathematical and Computational Applications, Vol. 16, No. 3, pp. 588-597, 2011.

AN APPROXIMATION APPROACH FOR RANKING FUZZY NUMBERS BASED ON WEIGHTED INTERVAL - VALUE

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Abstract- In the present paper, the researchers discuss the problem of weighted interval approximation of fuzzy numbers. This interval can be used as a crisp set approximation with respect to a fuzzy quantity. Then, by using this, the researchers propose a novel approach to ranking fuzzy numbers. This method can effectively rank various fuzzy numbers, their images and overcome the shortcomings of the previous techniques.
Key Words- Ranking, Fuzzy numbers, Weighted interval-value, Defuzzification.

1.INTRODUCTION

In decision analysis under fuzzy environment, ranking fuzzy numbers is a very important decision-making procedure. Since Jain, Dubis and Prade [17] introduced the relevant concepts of fuzzy numbers, many researchers proposed the related methods or applications for ranking fuzzy numbers. For instance, Bortolan and Degani [5] reviewed some methods to rank fuzzy numbers in 1985, Chen and Hwang [6] proposed fuzzy multiple attribute decision making in 1992, Choobineh and Li [7] proposed an index for ordering fuzzy numbers in 1993, Lee [15] ranked fuzzy numbers with a satisfaction function in 1994, Fortemps [8] presented ranking and defuzzification methods based on area compensation in 1996, and Raj [14] investigated maximizing and minimizing sets to rank fuzzy alternatives with fuzzy weights in 1999. In 1988, Lee and Li proposed the comparison of fuzzy numbers, for which they considered mean and standard deviation values for fuzzy numbers based on the uniform and proportional probability distributions. Although, Cheng overcame the problems from these comments and also proposed a new distance index to improve the method [16] proposed by Murakami et al., Chu and Tsao's still believed that Cheng's method contained some shortcomings. Furthermore, Cheng also proposed a distance method to rank fuzzy numbers for improving the method of Murakami et al. and the distance method often contradicts the CV index on ranking fuzzy numbers. To overcome these above problems, Chu and Tsao proposed a method to rank fuzzy numbers with an area between their centroid and original points. The method can avoid the problems Chu and Tsao mentioned; however, the researchers find other problems in their method. Also, Wang et al. [27] proposed an approach to ranking fuzzy numbers based on lexicographic screening procedure and summarized some limitations of the existing methods. Having reviewed the previous methods, this article proposes here a method to use the concept weighted interval-value of a fuzzy number, so as to find the order of fuzzy numbers. This method can distinguish the alternatives clearly. The main purpose of this article is that, this weighted interval-value of a fuzzy number can be used as a crisp set approximation of a fuzzy number. Therefore, by the means of this approximation, this article aims to present a new method for ranking of fuzzy numbers.

In addition to its ranking features, this method removes the ambiguities resulted and overcome the shortcomings from the comparison of previous ranking.

The paper is organized as follows: In Section 2, this article recall some fundamental results on fuzzy numbers. In Section 3, a crisp set approximation of a fuzzy number is obtained. In this Section some remarks are proposed and illustrated. Proposed method for ranking fuzzy numbers is in the Section 4. Discussion and comparison of this work and other methods are carried out in Section 5.

2. BASIC DEFINITIONS AND NOTATIONS

The basic definitions of a fuzzy number are given in [9, 10, 11, 12, 13, 19, 20] as follows.

Definition 1. Let be a universe set. A fuzzy set $\tilde{A}$ of is defined by a membership function $\mu_{\tilde{A}}(x): U \to [0,1]$, where $\mu_{\tilde{A}}(x), \forall x \in U$, indicates the degree of in $\tilde{A}$.

Definition 2. A fuzzy subset $\tilde{A}$ of universe set is normal iff $\sup_{x \in U} \mu_{\tilde{A}}(x) = 1$, where is the universe set.

Definition 3. A fuzzy subset $\tilde{A}$ of universe set is convex iff $\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq (\mu_{\tilde{A}}(x) \wedge \mu_{\tilde{A}}(y)), \forall x, y \in U, \forall \lambda \in [0,1]$. In this article symbols $\wedge$ and $\vee$ denotes the minimum and maximum operators, respectively.

Definition 4. A fuzzy set $\tilde{A}$ is a fuzzy number iff $\tilde{A}$ is normal and convex on .

Definition 5. A trapezoidal fuzzy number $\tilde{A}$ is a fuzzy number with a membership function $\mu_{\tilde{A}}$ defined by :

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{when } a_1 \leq x \leq a_2 \\ 1 & \text{when } a_2 \leq x \leq a_3, \\ \dfrac{a_4 - x}{a_4 - a_3}, & \text{when } a_3 \leq x \leq a_4, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

which can be denoted as a quartet .

Definition 6. The -cut of a fuzzy number $\tilde{A}$, where $0 < r \leq 1$ is a set defined as $\tilde{A} = \left\{x \in \mathcal{R} \middle| \mu_{\tilde{A}}(x) \geq r\right\}$. According to the definition of a fuzzy number it is seen at once that every -cut of fuzzy number is a closed interval. Hence, this article has $\tilde{A} = [\tilde{A}_L(r), \tilde{A}_R(r)]$, where

$$\tilde{A}_L(r) = \inf\{x \in \mathcal{R} | \mu_{\tilde{A}}(x) \geq r\} \text{ and } \tilde{A}_R(r) = \sup\{x \in \mathcal{R} | \mu_{\tilde{A}}(x) \geq r\}.$$

A space of all fuzzy numbers will be denoted by $\mathcal{F}$.

Definition 7. [4]. For two arbitrary fuzzy numbers $\tilde{A}$ and $\tilde{B}$ with -cut sets $\tilde{A} = [\tilde{A}_L(r), \tilde{A}_R(r)]$ and $\tilde{B} = [\tilde{B}_L(r), \tilde{B}_R(r)]$ respectively, the quantity

$$d_W(\tilde{A}, \tilde{B}) = \left(\int_0^1 f(r) d^2(\tilde{A}_r, \tilde{B}_r) dr\right)^{\frac{1}{2}}, \tag{2}$$

is the weighted distance between $\tilde{A}$ and $\tilde{B}$, where

$d^2(\tilde{A}_r, \tilde{B}_r) = (\tilde{A}_L(r) - \tilde{B}_L(r))^2 + (\tilde{A}_R(r) - \tilde{B}_R(r))^2$.

And the $f(r)$ is nonnegative and increasing on $[0,1]$ with $f(0)$ and $\int_0^1 f(r)dr = \frac{1}{2}$.

Definition 8. [25]. An operator is called an interval approximation operator if for any $\tilde{A} \in \mathcal{F}$

(a) $I(\tilde{A}) \subseteq \text{supp}\tilde{A}$,

(b) $\text{core}(\tilde{A}) \subseteq I(\tilde{A})$,

(c) $\forall(\varepsilon > 0)\ \exists(\delta > 0)$ s.t $d(\tilde{A}, \tilde{B}) < \delta \Rightarrow d\left(I(\tilde{A}), I(\tilde{B})\right) < \varepsilon$,

where $d: \mathcal{F} \to [0, +\infty[$ denotes a metric defined in the family of all fuzzy numbers.

Definition 9. [25]. An interval approximation operator satisfying in condition (c) for any $\tilde{A}, \tilde{B} \in \mathcal{F}$ is called the continuous interval approximation operator.

3. WEIGHTED INTERVAL - VALUE APPROXIMATION

Various authors in [18] and [25] have studied the crisp approximation of fuzzy sets. They proposed a rough theoretic definition of that crisp approximation, called the nearest ordinary set and nearest interval approximation of a fuzzy set. In this section, the researchers will propose another approximation called the weighted interval-value approximation. Let $\tilde{A}$ be an arbitrary fuzzy number and $[\tilde{A}_L(r), \tilde{A}_R(r)]$ be its -cut set. This article will try to find a closed interval $I_{d_w}(\tilde{A})$, which is the weighted interval to $\tilde{A}$ with respect to metric d_w. Since each interval with constant -cuts for all $r \in]0,1]$ is a fuzzy number, hence, suppose $I_{d_w}(\tilde{A}) = [I_L, I_R]$, i.e. $\left(I_{d_w}(\tilde{A})\right)_r = [I_L, I_R]$, $\forall r \in]0,1]$. So, this article has to minimize

$$d_w\left(\tilde{A}, I_{d_w}(\tilde{A})\right) = \left(\int_0^1 f(r) d^2\left(\tilde{A}_r, \left(I_{d_w}(\tilde{A})\right)_r\right)\right)^{\frac{1}{2}}. \tag{3}$$

with respect to I_L and I_R, where $d^2\left(\tilde{A}_r, \left(I_{d_w}(\tilde{A})\right)_r\right) = \left(\tilde{A}_L(r) - I_L\right)^2 + \left(\tilde{A}_R(r) - I_R\right)^2$.

In order to minimize d_w it suffices to minimize $\bar{D}_w(I_L, I_R) = d_w^2(I_L, I_R)$.

It is clear that, the parameters I_L and I_R which minimize Eq.(3) must satisfy in

$$\nabla\bar{D}_w(I_L, I_R) = \left(\frac{\partial\bar{D}_w}{I_L}, \frac{\partial\bar{D}_w}{I_R}\right) = 0.$$

Therefore, this article has the following equations:

$$\begin{cases} \dfrac{\partial\bar{D}_w(I_L, I_R)}{\partial I_L} = -2\displaystyle\int_0^1 f(r)\left(\tilde{A}_L(r) - I_L\right)dr = 0, \\ \dfrac{\partial\bar{D}_w(I_L, I_R)}{\partial I_R} = -2\displaystyle\int_0^1 f(r)\left(\tilde{A}_R(r) - I_R\right)dr = 0. \end{cases} \tag{4}$$

The parameter associated with the left bound and associated with the right bound of the weighted interval-value can be found by using Eq. (4) as follows:

$$\begin{cases} I_L = 2\displaystyle\int_0^1 f(r)\tilde{A}_L(r)dr, \\ I_R = 2\displaystyle\int_0^1 f(r)\tilde{A}_R(r)dr. \end{cases} \tag{5}$$

Remark 1. Since $\det\begin{bmatrix} \frac{\partial D^2(I_L, I_R)}{\partial I_L^2} & \frac{\partial D^2(I_L, I_R)}{\partial I_R \partial I_L} \\ \frac{\partial D^2(I_L, I_R)}{\partial I_L \partial I_R} & \frac{\partial D^2(I_L, I_R)}{\partial I_R^2} \end{bmatrix} = \det\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 > 0,$ and

, therefore

I_L and I_R given by (5), minimize $d_w(\tilde{A}, I_{d_w}(\tilde{A}))$. Therefore, the interval

$$I_{d_w}(\tilde{A}) = \left[2\int_0^1 f(r)\tilde{A}_L(r)dr, 2\int_0^1 f(r)\tilde{A}_R(r)dr\right], \tag{6}$$

is theweighted interval-value approximation of fuzzy number $\tilde{A}$ with respect to d_w.

Remark 2. In special case, if this article assume that $f(r) = r$, therefore $I_{d_w}(\tilde{A})$ is weighted interval-value possibilistic mean [26].

Then, let this article wants to approximate a fuzzy number by a crisp interval. Thus, the researchers have to use an operator which transforms fuzzy numbers into family of closed intervals on the real line.

Lemma 1. [4].

Theorem 1. The operator is an interval approximation operator, i.e. I_{d_w} is a continuous interval approximation operator.

Proof. It is easy to verify that the conditions $(\acute{a})$ and $(\acute{b})$ are hold. For the proof of $(\acute{c})$, let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers, then

$$d_w^2\left(I_{d_w}(\tilde{A}), I_{d_w}(\tilde{B})\right) = \int_0^1 f(r)\left(I_L(\tilde{A}) - I_L(\tilde{B})\right)^2 dr + \int_0^1 f(r)\left(I_R(\tilde{A}) - I_R(\tilde{B})\right)^2 dr$$

$$= \frac{1}{2}(I_L(\tilde{A}) - I_L(\tilde{B}))^2 + \frac{1}{2}(I_R(\tilde{A}) - I_R(\tilde{B}))^2$$

$$= 2\left(\int_0^1 f(r)(\tilde{A}_L(r) - \tilde{B}_L(r))dr\right)^2 + 2\left(\int_0^1 f(r)(\tilde{A}_R(r) - \tilde{B}_R(r))dr\right)^2.$$

Via to Lemma (1), there is

$$\leq 2\int_0^1 f(r)(\tilde{A}_L(r) - \tilde{B}_L(r))^2 dr + 2\int_0^1 f(r)(\tilde{A}_R(r) - \tilde{B}_R(r))^2 dr$$

$$= 2\int_0^1 f(r)[(\tilde{A}_L(r) - \tilde{B}_L(r))^2 + (\tilde{A}_R(r) - \tilde{B}_R(r))^2]dr = 2d_w^2(\tilde{A}, \tilde{B}).$$

It means that $\forall \varepsilon > 0, \exists \delta = \frac{\sqrt{2}\varepsilon}{2} > 0,$ when , then this article has $d_w = \left(I_{d_w}(\tilde{A}), I_{d_w}(\tilde{B})\right) < \varepsilon.$ It shows that our weighted interval-value approximation is continuous interval approximation.

4. COMPARISON FUZZY NUMBERS BY WEIGHTED INTERVAL

In this Section, the researchers will propose the ranking of fuzzy numbers associated with the weighted interval-value. Let $\tilde{\mathbf{A}}$ be an arbitrary fuzzy number that is characterized by (1) and $[\tilde{A}_L(r), \tilde{A}_R(r)]$ be its -cut and $I_{d_w}(\tilde{A})$ be its the weighted interval-value. Since ever weighted interval-value can be used as a crisp set approximation of a fuzzy number, therefore, the resulting interval is used to rank the fuzzy numbers. Thus, I_{d_w} is used to rank fuzzy numbers.

Definition 10. Let $\mathcal{F}[L]$ represents the set of closed intervals in $\mathcal{R}$. The interval order is the following:

$\forall\, \alpha_1, \beta_1, \alpha_2, \beta_2 \in \mathcal{R}$ verifying $\alpha_1 \le \beta_1$ & $\alpha_2 \le \beta_2$,

$$[\alpha_1, \beta_1] \le [\alpha_2, \beta_2] \Leftrightarrow \alpha_1 \le \alpha_2 \;\;\&\;\; \beta_1 \le \beta_2 .$$

Let, $\mathcal{A}$ and $\mathcal{B} \in \mathcal{F}[L]$, this article also defines $\mathcal{SUP}(\mathcal{A}, \mathcal{B})$ and $\mathcal{INF}(\mathcal{A}, \mathcal{B})$ the supremum and the infimum of two intervals as follows:

$$\mathcal{SUP}(\mathcal{A}, \mathcal{B}) = \mathcal{A} \bigvee \mathcal{B} \quad \text{and} \quad \mathcal{INF}(\mathcal{A}, \mathcal{B}) = \mathcal{A} \bigwedge \mathcal{B},$$

where the supremum and the infimum of the intervals are defined in the following way:

$\forall\, \alpha_1, \beta_1, \alpha_2, \beta_2 \in \mathcal{R}$ verifying $\alpha_1 \le \beta_1$ & $\alpha_2 \le \beta_2$,

$[\alpha_1, \beta_1] \vee [\alpha_2, \beta_2] = [\alpha_1 \vee \alpha_2, \beta_1 \vee \beta_2]$ and $[\alpha_1, \beta_1] \wedge [\alpha_2, \beta_2] = [\alpha_1 \wedge \alpha_2, \beta_1 \wedge \beta_2]$.

Let $\tilde{A}$ and $\tilde{B} \in \mathcal{F}$ be two arbitrary fuzzy numbers, and $I_{d_w}(\tilde{A}) = [a_1, a_2]$ and $I_{d_w}(\tilde{B}) = [b_1, b_2]$ be the weighted interval-value of $\tilde{A}$ and $\tilde{B}$, respectively. Define the ranking of $\tilde{A}$ and $\tilde{B}$ by on $\mathcal{F}$, i.e.

1. $I_{d_w}(\tilde{A}) \supseteq I_{d_w}(\tilde{B})$ **if only if** $\tilde{A} \succ \tilde{B}$,
2. $\mathcal{INF}(\tilde{A}, \tilde{B})$ $\mathcal{SUP}(\tilde{A}, \tilde{B})$ **if only if** $\tilde{A} \prec \tilde{B}$,
3. $\mathcal{INF}(\tilde{A}, \tilde{B}) = \mathcal{SUP}(\tilde{A}, \tilde{B})$ **if only if** $\tilde{A} \sim \tilde{B}$.

Then, this article formulates the order $\succsim$ and $\precsim$ as $\tilde{A} \succsim \tilde{B}$ if and only if $\tilde{A} \succ \tilde{B}$ or $\tilde{A} \sim \tilde{B}$, $\tilde{A} \precsim \tilde{B}$ if and only if $\tilde{A} \prec \tilde{B}$ or $\tilde{A} \sim \tilde{B}$.

Remark 3. For two arbitrary fuzzy numbers $\tilde{A}$ and $\tilde{B}$, this article has

$$I_{d_w}(\tilde{A} + \tilde{B}) = I_{d_w}(\tilde{A}) + I_{d_w}(\tilde{B}).$$

Proof. Let $\tilde{A} = [\tilde{A}_L(r), \tilde{A}_R(r)]$ and $\tilde{B} = [\tilde{B}_L(r), \tilde{B}_R(r)]$ be the -cut sets of $\tilde{A}$ and $\tilde{B}$, respectively. There is

$$\begin{aligned}
& I_{d_w}(\tilde{A}) + I_{d_w}(\tilde{B}) \\
&= \left[2\int_0^1 f(r)\tilde{A}_L(r)\,dr,\ 2\int_0^1 f(r)\tilde{A}_R(r)dr\right] + \left[2\int_0^1 f(r)\tilde{B}_L(r)\,dr,\ 2\int_0^1 f(r)\tilde{B}_R(r)dr\right] \\
&= \left[2\int_0^1 f(r)(\tilde{A}_L(r) + \tilde{B}_L(r))dr,\ 2\int_0^1 f(r)(\tilde{A}_R(r) + \tilde{B}_R(r))dr\right] \\
&= \left[2\int_0^1 f(r)\left(\tilde{A}_L + \tilde{B}_L\right)(r)dr,\ 2\int_0^1 f(r)\left(\tilde{A}_R + \tilde{B}_R\right)(r)dr\right] = I_{d_w}(\tilde{A} + \tilde{B}).
\end{aligned}$$

This article considers the following reasonable axioms that Wang and Kerre [21] proposed for fuzzy quantities ranking. Let $I_{d_w}(.)$ be an ordering method, the set of fuzzy quantities for which the method $I_{d_w}(.)$ can be applied, and $\mathcal{A}$ a finite subset of . The statement "two elements $\tilde{A}$ and $\tilde{B}$ in $\mathcal{A}$ satisfy that $\tilde{A}$ has a higher ranking than $\tilde{B}$ when $I_{d_w}(.)$ is applied to the fuzzy quantities in" will be written as " $\tilde{A} \succ \tilde{B}$ by $I_{d_w}(.)$ on $\mathcal{A}$ ", "$\tilde{A} \sim \tilde{B}$ by $I_{d_w}(\cdot)$ on $\mathcal{A}$ ", and "$\tilde{A} \succsim \tilde{B}$ by $I_{d_w}(\cdot)$ on $\mathcal{A}$ " are similarly interpreted. The axioms as the reasonable properties of ordering fuzzy quantities for an ordering approach $I_{d_w}(.)$ are as follows:

A-1 For an arbitrary finite subset $\mathcal{A}$ of S and $\tilde{A} \in \mathcal{A}$; $\tilde{A} \succsim \tilde{A}$.

A-2 For an arbitrary finite subset $\mathcal{A}$ of S and $(\tilde{A}, \tilde{B}) \in \mathcal{A}^2$; $\tilde{A} \succsim \tilde{B}$ and $\tilde{B} \succsim \tilde{A}$, this method should have $\tilde{A} \sim \tilde{B}$.

A-3 For an arbitrary finite subset $\mathcal{A}$ of S and $(\tilde{A}, \tilde{B}, \tilde{C}) \in \mathcal{A}^3$; $\tilde{A} \succsim \tilde{B}$ and $\tilde{B} \succsim \tilde{C}$, this method should have $\tilde{A} \succsim \tilde{C}$.

A-4 For an arbitrary finite subset $\mathcal{A}$ of and $(\tilde{A}, \tilde{B}) \in \mathcal{A}^2$; $\inf \operatorname{supp}(\tilde{A}) > \sup \operatorname{supp}(\tilde{B})$, this method should have $\tilde{A} \succsim \tilde{B}$.

A'- 4 For an arbitrary finite subset $\mathcal{A}$ of S and $(\tilde{A}, \tilde{B}) \in \mathcal{A}^2$; $\inf \operatorname{supp}(\tilde{A}) > \sup \operatorname{supp}(\tilde{B})$, this method should have $\tilde{A} \succ \tilde{B}$.

A- 5 Let S and S' be two arbitrary finite sets of fuzzy quantities in which $I_{d_w}(.)$ can be applied and $\tilde{A}$ and $\tilde{B}$ are in $S \cap S'$. This method obtain the ranking order $\tilde{A} \succsim \tilde{B}$ by $I_{d_w}(.)$ on S' iff $\tilde{A} \succsim \tilde{B}$ by $I_{d_w}(.)$ on .

A-6 Let $\tilde{A}$, $\tilde{B}$, $\tilde{A} + \tilde{C}$ and $\tilde{B} + \tilde{C}$ be elements of S . If $\tilde{A} \succsim \tilde{B}$, then $\tilde{A} + \tilde{C} \succsim \tilde{B} + \tilde{C}$ by on $\tilde{A} + \tilde{C}$ and $\tilde{B} + \tilde{C}$.

A'- 6 Let $\tilde{A}$, $\tilde{B}$, $\tilde{A} + \tilde{C}$ and $\tilde{B} + \tilde{C}$ be elements of . If $\tilde{A} \succ \tilde{B}$ by on $\tilde{A}$ and $\tilde{B}$, then $\tilde{A} + \tilde{C} \succ \tilde{B} + \tilde{C}$ by on $\tilde{A} + \tilde{C}$ and $\tilde{B} + \tilde{C}$.

A-7 For an arbitrary finite subset $\mathcal{A}$ of S and $\tilde{A} \in \mathcal{A}$; the must belong to its support.

Theorem 2. The function I_{d_w} has the properties (A-1), (A-2), ..., (A-7).

Proof. It is easy to verify that the properties (A-1), (A-2), ..., (A-5) and (A-7) are hold. For the proof of (A-6), this article consider the fuzzy numbers $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$. Let $\tilde{A} \succsim \tilde{B}$, from the relation (7), there is

$$I_{d_w}(\tilde{A}) \geq I_{d_w}(\tilde{B}),$$

by adding $I_{d_w}(\tilde{C})$,

$$I_{d_w}(\tilde{A}) + I_{d_w}(\tilde{C}) \geq I_{d_w}(\tilde{B}) + I_{d_w}(\tilde{C}),$$

and by Remark (3),

$$I_{d_w}(\tilde{A} + \tilde{C}) \geq I_{d_w}(\tilde{B} + \tilde{C}),$$

therefore

$$\tilde{A} + \tilde{C} \succsim \tilde{B} + \tilde{C}.$$

With which the proof is complete. Similarly (A'- 6) is hold.

Remark 4. If $\tilde{A} \precsim \tilde{B}$, then $-\tilde{A} \succsim -\tilde{B}$.

Hence, this article can infer ranking order of the images of the fuzzy numbers.

5. NUMERICAL EXAMPLES

In this section, the researchers compare proposed method with others in [1, 2, 3, 7, 22, 23, 24]. Throughout this section the researchers assumed that $\mathbf{f(r)} =$

Example 1. Consider the following sets, see Yao and Wu [27].

Set 1: $\tilde{\mathbf{A}}$=(0.4,0.5,1), $\tilde{\mathbf{B}}$=(0.4,0.7,1), $\tilde{\mathbf{C}}$=(0.4,0.9,1).

Set 2: $\tilde{\mathbf{A}}$=(0.3,0.4,0.7,0.9) (trapezoidal fuzzy number), $\tilde{\mathbf{B}}$=(0.3,0.7,0.9), $\tilde{\mathbf{C}}$=(0.5,0.7,0.9).

Set 3: $\tilde{\mathbf{A}}$=(0.3,0.5,0.7), $\tilde{\mathbf{B}}$=(0.3,0.5,0.8,0.9) (trapezoidal fuzzy number), $\tilde{\mathbf{C}}$=(0.3,0.5,0.9).

Set 4: $\tilde{\mathbf{A}}$=(0.4,0.7,0.4,0.1) (trapezoidal fuzzy number), $\tilde{\mathbf{B}}$=(0.5,0.3,0.4), $\tilde{\mathbf{C}}$=(0.6,0.5,0.2).

To compare with other methods authors refer the reader to Table 1.

Table 1. Comparative results of Example 1.

Authors	Fuzzy umber	Set1	Set2	Set3	Set4
Proposed method	$\tilde{A}$	[0.46,0.66]	[0.36,0.76]	[0.43,0.56]	[0.26,0.73]
	$\tilde{B}$	[0.60,0.80]	[0.56,0.76]	[0.43,0.83]	[0.40,0.63]
	$\tilde{C}$	[0.73,0.93]	[0.63,0.76]	[0.43,0.63]	[0.43,066]
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{B} \prec \tilde{C} \prec \tilde{A}$
Sing Distance method with p=1	$\tilde{A}$	1.2000	1.1500	1.0000	0.0950
	$\tilde{B}$	1.4000	1.3000	1.2500	1.0500
	$\tilde{C}$	1.6000	1.4000	1.1000	1.0500
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \sim \tilde{C}$
Sing Distance method with p=2	$\tilde{A}$	0.8869	0.8756	0.7257	0.7853
	$\tilde{B}$	1.0194	0.9522	0.9416	0.7958
	$\tilde{C}$	1.1605	1.0033	0.8165	0.8386
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Distance Minimization	$\tilde{A}$	0.6	0.575	0.5	0.475
	$\tilde{B}$	0.7	0.65	0.625	0.525
	$\tilde{C}$	0.9	0.7	0.55	0.525
Result		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \sim \tilde{C}$
Abbasbandy and Hajjari (Magnitude method)	$\tilde{A}$	0.5334	0.5584	0.5000	0.5250
	$\tilde{B}$	0.7000	0.6334	0.6416	0.5084
	$\tilde{C}$	0.8666	0.7000	0.5166	0.5750
Result		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{B} \prec \tilde{A} \prec \tilde{C}$
Choobineh and Li	$\tilde{A}$	0.3333	0.5480	0.3330	0.5000
	$\tilde{B}$	0.5000	0.5830	0.4164	0.5833
	$\tilde{C}$	0.6670	0.6670	0.5417	0.6111
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Yager	$\tilde{A}$	0.6000	0.5750	0.5000	0.4500
	$\tilde{B}$	0.7000	0.6500	0.5500	0.5250
	$\tilde{C}$	0.8000	0.7000	0.6250	0.5500
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Chen	$\tilde{A}$	0.3375	0.4315	0.3750	0.5200
	$\tilde{B}$	0.5000	0.5625	0.4250	0.5700

	$\tilde{C}$	0.6670	0.6250	0.5500	0.6250
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$
Baldwin and Guild	$\tilde{A}$	0.3000	0.2700	0.2700	0.4000
	$\tilde{B}$	0.3300	0.2700	0.3700	0.4200
	$\tilde{C}$	0.4400	0.3700	0.4500	0.4200
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \sim \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \sim \tilde{C}$
Chu and Tsao	$\tilde{A}$	0.2990	0.2847	0.2500	0.2440
	$\tilde{B}$	0.3500	0.3247	0.3152	0.2624
	$\tilde{C}$	0.3993	0.3500	0.2747	0.2619
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Yao and Wu	$\tilde{A}$	0.6000	0.5750	0.5000	0.4750
	$\tilde{B}$	0.7000	0.6500	0.6250	0.5250
	$\tilde{C}$	0.8000	0.7000	0.5500	0.5250
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{B} \sim \tilde{C}$
Cheng Distance	$\tilde{A}$	0.7900	0.7577	0.7071	0.7106
	$\tilde{B}$	0.8602	0.8149	0.8037	0.7256
	$\tilde{C}$	0.9268	0.8602	0.7458	0.7241
Results		$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{B} \prec \tilde{C}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$
Cheng CV uniform distribution	$\tilde{A}$	0.0272	0.328	0.0133	0.0693
	$\tilde{B}$	0.0214	0.0246	0.0304	0.0385
	$\tilde{C}$	0.0225	0.0095	0.2750	0.0433
Results		$\tilde{B} \prec \tilde{C} \prec \tilde{A}$	$\tilde{C} \prec \tilde{B} \prec \tilde{A}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{B} \prec \tilde{C} \prec \tilde{A}$
Cheng CV proportional distribution	$\tilde{A}$	0.1830	0.0260	0.0080	0.0471
	$\tilde{B}$	0.0128	0.0146	0.0234	0.0236
	$\tilde{C}$	0.0137	0.0057	0.0173	0.0255
Results		$\tilde{B} \prec \tilde{C} \prec \tilde{A}$	$\tilde{C} \prec \tilde{B} \prec \tilde{A}$	$\tilde{A} \prec \tilde{C} \prec \tilde{B}$	$\tilde{B} \prec \tilde{C} \prec \tilde{A}$

Note that, in Table (1) and in set (4), for Sign Distance(p=1), Distance Minimization, Chu-Tsao and Yao-Wu methods, the ranking order for fuzzy numbers $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{C}}$ is $\tilde{\mathbf{B}} \sim \tilde{\mathbf{C}}$, which is unreasonable. But this method has the same result as other techniques(Cheng distribution).

Example 2. The two symmetric triangular fuzzy numbers $\tilde{\mathbf{A}} = (1, 3, 5)$ and $\tilde{\mathbf{B}} = (2, 3, 4)$, taken from paper [24]. To compare with other methods this article refer the readers to Table 2. In this Table, $\tilde{\mathbf{A}} \sim \tilde{\mathbf{B}}$ is the result of Sign Distance method with p=1, Magnitude method, Distance Minimization and Chen method, which is unreasonable. The results of proposed method is the same as Sign Distance method with p=2, i.e. $\tilde{\mathbf{A}} \succ \tilde{\mathbf{B}}$.

Table 2. Comparative results of Example 2.

fuzzy number	new approach	Magnitude method	Sign Distance With p=1	Sign Distance with p=2	Distance Minimization	Chen Max-Min
$\tilde{A}$	[2.33,3.50]	3	6	4.546	3	0.5
$\tilde{B}$	[2.66,3.33]	3	6	4.32	3	0.5
Results	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \succ \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$

Example 3. Consider the three fuzzy numbers $\tilde{A}$=(1,2,5), $\tilde{B}$=(0,3,4) and $\tilde{C}$=(2,2.5,3). By using this new approach $I_{d_w}(\tilde{A})$=[1.66,3.00], $I_{d_w}(\tilde{B})$=[2.00,3.33] and $I_{d_w}(\tilde{C})$=[2.33,2.66]. Hence, the ranking order is $\tilde{C} \prec \tilde{A} \prec \tilde{B}$ too. Obviously, the results obtained by "Sign distance" and "Distance Minimization" methods are unreasinable. To compare with some of the other methods in [23], the reader can refer to Table 3.

Furthermore, to aforesaid example $I_{d_w}(-\tilde{A})$=[-3.00,-1.66], $I_{d_w}(-\tilde{B})$=[-3.33,-2.00] and $I_{d_w}(-\tilde{C})$=[-2.66,-2.33], consequently the ranking order of the images of three fuzzy number is $-\tilde{B} \prec -\tilde{A} \prec -\tilde{C}$. Clearly, this proposed method has consistency in ranking fuzzy numbers and their images, which could not be guaranteed by CV-index method.

Table 3. Comparative results of Example 3.

Fuzzy number	new approach	Sign Distance With p=1	Sign Distance with p=2	Distance Minimization	Chu and Tsao
$\tilde{A}$	[1.66,3.00]	5	3.9157	2.5	0.7407
$\tilde{B}$	[2.00,3.33]	5	3.9157	2.5	0.7407
$\tilde{C}$	[2.33,2.66]	5	3.5590	2.5	0.75
Results	$\tilde{C} \prec \tilde{A} \prec \tilde{B}$	$\tilde{C} \sim \tilde{A} \sim \tilde{B}$	$\tilde{C} \prec \tilde{A} \sim \tilde{B}$	$\tilde{C} \sim \tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B} \prec \tilde{C}$

All the above examples show that the results of this method are reasonable results. This method can overcome the shortcomings of "Magnitude method" and "Distance Minization" method.

6. CONCLUSION

In this paper, the researchers proposed a method to rank fuzzy numbers. This method used a crisp set approximation of a fuzzy number that this operator leads to the interval which is the best one with respect to a certain measure of distances between fuzzy numbers. The method can effectively rank various fuzzy numbers and their images (normal/ nonnormal/trapezoidal/general), and overcome the shortcomings which are found in the other methods.

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