

HARD FLING OBJECTOR CONTACT WITH SURFACE OF FLUID AS A RICOCHET

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Abstract- For a hard flying objector contacts with the surface of fluid, there exists a critical contact angle between the flying direction and the surface direction. When the actual contact angle is less than the critical angle, the flying objector will get additional moment in flying direction and be rebounded up to tens grades. This critical angle is determined by the flying velocity and elastic constants of fluid. These phenomena are named as ricochet which essentially is due to the dynamic pressure of the fluid acting upwards on the flying objector to overcome its gravity force. Although the skipping of a flat stone on water surface is well known practices, the essential theoretic interpretation is not suitably formulated. In this research, it is shown that the fluid has two typical deformation modes: one is orthogonal rotational deformation (which is related with conventional contact), another is orthogonal rotation with intrinsic volume expansion. For the first kind of deformation, the dynamic pressure is inward direction, so the objector will sink into the fluid. However, for the second of deformation, the dynamic pressure is upward direction, so the flying objector will be raised up. It is this mechanism that produces the ricochet phenomenon. In this paper, the dynamic stress is determined by the fluid deformation. Then the contact condition equations are used to establish the related phenomenon. Based on these formulations, the critical angle is expressed by the flying velocity, mass and the fluid viscosity parameters. The related mechanic equations are formulated also. These results may promote the researches on the dynamic contact problem with bifurcation, such as ricochet and/or emerging.

Keywords- Critical angle, ricochet, dynamic contact, dynamic stress, local rotation, intrinsic strain, rational mechanics

1. INTRODUCTION

For a hard flying objector contacts with the surface of water in near surface tangent direction, the flying distance of objector (by means of several rebounds) is bigger than the theoretic distance calculated by the initial velocity of the flying object. This phenomenon can be observed by throwing a near flat small stone along the river surface. It can be seen that when the stone contacts with the water surface in small angle it can fly a much longer distance than normal case and be bounced up several times. There exists a critical angle between the flying direction and the surface direction. In mechanics, this phenomenon is named as ricochet ^[1-2]. Until now, its theoretical formulation is still been required. This topic plays an important role for high-velocity contact problem in theoretic sense.

For this topic, the papers [1] and [2] give a good introduction. The resent situation can be found in the review paper [3] and others ^[4-6]. As the dynamic contact surface

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behaviour is highly non-linear, the problem is extremely complex and difficult to solve analytically. The central problem is the dynamic interaction process in the contact surface. For a hard flying objector, the dynamic contact feature is mainly controlled by the water deformation feature which is determined by the water flow near the contact surface.

Based on the point of rational mechanics, the water will suffer large deformation and rotation while the hard flying objector can be viewed as a non-deformable body. For large deformation and rotation problems, there are several theoretic formulas. The widely accepted one is the strain defined on Finger-Truesdell polar decomposition theorem of deformation gradient ^[1]. The main problem for using this strain definition is how to determine the unit orthogonal rotation tensor. As the rotation plays an important role in this kind of contact problem, the problem cannot be solved by strain defined without rotation consideration.

This research shows that the strain defined by Chen Zhida based on his S-R additive decomposition formula of deformation gradient ^[7-14] can be used to solve this kind of contact problems. For this purpose, the paper introduces the related equations of deformation and rotation for fluid near contact surface. Finally, it shows the theoretic formula of the critical angle determined by the flying velocity and elastic constants of water.

2. STRAIN DEFINITION

Based on the concept of point set transformation, Chen Zhida introduces the concept of points set transformation between original configuration and present configuration of continuum ^[7-10]. Taking co-moving coordinators defined by original configuration, the deformation of medium can be expressed by base vector transformation tensor F_j^i which is defined as:

$$\vec{g}_i = F_i^i \vec{g}_i^0 \tag{1}$$

Where, \vec{g}_i and \vec{g}_i^0 are base vectors for present configuration and original configuration, respectively. The transformation tensor can be expressed by the gradient of displacement u^i as following:

$$F_j^i = u^i \Big|_j + \delta_j^i \tag{2}$$

Where, $u^i \Big|_j$ express the covariant differentiate of displacement fields; δ^i_j is Kronecher delta.

Unlike elastic deformation (where the deformation mode is expressed as the addition of a symmetrical stretching and an unit orthogonal rotation), for water under high-velocity impact, the cracking or bubbling deformation mode^[11-12] is expressed as:

$$F_j^i = \tilde{S}_j^i + (\cos\theta)^{-1} \tilde{R}_j^i \tag{3}$$

Where, the related items are:

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$$\tilde{S}_{j}^{i} = \frac{1}{2} (u^{i} \big|_{j} + u^{j} \big|_{i}) - (\frac{1}{\cos \theta} - 1) (\tilde{L}_{k}^{i} \tilde{L}_{j}^{k} + \delta_{j}^{i})$$

$$\tag{4}$$

$$(\cos\theta)^{-1}\tilde{R}^{i}_{j} = \delta^{i}_{j} + \frac{\sin\theta}{\cos\theta}\tilde{L}^{i}_{j} + (\frac{1}{\cos\theta} - 1)(\tilde{L}^{i}_{k}\tilde{L}^{k}_{j} + \delta^{i}_{j})$$
(5)

$$\tilde{L}_{j}^{i} = \frac{\cos\theta}{2\sin\theta} \left(u^{i} \Big|_{j} - u^{j} \Big|_{i} \right)$$
(6)

$$(\cos\theta)^{-2} = 1 + \frac{1}{4} [(u^{1}|_{2} - u^{2}|_{1})^{2} + (u^{2}|_{3} - u^{3}|_{2})^{2} + (u^{3}|_{1} - u^{1}|_{3})^{2}]$$
(7)

Here, the \tilde{S}_{j}^{i} is the intrinsic strain tensor; \tilde{R}_{j}^{i} is an unit orthogonal rotation tensor; \tilde{L}_{j}^{i} is the rotation direction tensor; θ is the local rotation angular.

3. CONTACT EQUATIONS FOR RICOCHET

For the water under high velocity objector impact, the intrinsic relation among material elements is broken down in some scales. This can be proven by the splashing and bubbling near the contact surface. Hence, as an idea simplification, the intrinsic strain of water under high velocity impact surface can be approximated as being zero. This condition is expressed as:

$$\tilde{S}_{i}^{i} = 0$$
, on high velocity impact surface (8)

This means that, on high velocity impact surface, the deformation tensor is:

$$F_j^i = \frac{1}{\cos\theta} \tilde{R}_j^i(\theta) \tag{9}$$

Where, $\tilde{R}_{j}^{i}(\theta)$ is an unit orthogonal rotation tensor with rotation direction on the contact surface normal and intrinsic rational angle θ .

Taking the water surface as (x^1, x^2) plane, the objector flying direction on x^1 direction, and the x^3 on the depth direction of water. For a flat flying objector with contact surface area A and velocity v_0 , if ricochet phenomenon is produced, the shear component of fluid local velocity gradient tensor is expressed as:

$$\frac{\partial V^1}{\partial x^2} = \frac{V_0}{L} \cdot \cos \alpha , \quad \text{others are zero}$$
(10)

Where, α is the impact angular between the objector flying direction and the water surface direction; *L* is a suitable characteristic length parameter. It defines the distance between the contact surface forced flow region and the undisturbed water region on the contact surface direction. $V_0 \cos \alpha$ is the water flow velocity on surface direction at the contact surface. Here, the complex non-linear flow is simplified in some extent for simplicity.

Based on Equ.(7), the local intrinsic rotation angle is determined by the flying velocity and contact angle α between the surface direction and the velocity direction by the following equation:

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$$\frac{1}{\cos\theta} = \sqrt{1 + \frac{V_0^2 \cdot \cos^2 \alpha}{L^2}}$$
(11)

Based on the definition, the local rotation direction is on the water surface normal direction (x^3 . direction).

For water flow, the classical stress on contact surface is defined as:

$$\sigma_{ij} = -p_0 \delta_{ij} + 2\mu \varepsilon_{ij} \tag{12}$$

Where, p_0 is the water initial pressure; μ is the viscosity of water.

By Eqs.(4) and (8), the classical strain rate is found out to be:

$$\varepsilon_{ij} = \frac{1}{2} (u^i \big|_j + u^j \big|_i) = (\frac{1}{\cos \theta} - 1) (\tilde{L}^i_k \tilde{L}^k_j + \delta^i_j)$$
(13)

As the water intrinsic local rotation direction is on the x^3 direction, the only non-zero component is:

$$\varepsilon_{33} = \frac{1}{\cos\theta} - 1 \tag{14}$$

The graze condition is that the dynamic pressure on the contact surface is bigger than the gravity force of the object. Based on Equs.(11), (12), and (14), this condition is expressed by equation:

$$2\mu \left(\frac{1}{\cos\theta} - 1\right) - p_0 \ge \frac{Mg}{A} \tag{15}$$

That is:

$$2\mu \cdot \sqrt{1 + \frac{V_0^2 \cdot \cos^2 \alpha}{L^2}} \ge \frac{Mg}{A} + p_0 + 2\mu$$
 (16)

Where, M is the mass of objector; the g is the gravity constant. It says that, for the objector, the velocity must be high enough and have a small incident angle.

This gives out the critical impact angle α_c as:

$$\cos \alpha_{c} = \frac{L}{V_{0}} \cdot \sqrt{\left(1 + \frac{1}{2\mu} (\frac{Mg}{A} + p_{0})\right)^{2} - 1}$$
(17)

For $\alpha \le \alpha_c$ case, the ricochet phenomenon will be produced. The higher is the impact velocity, the bigger the critical angle is.

On the other hand, the physical requirement for the existence of such an angle is:

$$\frac{L}{V_0} \cdot \sqrt{\left(1 + \frac{1}{2\mu} \left(\frac{Mg}{A} + p_0\right)\right)^2 - 1} \le 1$$
(18)

So, to produce a ricochet, the impact velocity condition can be obtained as:

$$V_0 \ge L \cdot \sqrt{\left(1 + \frac{1}{2\mu} (\frac{Mg}{A} + p_0)\right)^2 - 1}$$
(19)

Hence, the ricochet condition is formulated by the Equs.(17) and (19). So, the minimum ricochet velocity can be defined as V_c by equation:

$$V_{c} = L \cdot \sqrt{\left(1 + \frac{1}{2\mu} \left(\frac{Mg}{A} + p_{0}\right)\right)^{2} - 1}$$
(20)

Summing up above results, the production condition of ricochet for stone-skipping is:

$$V \ge V_c$$
 and $\alpha \le \alpha_c$ (21)

Where, the critical velocity v_c is completely determined by the fluid mechanic features (static pressure and viscosity), the flying objector mass and its geometrical factors (contact surface area and characteristic length). For the velocity bigger than the critical value, the critical incident angle α_c is determined by fluid features, objector geometry, and the incident velocity. The only needed parameter to be determined by experiments or a set of complete dynamic motion equations is the characteristic length *L*. Now, it is time to consider the graze. By experimental observations, the impact angle is near invariant for ricochet process, so it is reasonable to calculate the velocity loss for each graze as:

$$\Delta V = \frac{2\mu A}{M} \cdot \left(\sqrt{1 + \frac{V^2 \cos^2 \alpha}{L^2}} - 1 \right) \cdot \sin \alpha$$
(22)

Hence, the numbers of grazes can be determined by using the Eqs. (17)-(22).

Observing the Eq.(22), when the $V_0 \rightarrow \infty$, the critical angle tends to 90 degrees. For water, it is not true. This problem is caused by the simplification that omitting the thermo effects and simplifying the fluid feature. In fact, for very high impact velocity, the compressibility of water must be taken into consideration. At the same time, the shear in depth direction should be taken into consideration. Further more, the local rotation direction is not on the normal direction of water surface, although it may be near this direction.

This research shows that: there is an instability region between the ricochet and conventional impact. Therefore, to formulate a dynamic problem, only focusing on the dynamic equations are not enough. The dynamic boundary problems must be taken into consideration seriously.

4. CONCLUSION

Based on the concept of rational mechanics, the continuous of deformation does not mean the continuous of classical stress. Based on the condition that the intrinsic stretching strain should be zero for water, the fluid deformation near the contact surface is formulated. So, the related stress condition is formulated for ricochet. The results show that: the critical velocity V_c is completely determined by the fluid mechanic features (static pressure and viscosity), the flying objector mass and its geometrical factors (contact surface area and characteristic length). For the velocity bigger than the critical value, the critical incident angle α_c is determined by fluid features, objector geometry, and the incident velocity.

However, to determine the characteristic length parameters, a detailed fluid flow region should be determined exactly or experimentally. This problem should be solved by further research. Here, the result is only a simplified scheme. Furthermore, the instability problem is discussed for high velocity impact on elastic-fluid materials. These results may cause the interest on ricochet for its theoretic significance on formulating dynamic boundary condition. Anyway, these topics play important role in theoretic sense and actual applications.

5. REFERENCES

- 1. Johnson, W. Ricochet of non-spining projectiles, mainly from water, part 1: some historical contributions, *Int. J. Impact Engng.* **21**(1998): 15-24
- 2. Johnson, W. Ricochet of non-spining projectiles, mainly from water, part 2: an outline of theory and warlike applications, *Int. J. Impact Engng.* **21**(1998): 25-34
- C.M. Seddon, M. Moatamedi, Review of water entry with applications to aerospace structures, *Int. J. Impact Engng* 32(2006): 1045–1067
- 4. Clanet, C., Hersen, F. and Bocquet, L., Secrets of Successful stone-Skipping, *Nature* **427**(2004): 29
- Man-Sung Park, Young-Rae Jung, Warn-Gyu Park, Numerical study of impact force and ricochet behavior of high speed water-entry bodies, *Computers & Fluids* 32(2003): 939–951
- 6. Shin-ichiro Nagahiro* and Yoshinori Hayakawa, A theoretical and numerical approach to "magic angle" of stone skipping, *arXiv:physics*/0411125V2 [physics:flu-dyn], 2004
- 7. Chen Zhida. Rational Mechanics—Non-linear Mechanics of Continuum. Xizhou: China University of Mining & Technology Publication, 1987 (In Chinese)
- 8. Chen Zhida. Rational Mechanics. Chongqing Publication, 2000 (In Chinese)
- 9. Chen Zhida. Geometric Theory of Finite Deformation Mechanics for Continuum. *Acta Mechanica Sinica*, **2**(1979): 107-117 (In Chinese)
- 10. Chen Zhida. Limit Rotation Expression in Non-linear Field Theory of Continuum. *Applied Mathematics and Mechanics*, **7**(1986): 959-968 (In Chinese)
- 11. Xiao Jianhua. Decomposition of Displacement Gradient and Strain Definition, J. Central South University of Technology, **14** (S.1)(2007), 401-404
- 12. Xiao Jianhua, Bubble dynamics equations for Newton fluid, ISND2007, *Journal of Physics: Conference Series* **96**(2008), 012134, doi:10.1088/1742-6596/96/1/012134