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NUMERICAL SIMULATIONS OF YARN UNWINDING FROM PACKAGES

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Abstract- We derive a system of coupled nonlinear differential equations that govern the motion of yarn in general. The equations are written in a (non-uniformly) rotating observation frame and are thus appropriate for description of over-end unwinding of yarn from stationary packages. We comment on physical significance of virtual forces that appear in a non-inertial frame and we devote particular attention to a lesser known force, that only appears in non-uniformly rotating frames. We show that this force should be taken into account when the unwinding point is near the edges of the package, and the quasi-stationary approximation is not valid because the angular velocity is changing with time. The additional force has an influence on the yarn dynamics in this transient regime where the movement of yarn becomes complex and can lead to yarn slipping and even breaking.

Keywords- yarn dynamics, eqations of motion, virtual forces, nunerical simulations

1. INTRODUCTION

In the production of garments thread unwinding exist in a sewing process. In order to achieve low and constant tension of thread or yarn it is necessary to optimize the process of unwinding. Computer simulation are now in use for this purpose, so it is important to obtain a mathematical description of yarn motion [1,2].

2. KINEMATICS

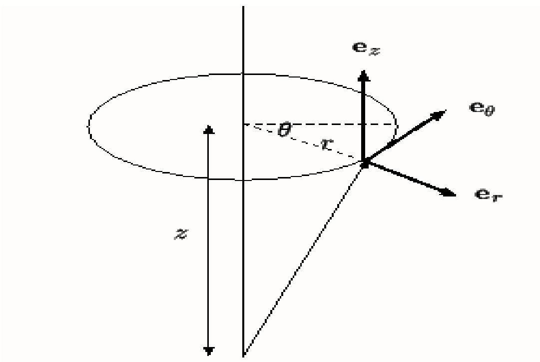

Fig.1 Cylindrical coordinate system rotates around the z axis with an angular velocity ω

We introduce a cylindrical coordinate system that rotates around the z axis with an angular velocity ω. The yarn is parametrised with arc length s (s is therefore the length of yarn from the origin of the coordinate system to the given point on the yarn). The coordinates of a point are given by r, the radial distance from the axis, θ, the polar angle and z, the vertical distance from the origin [3,4].

It should be kept in mind that each point has its own triplet of base vectors e_z, e_θ, e_r, respectively pointing in vertical, tangential and radial direction. The radius vector pointing to a point on the yarn can be decomposed along radial and vertical directions (the polar angle dependence is hidden in the e_r vector):

$$r(s,t) = r(s,t)e_r(\theta(s,t),t) + z(s,t)e_z \tag{1}$$

We have emphasized that coordinates of a point depend explicitelyon both the time of observation t and on the arc length s, where the point is located at given time t.The velocity of a point on a yarn that is being withdrawn (with withdrawing speed V) is given by the total time derivative:

$$v = \frac{dr}{dt} = \frac{\partial r}{\partial t} + \frac{\partial r}{\partial s}\frac{\partial s}{\partial t} \tag{2}$$

It is important to note that the velocity is not given by the local (partial) time derivative, denoted by ∂r/∂t. This derivative does not take into account that in the infinitesimal time Δt the point moves to a different position along the yarn (i.e. to a different arc length s). The contribution to velocity due to this movement is described by the additional term ∂r/∂s ∂s/∂t. The withdrawing speed is equal to V=-∂s/∂t and we obtain the following expression:

$$v = \dot{r} - V\frac{\partial r}{\partial s} \tag{3}$$

where the dot denotes the partial derivative with respect to time. It's worth noting that t=∂r/∂s is the unit tangential vector to the yarn. Indeed the direction of the withdrawing velocity at a given point should be in the direction of the yarn.
To calculate the time derivative of the radius vector we make use of a relation between derivatives in an inertial and a rotating frame:

$$\left(\frac{\partial}{\partial t}\right)_K = \left(\frac{\partial}{\partial t}\right)_{K'} + \omega x\,. \tag{4}$$

When applied to a base vector, that is rotating around the Z axis together with the yarn, this equations gives

$$\frac{\partial e_i(t)}{\partial t} = \omega \times e_i(t) \tag{5}$$

The partial time derivative of the radius vector is then found to be: $\dot{r} = \dot{r}e_r + r\dot{e}_r + \dot{z}e_z + z\dot{e}_z = \dot{r}e_r + r\dot{\theta}e_\theta + \dot{z}e_z + \omega \times (re_r + ze_z) = v_{rel} + \omega \times r.$ (6)

The final expression for the velocity of a point is of the form

$$v = v_{rel} + \omega \times r - V\frac{\partial r}{\partial s} \tag{7}$$

The three contributions to the velocity of the point have very simple physical interpretations. The first term is the relative velocity in the non-intertial frame; it

describes how the form of the yarn is changing from the point of view of an observer that is rotating together with the yarn, but it is not equal to the velocity of a given point in the non-intertial frame. (This term is dropped in the quasi-stationary approximation that we describe below.) The second term is the circular velocity of the point due to the rotation of the frame; this is the velocity of a point that is fixed in the non-inertial frame. Finally, the last term is the withdrawing velocity that we introduced above.
By analogy, the acceleration of a point is given by the total time derivative of the velocity. By a lengthy but straight-forward calculation we obtain the following expression:

$$a = a_{rel} + 2\omega \times v_{rel} - 2V\omega \times \frac{\partial r}{\partial s} + \omega \times (\omega \times r) + \dot{\omega} \times r - 2V\frac{\partial v_{rel}}{\partial s} + V^2 \frac{\partial^2 r}{\partial s^2} \tag{8}$$

This complex expression can be given more compact form if we introduce a differential operator D, which follows the motion of the point in the rotating frame[5]:

$$D = \frac{\partial}{\partial t} - V\frac{\partial}{\partial s} \tag{9}$$

The fact that this operator "follows the motion of the point in the rotating frame" means, that the partial time derivative operator only operates on the coordinates of the point (r, θ, z), but it gives zero when applied on the base vectors e_z, e_θ, e_r.
The simplified expression for the acceleration is

$$a = D^2 r + 2\omega \times (Dr) + \omega \times (\omega \times r) + \dot{\omega} \times r \tag{10}$$

This expression is reminiscent of an analogous expression for acceleration of a point object in a rotating frame, with partial time derivatives replaced by the differential operator D.

3. DINAMICS

Newton's law in the form of **F**=m**a**, where **F** is the force on the body, **a** the acceleration and m the mass of the body, can be used to describe the motion of point bodies and the centre-of-mass motion of rigid bodies. Here we are dealing with yarn, which is a deformable body, and we want to describe not only the motion of the yarn as a whole, but also it's shape itself [6]. For this reason we partition the yarn in a large number of short (infinitesimal) segments of length δs and we apply Newton's law for each individual segment (Fig.2).

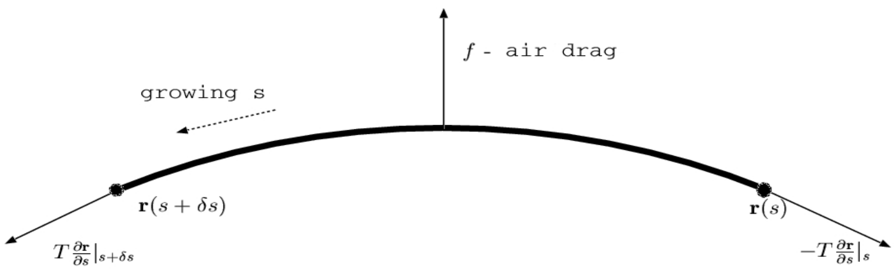

Fig. 2 A segment of yarn and forces that act on it.

The three largest forces that act on each segment are:

- the air drag for that part of the yarn that forms the balloon (or the force of friction for the part of yarn between unwinding and lift-off point on the package, which is sliding on lower layers of yarn)
- the force imparted to the segment by the yarn "attached" to the right end point (at arclength s), -T $\partial\mathbf{r}/\partial s(s)$. Scalar T is the yarn tension, and the force is obviously directed along the yarn.
- the force imparted to the segmend by the yarn "attached" to the left end point (at arclength s+∂s), T $\partial\mathbf{r}/\partial s(s+\partial s)$.

The last two forces are due to internal elastic stress which appear because the yarn is being strained. In tridimensional bodies the elastic state is described by a tensor (stress tensor), while in a one-dimensional body such as yarn a scalar quantity T (tension) is sufficient. It is measured in units of force [N].
We can thus write the second Newton's law for the yarn segment as

$$ma = \left(T\frac{\partial r}{\partial s}\right)(s+\delta s) - \left(T\frac{\partial r}{\partial s}\right)(s) + F \tag{11}$$

The mass of a segment is m=ρδs, where ρ is the linear density of mass (i.e. mass per unit length). We write the external force **F** as **F**=**f**δs, where **f** is the linear density of external force (i.e. external force per unit length). We divide the previous equation by δs and we go the limit of infinitesimal length of the segment, δs→0:

$$\rho a = \lim_{\delta s \to 0} \frac{\left(T\frac{\partial r}{\partial s}\right)(s+\delta s) - \left(T\frac{\partial r}{\partial s}\right)(s)}{\delta s} + f \tag{12}$$

The limit in this expression is by definition the derivative of function T∂**r**/∂s with respect to arc-length s. The final result, the equation of motion for an infinitesimal yarn segment, can be written as

$$\rho a = \frac{\partial}{\partial s}\left(T\frac{\partial r}{\partial s}\right) + f \tag{13}$$

or, if we take into account the expression for the acceleration,

$$\rho(D^2 r + 2\omega \times Dr + \omega \times (\omega \times r) + \dot{\omega} \times r) = \frac{\partial}{\partial s}\left(T\frac{\partial r}{\partial s}\right) + f \tag{14}$$

4. VIRTUAL FORCES IN A NON-UNIFORMALY ROTATING FRAME

The D^2r term in the equation of motion can be interpreted as the acceleration of a point in the rotating coordinate system. The other three terms on the left hand can be moved to the right side of the equation and reinterpreted as virtual forces that appear due to the non-inertial character of this observation frame. These are not »real« physical forces, but rather forces that an observer in a non-inertial frame would feel because of inertial effects. To emphasize the difference the virtual forces are also called system forces, inertial forces or pseudo-forces. It should be kept in mind that these forces do

not appear in equations of motion if they are written in an inertial frame, even if the motion of the body itself is accelerated. They only appear when the equations are expressed in the form appropriate for a non-inertial obsertvation system.
The three virtual forces that we're dealing with are:

1) $-\rho 2\omega \times Dr$ the Coriolis force

2) $-\rho\omega \times (\omega \times r)$ the centrifugal force

3) $-\rho\dot{\omega}\times r$ an additional force due to changes of the rotational velocity.

In most of the introductiory textbooks on mechanics the only case that is considered is that in which the angular velocity is constant, so that only Coriolis and the centrifugal forces appear. For this reason the third force is less known and unfortunately it is often neglected even when it plays some role. We were unable to find any mention of this virtual force in the available litterature on yarn unwinding and the balloon theory.
It is interesting to describe how an observer standing on a merry-go-round would feel each of these forces. Usually we first notice the centrifugal force; this force »tries« to »eject« us from the merry-go-round. Coriolis force can be seen at work when we throw an object in the radial direction. As seen from our point of view, the object work fly in a straight line as in an inertial frame, but it will deviate in a direction that is perpendicular to its velocity. The third force could be felt if the merry-go-round would suddenly come to a halt. As our experience tells us, we would most likely fall in this event. This force therefore isn't always negligible: it has very sensible effects when the angular velocity suddenlty changes. We will now show when this force should be taken into account in the balloon theory.

On cylindrical packages the angular velocity depends on the unwinding speed V, the package radius c and on the winding angle ϕ:

$$\omega = \frac{V}{c}\left(\frac{1}{\cos\phi} - \tan\phi\right)^{-1} = \frac{V}{c}\frac{\cos\phi}{1-\sin\phi} \tag{15}$$

The unwinding speed and the package radius are approximately constant in the time interval required to unwind a few layers of yarn. On the other hand, the winding angle ϕ is different in each layer of a cross-wound package: it is approximately constant when the unwinding point is in the middle of the package and it changes sign near the edges of the package. Variations of ϕ lead to sudden changes of angular velocity near the edges.

We have performed numerical simulations of unwinding from both cylindrical and conic packages. We show the time dependence of the position of the unwinding point z(t), the radius of the package at the unwinding point c(t), the winding angle $\phi(t)$ and the dimensionless angular velocity $\Omega(t) = c\,\omega/V$ for a cylindrical (full line) and a conic package (dashed line, see Fig.3.

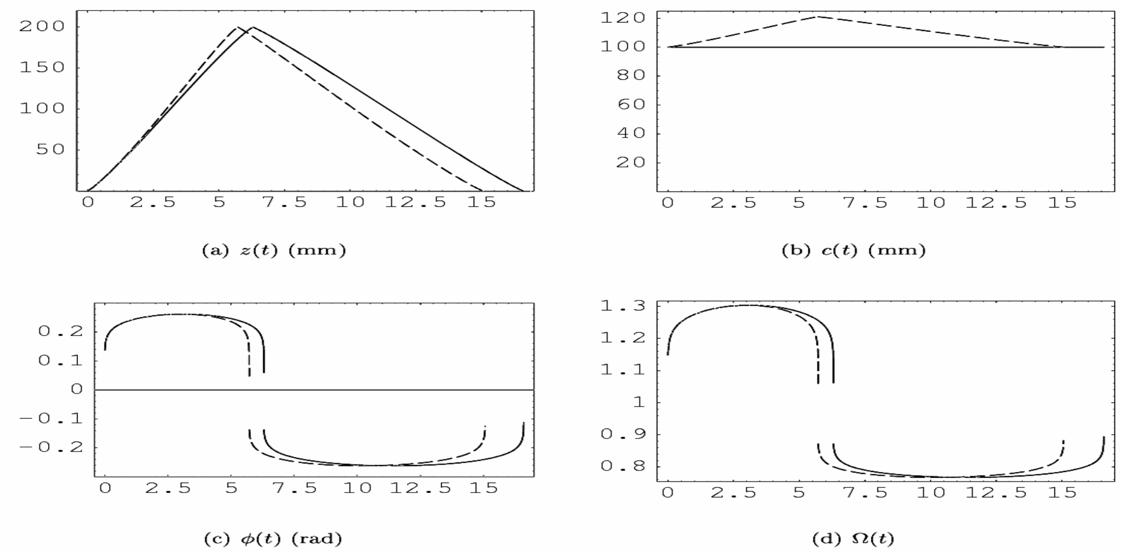

Fig. 3 Results of simulation of unwinding from cylindrical and conic packages.

We indeed see that when the unwinding point is in the middle of the package, the conditions are changing slowly with time. The quasi-stationary approximation can then be applied, as is done in most of the theoretical works devoted to yarn unwinding. The time dependence is shifted to the boundary conditions while the equation of motion is simplified to:

$$\rho\left(V^2\frac{\partial^2 r}{\partial s^2}-2V\omega\times\frac{\partial r}{\partial s}+\omega\times(\omega\times r)\right)=\frac{\partial}{\partial s}\left(T\frac{\partial r}{\partial s}\right)+f \tag{16}$$

Near the front and read end of the package the conditions quickly change from two different quasi-stationary regimes. The third non-inertial force can then become a very large quantity. In the figure 4 we show directions of virtual forces at both edges of a cylindrical package:

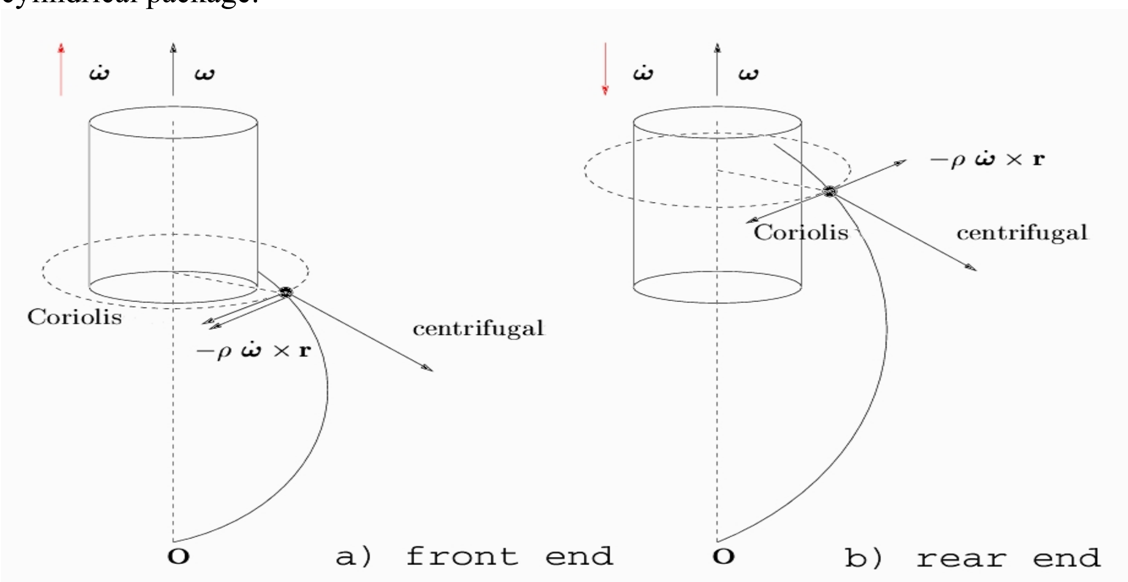

Fig. 4 Virtual forces on the yarn during unwinding

Using a simple calculation we can show that this force has larger effect when the winding angle ϕ_0 is high:

$$\Delta\omega = \omega_{\max} - \omega_{\min} = \frac{V}{c}\frac{\cos\phi_0}{1-\sin\phi_0} - \frac{V}{c}\frac{\cos(-\phi_0)}{1-\sin(-\phi_0)} = \frac{2V}{c}\tan\phi_0 \qquad (17)$$

We can thus safely neglect this force in parallely-wound packages, however we should be careful when using quasi-stationary approximation to describe cross-wound packages.

5. CONCLUSIONS

We have shown crucial steps in the derivation of the equation of motion of yarn: the introduction of the non-uniformly rotating observation frame, the calculation of velocity and acceleration and the application of Newton's second law to an infinitesimal segment of yarn. The origin of the virtual (system) forces was described. We've emphasized the role of the lesser known virtual force that can have important effects near the edges of a package.

6. REFERENCES

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