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PERISTALTIC MECHANISM IN AN ASYMMETRIC CHANNEL WITH HEAT TRANSFER

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Abstract- This study reports the effects of velocity and thermal slip parameters on the peristaltic motion of variable viscosity and magnetohydrodynamic (MHD) fluid in an asymmetric channel. Heat transfer coefficient and temperature are given due attention with respect to embedded parameters in the problem.

Keywords- Peristaltic transport, Heat transfer, MHD fluid, Slip condition, Variable Viscosity.

1. INTRODUCTION

The peristaltic flows are now being widely studied in the recent years. Interest in such flows is inspired because of their occurrence in urine transport from kidney to the bladder, chyme motion in the gastrointestinal tract, vasomotion of small blood vessels, roller and finger pumps etc. Various aspects of peristalsis for constant viscosity fluid in a symmetric channel have been studied by Mekheimer [1-3], Elshahed and Haroun [4], Srivastava and Srivastava [5], Hayat et al.[6-10] and many others. The studies on peristaltic flow of a constant viscosity fluid in an asymmetric channel have been carried out by Misra and Rao [11] and Hayat et al.[12].Ali et al.[13] recently discussed the peristaltic flow of variable viscosity MHD viscous fluid in a symmetric channel.

The aim of present study is to extend the flow analysis of study [13] in the three directions. Firstly to describe the flow in an asymmetric channel.

Secondly to predict the heat transfer effects. Thirdly to examine the velocity and thermal slip effects. Proper mathematical formulation is carried out. The resulting problems for the stream function and temperature are solved using long wavelength approximation. Important flow quantities are analyzed.

2. PROBLEM STATEMENT

Let us examine the flow of viscous fluid with variable viscosity in an asymmetric channel with insulating walls and width $d_1 + d_2$.The fluid is electrically conducting under the action of a uniform magnetic field $\mathbf{B}_0$ applied in the perpendicular direction to the flow. The effects of induced and electric fields are not taken into consideration. The temperature of upper and lower walls are characterized by T_0 and T_1 respectively. Both velocity and thermal slips are considered. Asymmetry in the flow is generated by the waves with different amplitudes and phases. The waves propagating on the channel walls with velocity c are described as follows:

$$\begin{aligned} H_1(\bar{X},\bar{t}) &= d_1 + a_1 \sin\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}), \quad \text{upper wall,} \\ H_2(\bar{X},\bar{t}) &= -d_2 - b_1 \sin\left(\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}) + \phi\right), \quad \text{lower wall,} \end{aligned} \tag{1}$$

where a_1, b_1 are the wave amplitudes, λ is the wavelength, the phase difference ϕ varies in the range $0 \le \phi \le \pi$ and a_1, b_1, d_1, d_2 and ϕ satisfies the condition

$$a_1^2 + b_1^2 + 2a_1 b_1 \cos\phi \le (d_1 + d_2)^2. \tag{2}$$

It should be noted that $\phi = 0$ corresponds to symmetric channel with waves out of phase and for $\phi = \pi$, the waves are in phase.

Denoting the velocity components $(\bar{U}, \bar{V})$ and $(\bar{u}, \bar{v})$ in the laboratory $(\bar{X}, \bar{Y})$ and wave frames $(\bar{x}, \bar{y})$ one can express that

$$\bar{x} = \bar{X} - c\bar{t}, \quad \bar{y} = \bar{Y}, \quad \bar{u} = \bar{U} - c, \quad \bar{v} = \bar{V}, \quad \bar{p}(\bar{x},\bar{y}) = \bar{P}(\bar{X},\bar{Y},\bar{t}) \tag{3}$$

in which $\bar{t}$ is the time and $\bar{p}$ and $\bar{P}$ are the pressures in the wave and laboratory frames respectively.

With the aid of Eq (3), continuity, motion and energy equations give

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{4}$$

$$\begin{aligned} \rho\left(\bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{u} = -\frac{\partial \bar{p}}{\partial \bar{x}} + 2\frac{\partial}{\partial \bar{x}}\left(\bar{\mu}(\bar{y})\frac{\partial \bar{u}}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left[\bar{\mu}(\bar{y})\left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)\right] \\ -\sigma B_0^2(\bar{u} + c), \end{aligned} \tag{5}$$

$$\rho\left(\bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}}\right)\bar{v} = -\frac{\partial \bar{p}}{\partial \bar{y}} + 2\frac{\partial}{\partial \bar{y}}\left(\bar{\mu}(\bar{y})\frac{\partial \bar{v}}{\partial \bar{y}}\right) + \frac{\partial}{\partial \bar{x}}\left[\bar{\mu}(\bar{y})\left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}}\right)\right], \tag{6}$$

$$\rho\zeta\left(\bar{u}\frac{\partial}{\partial\bar{x}}+\bar{v}\frac{\partial}{\partial\bar{y}}\right)T=\bar{\mu}(\bar{y})\left[2\left\{\left(\frac{\partial\bar{u}}{\partial\bar{x}}\right)^2+\left(\frac{\partial\bar{v}}{\partial\bar{y}}\right)^2\right\}+\left(\frac{\partial\bar{v}}{\partial\bar{x}}+\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2\right]$$
$$+k\left[\frac{\partial^2T}{\partial\bar{x}^2}+\frac{\partial^2T}{\partial\bar{y}^2}\right], \tag{7}$$

where ρ is the density, T is temperature, σ is the electrical conductivity, k is the thermal conductivity and $\bar{\mu}(\bar{y})$ is the viscosity function. Letting

$$x=\frac{2\pi\bar{x}}{\lambda},\quad y=\frac{\bar{y}}{d_1},\quad u=\frac{\bar{u}}{c},\quad v=\frac{\bar{v}}{c\delta},\quad \delta=\frac{2\pi d_1}{\lambda},$$
$$h_1=\frac{H_1}{d_1},\quad h_2=\frac{H_2}{d},\quad d=\frac{d_2}{d_1},p=\frac{2\pi d_1^2\bar{p}}{c\lambda\mu_0},$$
$$\theta=\frac{T-T_0}{T_1-T_0},\quad \mu(y)=\frac{\bar{\mu}(\bar{y})}{\mu_0},\quad M=\left(\frac{\sigma}{\mu_0}\right)^{1/2}B_0d_1,\quad Pr=\frac{\mu_0\varsigma}{k},$$
$$E=\frac{c^2}{\varsigma(T_1-T_0)},\quad \upsilon_0=\frac{\mu_0}{\rho}.$$
$$a=\frac{a_1}{d_1},\quad b=\frac{b_1}{d_1},Re=\frac{\rho c d_1}{\mu_0},\quad t=\frac{c\bar{t}}{\lambda},u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x},$$

where ζ is the specific heat, ψ is the stream function, υ_0 is the kinematic viscosity, M is the Hartman number, Re is the Reynolds number, δ is the wave number, Pr is the Prandtl number, E is the Eckret number, μ_0 is the constant viscosity and θ is the dimensionless temperature. Adopting the long wavelength approximation one obtains:

$$0=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu(y)\frac{\partial^2\psi}{\partial y^2}\right)-M^2\left(\frac{\partial\psi}{\partial y}+1\right) \tag{8}$$

$$0=-\frac{\partial p}{\partial y} \tag{9}$$

$$0=\frac{\partial^2\theta}{\partial y^2}+Br\mu(y)\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \tag{10}$$

in which the Brinkman number $Br=Pr\ E$ and continuity equation is automatically satisfied.

Following the analysis of ref [13], the subjected boundary conditions can be written as

$$\psi=\frac{F}{2},\quad \frac{\partial\psi}{\partial y}+\beta\frac{\partial^2\psi}{\partial y^2}=-1,\quad \theta+\gamma\frac{\partial\theta}{\partial y}=0,\quad at\quad y=h_1, \tag{11}$$

$$\psi=-\frac{F}{2},\ \ \frac{\partial\psi}{\partial y}-\beta\frac{\partial^2\psi}{\partial y^2}=-1,\ \ \theta-\gamma\frac{\partial\theta}{\partial y}=1,\ \ at\ \ y=h_2, \tag{12}$$

$$h_1(x){=}1+a\sin(2\pi x),\quad h_2(x)=-d-b\sin(2\pi x+\phi),\ \ F=\int_{h_2}^{h_1}\frac{\partial\psi}{\partial y}dy, \tag{13}$$

$$a^2+b^2+2ab\cos\phi=(1+d)^2, \tag{14}$$

where $\mu(y)=e^{-\alpha y}$ or $\mu(y)=1-\alpha y$ for $\alpha\ll 1$; α is the viscosity parameter and β and γ are the non-dimensional velocity and thermal slip parameters respectively.

Dimensionless expression of pressure rise per wavelength is

$\Delta p_\lambda=\int 0^{2\pi}\frac{\partial p}{\partial x}dx.$

3. SOLUTION OF PROBLEM

Using Eqs (8) - (12) and then employing similar procedure as used in [13], we have the following solutions:

3.1. Case 1 ($M=0$)

$$\psi=\frac{\left[\begin{array}{c}(2A_1(h_1-y)(h_2-y)(A_1(A_2-2y+(h_1^2+h_1h_2+h_2^2\\ -A_2y-y^2)\alpha)+2(A_2+h_1h_2\alpha-y(2+\alpha y))\beta)-F(A_1(h_2^3\\ -6h_2y^2+h_1^4\alpha+h_2^4\alpha-4h_2^2y^2\alpha+h_1^3(1-2h_2\alpha)\\ -h_1(h_2^2-4h_2y+2y^2)(3+2h_2\alpha)+h_1^2(-3h_2-4(h_2-y)^2\alpha)\\ +2y^3(2+\alpha y))+4(-3h_2^2y-3h_1(h_2^2-4h_2y+y^2)\\ +h_1^4\alpha+h_2^4\alpha-h_1y(-3h_2^2+y^2)\alpha-3h_1^2(h_2+y+h_2(h_2-y)\alpha)\\ +(2-2y\alpha)(h_1^3+h_2^3)+y^3(2+2y\alpha)\\ -h_2y^2(3+\alpha y))\beta+12A_1(A_2-2y)\beta^2))\end{array}\right]}{(2A_1^2(A_2^2(1+A_2\alpha)+4A_1(2+A_2\alpha)\beta+12\beta^2))}, \tag{15}$$

$$\frac{dP}{dx}=-\frac{(6(F+A_1)(A_1(2+\alpha A_2)+4\beta))}{(A_1^2(A_1^2(1+\alpha A_2)+4A_1(2+\alpha A_2)\beta+12\beta^2))}, \tag{16}$$

$$\theta=\frac{1}{L_1}(L_2+\gamma(-23h_1^6\alpha+h_2L_3+L_4+L_5+L_6+L_7\gamma)). \tag{17}$$

$$
\begin{aligned}
L_1 &= 5A_2^4(A_2+6\beta)(A_2^2(1+2A_1\alpha+8A_2(1+A_1\alpha)\beta+12\beta^2)(A_2+2\gamma)),\\
L_2 &= (A_2^4(A_2+6\beta)(A_2^2(1+2A_1\alpha)+8A_2(1+A_1\alpha)\beta+12\beta^2)(h_1-y+\gamma)\\
&\quad -2(F+A_2)^2Br(A_2(h_1-y)(h_2-y)(A_2(15(h_1^2+h_2^2-2A_1y+2y^2)\\
&\quad +(23(h_1^3+h_2^3)+13h_1h_2(A_1)(37(h_1^2+h_2^2)+52h_1h_2)y+18A_1y^2\\
&\quad +18y^3)\alpha)+6(h_1^2+h_2^2+2y(-A_1+y)(5+(A_1+3y)\alpha)\beta),\\
L_3 &= 120h_2y^3+23h_2^5\alpha+15h_2^4(1+3y\alpha)-12y^4(5+3y\alpha)-30h_2^3y(-1\\
&\quad +4y\alpha)+10h_2^2y^2(-9+11y\alpha),\\
L_4 &= -6(h_2^4+2h_2^3y-6h_2^2y^2+8h_2y^3-4y^4)(5+h_2\alpha+3y\alpha)\beta+3h_1^5(-5\\
&\quad +46h_2\alpha-15\alpha y-2\alpha\beta)-15h_1^4(9h_2^2\alpha+2y\alpha(-4y+\beta)+2(y+\beta)\\
&\quad +h_2(7(-1+y\alpha)-2\alpha\beta))+3h_1(-46h_2^5\alpha+10h_2^2y(-3y\alpha)-6\beta\\
&\quad +40h_2y^2(3+y\alpha)\beta+52h_2^4(7(-1+y\alpha)+2\alpha\beta))+20h_2^3(y(1-2y\alpha)\\
&\quad +3\beta+y\alpha\beta)+4y^3(y(5+3y\alpha)-10(2+y\alpha)\beta)),\\
L_5 &= 15h_1^2(12h_2^3+9h_2^4\alpha-2h_2y(y(-3y\alpha)+6\beta)-12h_2^2(\beta+y\alpha\beta)\\
&\quad +4y^2(3\beta+y(-2+\alpha\beta))),\\
L_6 &= -10h_1^3(18h_2^2+y(y(-9+11y\alpha)+6\beta)-6h_2^2(3\beta+y(-1+2\alpha y\\
&\quad +\alpha\beta))),\\
L_7 &= -15A_2^3(A_2(2+3A_1\alpha)+2(2+A_1\alpha)\beta)
\end{aligned}
$$

3.2. Case 2 ($M \neq 0$)

$$
\psi = \begin{array}{l} \frac{1}{2A_5}\left[2e^{M(A_1-y)}(F+A_2)-2e^{My}(F+A_2)+e^{h_2M}(A_1-2y)\right. \\ \left.(2+FMA_4)\right]-e^{h_1M}(A_1-2y)(2+FMA_3)+\alpha\left[m_1+m_2\right] \end{array} \tag{18}
$$

$$
m_1 = \left[\begin{array}{c}
-\frac{1}{8A_5^2}(\frac{1}{MA_6}(e^{-M(2A_1+y)}(A_5(A_6(e^{2M(A_1+y)}(F+A_2-8F_1M\\
-2(F+A_2)My+2(F+A_2)M^2y^2)+e^{3A_1M}(F+A_2+8F_1M\\
+2(F+A_2)My+2(F+A_2)M^2y^2)+4e^{M(2h_1+3h_2+y)}F_1A_1M^2A_4\\
-4e^{M(3h_1+2h_2+y)}F_1A_1M^2A_3-8A_4F_1Me^{M(3h_1+4h_2)}\\
+2A_4e^{M(2h_1+3h_2+2y)}-A_1A_4^2Me^{M(2h_1+4h_2+y)}\\
-2A_3e^{M(4h_1+3h_2)}+2A_3e^{M(3h_1+2(h_2+y))}+A_1A_3^2Me^{M(4h_1+2h_2+y)}))\\
+(F+A_2)(-8A_6e^{M(3A_1+y)}A_2A_1M^3y\\
+2e^{4A_1M}(2-M(A_1+A_2^2M)(2+A_1M)+A_2^2M^4(-1+A_1M)\beta^2))
\end{array}\right]
$$

$$m_2 = \left[\begin{array}{c} -2e^{3A_1M+2My}(-2+M(-2A_1+3M(h_1^2+h_2^2)-2h_1h_2M- \\ M^2(h_1^3+h_2^3)+M^2h_1h_2A_1+A_2^2M^3(1+A_1M)\beta^2)) \\ +e^{M(3h_1+5h_2)}(-2+M(h_2+h_1(-1+2M(A_1+h_1A_2M))-2\beta- \\ 4h_1M(1+h_1M(1+h_1M-h_2M))\beta+A_2M^2(1+2h_1M \\ (1+h_1M))\beta^2))+A_7e^{2M(h_1+2h_2+y)}+2A_8e^{M(2h_1+5h_2+y)}- \\ 2A_9e^{M(5h_1+2h_2+y)}+A_{10}e^{2M(2h_1+h_2+y)}+2A_{11}e^{M(3h_1+4h_2+y)}+ \\ A_{12}e^{M(5h_1+3h_2)}+2A_{13}e^{M(4h_1+3h_2+y)}))) \\ +8e^{A_1M}(F+A_2)A_1My\sinh[A_2M] \end{array}\right]$$

$$\frac{dP}{dx} = -\frac{\left[\begin{array}{c}(F+A_2)M^3(A_1(-e^{2h_1M}+e^{2h_2M}+2e^{A_1M}A_2M)\alpha \\ -2A_5e^{h_2M}A_4+2A_5e^{h_1M}A_3\end{array}\right]}{2A_5(e^{h_2M}(2-A_2MA_4)+e^{h_1M}(-2+A_2MA_3))}, \tag{19}$$

$$\begin{array}{c}\theta = \frac{1}{24A_5}(N_1+\frac{1}{2}Br(F+A_2)M\alpha(N_2+\frac{1}{A_5A_6}N_3+N_4+N_5+-48e^{MA_1} \\ F_1M(\cosh[A_2M]-\cosh[(A_2-2y)M]+2M\gamma\sinh[A_2M])))\end{array} \tag{20}$$

$$\begin{aligned}
N_1 &= \frac{24A_5^2(h_1-y+\gamma)}{A_2+2\gamma}+12Bre^{MA_1}(F+A_2)^2M^2(\cosh[A_2M] \\
&\quad -\cosh[A_2M-2y]+2M(M(h_1-y)(h_2-y)-A_2M\gamma \\
&\quad +\gamma\sinh[A_2M])) \\
N_2 &= -6e^{2My}(F+A_2)(1-My+M^2y^2)+6e^{2M(A_1-y)}(F+A_2) \\
&\quad (1+My(1+My))+\frac{1}{A_5A_6}(3e^{-2My}(F+A_2)(2e^{3MA_1}A_{14} \\
&\quad +e^{2M(h_1+2h_2)}A_{15}-e^{2M(2h_1+h_2)}A_{16}+2e^{M(A_1+4y)}A_{17} \\
&\quad -e^{2M(h_2+2y)}A_{18}+e^{2M(h_1+2y)}A_{19})), \\
N_3 &= 6e^{A_1M}M^2(2e^{A_1M}(F-(-A_{14}+A_{17})+A_2(A_{14}+A_{17})) \\
&\quad -e^{2h_2M}((F-A_2)A_{15}+(F+A_2)A_{18})+e^{2h_1M}((F-A_2)A_{16} \\
&\quad +(F+A_2)A_{19}))((h_1-y)(h_2-y)-A_2\gamma)), \\
N_4 &= \frac{1}{A_2+2\gamma}(8e^{A_1M}M^3(A_2(h_1-y)(h_2-y)(-12F_1+(F+A_2) \\
&\quad (A_1+y))-(h_1^2-4h_1h_2+h_2^2+2A_1y-2y^2)(-12F_1 \\
&\quad +(F+A_2)(A_1+y))\gamma-3A_2(-8F_1+(F+A_2)A_1)\gamma^2)) \\
&\quad -\frac{1}{A_6}(24F_1M(e^{h_2M}A_4+e^{h_1M}A_3)(e^{2M(A_1-y)}-e^{2My} \\
&\quad +4e^{MA_1}M^2((h_1-y)(h_2-y)-A_2\gamma)))-\frac{1}{A_2+2\gamma}
\end{aligned}$$

$$(6e^{2h_2M}(F+A_2)(-h_2(1+My-2M\gamma)-2y(-1+M\gamma)+h_1^2M^2(h_2-y-\gamma)$$
$$(-1+2M\gamma)-h_2^2M^2(y-\gamma)(-1+2M\gamma)+h_1(-1+My-2M\gamma$$
$$+h_2^2M^2(1+2M\gamma)))),$$

$$\begin{aligned}
N_5 \;=\; & -\frac{1}{A_2+2\gamma}(6e^{2h_1M}(F+A_2)(h_1^2M^2(h_2-y-\gamma)(1+2M\gamma) \\
& -h_2^2M^2(y-\gamma)(1+2M\gamma)+h_2(1-My+2M\gamma)-2(y+My\gamma) \\
& +h_1(1+My+2M\gamma+h_2^2M^2(1+2M\gamma))))+\frac{1}{A_5A_6(A_2+2\gamma)} \\
& (3(8A_5F_1M(e^{h_2M}A_4+e^{h_1M}A_4)(A_2-2\gamma)(e^{2h_2M}(1-2M\gamma)+e^{2h_1M}(1 \\
& +2M\gamma)-(F+A_2)(-2e^{M(h_1+3h_2)}(-1+2M\gamma)(-h_2A_{14}+y(A_{14}-A_{17}) \\
& +h_1A_{17}+(A_{14}+A_{17})\gamma)+2e^{M(3h_1+h_2)}(1+2M\gamma)(h_1A_{14}-h_2A_{17} \\
& +h_2A_{19}-y(A_{16}+A_{19})+\gamma A_{16}-\gamma A_{19})+e^{4h_1M}(-1+2M\gamma)(h_2A_{15} \\
& -y(A_{15}+A_{18})A_{15}y(-A_{14}+A_{17})+(A_{14}+A_{17})\gamma- \\
& -e^{4h_1M}(1+2M\gamma)(h_1A_{16}-+A_{18}(h_1+\gamma)) \\
& +e^{2MA_1}(h_2(A_{16}+A_{18})-y(A_{15}+A_{16}+A_{18}+A_{19})-2My(A_{15}-A_{16} \\
& +A_{18}-A_{19})\gamma)+\gamma(A_{15}-A_{16}-2h_2MA_{16}-A_{18} \\
& +2h_2MA_{18}+A_{19}+2M(A_{15}+A_{16}-A_{18}-A_{19}))))).
\end{aligned}$$

The heat transfer coefficient (Z) at the upper wall is

$$Z_1 = h_{1x}\theta_y \;, \tag{21}$$

which upon using Eq (20) give

$$\begin{aligned}
Z_1 \;=\; & -\frac{1}{24A_5^2}(a\cos[x](-\frac{24A_5^2}{A_2+2\gamma}+12Bre^{MA_1}(F+A_2)^2M^2 \\
& (2M(-M(h_1-y)-M(h_1-y))+2M\sin[M(A_1-2y)])\frac{1}{2}Br(F \\
& +A_2)M\alpha(-6e^{2My}(F+A_2)(-M+2M^2y)-12e^{2My}(F+A_2) \\
& M(1-My+M^2y^2)+6e^{2M(A_1-y)}(F+A_2)(M^2y+M(1+My)) \\
& -12e^{2M(A_1-y)}(F+A_2)M(1+My(1+My))-\frac{1}{A_5A_6}(6e^{-2My}(F \\
& +A_2)M(2e^{3MA_1}A_{14}+e^{2M(h_1+2h_2)}A_{15}-e^{2M(2h_1+h_2)}A_{16} \\
& +2e^{M(A_1+4y)}A_{17}-e^{2M(h_2+2y)}A_{18}+e^{2M(h_1+2y)}A_{19}))+\frac{1}{A_5A_6} \\
& (3e^{-2My}(F+A_2)(8e^{M(A_1+4y)}MA_{17}-4e^{2M(h_2+2y)}MA_{18}
\end{aligned} \tag{22}$$

$$
\begin{aligned}
&+4e^{2M(h_1+2y)}A_{19}))+\frac{1}{A_5A_6}(6e^{MA_1}M^2(-A_1+2y)(2e^{MA_1}(F(-A_{14}\\
&+A_{17})+A_2(A_{14}+A_{17}))-e^{2Mh_2}((F-A_1)A_{15}+(F+A_2)A_{18})\\
&+e^{2Mh_1}((F-A_1)A_{16}+(F+A_2)A_{19})))-\frac{1}{A_6}(24F_1M(-2e^{2M(A_1-y)}\\
&M-2e^{2My}M+4e^{MA_1}M^2(-A_2+2y))(e^{Mh_2}A_4+e^{Mh_1}A_3))\\
&+\frac{1}{A_2+2\gamma}(8e^{MA_1}M^3(A_2(F+A_2)(h_1-y)(h_2-y)-A_2(h_1-y)\\
&(-12F_1+(F+A_2)(A_1+y))-A_2(h_2-y)(-12F_1+(F+A_2)\\
&(A_1+y))-(F+A_2)(h_1^2-4h_1h_2+h_2^2+2A_1y-2y^2)\gamma\\
&-2(2A_1-4y)(-12F_1+(F+A_2)(A_1+y))\gamma))-\frac{1}{A_2+2\gamma}\\
&(6e^{2Mh_2}(F+A_2)(h_1M-h_2M-2(-1+M\gamma)-h_1^2M^2(-1+2M\gamma)\\
&-h_2^2M^2(-1+2M\gamma)))-\frac{1}{A_2+2\gamma}(e^{2Mh_1}(F+A_2)(h_1M-h_2M\\
&-2(1+M\gamma)-h_1^2M^2(1+2M\gamma)-h_2^2M^2(1+2M\gamma)))\\
&-\frac{1}{A_5A_6(A_2+2\gamma)}(3(F+A_2)(-2e^{(h_1+3h_2)M}(-A_{14}+A_{17})(-1+2M\gamma)\\
&+e^{4h_2M}(-A_{15}-A_{18})(-1+2M\gamma)+2e^{M(3h_1+h_2)}(A_{14}-A_{17})\\
&(1+2M\gamma)-e^{4Mh_1}(-A_{16}-A_{19})(1+2M\gamma)+2e^{MA_1}(-A_{15}\\
&-A_{16}-A_{18}-A_{19}-2M(A_{15}-A_{16}+A_{18}-A_{19})\gamma)))\\
&-96e^{MA_1}F_1M^2\sinh[(A_1-2y)])))
\end{aligned}
$$

4. RESULTS AND DISCUSSION

The purpose of this section is to see the salient features of temperature θ, heat transfer coefficient Z and stream lines for the velocity slip β, thermal slip γ, flow rate η, viscosity parameter α and Brinkman number Br.

Figs. 1 (a)-(e) show the behavior of temperature. Fig 1(a) explains that an increase in the velocity slip β decreases the temperature. Fig. 1(b) illustrates that the temperature increases with an increase in flow rate η. Temperature increases by increasing in Br and γ see Figs. 1(c) and 1(d). Fig. 1(e) demonstrates the effect of viscosity parameter on the temperature. Obviously there is an increase in the temperature when the value of viscosity parameter increases.

Fig. 2 represents the behavior of streamlines for the different values of α and β. Fig. 2 (b) shows that the size of trapped bolus increases with an increase in the viscosity parameter (α). Figs. 2 (a) and (b) examine that size of trapped bolus decreases when β increases.

Figs. 3-5 represent the behavior of heat transfer coefficient at the upper wall (h_1). Heat transfer coefficient has an oscillatory behavior due to peristalsis. Absolute value of heat transfer coefficient decreases with an increase in β (see Fig. 3).

Fig 4 shows that absolute value of heat transfer coefficient increases by increasing Br. Fig 5 represents that heat transfer coefficient increases with an increase in γ. By comparison of left and right panels, we conclude that the heat transfer coefficient at the upper wall increases with an increase in α.

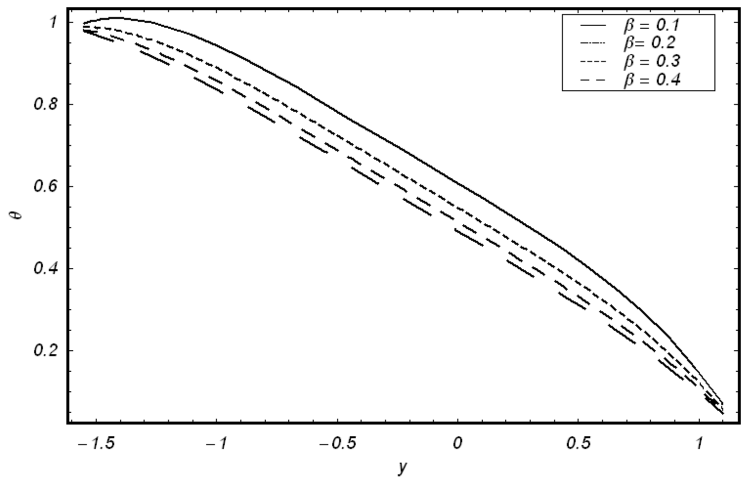


Figure 1a. Variation of β on the temperature when $d = 1.1$; $a = 0.5$; $b = 0.7$; $M = 1.0$; $\phi = \frac{\pi}{6}$; $x = 0$; $\gamma = 0.2$; $Br = 0.5$ and $\eta = 1.4$.

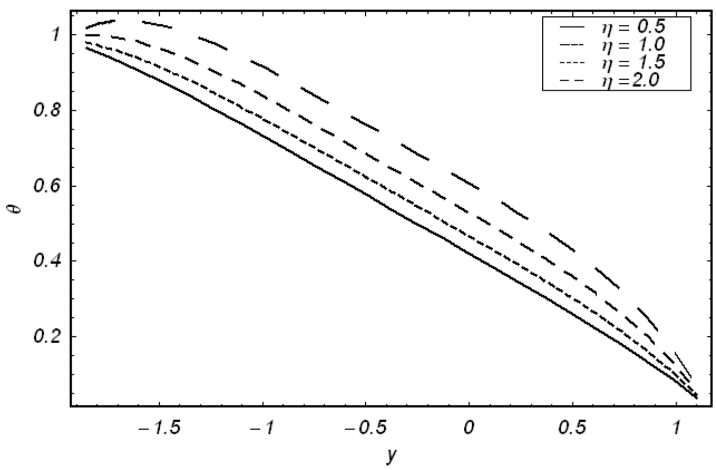


Figure 1b. Variation of η on the temperature for $d = 1.1$; $a = 0.5$; $b = 0.7$; $\beta = 0.2$; $\phi = \frac{\pi}{6}$; $x = 0$; $M = 1$; $Br = 0.5$ and $\gamma = 0.2$.

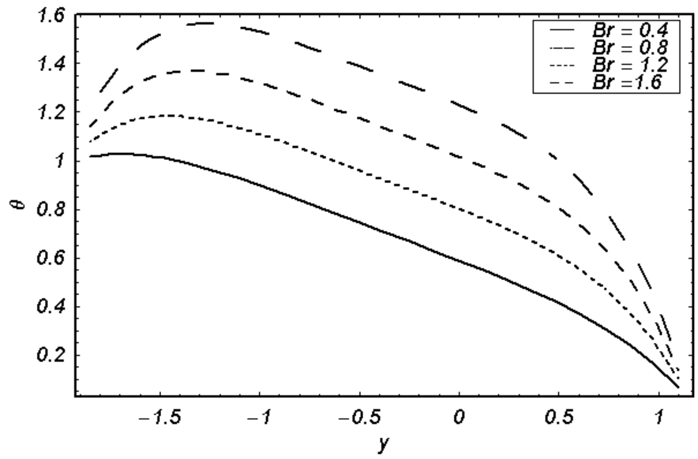


Figure 1*c*. Variation of Br on the temperature when $d = 1.1$; $a = 0.5$; $b = 0.7$; $\beta = 0.2$; $\phi = \frac{\pi}{6}$; $x = 0$; $\eta = 2.2$; $M = 1$ and $\gamma = 0.2$.

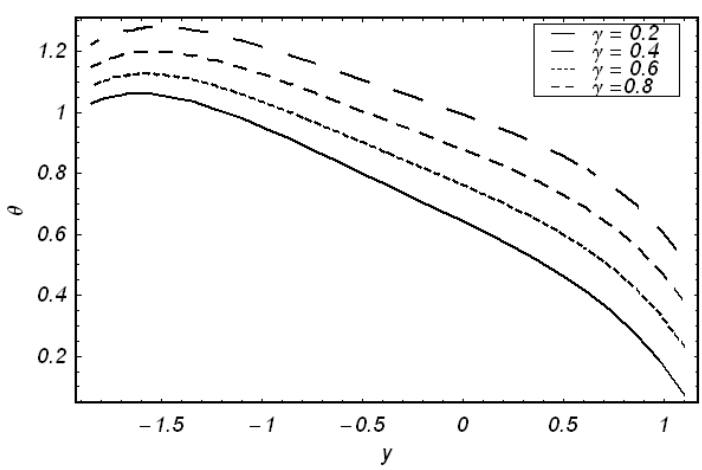


Figure 1*d*. Variation of γ on the temperature when $d = 1.1$; $a = 0.5$; $b = 0.7$; $\beta = 0.2$; $\phi = \frac{\pi}{6}$; $x = 0$; $\eta = 2.2$; $Br = 0.5$ and $\eta = 1.4$.

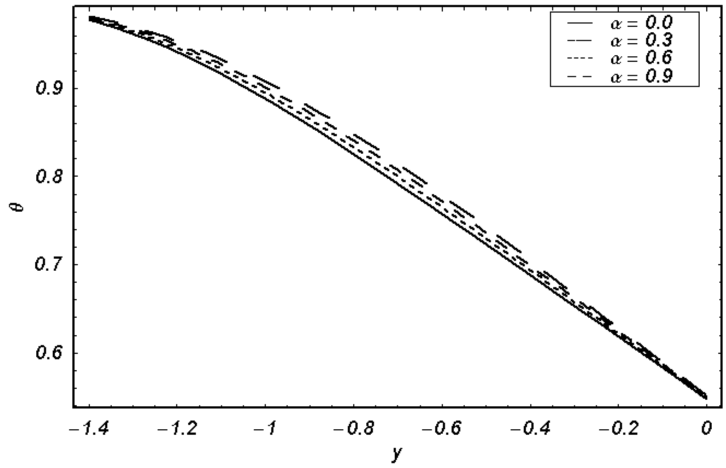


Figure 1e. Variation of α on the temperature when $d = 1.1$; $\beta = 0.2$; $b = 0.7$; $\beta = 0.2$; $\phi = \frac{\pi}{6}$; $x = 0$; $\eta = 2.2$; $M = 1$ and $\gamma = 0.2$.

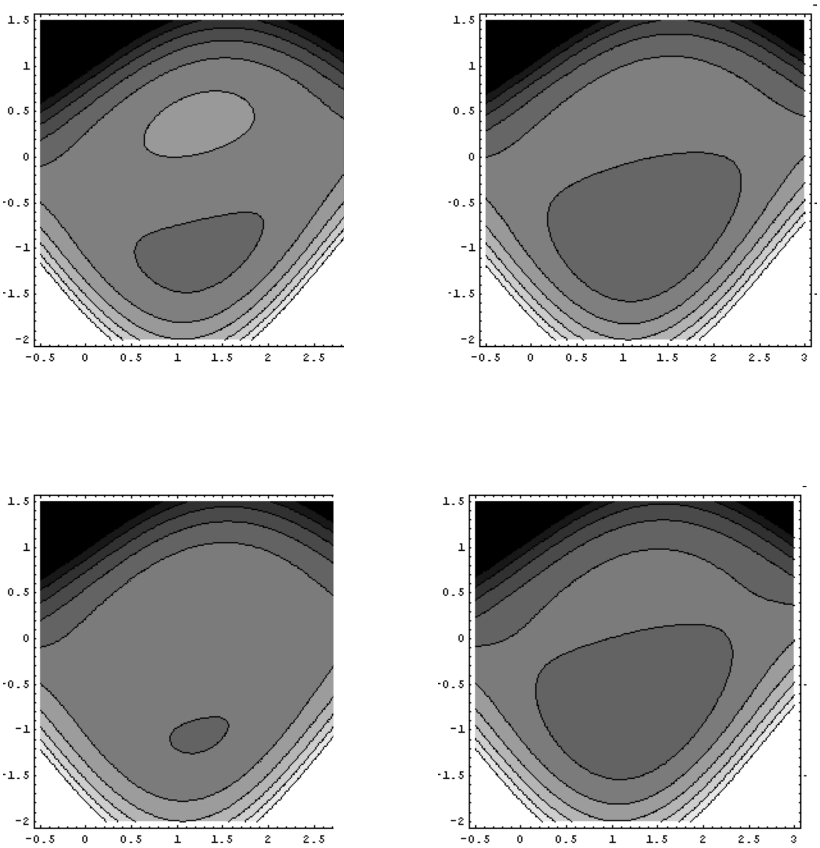
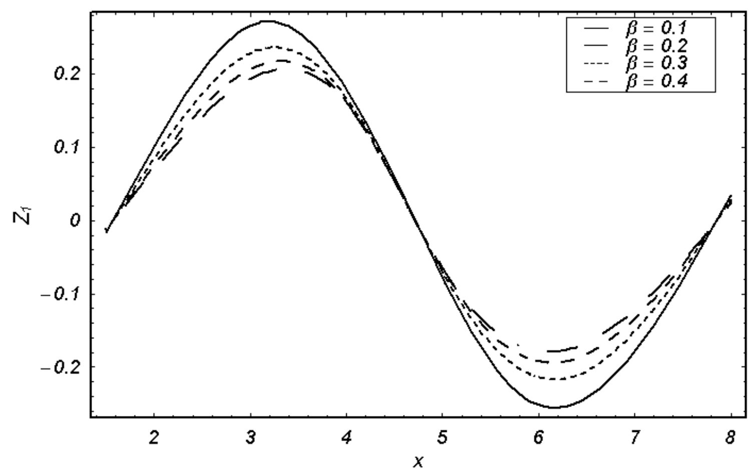


Figure 2. Effect of β on the stream lines (left panels are for $\alpha = 0$,and right panels are for $\alpha = 0.2$), when $a(\beta = 0.4)$, $b(\beta = 0.08)$ and $d = 1.2$; $a = 0.7$; $b = 1.2$; $\phi = \frac{\pi}{6}$; $\eta = 1.4$; $M = 1.0$.



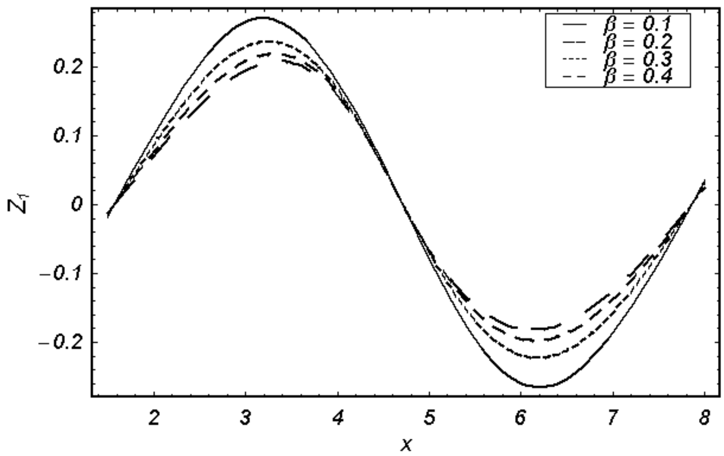
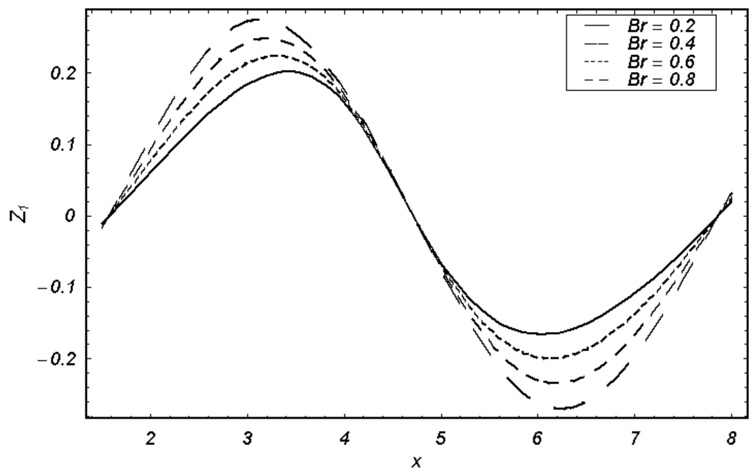


Figure 3. Effect of β on the heat transfer coefficient (Z_1) at the upper wall for $d = 1.4$; $a = 0.4$; $b = 0.8$; $\phi = \frac{\pi}{6}$; $\eta = 1.5$; $\gamma = 0.2$; $Br = 0.5$; and $M = 1$.



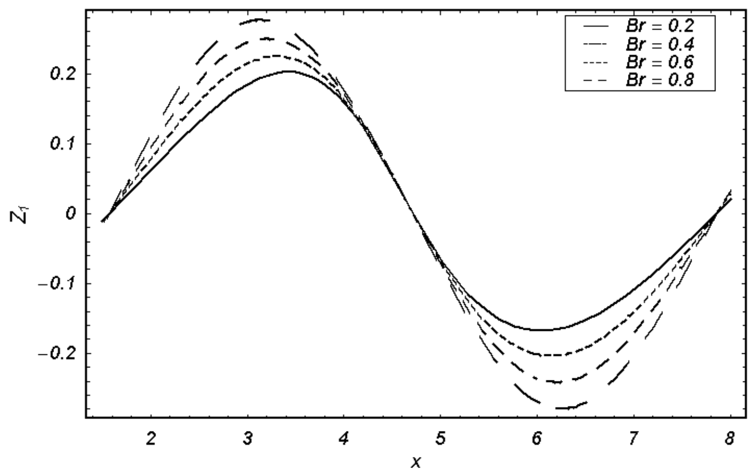
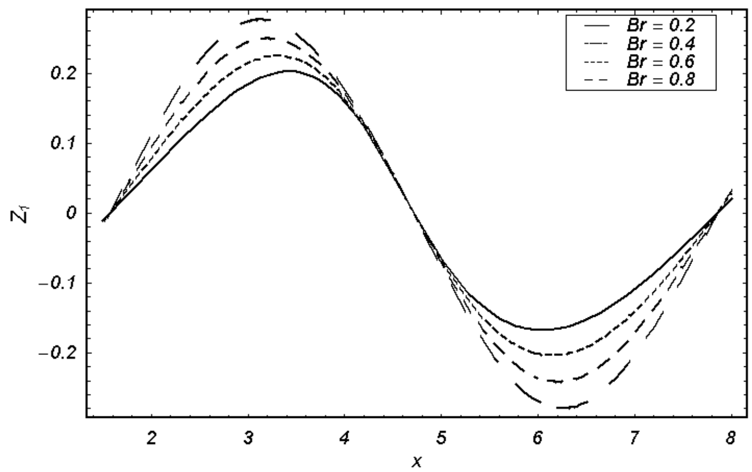


Figure 4. Effect of Br on the heat transfer coefficient (Z_1) at the upper wall for $d = 1.4$; $a = 0.4$; $b = 0.8$; $\phi = \frac{\pi}{6}$; $\eta = 1.5$; $\gamma = 0.2$; $\beta = 0.2$; and $M = 1$.



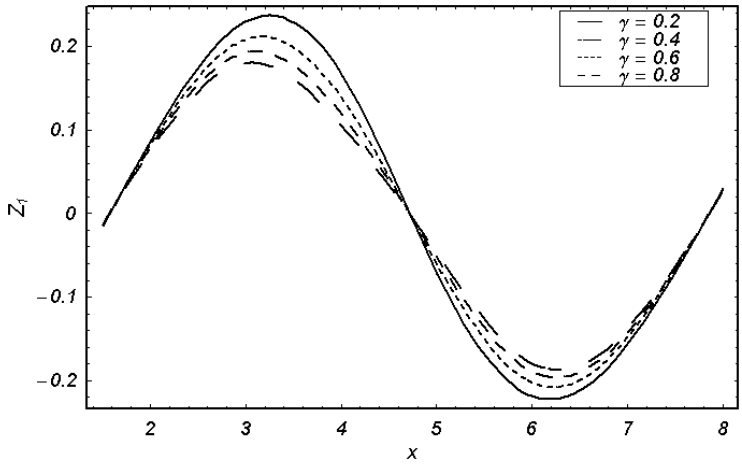


Figure 5. Effect of γ on the heat transfer coefficient (Z_1) at the upper wall when $d = 1.4$; $a = 0.4$; $b = 0.8$; $\phi = \frac{\pi}{6}$; $\eta = 1.5$; $Br = 0.5$; $\beta = 0.2$;and $M = 1$.

5. CONCLUSION

Peristalsis of variable viscosity fluid in an asymmetric channel has been studied in the presence of slip condition. The following observations are noted.

- There is a decrease in temperature when β increases.
- The effects of γ, Br and η on temperature are quite opposite to that of β.
- An increases in β reduces the size of trapped bolus.
- The magnitude of the heat transfer coefficient at the upper wall increases when thermal slip parameter increases.
- The no-slip results can be recovered by choosing $\beta = \gamma = 0$.

5.1. Appendix

Here, we present the involved values in solution expressions.

$A_1 = h_1 + h_2$, $A_2 = h_1 - h_2$, $A_3 = 1 + M\beta, A_4 = -1 + M\beta$, $A_5 = e^{h_2 M}(2 - MA_2A_4) + e^{h_1 M}(-2 + MA_2A_3)$, $A_6 = e^{h_2 M}A_4 + e^{h_1 M}A_3$, $A_7 = -2 + M(-5h_1 + h_2 + 2Mh_1h_2 + 2Mh_1^2 + 2M^2h_1h_2^2 - 2M^2h_2^3 - 2\beta + 4Mh_2(1 + Mh_2(-1 - Mh_1 + Mh_2))\beta + A_2M^2(1 + Mh_2(-1 + Mh_2))\beta^2)$, $A_8 = 2 + h_2M(-3 + M\beta) + Mh_1(3 - M\beta + 4Mh_2A_4)$, $A_9 = -2 - h_2M(3 + M\beta) + Mh_1(3 + M\beta + 4Mh_2A_3)$, $A_{10} = -2 + M(2\beta - 2M^2h_1^3A_3^2 + 2Mh_1^2A_3^2(1 + Mh_2) + h_2(-5 + M^2\beta^2) + h_1(1 + M(2h_2 -$

$4\beta - M(1+2Mh_2)\beta^2))),\ A_{11} = -2 + M(h_2 + 2M^2h_1^3A_4 - h_1(-1+2M^2h_2^2)A_4 + 2Mh_1^2(A_3 + Mh_2A_4) + Mh_2(-\beta + 2h_2(1 + M(h_2 + \beta - \beta Mh_2)))),\ A_{12} = -2 + M(2\beta + h_2(-1 + M(2h_2A_3^2 + \beta(4+M\beta) + 2Mh_2^2A_3^2)) - h_1(-5 + M(M\beta^2 + 2Mh_2^2A_3^2 + 2h_2(-1+M^2\beta^2)))),\ A_{13} = -2 + M(2 + h_1^3M^2A_3 - h_1(-1+2M^2h_2^2)A_3 + 2Mh_1^2(1 + h_2M + M(-1+h_2M)\beta) - h_2(1 + M(\beta 2h_2(-1 + M(h_2 + \beta + h_2M\beta))))),\ A_{14} = 2 - M(A_1 + A_2^2M)(2 + MA_1) + A_2^2M^4(-1+A_1M)\beta^2,\ A_{15} = 2 + M(5h_2 + h_1(-1 + 2M(A_1 + h_1A_2M)) - 2\beta - 4Mh_1(1 + h_1M(1 + h_1M - h_2M))\beta + A_2M^2(1 + 2h_1M(1 + h_1M))\beta^2),\ A_{16} = 2 + M(h_2 - 2\beta + h_2M(-2h_2A_3^2 - \beta(4+M\beta) - 2Mh_2^2A_3^2) + h_1(-5 + M(M\beta^2 + 2M(h_2 + Mh_2\beta)^2 + 2h_2(-1+ M^2\beta^2)))),\ A_{17} = -2 + M(-2A_1 + 3Mh_1^2 - 2Mh_1h_2 + 3Mh_2^2 - M^2h_1^3 + M^2h_1^2h_2 + M^2h_2^2h_1 - M^2h_2^3 - M^3A_2^2(1+A_1M)\beta^2),\ A_{18} = -2 + M(-5h_1 + h_2 + 2Mh_1h_2 + 2Mh_2^2 + 2M^2h_2^2h_1 - 2M^2h_2^3 - 2\beta + 4Mh_2(1 + Mh_2(-1 - Mh_1 + Mh_2))\beta + A_2M^2(1 + 2Mh_2(-1+Mh_2)\beta^2,\ A_{19} = 2 + M(5h_2 - 2\beta - M^2\beta^2h_2 + 2M^2h_1^3A_3^2 - 2M(1+Mh_2)h_1^2A_3^2 + h_1(-1 + M(-2h_2 + 4\beta + M(1+2h_2M)\beta^2))).$

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