

THREE DIMENSIONAL CUTTING PROBLEM: AN INTEGER PROGRAMMING APPROACH

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Abstract- Cutting problem is one of the major problems faced in marble factories. Huge marble blocks coming from marble quarries are first cut into planes according to their depths. Then these planes are sized according to the length and width of the demanded products. In this study, cutting problem of a marble factory is handled. The two problems, which are cutting marble blocks into planes and sizing these planes, are integrated together. An integer programming model is built for the integrated problem. The objective of the model is to minimize total waste of marble while finding the cutting design of the marble block and marble planes. The model is used to solve the cutting problem of a real life marble processing factory. It is shown that the mathematical model decreases wasted amount of marble considerably compared to the industry average.

Key Words- Cutting problem, Integer Programming, Marble Processing Industry

1. INTRODUCTION

Cutting plane problems have very many applications in industry and are well known problems in literature [1]. One of the industries which face cutting plane problems in a considerable way is the marble processing industry. These factories supply huge marble blocks from marble quarries and turn these into fine stones to be covered on house, office or outdoor floors and surfaces. The two main operations in marble factories are cutting the marble blocks into marble planes (Figure 1) and cutting marble planes according to the length and width of the final products (Figure 2).



Figure 1. Cutting marble block into planes.



Figure 2. Cutting marble planes into finished products

Due to the nature of cutting operations, spoilage is of major concern in marble factories. Spoilage of marbles cause many problems to the managers of marble factories. Firstly, material costs of products increase as waste increases. In addition, spoiled marbles consume quite a large amount of area. Hence the factory needs to hire or buy more area to do its operations properly. Finally, more costs are incurred in order to remove wastes from the factory site (Figure 3).

In this paper, both cutting operations; cutting the marble blocks into planes and cutting planes into finished products; are handled together. Since the output of the first operation feeds the second operation, the two cutting operations should be planned together to minimize waste. The purpose of this paper is to develop a procedure to prepare cutting plans of marble blocks and planes to minimize waste of marble. In order to do so, an integer programming (IP) model is built, which integrates both operations together. The model handles the classical cutting plane problem but also considers the third dimension which is the depth. The objective of the model is to minimize total waste of marble while giving the cutting patterns of the marble blocks and planes. Also, a real life cutting problem of a marble factory is solved using the proposed approach.



Figure 3. Waste of marble in a marble factory.

The rest of the paper is organized as follows. In section two the problem is defined. In section three, proposed approach is explained in detail. Section four gives the real life marble cutting problem and computational results. Finally, conclusions and future research are given in section five.

2. PROBLEM OVERVIEW

Turkey owns 5,2 billion m^3 marble reserves which make up 40% of the worlds marble reserves. Marble production in Turkey has started to grow since 1980s. Today, Turkey is the 7th marble producer of the world. All around the country, there exists 563 marble block producers and over 1000 fine stone producers [2].

Within these factories, 1150000 m^3 marble blocks are processed to produce fine stones in 2002. The level of waste in these factories turns out to be around %20 [2]. That means 230000 m^3 of marble is wasted only in 2002. Most of this amount is caused by selecting wrong cutting patterns when cutting marble blocks into planes and cutting marble planes into final dimensioned products.

In this study, in order to minimize wastes occurring from cutting operations, an IP model is built. The model handles both cutting operations and finds cutting patterns to both of them together while minimizing total waste of marble.

3. PROPOSED APPROACH

In marble processing factories huge marble blocks coming from quarries are first prepared as a rectangular prism with uniform dimensions. Then these blocks go through the first cutting operation where the block is cut into marble planes through the depth dimension.

During this operation some of the block is wasted. For example consider a block with 200cmx200cmx100cm dimensions. If the depth of the block is 100cm and the depth of planes to be cut is 3 cm then, 33 planes will be cut out of the block leaving 1cm back. This last 1cm plane cannot be used for any other product (too thin) so that it brings out a waste of 40000 cm^3 (200cmx200cmx1cm) marble. This situation is also represented in Figure 4 below. The dashed lines represent the planes to be cut out of the block and the remaining volume in the block (wasted marble) is shown by the shaded region.

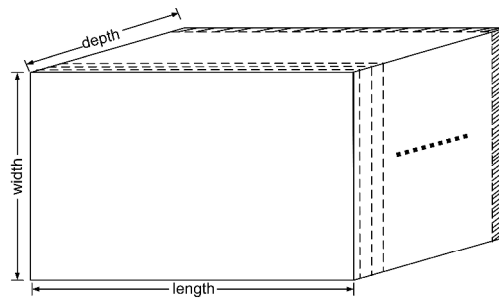


Figure 4. Sample marble block cutting pattern

After cutting the marble block into planes comes the second cutting operation. This operation consists of cutting marble planes according to the final products dimensions. Similar to block cutting, more wastes are incurred in this operation to the cutting pattern of the plane. An example cutting pattern can be seen in Figure 5. In the figure, 15 products are cut out of the plane and the shaded region is the wasted marble.

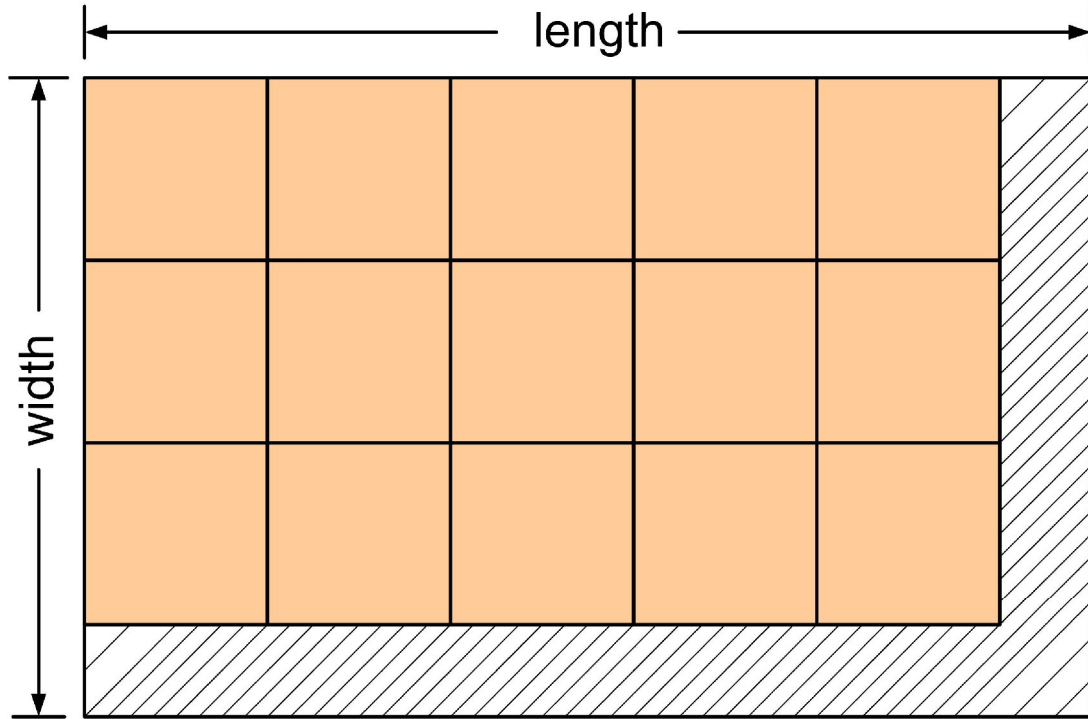


Figure 5. Sample marble plane cutting pattern

In real life, most of the time a factory has to deal with more than one type of a product. When this is the case, looking at Figures 4 and 5, it can be said that it is possible to lower wasted marble amount by applying different cutting patterns. In this study, an integer programming model is built to deal with both cutting operations. The model outputs the cutting patterns of marble blocks and planes while minimizing total wasted amount of marble. The notation used in the model is given in Table 1.

The model built is given with Equations 1 to 6 below. Equation 1 is the objective function which states that the total wasted amount of marble from the two cutting operations should be minimized. Equation 2 is the set of constraints which guarantees that the demand is satisfied. Equation 3 builds the relation between cutting blocks and cutting planes operations. It states that the number of planes cut from the block should not be exceeded in the cutting planes operation. Equation 4 states the available number of blocks to be used. Finally, equation 5 set the decision variables X and Y to be integers.

Table 1. Notation used in the model.

Sets
P : Number of products
I : Number of alternative cutting patterns of a marble block.
J : Number of alternative cutting patterns of a marble plane.
K : Number of different depth dimension considering all products.
Parameters
Demand _p : Demand for product p, p : 1..P.
BlockPattern _{ik} : Number of times dimension k can be cut out of a marble block in pattern i , i : 1..I, k : 1..K.
PlanePattern _{jp} : Number of times product p can be cut out of a marble block in pattern j, j : 1..J, p : 1..P.
WB _i : Waste of marble block when it is cut according to pattern i, i : 1..I.
WP _j : Waste of marble plane when it is cut according to pattern j, j : 1..J.
PlaneDepth _{jk} : $\begin{cases} 1 & \text{if depth of plane pattern j equals to} \\ & \text{depth k, j : 1..J, k : 1..K.} \\ 0 & \text{otherwise} \end{cases}$
Decision Variables
$X_i : \begin{cases} 1 & \text{if pattern i is selected to cut the} \\ & \text{marble block} \\ 0 & \text{otherwise} \end{cases}, i : 1..I$
$Y_j : \text{Number of times pattern j is selected to cut a marble plane, j : 1..J.}$

$$\text{minimize } \sum_{i=1..N} WB_i X_i + \sum_{j=1..M} WP_j Y_j \quad (1)$$

subject to

$$\sum_{j=1..J} \text{PlanePattern}_{jp} Y_j \geq \text{Demand}_p \quad \forall p = 1..P \quad (2)$$

$$\sum_{j=1..J} \text{PlaneDepth}_{jk} Y_j \leq \sum_{i=1..I} \text{BlockPattern}_{ik} X_i \quad \forall k = 1..K \quad (3)$$

$$\sum_{i=1..I} X_i \leq \text{Number of blocks} \quad (4)$$

$$X_i, Y_j \in \{0, 1, 2, 3, \dots\} \quad \forall i = 1..I, \forall j = 1..J. \quad (5)$$

When the model is solved, optimum cutting patterns and the corresponding amount of marble waste is obtained.

4. REAL LIFE MARBLE CUTTING PROBLEM

In this paper the cutting problem of a marble processing factory operating in Afyon region is handled. The factory has received an order including four different products. The products, their dimensions and demanded units are given in Table 2 below. The dimensions of the marble blocks supplied to produce this order are 220cmx140cmx108cm.

Table 2. Products required.

	A	B	C	D
Dimensions (cm)	60×30×3	40×40×3	40×40×2	30×30×2
Demand (unit)	3200	900	6000	700

It can be seen from Table 2 that there exists two different products thicknesses. These are 3cm thick products (A and B) 2cm thick products (C and D). Considering these two thickness values and the depth of the marble blocks supplied, alternative cutting patterns are listed in Table 3. For example, pattern 1 in Table 3 means that 54 times 2cm-thick planes will be cut out of the block. Therefore there will be no wasted marble. Pattern 2 says, 52 times 2cm-thick planes and 1 3cm-thick plane will be cut out of the block. When this is the case, 30800cm³ marble would be wasted (220cm×140cm×1cm).

Similarly, alternative cutting patterns of a marble plane are generated and listed in Table 4. The first 11 patterns are for products A and B, which are 3cm-thick products, and the rest 16 patterns are for products C and D which are 2cm-thick products. The important point is that we cannot cut two products with different thicknesses out of the same plane. An example cutting pattern is given in Figure 6 in order to show how to read Table 4. Pattern 6 in Table 4 states to cut 8 of product A and 9 of product B from the plane. This cutting pattern can be seen in Figure 6. The shaded region in the figure shows the wasted amount of marble which is 6000cm³.

The model is written and solved in Lingo optimization software. In the solution, the selected cutting patterns for block cutting turned out to be pattern 3 and pattern 11. 13 blocks are cut according to pattern 3 and 11 blocks are cut according to pattern 35. For cutting planes operation the selected patterns and number of times to be performed are given in Table 5. As seen from Table 5, patterns 6, 18 and 20 are selected. That means 400 planes are cut according to pattern 6; 663 planes are cut according to pattern 18 and 3 planes are cut according to pattern 20. When these patterns are applied, the total wasted marble amount turns out to be around 5,07m³. This corresponds to about 6,3% waste which is far less than 20%.

Table 3. Alternative cutting patterns of a marble block.

Pattern	Depth		Waste (cm ³)
	2cm	3cm	
1	54	0	0
2	52	1	30800
3	51	2	0
4	49	3	30800
5	48	4	0
6	46	5	30800
7	45	6	0
8	43	7	30800
9	42	8	0
10	40	9	30800
11	39	10	0
12	37	11	30800
13	36	12	0
14	34	13	30800
15	33	14	0
16	31	15	30800
17	30	16	0
18	28	17	30800
19	27	18	0
20	25	19	30800
21	24	20	0
22	22	21	30800
23	21	22	0
24	19	23	30800
25	18	24	0
26	16	25	30800
27	15	26	0
28	13	27	30800
29	12	28	0
30	10	29	30800
31	9	30	0
32	7	31	30800
33	6	32	0
34	4	33	30800
35	3	34	0
36	1	35	30800
37	0	36	0

Table 4. Alternative cutting patterns of a marble plane.

Pattern	A	B	C	D	Waste(cm ³)
1	14	0	0	0	16800
2	13	2	0	0	12600
3	12	3	0	0	13200
4	10	5	0	0	14400
5	9	7	0	0	10200
6	8	9	0	0	6000
7	6	11	0	0	7200
8	4	12	0	0	13200
9	3	13	0	0	13800
10	2	14	0	0	14400
11	0	15	0	0	20400
12	0	0	15	0	13600
13	0	0	14	4	9600
14	0	0	13	6	9200
15	0	0	12	8	8800
16	0	0	11	10	8400
17	0	0	10	12	8000
18	0	0	9	16	4000
19	0	0	8	18	3600
20	0	0	7	19	5000
21	0	0	6	20	6400
22	0	0	5	21	7800
23	0	0	4	22	9200
24	0	0	3	24	8800
25	0	0	2	26	8400
26	0	0	1	27	9800
27	0	0	0	28	11200

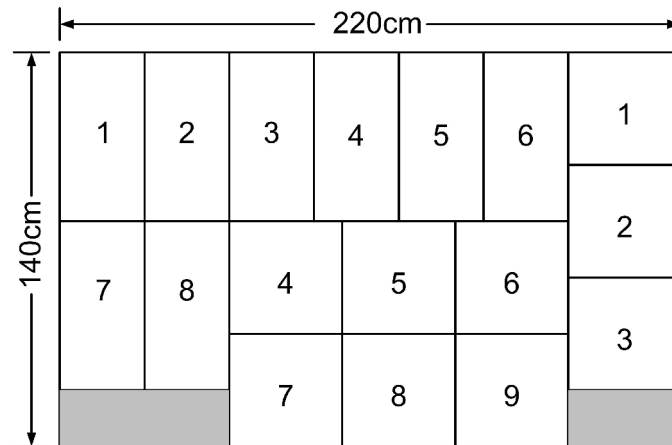


Figure 6. Cutting pattern 6 from Table 4.

Table 5. Selected cutting patterns of planes

Selected Patterns	Number of planes to be cut in this pattern
6	400
18	664
20	3

5. CONCLUSION AND FUTURE RESEARCH

In this study, the cutting problem which is a major problem of marble processing industry is handled. Today, Turkish marble processing industry operates with 20% waste on the average. Most of this amount is caused by selecting wrong cutting patterns in cutting operations. The two main cutting operations in a marble processing factory which are cutting marble blocks into planes and cutting marble planes into final products dimensions, are handled together. An IP model is built to minimize total wasted amount of marble and to find the optimum cutting patterns for both operations. The proposed approach is employed to solve the cutting problem of a marble processing factory in Afyon region. The results showed that average amount of waste decreases to 6,3% which is much better than the industry average, 20%.

6. REFERENCES

1. Winston, L. W, *Operations research applications and algorithms*, Duxbury Press USA, 1994.
2. Ozturk, M, *Pollution caused by marble cutting*, Turkish National Assembly Environment Commission, 2009.