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AN ALGEBRAIC APPROACH FOR DETERMINING THE OPTIMAL LOT SIZE FOR EPQ MODEL WITH REWORK PROCESS

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Abstract- This paper presents a simple algebraic approach for deriving the optimal lot size for economic production quantity (EPQ) model with rework process. Conventional methods for solving lot size problems are by using differential calculus on the long-run average production-inventory cost function with the need to prove optimality first. A few recent articles proposed the algebraic approach to the solution of classic economic order quantity (EOQ) and EPQ model without reference to the use of derivatives. This paper extends it to an EPQ model with reworking of defective items. We demonstrate that optimal lot size and optimal production-inventory cost for such an imperfect EPQ model can be derived without derivatives. As a result, it may enable the practitioners or students who with little knowledge of calculus to understand or handle with ease the realistic production systems.

Keywords- Operations management; Inventory control; Manufacturing; Lot sizing; EPQ

1. INTRODUCTION

The mathematical modeling and analysis was employed by the EOQ model several decades ago [1] to balance the setup and holding costs and to derive the optimal order quantity that minimizes overall inventory costs. A considerable amount of research has since been carried out to enhance the classical EOQ model by relaxing its unrealistic assumptions [2-4]. In the manufacturing sector, the economic production quantity (EPQ) model (also known as economic manufacturing quantity (EMQ) model) is often adopted to determine the optimal production lot size for items that are produced internally instead of being obtained from an outside supplier. The classical EPQ model assumes that production process functions perfectly at all times. However, in real-life situations generation of imperfect quality items during a production run is inevitable. Sometimes, these defective items can be reworked and repaired; therefore the overall production- inventory costs can be reduced significantly. For examples, the printed circuit board assembly (PCBA) in PCBA manufacturing, plastic goods in the plastic injection molding process, and production process in other industries, such as metal components, textiles, etc., sometimes employ rework as an acceptable process in terms of level of quality. Many studies were carried out to address imperfect quality issue of production systems [5-14]. Effect of random defective rate and the reworking of defective items on EPQ model were studied by Chiu and Chiu [15] using conventional methodology. They employed the differential calculus on the long-run average production-inventory cost function with the need to prove optimality first. A few recent articles for example, Grubbström AND Erdem [16] and Cárdenas-Barrón [17] presented algebraic approaches for solving classic EOQ and EPQ model without reference to the use of derivatives (neither applying the first-order nor second-order differentiations). This paper extends it to a prior research [15] which takes the reworking of random defective items into consideration.

2. THE BASIC MODEL

Assuming an imperfect production process [15] may randomly generate x percent of defective items at a production rate d and all of the defective items produced are assumed to be repairable through a rework process. The production rate P is a constant, and is much larger than the demand rate λ. The production rate d of the imperfect quality items can be expressed as the product of the production rate P times the percentage of defective items produced x. Therefore, d can be written as $d=Px$. The inspection cost per item is included in unit production cost C. Both repairing cost C_R and holding cost h_1 per reworked item are also included in the proposed cost analysis. Additional notation used is given in the section of Nomenclature.

Because the proposed EMQ model assumes that no shortages are permitted, this implies the production rate must always greater than or equal to the sum of the demand rate and the rate at which defective items are produced. Hence, we must have $P-d-\lambda \geqq 0$. The following derivations are similar to that were given by [8,15]. The expressions of production uptime t_1; the time t_2 needed to rework defective items; production downtime t_3; on-hand inventory level H_1 and H, and cycle length T are as follows (see Figure 1).

$$t_1 = \frac{Q}{P} \tag{1}$$

$$t_2 = \frac{x \cdot Q}{P_1} \tag{2}$$

$$t_3 = \frac{H}{\lambda} \tag{3}$$

$$H_1 = (P-d-\lambda)t_1 = Q\left(1-x-\frac{\lambda}{P}\right) \tag{4}$$

$$H = H_1 + (P_1-\lambda)\ t_2 \tag{5}$$

$$T = \frac{Q}{\lambda} = t_1 + t_2 + t_3 \tag{6}$$

Solving the inventory cost per cycle [15], *TC(Q)* is

$$TC(Q)=C\cdot Q+C_R(x\cdot Q)+K+h\left[\frac{H_1+d\cdot t_1}{2}(t_1)+\frac{(H_1+H)}{2}(t_2)+\frac{H}{2}(t_3)\right]+h_1\left[\frac{P_1\cdot t_2}{2}(t_2)\right] \tag{7}$$

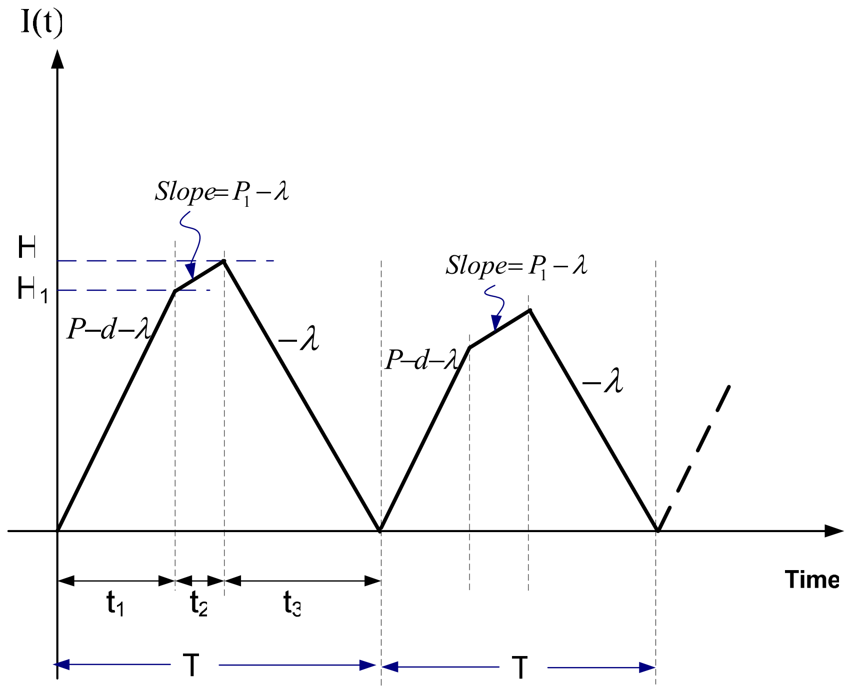

Figure 1: On-hand inventory of perfect quality items [15]

In this study, the proportion of defective items produced is considered to be a random variable with a known probability density function. Although the randomness of defective rate will not affect the production cycle length, it will change the total repairing time and amount of extra costs required for the rework process in each production cycle. Thus, one must take the randomness of defective rate into account and utilize the expectation values of x in the inventory cost analysis. Hence, the long-run expected values of production-inventory cost *E[TCU(Q)]=E[TC(Q)/T]* can be derived below [15]:

$$E[TCU(Q)]=\lambda\cdot[C+C_R\cdot E[x]]+\frac{K\lambda}{Q}+\frac{hQ}{2}\left(1-\frac{\lambda}{P}\right)+(h_1-h)\frac{\lambda Q}{2P_1}\cdot E[x]^2 \tag{8}$$

3. OPTIMAL LOT SIZE DERIVED WITHOUT DERIVATIVES

Instead of using conventional differential calculus on the long-run cost function *E*[*TCU(Q)*], this study employs algebraic approach to the solution of lot size problem of such an imperfect quality EMQ model. From Equation (8), one has the following:

$$E\left[TCU(Q)\right] = \lambda\left[C + C_R E[x]\right] + \frac{1}{2Q}\left[2K\lambda + hQ^2\left(1-\frac{\lambda}{P}\right) + \left(h_1 - h\right)\frac{\lambda Q^2}{P_1}E[x]^2\right] \tag{9}$$

$$\begin{aligned} E\left[TCU(Q)\right] = {} & \lambda\left[C + C_R E[x]\right] + \frac{1}{2Q}\left[\left(\sqrt{2K\lambda}\right)^2 + Q^2\left(\sqrt{h\left(1-\frac{\lambda}{P}\right) + \left(h_1 - h\right)\frac{\lambda}{P_1}E[x]^2}\right)^2\right] \\ & - \frac{1}{2Q}\left[2\sqrt{2K\lambda}\left(Q\sqrt{h\left(1-\frac{\lambda}{P}\right) + \left(h_1 - h\right)\frac{\lambda}{P_1}E[x]^2}\right)\right] \\ & + \frac{1}{2Q}\left[2\sqrt{2K\lambda}\left(Q\sqrt{h\left(1-\frac{\lambda}{P}\right) + \left(h_1 - h\right)\frac{\lambda}{P_1}E[x]^2}\right)\right] \end{aligned} \tag{10}$$

$$\begin{aligned} E\left[TCU(Q)\right] = {} & \lambda\left[C + C_R E[x]\right] + \frac{1}{2Q}\left[\sqrt{2K\lambda} - \left(Q\sqrt{h\left(1-\frac{\lambda}{P}\right) + \left(h_1 - h\right)\frac{\lambda}{P_1}E[x]^2}\right)\right]^2 \\ & + \sqrt{2K\lambda}\sqrt{h\left(1-\frac{\lambda}{P}\right) + \left(h_1 - h\right)\frac{\lambda}{P_1}E[x]^2} \end{aligned} \tag{11}$$

From Equation (11), if the following term is zero, then $E[TCU(Q)]$ can be minimized:

$$\left[\sqrt{2K\lambda} - \left(Q\sqrt{h\left(1-\frac{\lambda}{P}\right) + \left(h_1 - h\right)\frac{\lambda}{P_1}E[x]^2}\right)\right]^2 = 0 \tag{12}$$

Hence,

$$Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right) + \left(h_1 - h\right)\frac{\lambda}{P_1}E[x]^2}} \tag{13}$$

Equation (13) yields the same result as what was derived by using the differential calculus [15] (see Appendix). Further, suppose the manufacturing process produces no defective items, then x=0, one confirms that Equation (13) becomes the same equation as that given by classic EPQ model [3]:

$$Q^* = \sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right)}} \tag{14}$$

The optimal production-inventory cost $E[TCU(Q^*)]$ can be obtained by substituting Q^* into Equation (11):

$$E\left[TCU(Q^*)\right] = \lambda\left[C + C_R E[x]\right] + \sqrt{2K\lambda}\sqrt{h\left(1-\frac{\lambda}{P}\right) + \left(h_1 - h\right)\frac{\lambda}{P_1}E[x^2]} \tag{15}$$

3.1. Discussion

Conventional methods for solving lot size problems are by using differential calculus on the long-run average production-inventory cost function with the need to prove optimality first (see for example [8] and Appendix [15]). This paper demonstrates that the optimal lot size and the optimal production-inventory cost (i.e. Eqs. (13 and (15)) can be derived effortlessly.

4. CONCLUSIONS

This paper presents a simple algebraic approach to replace the conventional use of differential calculus for determining the optimal lot size for an imperfect EPQ model with rework process. The proposed method uses algebraic derivation, through forming a square term in the long-run average cost function, then setting it to zero in order to minimize this cost function. As a result, this paper demonstrates that the optimal lot size and the optimal production-inventory cost for such an imperfect EPQ model can be derived without derivatives. With this simplified approach, the practitioners or students who with little or no knowledge of calculus should be able to understand or handle with ease the realistic production systems.

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5. APPENDIX

Differentiation of E[TCU(Q)]: The first and the second derivatives of E[TCU(Q)] (Equation (8)) are as follows:

$$\frac{d\,E\left[TCU(Q)\right]}{d\,Q}=\frac{-K\lambda}{Q^2}+\frac{h}{2}\left(1-\frac{\lambda}{P}\right)+\left(h_1-h\right)\frac{\lambda}{2P_1}\cdot E[x^2] \tag{16}$$

$$\frac{d^2\,E\left[TCU(Q)\right]}{d\,Q^2}=\frac{2K\lambda}{Q^3}\ >0 \tag{17}$$

Since the second derivative of *E[TCU(Q)]* is greater than zero, it is convex. One can derive the optimal production quantity Q^* by setting the first derivative of *E[TCU(Q)]* equal to zero and obtains the following [15]:

$$\therefore Q^*=\sqrt{\frac{2K\lambda}{h\left(1-\frac{\lambda}{P}\right)+\left(h_1-h\right)\frac{\lambda}{P_1}E[x^2]}} \tag{18}$$

6. NOMENCLATURE

λ = annual demand rate (items/year),
P = annual production rate (items/year),
d = production rate of imperfect quality items,
h_1 = holding cost per reworked item (\$/item/unit time),
P_1 = rate of reworking of defective items (units per unit time); note that P_1 does not have to be greater than λ,
H_1 = the maximum level of on-hand inventory in units, when the regular production process stops,
H = the maximum level of on-hand inventory in units, when the rework process ends,
x = the proportion of defective items produced, a random variable with known probability density function,
K = setup cost for each production run,
h = holding cost (\$/item/unit time),
Q = production lot size for each cycle,
$TC(Q)$ = the total inventory costs per cycle,
$TCU(Q)$ = the total inventory costs per unit time.

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