

ON THE PROBLEM OF WIRE ROPE MODEL GENERATION WITH AXIAL LOADING

C. Erdem İmrak¹ and Cengiz Erdönmez²

¹Department of Mechanical Engineering, Istanbul Technical University, 34394
Gumussuyu, Istanbul, Turkey. imrak@itu.edu.tr

²Informatics Institute, Computational Science and Engineering Program,
Istanbul Technical University, 34469
Maslak, Istanbul, Turkey. erdonmez@be.itu.edu.tr,

Abstract- Independent wire rope core (IWRC) is one of the fundamental components of complex wire ropes. It is a complex geometry and constructed by wrapping wire strands over a straight wire strand. The outer wires of the IWRC are double helical shaped feature which can be only modeled using special treatment. The aims of this paper are to introduce a new technique of modeling wire rope with IWRC and to compare numerical results obtained from this model with available results in the literature. Therefore the model generation for a simple straight strand is explained and accuracy of this model is validated. Furthermore this modeling procedure is adapted to whole wire rope structure considering double helical geometry concept. The proposed rope model gives remarkable results by means of wire by wire analysis scheme. This model is found easier and more effective. The results show good agreement with other available results but with a simpler and more practical approach.

Key Words- Independent wire rope core, double helical geometry, relative rotation, modeling wire rope

1. INTRODUCTION

Theory of wire rope is based on the equilibrium equations introduced by the Love in a well-known classical treatise [1]. Most of the analytical studies are relied on the solutions of the equilibrium equations without frictional effects due to the difficulty of the theory. Even the geometry of the wire rope has a complicated nature and the equilibrium equations become nonlinear and it is difficult to solve without linearization processes mostly. A basic and simpler model considered for analytical applications is a simple straight strand. It is composed of a straight wire surrounded with six single helical wires. This simple straight strand is used as a core strand while constructing complicated wire ropes.

An independent wire rope core (IWRC) is a principle component of complex wire rope. It is used as a rope by itself in some rare applications but commonly it is used as a core for more complicated designs of wire ropes such as Seale IWRC and Warrington IWRC. On the design of IWRC, a simple straight strand as a core strand is wound by six outer strands. IWRCs are preferred when the wire rope is run under large lateral compressive loads and additional axial loading capacity is required [2].

In the literature a number of analytical studies are available for IWRC [3-10]. These studies are mostly used the superposition method which takes core strand and

outer strands as a straight wire and single helical wires respectively to model IWRC as if a simple straight strand and then solve the whole system. Static response of a simple straight strand is presented in [3] using the theory of Love. Costello has also presented variety of studies about the frictionless theory of wire ropes in his reference book [4]. It includes the static response of reduced rope rotation, bird caging, and bending theory of wire ropes. A wire rope with complex cross sections as a Seale IWRC is analyzed in [5] which discuss the prediction of the axial static response of a wire rope. A general nonlinear theory to analyze complex wire rope is developed as an extension of the frictionless strand theory of Costello for a Seale IWRC in [6]. A number of analytical models of twisted wire ropes under axis-symmetric loads are compared simultaneously with each including the available test results in [7]. Among them Phillips & Costello's model [8] is remarkable which represents excellent correlation with the available experimental results in the literature. In most of the theoretical studies over the IWRCs some kind of homogenization hypothesis used except the theoretical studies given in [9,10]. Outer double helical wire centerline of the IWRC can be obtained by using the parametric equations given in [9]. Using the analytical solutions of the wire rope theory it is not possible to analyze and get the results at any specific point of the generated model.

Therefore finite element approach has been adapted to model and analysis of this problem. An early finite element approach which intends to show the termination of the end effects and the frictional behavior of a wire strand is given in [11]. A concise finite element model which models a basic sector of the wire strand taking into account the helical symmetry features and three-layered straight helical wire ropes are presented respectively in [12,13]. Due to the need for huge computational capacity and using the symmetry most of the studies in the literature are carried over a simple straight strand with considering a basic sector or an arc length of the model in [12]. Meanwhile there is no available tool for creating a double helical solid geometry in commercial modeling computer software for the moment. For this reason parametric equation of the double helical geometry is used to construct solid wire geometry which is used while modeling an IWRC in this study.

While the general parametric equations of the double helical geometry are presented in the literature, there is no available numerical model by using the advantage of the double helical geometry at present. The aim of the present study is to propose a new modeling scheme for IWRC by using the double helical geometry. Proposed numerical model without and with frictional effects are considered and the boundary conditions are defined. The numerical results for IWRC are presented and discussed which takes into account wire by wire based analysis.

2. MODELING DOUBLE HELICAL GEOMETRY

An independent wire rope core (IWRC) is a complex geometry and constructed by wrapping wire strands over a straight wire strand. The outer wires of the IWRC are double helical shaped elements. Cross-section of IWRC model is illustrated in Figure 1-(a) with a simple straight strand as a core and six outer strands around it.

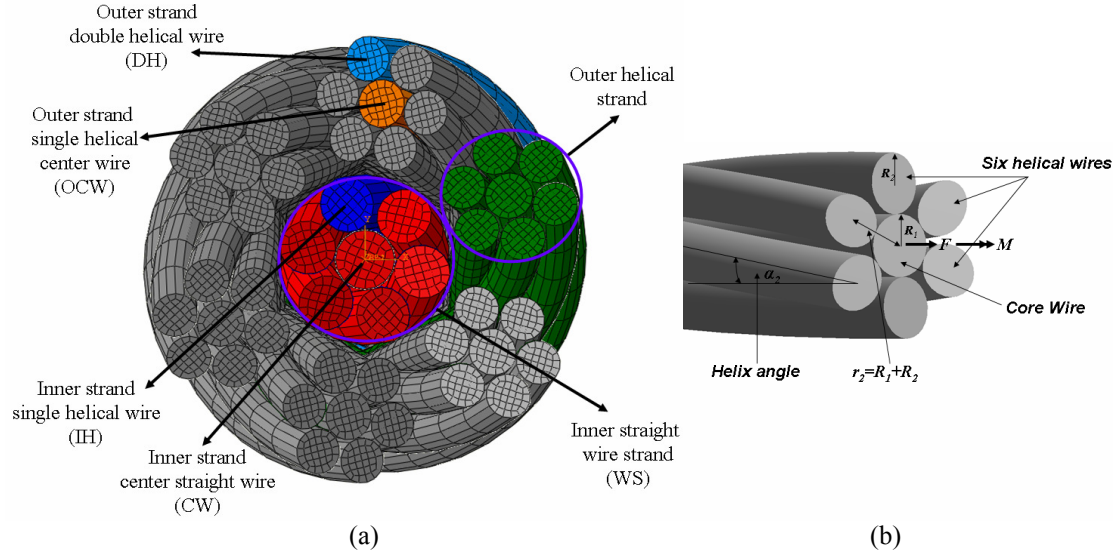


Figure 1. (a) (6x7) wire IWRC cross section, (b) (1+6) wire simple straight strand

A simple straight wire strand composed by a straight center wire of radius R_1 and it is surrounded by six single helical wires of radius R_2 with the helix angle α_2 as shown in Figure 1-(b). The helix angle α_2 is determined by $\tan \alpha_2 = p_2 / 2\pi r_2$, where p_2 is the pitch length of the single helical wires and $r_2 = R_1 + R_2$.

Outer double helical wire centerline of the IWRC can be obtained by using the parametric equations given in [9]. Each outer wire of the outer strand with a double helical geometry is separately modeled. To define the location of an outer single helix centerline, Cartesian coordinate system (x, y, z) is used with the Cartesian frame $\{e_x, e_y, e_z\}$ and the location along the centerline of a single helix is defined as,

$$x_s = r_s \cos(\theta_s), \quad y_s = r_s \sin(\theta_s), \quad z_s = r_s \tan(\alpha_s) \theta_s, \quad (1)$$

where e_z is the rope axis, r_s is the radius of the single helix, α_s is the single helix laying angle and θ_s is the angle showing the position of the wire within a strand which is given as $\theta_s = \theta_0 + \theta$. Free angle θ is used to define the location of the wire around the rope axis e_z , relative to e_x while single helix phase angle is defined by $\theta_0 = \theta_{(z=0)}$. The outer double helical wires are wound around a single helical wire by using the location along the centerline of a single helix given in equation (1) and the location of the double helices can be defined as [9],

$$\begin{aligned} x_d &= x_s(\theta_s) + r_d \cos(\theta_d) \cos(\theta_s) - r_d \sin(\theta_d) \sin(\theta_s) \sin(\alpha_s) \\ y_d &= y_s(\theta_s) + r_d \cos(\theta_d) \sin(\theta_s) + r_d \sin(\theta_d) \cos(\theta_s) \sin(\alpha_s) \\ z_d &= z_s - r_d \sin(\theta_d) \cos(\alpha_s) \end{aligned} \quad (2)$$

where r_d is the distance along the double helical wire centerline and single helical centerline and $\theta_d = \eta\theta_s + \theta_{d0}$ shows the rotation of the double helical wire around the single helical center wire of the outer strand, η is the construction parameter which shows the frequency of the wire along the helix length and θ_{d0} is the wire phase angle. The construction parameter η is a ratio of angle of double helical wire rotation to the angle of outer helical strand rotation. This ratio is dependent on the angles of both helices and computed as a constant value for the model. It is considered to be important in characterizing the rope structure, specifically the relationship between the wire and strand helices, and is called the "relative rotation". The relative rotation will be positive for lang lay ropes and negative for regular lay ropes. Centroidal axes of both strand and wire can be considered lying on right circular cylinders which can be developed into a plane as shown in Figure 2.

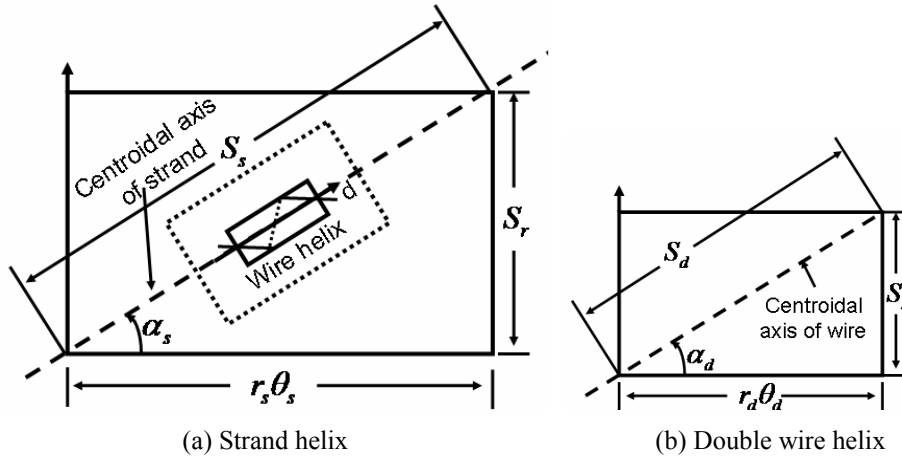


Figure 2: Developed view of strand helix and double wire helix

Using the developed view of the strand helix given in Figure 2-(a) the relationships between the length of rope S_r , length of strand S_s and the angle of strand rotation θ_s can be obtained as,

$$S_r = r_s \theta_s \tan \alpha_s, \quad (3)$$

$$S_s = \frac{r_s \theta_s}{\cos \alpha_s} \quad (4)$$

where r_s , θ_s and α_s shows radius of the strand, angle of the strand rotation and strand helix angle respectively [14]. Similarly, the relationships between the length of strand S_s , length of wire S_d and the angle of double helical wire rotation θ_d can also be obtained by using the developed view of the wire helix given in Figure 2-(b) as,

$$S_s = r_d \theta_d \tan \alpha_d, \quad (5)$$

$$S_d = \frac{r_d \theta_d}{\cos \alpha_d} \quad (6)$$

where r_d , θ_d and α_d shows radius of the wire helix, angle of the wire rotation and wire helix angle respectively. Because the length of strand obtained from the wire helix must equal that obtained from the strand helix for a given length of rope, a new term η is defined to be the ratio of the angle of double helical wire rotation θ_d to the angle of strand rotation θ_s , which can be obtained from equations (4) and (5) as [14],

$$\eta = \frac{\theta_d}{\theta_s} = \frac{r_s}{r_d \tan \alpha_d \cos \alpha_s}, \quad (7)$$

where η shows the ratio of the angle of double helical wire rotation to the angle of strand rotation. The ratio of η is important while modeling the double helical wires. This parameter computed and used as a constant parameter while modeling all the double helical wires along an outer strand. According to equation (2) a right lay double helix can be constructed. To construct a left lay double helix, it is enough to negate one of the coordinate values of x_d , y_d or z_d given in equation (2).

For the solid model generation issues using the preceding double helical geometry definition, Frenet-Serret frames constructed over the centerline of the double helical wire defined by equation (2). Using the Frenet-Serret frame a plane which is perpendicular to the centerline of the double helical geometry is defined. A circle is created over the generated plane which is designed to be perpendicular to the centerline of the double helical wire curve.

Both procedures for drawing the centerline of the double helical wire and the circle over a plane which is perpendicular to the centerline of the double helical wire are necessary to find control nodes. These control nodes are computed by a new generated algorithm and code with using Matlab® and written to a text file in an appropriate format. Then the control nodes are imported in Hypermesh® and a double helical centerline curve and a circle perpendicular to this curve is generated. A surface is defined over the created circle and this surface is swept along the centerline of the double helical wire to construct a meshed form of a double helical wire. This procedure is repeated for each wire. Center wire of the core strand, inner single helical wires and outer single helical wires can also build by similar generated codes and algorithms as well. All the generated meshed wires are imported in Abaqus/CAE® and assembled to compose IWRC. Even if these procedures seem to be a little bit complicated there is no other way to produce a good double helical geometry which can be used for analysis purposes. Also the advantage of this scheme is that the model has no length limitations because of the meshed form of the generated solid wires. This scheme can be adopted to generate a double helical wire using other solid generating commercial CAD applications but the produced solid parts include problematic surfaces and meshing process fails depending on the length of the model. The proposed scheme solves the meshing problems of this complicated geometry and helps to compose ready to analyze geometries by applying necessary boundary conditions. The meshed IWRCs generated by the proposed scheme for different lay types are presented in Figure 3.

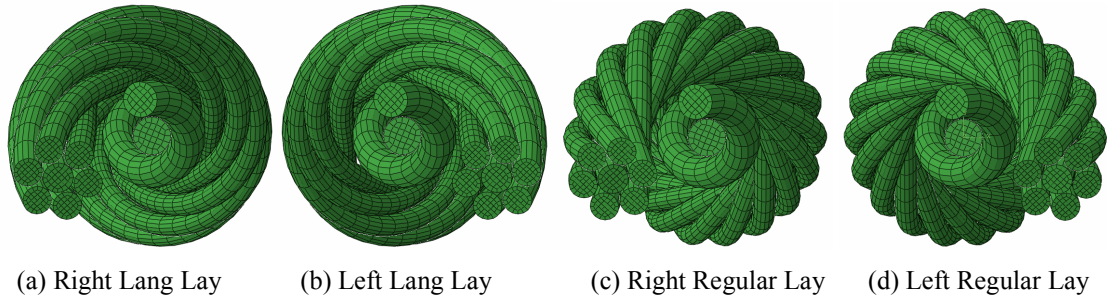


Figure 3: Meshed IWRC model with different lay types;

3. NUMERICAL MODELS AND SOLUTIONS

Modeling parameters for both simple straight strand and IWRC are prescribed in Table 1 and abbreviations are defined in Figure 1-(a). The wire material properties used in the elastic-plastic finite element analysis (FEA) are defined as; Young's modulus $E=188000 \text{ N/mm}^2$, plastic modulus $E_p=24600 \text{ N/mm}^2$, yield stress $R_{p0.2}=1540 \text{ N/mm}^2$, ultimate tensile stress $R_m=1800 \text{ N/mm}^2$, Poisson's ratio $\nu=0.3$ and friction coefficient $\mu=0.115$ [11]. Numerical models are created by the proposed scheme as described and solved using the finite element code Abaqus/CAE®. FEA results are compared with the available test and analytical results.

Table 1: Design parameters of a simple straight wire strand (WS) and IWRC

Simple straight strand		IWRC			
Parameter	Value	Parameter	Value	Parameter	Value
Strand radius	5.7mm	IWRC radius	29.80mm	Pitch length of IH, p_2	70mm
CW radius, R_1	1.97mm	CW radius, R_1	1.97mm	Pitch length of OCW, p_2^*	193mm
IH radius, R_2	1.865mm	IH radius, R_2	1.865mm	Pitch length of DH, p_4	70mm
Strand length, h	14mm	OCW radius, R_3	1.6mm	Helix angle of IH, α_2	71.01°
Pitch length, p	115mm	DH radius, R_4	1.5mm	Helix angle of OCW, α_2^*	71.46°
Helix angle, α	78.2°	IWRC length, h	18mm	Helix angle of DH, α_4	74.45°

The values of elastic and elasto-plastic reaction forces were calculated by finite element method, therefore Abaqus/CAE® software that is a general-purpose finite element code was used for solving wire rope problem. Throughout the finite element analysis solid brick elements are used. The solid (or continuum) elements can be used for linear analysis and for complex nonlinear analyses involving contact, plasticity, and large deformations. They are available for stress, heat transfer, acoustic, coupled thermal-stress, coupled pore fluid-stress, piezoelectric, and coupled thermal-electrical analyses. Element type of C3D20 was chosen as a 20-node quadratic brick element that is used for 3-D modeling of solid structures. Element type C3D20 has 27 integration points as shown in Figure 4. The element is defined by 20 nodes having three degrees of freedom at each node: translations in the nodal x, y and z directions. Isotropic material behavior is defined and material directions correspond to the element coordinate directions. The meshed model has total number of 92410 nodes, 17922 quadratic hexahedral elements of type C3D20 [15,16].

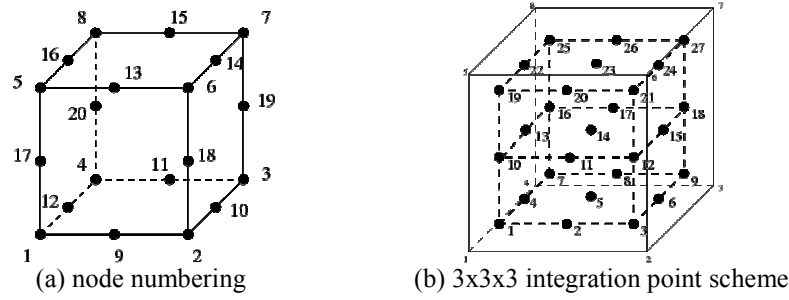


Figure 4: C3D20 type 20-node brick element node numbering and integration points

3.1. Validity of the proposed model of a simple straight strand

As a benchmark problem a wire strand is modeled and FEA results are compared with both analytical results of Costello [4] and the test results reported by Utting&Jones [17,18]. Numerical model without friction and with friction are developed over a simple straight strand respectively. Axial loading behavior is investigated by applying an axial strain ε of 0.015 to the free end of the strand with the increments of 0.001 while the other end of the strand is constraint to be fixed. Meanwhile the free end of the strand is restrained for zero rotation. The proposed numerical model is solved using FEA and the variation of axial force with the axial strain is presented in Figure 5.

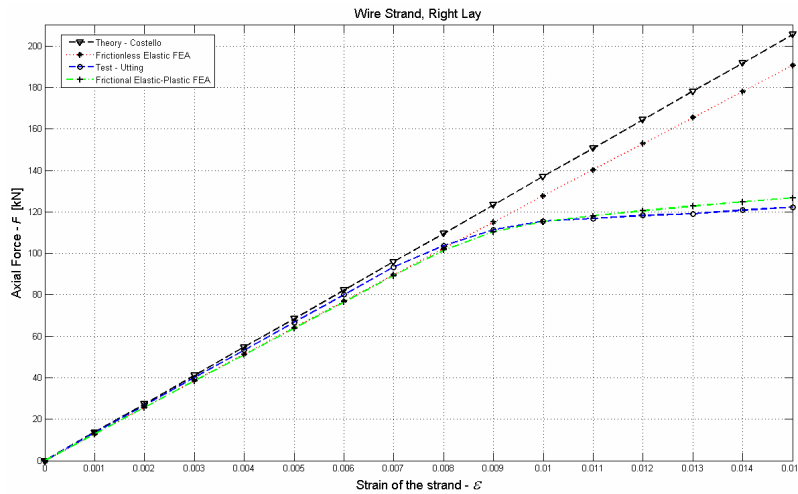


Figure 5: Variation of axial force with axial strain for the simple straight strand

It can be easily seen from this figure that the proposed numerical model solution with FEA is in good agreement with both the theory of Costello [4] and test results reported by Utting & Jones [17,18]. Plastic behavior of the material is also shows good agreement with the reported test results and this identity of the solution proofs the validity of the proposed numerical model.

3.2. Wire by wire elastic-plastic FEA of an IWRC

An 18mm length (6x7) wire right Lang lay IWRC is modeled using the double helical geometry with the developed modeling scheme proposed in this paper. The meshed model of the IWRC is solved using the prescribed boundary conditions. Fixed end of the model is constraint to be encastre boundary condition while the free end of

the model is restrained for zero rotation. An axial strain ε of 0.015 is applied to the free end of the model with the increments of 0.001. The proposed numerical model is solved using FEA and the variation of axial force with the axial strain is presented in Figure 6.

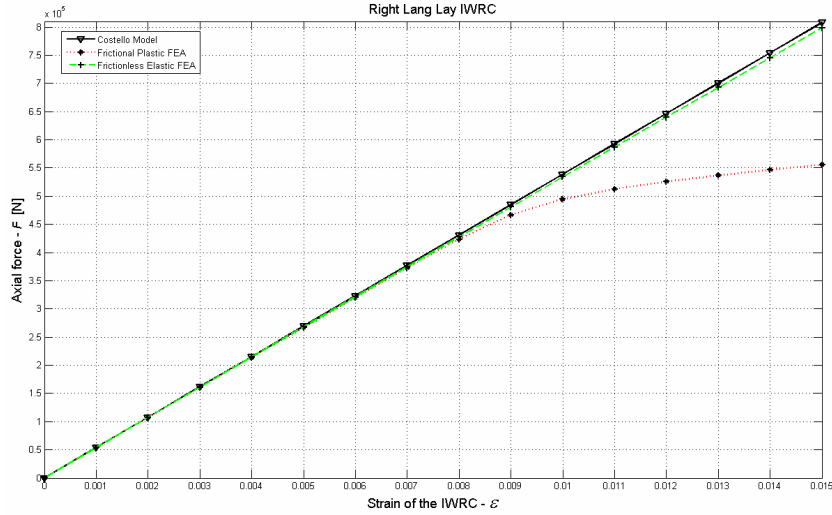


Figure 6: Variation of axial force with axial strain for RLL IWRC

It can be easily seen from the figure that the proposed numerical model results with FEA are in good agreement with the theory of Costello-Velinsky models [4,5] in elastic area and the plasticity results show similar perception as simple straight strand presented in Figure 5.

Variations of the axial force with the axial strain results are presented with wire by wire based insights for a right Lang lay IWRC in Figure 7. Core strand and outer strand wires are titled and shown in the figure and described in Table 1.

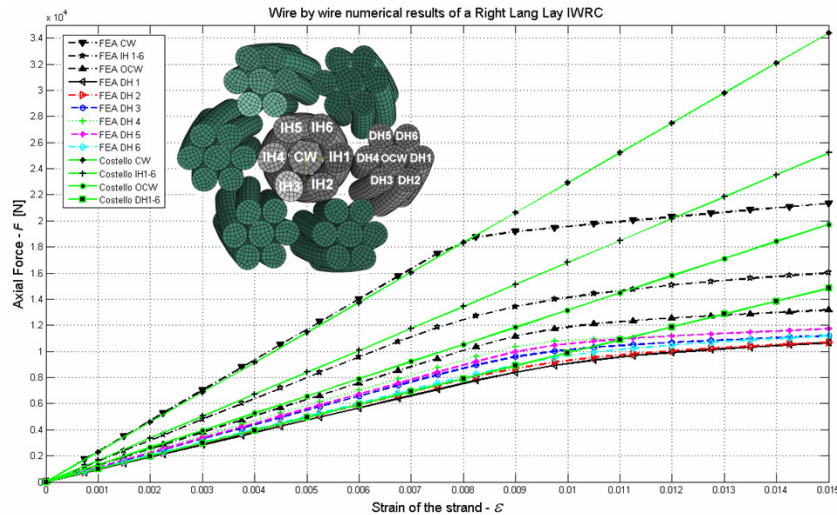


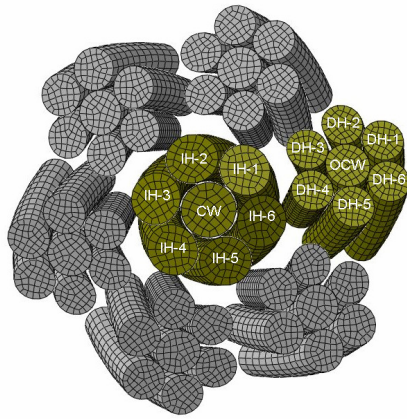
Figure 7: Wire by wire analysis, theory and FEA comparison of a RLL IWRC

The theoretical results of Costello-Velinsky model and FEA results show good agreement among the elastic area. The elastic behavior ends when the value of the

applied axial strain ε reached to 0.008 and then the plastic behavior of the model exists after 0.008 axial strains as in Figure 7.

A wire by wire load distribution is investigated by the analysis of a right Lang lay IWRC and load percentages for each wire are listed in Table 2. When the loads are sorted according to their percentages, center wire of the core strand carries the major portion of axial load. Load distribution over the outer strand is calculated from wire by wire analysis. It can be seen that double helical wires (labeled as DH3, DH4, and DH5) close to the core strand carries higher amount of the outer strand's axial load. If the axial load of the core strand is compared with the outer strand of the IWRC, core strand carries an average 20.18% while one of the outer strands carries an average 13.30% of the total axial load.

Table 2: Wire by wire axial load distribution of the IWRC



Wire	Load distribution
Core Strand	20.18%
CW	3.86%
1 of 6 IH	2.72%
1 of 6 outer strands	13.30%
OCW	2.21%
DH 1	1.69%
DH 2	1.74%
DH 3	1.90%
DH 4	2.02%
DH 5	1.96%
DH 6	1.79%
Total wires =	100.00 %

To get cost effective wire rope, the optimization of wire composition in the rope can be valuable issue. Therefore the load distribution presented in this paper can be used for optimization of the wire rope.

4. CONCLUDING REMARKS

A wire rope model generation is accomplished by using the double helical geometry proposed in this paper. Generated model of a simple straight strand shows good agreement with the available test and theoretical results for both frictionless and with frictional effects respectively. Furthermore a complete independent wire rope core is analyzed by means of wire by wire scheme. The results for IWRC are compared with those obtained by theoretical ones. The numerical results show reasonable agreement with other available results, with a simpler and more practical approach. Also the proposed modeling issue gives future opportunity to analyze more complicated problems such as bending over a sheave.

Acknowledgements-We are grateful to Professor M. Pakdemirli for many stimulating correspondences. The authors would like to thank the referee for valuable comments.

5. REFERENCES

1. A. E. H. Love, *A treatise on the mathematical theory of elasticity*. 4th ed., New York: Dover Publications, Chapter XVIII-XIX, pp. 381-426, 1944.
2. S. A. Velinsky, On the design of wire rope, *Transactions of the ASME, Journal of Mechanics, Transmissions, and Automation in Design*, **111**, 382-388, 1989.
3. G. A. Costello and S. K. Sinha, Static Behaviour of Wire Rope, *Proceedings ASCE, Journal of Engineering Mechanical Division*, **103**(No.EM6):1011-1022, 1977.
4. G. A. Costello, *Theory of wire rope*. Berlin: Springer, 1990.
5. S. A. Velinsky, G. L. Anderson, G. A. Costello, Wire rope with complex cross sections, *Journal of Engineering Mechanics*, **110**(3):380-391, 1984.
6. S. A. Velinsky, General nonlinear theory for complex wire ropes, *International Journal of Mechanical Science*, **27**:497-507, 1985.
7. C. Jolicoeur and A. Cardou, A numerical Comparison of current mathematical models of Twisted wire cables under axisymmetric loads, *Journal of Energy Resources Technology*, **113**:241-249, 1991.
8. J.W. Phillips and G. A. Costello, Analysis of wire ropes with internal-wire-rope cores, *Transactions of the ASME*, **52**: 510-516, 1985.
9. D. Elata et.al., The mechanical behavior of a wire rope with an independent wire rope core. *International Journal of Solids and Structures* **41**:1157-1172, 2004.
10. H. Usabiaga and J. M. Pagalday, Analytical procedure for modelling recursively and wire by wire stranded ropes subjected to traction and torsion loads, *International Journal of Solids and Structures*, doi:10.1016/j.ijsolstr.2008.04.009, 2008.
11. W. G. Jiang, J. L. Henshall, The analysis of termination effects in wire strand using finite element method. *Journal of Strain Analysis* **34**:1, 31-38, 1999.
12. W. G. Jiang, M. S. Yao and J. M. Walton, A concise finite element model for simple straight wire rope strand. *Int. Journal of Mechanical Sciences*, **41**:143-61, 1999.
13. W. G. Jiang et.al., A concise finite element model for three-layered straight wire rope strand. *International Journal of Mechanical Sciences*, **42**:63-86, 2000.
14. R. C. Wang, A. J. Miscoe and W. M. McKewan, Model for the Structure of Round-Strand Wire Ropes, *U.S. Department of Health and Human Services, Public Health Service, Centers for Disease Control and Prevention, National Institute for Occupational Safety and Health, DHHS (NIOSH)*, Publication No. 98-148, Report of Investigations **9644**:1-19, 1998.
15. Abaqus Version 6.8 Documentation, Abaqus Analysis User's Manuel (v6.8), Elements, Solid (continuum) elements, Dassault Systemes, 2008
16. F. Sen and M. Sayer, Elasto-plastic thermal stress analysis in a thermoplastic composite disc under uniform temperature using FEM, *Mathematical and Computational Applications* **11**(1):31-39, 2006.
17. W. S. Utting and N. Jones, The response of wire rope strands to axial tensile loads: Part I. Experimental results and theoretical predictions. *International Journal of Mechanical Science*, **29**(9): 605-619, 1987.
18. W. S. Utting, N. Jones, The response of wire rope strands to axial tensile loads: Part II. Experimental results and theoretical predictions, *International Journal of Mechanical Science*, **29**(9):621-636, 1987.