Mathematical and Computational Applications, Vol. 15, No. 3, pp. 428-438, 2010.
© Association for Scientific Research

E(5) BEHAVIOUR OF THE Ge ISOTOPES

Nureddin Turkan[1], Ismail Maras[2]
[1]Bozok University, Department of Physics, Yozgat, 66200 Turkey,
nurettin_turkan@yahoo.com
[2]Celal Bayar University, Department of Physics, Manisa, 45040 Turkey
maras.ismail@yahoo.com

Abstract- The sufficient aspects of model leading to the E(5) symmetry have been proved by presenting E(5) characteristic of the transitional nuclei $^{64-80}$Ge. The positive parity states of even-mass Ge nuclei within the framework of Interacting Boson Model have been calculated and compared with the Davidson potential predictions along with the experimental data. It can be said that the set of parameters used in an calculations is the best approximation that has been carried out so far. Hence, Interacting Boson Approximation (IBA) is fairly reliable for the calculation of spectra in such set of Ge isotopes.

Keywords- Davidson-like potentials, nuclear phase transition, critical symmetry, even Ge, Interacting Boson Model.

1. INTRODUCTION

This paper presents a computational study in the field of nuclear structure. The declared goal of the paper is to identify features of the E(5) dynamical symmetry in the critical point of the phase/shape transition for the even-even Ge isotopes with A in the range 64 to 80. The topic of dynamical symmetries (E(5) and X(5)) was theoretically predicted by Iachello [1] and found in real nuclei by Casten et al. [2], just beside experiment [3-5]. A systematic search of nuclei exhibiting this features along the nuclide chart is underway in several structure labs around the world. This subject is of highly scientific interest in the present days investigations. Three dynamical symmetry limits known as Harmonic oscillator, deformed rotor and asymmetric deformed rotor are labeled by U(5), SU(3) and O(6) respectively [6], and they form a triangle known as the Casten triangle representing the nuclear phase diagram [7].

In the original Bohr Hamiltonian, the U(5) and O(6) symmetry limits are connected by assuming potentials only depend on β. On the other hand, the U(5) and SU(3) limits are connected by separating the $V(\beta,\gamma)$ potentials into variables in the Hamiltonian. In those cases, one can use the Davidson-like potentials instead of β-dependent part of the potential.

As it was also described in [8,9], most of the shell-model studies of nuclei with Z, N≤50 assume a $^{88}_{38}$Sr inert core and restrict the valence-proton particles or neutron holes to the $p_{1/2}$ and $g_{9/2}$ orbital [10]. In principle, the inclusion of these orbital will also make the model space adequate in principle to describe nuclei with Z≤38 as well as to possibly account for the high spin states that have recently been established in the Zr isotopes. Another motivation for considering such a large model space is to allow from one to more accurate calculation of double-beta decay transitions in Kr, Se and Ge nuclei [10]. For the nuclei with Z>28, N<50 protons and neutrons are allowed to occupy

$g_{9/2}$, $p_{1/2}$, $p_{3/2}$ and $f_{5/2}$ orbital. It is obvious that in such a description the use of $^{88}_{38}$Sr as a core is no longer convenient. A theoretical explanation of the shape coexistence phenomena has been given by the presence of intruder levels in the neutron or the proton valence shell [11]. The evidence for an extensive region of nuclei near A~80 is consistent with the definition of three dynamical symmetry limits. The neutron-proton (IBM-2) version of the model has previously been applied successfully to the light isotopes of Se, Kr and Sr [9,12-18] with emphasis primarily on the description of energy levels. The even-even Ge isotopes are the members of the chain situated away from both the proton closed shell number at 28 and neutron closed shell at 50.

In this study, we have carried out the level scheme of the transitional nuclei $^{64-80}$Ge showing the characteristic E(5) pattern in its some low-lying bands. The positive parity states of even-mass Ge nuclei also stated within the framework of the Interacting Boson Model. By comparing transitional behavior in the Ge nuclei with the predictions of an E(5) critical symmetry, an achievable degree of agreement has been investigated.

The outline of the remaining part of this paper is as follows; starting from giving a basic solution of Bohr Hamiltonian with Davidson-like potentials in the E(5) limit in Section 2, we have reviewed the comparison of previous experimental and theoretical [19] data with calculated values in Section 3. The last section contains some concluding remarks.

There are three tables; the first table compares theoretical predictions of Davidson potential (labeled by Dav.) with IBM-1 and with IBM-2 along with the experimental data for $^{64-80}$Ge. The second contains the IBM-1 parameters while the third one describes the IBM-2 parameters used in the present study together with previous parameters of some neighboring Kr and Se nuclei.

There are two figures; the first one shows ground state band energy ratio $E(J^{\pi}_{i})/E(2^{+}_{1})$ predictions of the present model for $R_{4/2}=E(4^{+}_{1})/E(2^{+}_{1})$, $R_{6/2}=E(6^{+}_{1})/E(2^{+}_{1})$, $R_{8/2}=E(8^{+}_{1})/E(2^{+}_{1})$ and compare them with the previous experimental results as a function of neutron number in $^{64-80}$Ge nuclei. The second figure compares change of ratio for the gamma band energies $E(J^{\pi}_{\gamma})/E(2^{+}_{1})$ of the present model with the experimental ones for the $J^{\pi}_{\gamma} = 2^{\pi}_{\gamma}$, 3^{π}_{γ} and 4^{π}_{γ} levels in the same nuclei.

2. E(5) SOLUTION OF THE DAVIDSON-LIKE POTENTIALS

As it was also pointed out in [8], the original Bohr Hamiltonian [20] is

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_k \frac{Q_k^2}{\sin(\gamma - \frac{2\pi}{3}k)^2}\right] + V(\beta,\gamma) \tag{1}$$

β and γ are collective coordinates that identify the shape of the nuclear surface. B refers to a mass parameter and Q_k $(k = 1,2,3)$ represents angular momentum components. If one assumes that the potential depends only on β, that is

$V(\beta,\gamma)=V(\beta)$, the Schrödinger equation can easily be separated by using the wave function of the form $\psi(\beta,\gamma,\theta_j)=\chi(\beta)\xi(\gamma,\theta_j)$ [20,21].

Introducing the reduced energies $\varepsilon=\frac{2B}{\hbar^2}E$ and reducing the potential $u=\frac{2B}{\hbar^2}V$ [1], the radial β-dependent equation can be defined as follows;

$$\left[-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{\Lambda}{\beta^2}+u(\beta)\right]\chi(\beta)=\varepsilon\chi(\beta) \tag{2}$$

The equation is including Euler angles and the eigenvalues of the second order Casimir operator of SO(5) occur. Here, it has the form $\Lambda=\tau(\tau+3)$, where $\tau=0,1,2,...$ is the quantum number characterizing the irreducible representations of SO(5) [22] and the Davidson form with no gamma dependence is used. By using the exact numerical diagonalization of the Bohr Hamiltonian [23], the gamma independent Davidson potential is given by D.Bonatsos et. Al. [24],

$$u(\beta)=\beta^2+\frac{\beta_0^4}{\beta^2} \tag{3}$$

where β_0 is the potential's minimum position. By putting Eq. (3) into Eq. (2), one can obtain

$$\left[-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{\tau(\tau+3)+\beta_0^4}{\beta^2}+\beta^2\right]\chi(\beta)=\varepsilon\chi(\beta) \tag{4}$$

The exact solution of this equation can be done [25,26], in which the eigenfunctions are being the Laguerre polynomials of the form

$$F_m^\tau(\beta)=\left[\frac{2m!}{\Gamma(m+d+5/2)}\right]^{1/2}\beta^d L_m^{d+3/2}(\beta^2)e^{-\beta^2/2} \tag{5}$$

where $\tau(\tau+3)+\beta_0^4=d(d+3)$ and $\Gamma(m)$ represents the Γ-function. So, one can find d and energy eigenvalues (in $\hbar\omega=1$ units) as follows [25,26];

$$d=-\frac{3}{2}+\left[\left(\tau+\frac{3}{2}\right)^2+\beta_0^4\right]^{1/2} \tag{6}$$

$$E_{m,\tau}=2m+d+\frac{5}{2}=2m+1+\left[\left(\tau+\frac{3}{2}\right)^2+\beta_0^4\right]^{1/2} \tag{7}$$

Each irreducible representation of SO(5) contains values of angular momentum and they are given by the algorithm [27] $\tau=3\nu_\Delta+\lambda$, where $\nu_\Delta=0,1,2,...$ is the missing quantum number in the reduction $SO(5)\supset SO(3)$ and $L=\lambda,\lambda+1,...,2\lambda-2,2\lambda$ (with $2\lambda-1$ missing). The levels of the ground state band are characterized by $L=2\tau$, $m=0$ and the energy levels of the ground state bands are given by [8]

$$E_{0,L}=1+\frac{1}{2}\left[(L+3)^2+4\beta_0^4\right]^{1/2} \tag{8}$$

whereas the excitation energies of the levels of the ground state band relative to the ground state are

$$E_{0,L,exc} = E_{0,L} - E_{0,0} = \frac{1}{2}\left(\left[(L+3)^2 + 4\beta_0^4\right]^{1/2} - \left[9 + 4\beta_0^4\right]^{1/2}\right) \tag{9}$$

The energy level ratios with respect to E(2_1^+) (labeled by $E_{0,2}$) are defined by

$$R_{L/2} = \frac{E_{0,L} - E_{0,0}}{E_{0,2} - E_{0,0}} \tag{10}$$

3. THEORETICAL PREDICTIONS

The collective quantities including the ground state band ratios $R_{4/2}=E(4_1^+)/E(2_1^+)$, $R_{6/2}= E(6_1^+)/E(2_1^+)$, $R_{8/2}=E(8_1^+)/E(2_1^+)$; the bandheads of the β_1 and γ_1 bands, $E(0_\beta^+)$ and $E(2_\gamma^+)$, normalized to $E(2_1^+)$, the spacing within the β_1 band relative to these of the ground state band [28,29],

$$R_{2,0,\beta,g} = \frac{E(2_\beta^+) - E(0_\beta^+)}{E(2_1^+)} \quad , \quad R_{4,2,\beta,g} = \frac{E(4_\beta^+) - E(2_\beta^+)}{E(4_1^+) - E(2_1^+)} \tag{11}$$

and the spacing within the γ_1 and relative to that of the ground state band,

$$R_{4,2,\gamma,g} = \frac{E(4_\gamma^+) - E(2_\gamma^+)}{E(4_1^+) - E(2_1^+)} \tag{12}$$

are all given in Table 1.

Table 1. Comparing the theoretical predictions of Davidson potential (labeled by Dav.) with IBM-1 and IBM-2 along with the experimental data for $^{64-80}$Ge. The band heads of β_1 and γ_1 bands normalized to $E(2_1^+)$ and the experimental values have been taken from Ref. [30]. The numerical results of $R_{2,0,\beta,g}$, $R_{4,2,\beta,g}$, $R_{4,2,\gamma,g}$, $0_\beta^+/2_1^+$, $2_\beta^+/2_1^+$, $4_\beta^+/2_1^+$, $2_\gamma^+/2_1^+$, $3_\gamma^+/2_1^+$ and $4_\gamma^+/2_1^+$ are also given.

N	β_o	$R_{4/2}$				$R_{6/2}$				$R_{8/2}$			
		Dav.	IBM		Ex.	Dav.	IBM		Ex.	Dav.	IBM		Ex.
			1	2			1	2			1	2	
32	1.7	2.3	2.3	2.2	2.3	3.7	3..9	3.6	3.8	5.3	5.9	5.2	5.7
34	1.7	2.3	2.3	2.3	2.3	3.7	3.9	3.7	3.8	5.2	5.8	5.4	-
36	1.6	2.2	2.3	2.3	2.2	3.6	3.7	3.8	3.6	5.0	5.5	5.6	4.8
38	1.0	2.1	2.7	2.5	2.1	3.2	5.0	4.5	-	4.3	7.9	6.8	-
40	1.0	2.1	3.0	2.6	2.1	3.2	6.0	4.6	3.3	4.3	10.1	7.2	4.5
42	3.0	2.5	3.3	2.4	2.5	4.3	7.0	4.1	-	6.6	11.9	6.2	-
44	10	2.5	1.2	1.9	2.5	4.5	0.6	2.6	-	7.0	-0.9	3.2	-
46	10	2.5	1.8	2.4	2.5	4.5	2.4	4.2	-	7.0	2.7	6.4	-
48	10	2.5	2.5	2.6	2.6	4.5	4.6	4.9	-	7.0	-	-	-

N	$R_{2,0,\beta,g}$				$R_{4,2,\beta,g}$				$R_{4,2,\gamma,g}$			
	Dav.	IBM		Ex.	Dav.	IBM		Ex.	Dav.	IBM		Ex.
		1	2			1	2			1	2	
32	1.6	1.1	1.4	-	0.0	-	1.2	-	1.1	1.0	1.2	-
34	1.5	1.1	1.4	-	0.0	1.1	1.2	-	1.1	1.1	1.2	-
36	1.4	1.1	1.2	1.2	0.0	1.1	1.1	-	1.1	1.1	1.2	0.8
38	1.1	1.2	1.3	0.9	0.0	1.1	1.1	0.8	1.0	1.1	1.1	1.0

40	1.1	1.3	1.3	2.1	0.0	1.1	1.2	-	1.0	1.1	1.1	1.1
42	2.2	1.0	1.2	1.2	0.0	0.9	0.8	0.4	1.3	1.0	1.0	1.1
44	2.50	1.2	1.0	1.2	0.0	2.0	0.3	0.2	1.3	1.6	1.0	1.1
46	2.5	1.2	1.6	0.5	0.0	1.2	1.0	0.9	1.3	1.1	1.2	0.5
48	2.5	1.3	1.8	-	0.0	-	-	-	1.3	1.1	1.2	-
N	$0_\beta^+/2_1^+$				$2_\beta^+/2_1^+$				$4_\beta^+/2_1^+$			
	Dav.	IBM		Ex.	Dav.	IBM		Ex.	Dav.	IBM		Ex.
		1	2			1	2			1	2	
32	3.7	1.7	2.5	-	5.3	2.8	3.9	-	5.3	-	5.4	-
34	3.7	1.7	2.5	-	5.2	2.8	3.9	-	5.2	4.2	5.4	-
36	3.6	1.7	1.9	1.7	5.0	2.9	3.1	2.9	5.0	4.2	4.5	-
38	3.2	1.2	1.6	1.2	4.3	2.3	2.9	2.1	4.3	4.1	4.5	2.9
40	3.2	0.8	1.8	0.8	4.3	2.1	3.1	2.9	4.3	4.3	4.9	-
42	4.3	0.9	2.0	2.5	6.6	1.9	3.1	2.7	6.6	4.1	3.7	4.3
44	4.5	3.4	2.6	3.4	6.9	4.6	3.6	4.6	6.9	4.9	3.9	4.5
46	4.5	2.5	3.0	2.5	6.9	3.7	4.7	3.0	6.9	4.6	6.1	4.3
48	4.5	1.7	2.7	-	6.9	3.1	4.5	-	6.9	-	-	-
N	$2_\gamma^+/2_1^+$				$3_\gamma^+/2_1^+$				$4_\gamma^+/2_1^+$			
	Dav.	IBM		Ex.	Dav.	IBM		Ex.	Dav.	IBM		Ex.
		1	2			1	2			1	2	
32	2.3	1.9	2.3	-	3.7	3.0	3.7	-	3.7	3.3	3.7	-
34	2.3	1.9	2.3	-	3.7	3.1	3.7	-	3.7	3.3	3.7	-
36	2.2	2.0	2.3	1.8	3.6	3.1	3.7	-	3.6	3.3	3.8	2.8
38	2.1	1.7	2.3	1.6	3.2	2.8	3.8	2.4	3.2	3.5	4.9	2.7
40	2.1	1.7	2.6	1.8	3.2	2.8	4.5	2.5	3.2	3.8	4.4	2.9
42	2.4	1.6	2.2	2.0	4.3	2.6	3.0	2.8	4.3	3.8	3.6	3.6
44	2.5	2.8	2.1	2.0	4.4	4.0	3.0	2.7	4.4	3.2	3.1	3.5
46	2.5	2.3	2.3	1.9	4.4	3.5	4.0	2.7	4.4	3.2	4.1	2.7
48	2.5	2.1	2.4	2.4	4.4	3.4	4.1	-	4.4	3.8	4.4	-

This table also contains the $R_{2,0,\beta,g}$, $R_{4,2,\beta,g}$, $R_{4,2,\gamma,g}$, $0^+_\beta/2^+_1$, $2^+_\beta/2^+_1$, $4^+_\beta/2^+_1$, $2^+_\gamma/2^+_1$, $3^+_\gamma/2^+_1$, $4^+_\gamma/2^+_1$ numerical results of the present model, in which the results are compared with experimental ones. The ratio $E(J^\pi_i)/E(2^+_1)$ predictions of the present models for the $J^\pi_i = 4^+_1, 6^+_1$ and 8^+_1 levels are also given in Figure 1 as a function of neutron numbers changing from 32 to 48 for Ge. The Figure 1 compares the theoretical (IBM-1, IBM-2 and Davidson potential (labeled by Dav.)) predictions of $R_{4/2} = E(4^+_1)/E(2^+_1)$ with those of the experimental ones. Here, the standard forms of the IBM-1 and IBM-2 Hamiltonians are used (as they given in Ref. [30,27]) to calculate energies of $^{64\text{-}80}$Ge nuclei and it can easily be seen that Davidson potential predictions are successful for all Ge isotopes. We got the IBM-1 calculations using the code PHINT [30] with our own parameters and saw the transitional nuclei of the mass region around A~80 exhibit clear triaxial features. The model parameters are the free parameters and they have been determined so as to reproduce as closely as possible the excitation-energy of all positive parity levels for which a clear indication of the spin value exists and their values were estimated by fitting to the measured level energies. IBM-1 is the original form of the model and in this model; proton- and neutron–boson degrees of freedom are not distinguished. The Table 2 and 3 contain the best fitted IBM-1 and IBM-2 Hamiltonian parameters used in the present work, respectively. Here, ε value is dominated in SU(5)-like nuclei spectrum while κ value is dominated in O(6) like [31].

Table 2. The used Hamiltonian parameters set for the IBM-1 calculations of $^{64\text{-}80}$Ge nuclei.

${}^{A}_{Z}X$	N	EPS	ELL	QQ	CHQ	OCT	HEX
${}^{64}_{32}Ge_{32}$	4	0.900	0.050	-0.045	-1.750	-0.003	-0.0078
${}^{66}_{32}Ge_{34}$	5	1.000	0.050	-0.045	-1.900	-0.003	-0.0074
${}^{68}_{32}Ge_{36}$	6	1.134	0.042	-0.045	-1.900	-0.0035	-0.0074
${}^{70}_{32}Ge_{38}$	7	0.914	0.148	-0.045	-1.900	-0.004	-0.0074
${}^{72}_{32}Ge_{40}$	7	0.619	0.184	-0.045	-1.900	-0.004	-0.0074
${}^{74}_{32}Ge_{42}$	6	0.001	0.181	-0.040	-1.900	-0.004	-0.0074
${}^{76}_{32}Ge_{44}$	5	1.165	-0.135	-0.045	-1.900	-0.003	-0.0074
${}^{78}_{32}Ge_{46}$	4	0.925	-0.053	-0.045	-1.750	-0.003	-0.0078
${}^{80}_{32}Ge_{48}$	3	0.607	0.050	-0.090	-1.750	-0.003	-0.0078

Table 3. The used Hamiltonian parameters set for the IBM-2 calculations of $^{64\text{-}80}$Ge nuclei.

$^{A}_{Z}X$	N_π	N_ν	ε (ED)	κ (RKAP)	χ_ν (CHN)	χ_π (CHP)	$C_{L\nu}$ (CLN)	$C_{L\pi}$ (CLP)
$^{76}_{36}Kr_{40}$*	4	5	1.00	-0.16	0.28	-0.50	-1.20, 0.20, 0.10	-1.20, 0.30, 0.30
$^{78}_{36}Kr_{42}$*	4	4	0.96	-0.18	0.50	-0.50	-0.60, 0.25, 0.10	-1.20, 0.30, 0.30
$^{80}_{36}Kr_{44}$*	4	3	1.05	-0.18	0.0.71	-0.50	-0.20, 0.10, 0.10	-0.45, -0.20, 0.05
$^{82}_{36}Kr_{46}$*	4	2	1.15	-0.19	0.93	-0.50	0.10, -0.80, -0.35	1.40, -0.15, 0.07
$^{64}_{32}Ge_{32}$	2	2	1.25	-0.19	0.9	-0.2	0.00, 0.00, -0.35	0.00, 0.00, 0.00
$^{66}_{32}Ge_{34}$	2	3	1.35	-0.19	0.9	-0.2	0.00, 0.00, 0.00	0.00, 0.00, 0.00
$^{68}_{32}Ge_{36}$	2	4	1.40*	-0.20*	1.00*	-0.2	-1.60, 0.00, 0.20	-1.00, -1.00, -1.30
$^{70}_{32}Ge_{38}$	2	5	1.40*	-0.20*	1.20*	-0.2	-2.00, -0.40, 0.15	-1.30, -1.30, -0.50
$^{72}_{32}Ge_{40}$	2	5	1.20*	-0.21*	1.10*	-0.2	-2.50, 0.00, 0.15	0.50, 0.00, 0.50
$^{74}_{32}Ge_{42}$	2	4	1.20*	-0.21*	1.00*	-0.2	-1.20, -1.20, -0.25	0.00, 0.00, 0.00
$^{76}_{32}Ge_{44}$	2	3	1.20*	-0.21*	1.00*	-0.2	0.00, -1.00, -0.80	0.00, 0.00, -1.50
$^{78}_{32}Ge_{46}$	2	2	0.90	-0.23	1.00	-0.2	-0.40, 0.00, -0.20	0.00, 0.00, 0.20
$^{80}_{32}Ge_{48}$	2	1	0.85	-0.23	1.00	-0.2	0.00, 0.00, 0.00	0.00, -0.15, 0.45
$^{70}_{34}Se_{36}$*	3	7	1.16	-0.045	0.02	-1.20	-0.99, 0.00, 0.00	0.41, 0.00, 0.00
$^{72}_{34}Se_{38}$*	3	6	1.18	-0.083	-1.20	-1.20	0.09, 0.00, 0.00	0.30, 0.00, 0.00
$^{74}_{34}Se_{40}$*	3	5	0.90	-0.068	0.26	-1.20	-0.72, 0.00, 0.00	0.25, 0.00, 0.00
$^{76}_{34}Se_{42}$*	3	4	0.84	-0.093	0.38	-1.20	-0.34, 0.00, 0.00	0.22, 0.00, 0.00
$^{78}_{34}Se_{44}$*	3	3	0.866	-0.103	0.63	-1.20	0.00, 0.00, 0.00	0.19, 0.00, 0.00
$^{80}_{34}Se_{46}$*	3	2	0.952	-0.139	0.79	-1.20	-0.35, 0.00, 0.00	0.13, 0.00, 0.00
$^{82}_{34}Se_{46}$*	3	1	0.945	-0.135	0.78	-1.20	0.31, 0.00, 0.00	0.13, 0.00, 0.00

*From Ref. [9]

As it can be seen from the Figures 1 and 2, the agreement between the experimental [30] and theoretical results are quite good and the general features are reproduced well, especially for the members of the ground-state band. The value of $R_{4/2}$ ratio has the limiting value 2 for a quadrupole vibrator, 2.5 for a non-axial gamma-soft rotor and 3.33 for an ideally symmetric rotor. As it is seen in Table 1 it increases gradually from about 2.28 to 2.60. The agreement between the experimental values and calculated Davidson potential results is especially very good for $E_{4/2}$ ratios of all Ge isotopes and the results show that $R_{4/2}>2$ for all Ge isotopes. It means that their structure seems to be varying from Harmonic Vibrator (HV) to along gamma soft rotor. So, the energy spectrum of the $^{64-80}$Ge nuclei can be situated between the pure vibrational and rotational limit. The presented IBM calculation method in this paper is also used in Refs. [29,32]. Moreover, Boztosun and his collaborators [33] are also trying to get a solution of potentials for the *E*(5) and *X*(5) models of the Bohr Hamiltonian by comparing the findings with the experimental data as well as the previous results.

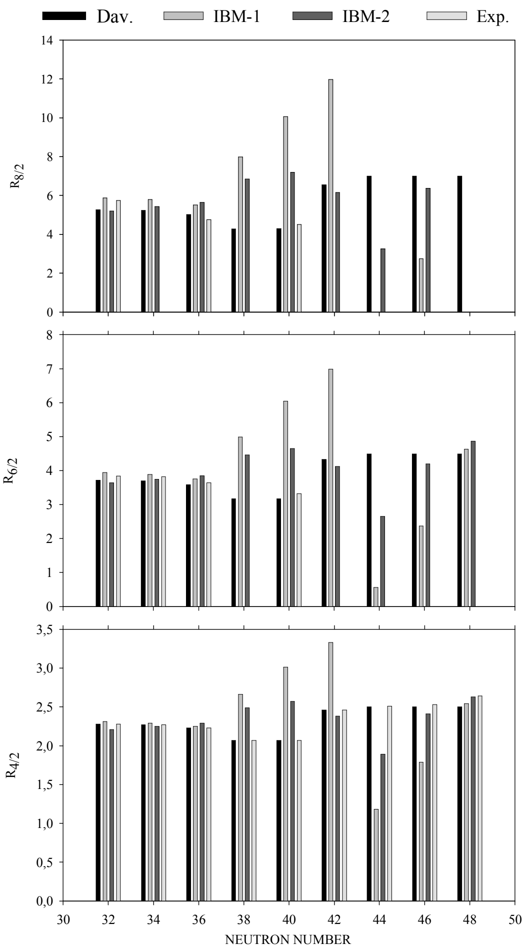

Figure 1. The Ground State Energy Ratio $E(J^{\pi}_{i})/E(2^{+}_{1})$ predictions of the present models for the $J^{\pi}_{i} = 4^{+}_{1}, 6^{+}_{1}$ and 8^{+}_{1} levels.

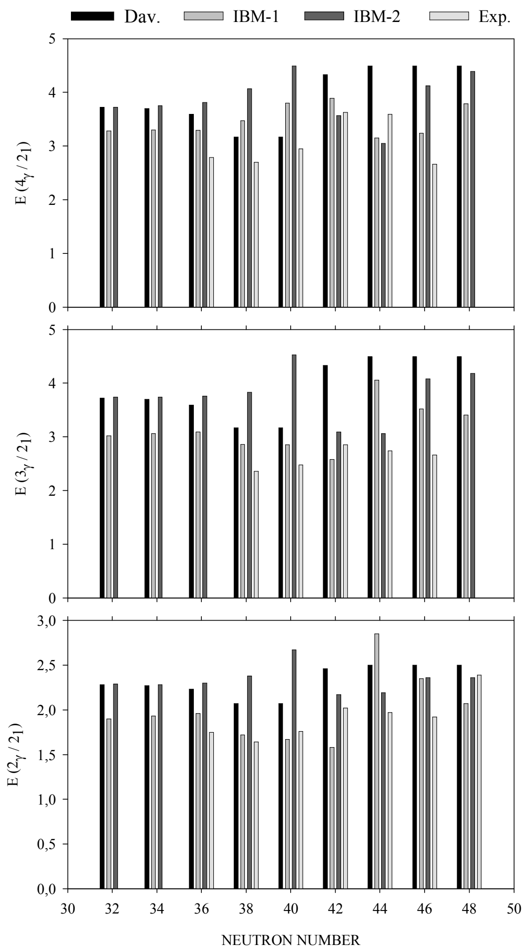

Figure 2. The Gamma Band Energy Ratio $E(J^{\pi}_{\gamma})/E(2^{+}_{1})$ predictions of the present models for the $J^{\pi}_{\gamma} = 2^{\pi}_{\gamma}$, 3^{π}_{γ} and 4^{π}_{γ} levels.

4. CONCLUSIONS

In this paper, it has been searched that the level scheme of the nuclei $^{64\text{-}80}$Ge shows the characteristic E(5) pattern or not in the ground state and some other low-lying bands by using two different approaches. Transitional behavior in Ge nuclei is compared with the results of E(5), critical symmetry and then an acceptable degree of agreement is proved. As it is seen from Figures 1 and 2, Davidson-like potentials give better results for almost all the ground and gamma band ratios. On the other hand, the validity of the presented parameters in IBM formulations has been investigated and it is seen that there is a existence of a satisfactory agreement between the presented results and experimental data. We may conclude that the general characteristics of the Ge isotopes are well satisfied in this study and are not expected to be deformed. We have investigated an acceptable degree of agreement between the predictions of the model and experiment. The shown systematic in the related tables and schemes are almost similar to the previous experimental and theoretical data. Moreover, the elegance of Figures. 1 and 2 suggest the success of the guess in parameterization in this paper. The obtained results in this study confirm that these two used methods are worth extending for investigating the nuclear structure of other nuclei existing around the mass of A~80.

Acknowledgements-This work was supported by The Scientific and Research Council of Turkey under grant TUBITAK-2219.

5. REFERENCES

1. F. Iachello, Dynamic Symmetries at the Critical Point, *Phys. Rev. Lett.* **85**, 3580-3583, 2000.
2. F. Iachello, Analytic Description of Critical Point Nuclei in a Spherical-Axially Deformed Shape Phase Transition, *Phys. Rev. Lett.* **87**, 052502, 2001.
3. R.F.Casten and N.V.Zamfir, Evidence for a Possible E(5) Symmetry in ^{134}Ba, *Phys.Rev. Lett.* **85**, 3584-3586, 2000.
4. J.M.Arias, E2 transitions and quadrupole moments in the E(5) symmetry, *Phys. Rev. C* **63**, 034308, 2001.
5. R.F.Casten and N.V.Zamfir, Empirical Realization of a Critical Point Description in Atomic Nuclei, *Phys. Rev. Lett.* **87**, 052503, 2001.
6. A.Arima and F.Iachello, Interacting boson model of collective states I. The vibrational limit, *Ann. Phys. (N.Y.)* **99**, 253-317, 1976 ; Interacting boson model of collective nuclear states II. The rotational limit, *Ann. Phys. (N.Y.)* **111**, 201-238, 1978 ; Interacting boson model of collective nuclear states IV. The O(6) limit, *Ann. Phys. (N.Y.)* **123**, 468-492, 1979.
7. D. Warner, A triple point in nuclei, *Nature* **420**, 614-615, 2002.
8. N. Turkan and I. Inci, Comparing some predictions between Davidson-like potentials and interacting boson model: X (5) behavior of even-even $^{128\text{–}140}$Nd isotopes, *Phys. At. Nucl.* **71 (11)**, 1918-1925, 2008.
9. N. Turkan, D. Olgun and I. Uluer, IBM-2 calculations of some even-even selenium nuclei, *Cent. Eur. J. Phys.* **4(1)**, 124-154, 2006.

10. J.Sinatkas, L.D.Skouras, D.Strottman and J.D.Vergados, Shell-model calculations in the A=80-100 mass region: I. A study of the N=50 nuclei, *J.Phys. G: Nucl. And Part. Phys.* **18**, 1377-1400, 1992 ; Shell-model calculations in the A=80-100 mass region: II. A study of the N=49, 48 nuclei, *J.Phys. G: Nucl. And Part. Phys.* **18**, 1401-1418, 1992.
11. K. Heyde, J. Jolie, J. Moreau, J. Ryckebusch, M. Waroquier, P. Van Duppen, M. Husye and J. L. Wood, A shell-model description of 0^+ intruder states in even-even nuclei, *Nucl. Phys. A* **466**, 189-226, 1987.
12. A.F. Barfield and K.P.Lieb, Boson effective charges for light Se, Kr, and Sr isotopes, *Phys. Rev. C* **41**, 1762-1767, 1990.
13. U.Kaup, C.Möntkemeyer and P.von Brentano, Band structure of even-even selenium isotopes in the proton-neutron interacting Boson model *, Z. Phys. A* **310** , 129-133, 1983.
14. U.Kaup and A.Gelberg, Description of even-even Krypton isotopes by the Interacting Boson Approximation, *Z.Phys. A* **293**, 311-313, 1979.
15. K.Heyde, J.Moreau and M.Waroquier, Evidence for very large deformation in neutron deficient nuclei with $Z\simeq 40$, *Phys. Rev. C* **29**, 1859-1871, 1984.
16. T.Matsuzaki and H.Taketani, Band structures of ^{76}Se and ^{78}Se, *Nucl. Phys. A* **390**, 413-448, 1982.
17. H.Higo, S.Matsuki and T.Yanabu, Level structure of ^{80,82}Sr via the ^{78,80}Kr(α, 2nγ) reactions, *Nucl. Phys. A* **393**, 224-236, 1983.
18. N. Turkan , D. Olgun, I. Uluer and S. Inan, Investigation of The Multipolarity of Electromagnetic Transitions in ^{88,90}Kr Nuclei, *Turk. J. Phys.* **30**, 89-94, 2006.
19. R.B.Firestone, *Table of Isotopes*, J.Wiley-Interscience, USA, 1996.
20. A.Bohr and B.Mottelson, *Nuclear Structure*, Vol. II, Benjamin, New York,1975.
21. L. Wilets and M. Jean, Surface Oscillations in Even-Even Nuclei, *Phys. Rev.* **102**, 788-796, 1956.
22. M.A.Caprio and F.Iachello, Analytic descriptions for transitional nuclei near the critical point, *Nucl. Phys. A* 781, 26-66, 2007.
23. M.A.Caprio, Effects of β-γ coupling in transitional nuclei and the validity of the approximate separation of variables, *Phys. Rev. C* **72**, 054323, 2005.
24. Dennis Bonatsos, D. Lenis, D. Petrellis, P. A. Terziev and I. Yigitoglu, Analytical Special Solutions of the Bohr Hamiltonian, *arXiv:nucl-th.* **1**, 0512046, 2005.
25. J. P. Elliott, J. A. Evans and P. Park, A soluble γ-unstable hamiltonian, *Phys. Lett. B* **169**, 309-312, 1986.
26. D. J. Rowe and C. Bahri, Rotation-vibrational spectra of diatomic molecules and nuclei with Davidson interactions , J. Phys. A 31, 4947-4961, 1998.
27. F.Iachello and A.Arima, *The Interacting Boson Model*, Cambridge University Press., Cambridge, 1987.
28. D. Bonatsos et al., Ground state bands of the E(5) and X(5) critical symmetries obtained from Davidson potentials through a variational procedure, *Phys. Lett. B* **584**, 40-47, 2004.
29. N. Turkan, Search for E(5) behavior: IBM and Bohr–Mottelson model with Davidson potential calculations of some even–even Xe isotopes, *J.Phys. G: Nucl. And Part. Phys.* **34**, 2235-2247, 2007.

30. O.Scholten, *The Program Package PHINT Manual Book*, National Superconducting Cyclotron Laboratory, Dept. of Physics, Michigan State University, East Lansing, Michigan 48824, 1982.
31. J. Stachel, P. Van Isacker and K. Heyde, Interpretation of the A≈100 transitional region in the framework of the interacting boson model, *Phys. Rev. C* **25**, 650-657, 1982.
32. N.Turkan, T.Bascetin and I.Inci, Quadrupole moments of some nuclei around the mass of A~80 ^{76,78,80,82,84,86,88}Kr and neighboring Se isotopes, Phys. At. Nucl., will be published in 2009.
33. I. Boztosun, D.Bonatsos and I. Inci, Analytical solutions of the Bohr Hamiltonian with the Morse potential, *Phys. Rev.C* **77**, 044302, 2008.