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NEW PERIODIC SOLITARY-WAVE SOLUTIONS TO THE (3+1)-DIMENSIONAL KADOMTSEV-PETVIASHVILI EQUATION

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Abstract- By the extended homoclinic test technique, explicit solutions of the (3+1)-dimensional Kadomtsev-Petviashvili(KP) equation are obtained. These solutions include doubly periodic wave solutions, doubly soliton solutions and periodic solitary-wave solutions. It is shown that the extended homoclinic test technique is a straightforward and powerful mathematical tool for solving nonlinear evolution equation.

Keywords- Extended homoclinic test, Doubly periodic, Soliton, Periodic solitary wave

1. INTRODUCTION

Recently many effective and powerful methods have been proposed to solve nonlinear evolution equations, such as the inverse scattering transform [1], the tanh function method [2], the homogeneous balance method [3], the auxiliary function method [4], the Exp-function method [5-8] and so on.

Very recently, a new technique called "extended homoclinic test technique"was proposed [9] and has been applied to seek periodic solitary wave solutions of integrable equations [10,11]. In this work, we apply the technique to the (3+1)-dimensional KP equation. New exact solutions including doubly periodic wave solutions, doubly soliton solutions and periodic solitary-wave solutions are obtained.

2. PROCEDURES FOR SOLVING THE (3+1)D KP EQUATION

The (3+1)-dimensional Kadomtsev-Petviashvili(KP) equation reads as [12]:

$$u_{xt} + 6u_x^2 + 6uu_{xx} - u_{xxxx} - u_{yy} - u_{zz} = 0 . \qquad (1)$$

By using Painlevé analysis we suppose

$$u = (2\ln F)_{xx} , \qquad (2)$$

where F (x, y, z, t) is an unknown real function.
Substituting (2) into (1), we have a Hirota bilinear equation:

$$(D_x D_t - D_x^4 - D_y^2 - D_z^2)F \cdot F = 0 . \qquad (3)$$

Then, by introducing different ansätz test function $F(x,y,z,t)$ to Eq. (3), we can obtain a series of exact solutions to the (3+1)D KP equation (1).

(1) Suppose that the test function $F(x,y,z,t)$ has the following ansätz:

$$F(x,y,z,t) = b_1 e^{-p(x+ky+lz-\omega t)} + b_2 \cos[q(x+ky+lz+\omega t)] + b_3 e^{p(x+ky+lz-\omega t)}, \qquad (4)$$

where b_1,b_2,b_3,p,q,k,l and ω are constants to be determined later.

On substituting (4) into (3), equating all the coefficients of different powers of $e^{ip(x+ky+lz-\omega t)}$ (i=-1,0,1) to zero yields a set of algebraic equations. Solving the resulting equations, simultaneously, we get

$$k^2 + l^2 = 2(q^2 - p^2), \quad \omega = p^2 + q^2, \quad b_1 = -\frac{b_2^2 q^2}{4 b_3 p^2} . \qquad (5)$$

Case (I). When $b_1 = b_2 = b_3 = b$, $p = iP$, we have $q = \pm 2P, k^2 + l^2 = 10P^2, \omega = 3P^2$.
Therefore, we get a doubly periodic wave solution,

$$u_1(x,y,z,t) = -\frac{2P^2\left(8 + \cos[3P(x+ky+lz+P^2t)] + 9\cos[P(x+ky+lz+9P^2t)]\right)}{\left(2\cos[P(x+ky+lz-3P^2t)] + \cos[2P(x+ky+lz+3P^2t)]\right)^2}.$$

where b, P, k,l are arbitrary real constants.

Case (II). When $b_1 = \mp b_2 = -b_3 = -b$, $p = P$, we have $q = \pm 2P, k^2 + l^2 = 6P^2, \omega = 5P^2$.
Therefore, we obtain a periodic solitary-wave solution,

$$u_{2,3}(x,y,z,t) = \frac{4P^2[-4 \pm 4\cosh(\xi)\sin(\eta) \mp 3\sinh(\xi)\cos(\eta)]}{[2\sinh(\xi) \pm \cos(\eta)]^2}, \qquad (6)$$

where $\xi = P(x+ky+lz-5P^2t), \eta = 2P(x+ky+lz-5P^2t)$ and $k^2 + l^2 = 6P^2$.

(2) Let the test function $F(x,y,z,t)$ be

$$F = b_1 e^{-p(x+ky+lz-\omega t)} + b_2 \cos[q(x+ky+lz)] + b_3 e^{p(x+ky+lz-\omega t)},$$

where b_1,b_2,b_3,p,q,k,l and ω are constants to be determined.

By using a similar procedures to derive (5), we obtain

$$k^2 + l^2 = -3p^2 + q^2, \quad \omega = 2(p^2 + q^2), \quad b_1 = -\frac{q^2 b_2^2}{4p^2 b_3} .$$

(i). Choosing $b_1 = b_3 = b$ and $p = iP, q = P$, we get $k^2 + l^2 = 2P^2, \omega = 0, b_2 = \pm 2b$.
Thus, a triangle function solution to equation (1) is given as

$$u_4 = -2P^2 \sec^2[P(x+ky+lz)].$$

(ii). Choosing $b_1 = b_2 = b_3 = b$ and $p = iP$, we have $k^2 + l^2 = 7P^2$, $q = \pm 2P$, $\omega = 6P^2$.
Therefore we get a doubly periodic wave solution,

$$u_5 = -\frac{2P^2\left[8+\cos[3P(x+ky+lz-2P^2t)]+9\cos[P(x+ky+lz+6P^2t)]\right]}{\left[2\cos[P(x+ky+lz-6P^2t)]+\cos[2P(x+ky+lz)]\right]^2}.$$

(iii).When $b_1 = b_2 = -b_3 = -b$ and $p = P$, we have $k^2 + l^2 = P^2$, $q = \pm 2P$, $\omega = 10P^2$. Then, we obtain periodic solitary-wave solutions,

$$u_{6,7}(x,y,z,t) = \frac{4P^2[-4 \pm 4\cosh(\xi)\sin(\eta) \mp 3\sinh(\xi)\cos(\eta)}{[2\sinh(\xi) \pm \cos(\eta)]^2},$$

where $\xi = P(x+ky+lz-10P^2t), \eta = 2P(x+ky+lz)$ and P is an arbitrary constant.
(3) We now suppose that the test function $F(x,y,z,t)$ has the following ansätz:

$$F = b_1 e^{-p(x+ky-\omega t)} + b_2\cos[q(x+lz+\lambda t)] + b_3 e^{p(x+ky-\omega t)},$$

where $b_1, b_2, b_3, p, q, k, l, \omega$ and λ are constants to be determined.
By using a similar procedures to derive (5), we get

$$\omega = \frac{-k^2p^2 + l^2q^2}{p^2+q^2} + 3p^2 - q^2,\ \lambda = \frac{-k^2p^2 + l^2q^2}{p^2+q^2} + 3q^2 - p^2, \tag{7}$$

$$b_1 = \frac{q^2[(k^2+l^2)p^2 - 3(p^2+q^2)^2]b_2^2}{4p^2[(k^2+l^2)+3(p^2+q^2)^2]b_3}.$$

By choosing $b_1 = b_2 = b_3 = b$, we have three types of solutions to Eq. (1).

Type (i). Choosing $p = iP, q = Q$ satisfies $Q^2 > \frac{4}{3}P^2$, we have

$$k^2 + l^2 = \frac{(P^2-Q^2)^2(3Q^2-4P^2)}{P^2Q^2},$$

$$\xi = P(x+ky-\omega t), \quad \eta = Q(x+lz+\lambda t). \tag{8}$$

Consequently, we obtain a doubly periodic wave solution,

$$u_8(x,y,z,t) = -\frac{8P^2 + 2Q^2 + 4(P^2+Q^2)\cos(\xi)\cos(\eta) + 8PQ\sin(\xi)\sin(\eta)}{[2\cos(\xi)+\cos(\eta)]^2}, \tag{9}$$

where η, ξ are given by (8) and ω, λ are given by (7).
Solution expressed by (9) is a doubly periodic wave solution with different period about different spatiotemporal variable (x,y,t) and (x,z,t), respectively.

Type (ii). Choosing $p = P, q = iQ$ satisfies $P^2 > 3/4Q^2$.
Then from solution (9) we have a doubly soliton solution,

$$u_9(x,y,z,t)=\frac{8P^2+2Q^2+4(P^2+Q^2)\cosh(\xi)\cosh(\eta)-8PQ\sinh(\xi)\sinh(\eta)}{[2\cosh(\xi)+\cosh(\eta)]^2}. \qquad (10)$$

where ξ,η are given by (8) and $k^2+l^2=\frac{(P^2-Q^2)^2(4P^2-3Q^2)}{P^2Q^2}$; P, Q are real constants.

Type (iii). Choosing $p=iP, q=iQ$, then from solution (9) we have a periodic solitary-wave solution,

$$u_{10}(x,y,z,t)=\frac{-8P^2+2Q^2-4(P^2-Q^2)\cos(\xi)\cosh(\eta)+8PQ\sin(\xi)\sinh(\eta)}{[2\cos(\xi)+\cosh(\eta)]^2}, \qquad (11)$$

where ξ,η are given by (8) and $k^2+l^2=\frac{(P^2+Q^2)^2(4P^2+3Q^2)}{P^2Q^2}$; P,Q are real constants.

(4) Finally, we suppose that the test function $F(x,y,z,t)$ has the following ansätz:

$$F=b_1e^{-p(x+ky-\omega t)}+b_2\cos[q(x+ky+lz+\omega t)]+b_3e^{p(x+ky-\omega t)},$$

where b_1,b_2,b_3,p,q,k,l and ω are constants to be determined.
By using a similar procedures to derive (5), we get

$$k^2=2(q^2-p^2), \omega=p^2+q^2+\frac{q^2l^2}{p^2+q^2}, b_1=\frac{q^2[p^2l^2-3(p^2+q^2)^2]b_2^2}{4p^2[q^2l^2+3(p^2+q^2)^2]b_3}. \qquad (12)$$

Choosing $b_1=b_2=b_3=b$ (b is an arbitrary constant), then (12) is reduced to

$$k^2=2(q^2-p^2), l^2=-\frac{(p^2+q^2)^2(4p^2+q^2)}{p^2q^2}, \omega=-\frac{(p^2+q^2)(3p^2+q^2)}{p^2}.$$

Therefore, we have

$$p=iP, q=Q, Q^2>4P^2,$$

$$k^2=2(P^2+Q^2), l^2=\frac{(P^2+Q^2)^2(4P^2-Q^2)}{P^2Q^2}, \omega=\frac{(P^2-Q^2)(3P^2-Q^2)}{P^2}$$

and

$$p=iP, q=iQ, Q^2>P^2,$$

$$k^2=2(P^2-Q^2), l^2=\frac{(P^2+Q^2)^2(4P^2+Q^2)}{P^2Q^2}, \omega=\frac{(P^2+Q^2)(3P^2+Q^2)}{P^2}.$$

Consequently, we obtain the following solutions,

$$u_{11}(x,y,z,t)=-\frac{8P^2+2Q^2+4(P^2+Q^2)\cos(\xi)\cos(\eta)+8PQ\sin(\xi)\sin(\eta)}{[2\cos(\xi)+\cos(\eta)]^2},$$

$$u_{12}(x,y,z,t)=\frac{-8P^2+2Q^2-4(P^2-Q^2)\cos(\varsigma)\cosh(\tau)+8PQ\sin(\varsigma)\sinh(\tau)}{[2\cos(\varsigma)+\cosh(\tau)]^2},$$

where

$$\xi=P[x+2(P^2+Q^2)y-\frac{(P^2-Q^2)(3P^2-Q^2)}{P^2}t],$$

$$\eta=Q[x+2(P^2+Q^2)y+\frac{(P^2+Q^2)^2(4P^2-Q^2)}{P^2Q^2}z+\frac{(P^2-Q^2)(3P^2-Q^2)}{P^2}t],$$

$$\varsigma=P[x+2(P^2-Q^2)y-\frac{(P^2+Q^2)(3P^2+Q^2)}{P^2}t],$$

$$\tau=Q[x+2(P^2-Q^2)y+\frac{(P^2+Q^2)^2(4P^2+Q^2)}{P^2Q^2}z+\frac{(P^2+Q^2)(3P^2+Q^2)}{P^2}t].$$

and P, Q are arbitrary real constants.

3. CONCLUSION

In this paper, the extended homoclinic test technique is applied to solve the (3+1)-dimensional Kadomtsev-Petviashvili(KP) equation. As a result, explicit solutions including doubly periodic wave solutions, doubly soliton wave solutions and periodic solitary-wave solutions are obtained, these solutions enrich the structures of exact solutions.

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