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# A STUDY FOR BORONIZING PROCESS WITHIN NONEXTENSIVE THERMOSTATISTICS

Özhan Kayacan[1], Salim Şahin[2], Filiz Taştan[2]

1. Department of Physics, Celal Bayar University, 45140 Muradiye, Manisa, Turkey, ozhan.kayacan@bayar.edu.tr
2. Department of Mechanical Engineering, Celal Bayar University, 45140 Muradiye, Manisa, Turkey

**Abstract-** In this study, diffusion mechanism of the boronizing process of AISI 1040 has been investigated. A nonlinear diffusion equation, which was proposed earlier, has been employed and compared with the experimental data. An interesting property of the diffusion equation used here is that it establishes a connection between the diffusion process and nonextensivity. The theoretical model also exhibits a possible connection between the exponents appeared in the differential equation and the entropic index. In this manner, the possible effects of nonextensivity on the boronizing process have been shown theoretically. We believe that a diffusion equation based on the nonextensive formalism is first applied to the boronizing process in literature.

**Keywords** *:* Boronizing, Tsallis formalism, Nonlinear diffusion

## 1. INTRODUCTION

The surface of industrial component may require treatment to enhance the surface characteristic. A number of different surface hardening process are commonly applied to metals in order to increase their surface performances. In general, two methods for that are known: (i) diffusing of small atoms on the metal surface leading to formation of an interstitial solid solution, (ii) a chemical reaction between the diffused atoms and those of basic metal forming of new compounds in the superficial layer [1,2].

Boronizing, is a thermochemical treatment that diffuses boron through the surface of metallic substrates. As boron is an element of relatively small size it diffuses into a variety of metals; including ferrous, nickel and cobalt alloys, metal-bonded carbides and most refractory alloys [3].

The introduced boron atoms react with the material and form a number of borides. According to the Iron–Boron equilibrium diagram, diffusing boron into the iron crystalline lattice leads to the formation of two kinds of iron borides (FeB and $Fe_2B$) [4].

Boronizing of ferrous materials is generally performed at temperatures ranging from 840 to 1050 $^{\circ}$C. The process can be carried out in solid, liquid or gaseous medium [5]. The powder-pack boronizing has the advantages of simplicity and cost-effectiveness in comparison with other boronizing processes. Metallic borides have relatively high hardness values ranging from 1600 to 2100HV. The hardness achieved by boronizing increases the resistance to abrasive wear [6].

Boronized layers have good tribological characteristics at high temperatures [7]. Very low coefficient of friction and high hardness are obtained on boronized surfaces. Since boron is reactive to oxygen, borides have thin oxide films on their surfaces, which lower the coefficient of friction. Dimensional increase is also observed on the surface of boronized materials [8,9].

In literature, there is growing interest in generalizations of nonlinear diffusion equation. These have been employed to study the physical system covering many fields. For example, the differential equation

$$\frac{\partial}{\partial t} P(x,t) = D \nabla^2 \left[P(x,t)\right]^{\nu} \quad (1)$$

has been applied in turbulent diffusion [10], percolation of gases through porous media [11], nonlinear diffusion in superconductors [12], etc. In Ref.[13], some mathematical properties of Eq.(1) has been investigated and the authors have established a connection between the diffusion equation given by Eq.(1) and Tsallis formalism [14]. In addition, this type of diffusion equation has been extensively studied; considering the diffusion coefficient to be spatial time-dependent, the solution of the diffusion equation is studied [15]; in Ref.[16], it is shown that the nonextensivity is consistent with the second law of thermodynamics when involved in nonlinear diffusion processes; in Ref.[17], the anomalous diffusion asociated with a nonlinear fractional Fokker-Planck equation with a diffusion coefficient $D \propto |x|^{-\theta}$ $\theta \in \Re$ . Other applications of Eq.(1) can be found in Ref.[18].

In this study, we apply a diffusion equation, which is based on Eq.(1) and has been proposed elsewhere [19], to the boronizing process. In doing so, it is established a connection between boronizing and Tsallis formalism. We believe that the nonlinear diffusion equation is first applied to the boronizing process, taking into account this possible connection. We also show that the generalized version of Eq.(1) could be useful to study the boronizing process. We, first of all, begin to summarize the model which was proposed in [19].

## 2. EXPERIMENTAL PROCEDURE

In this study AISI 1040 steel was chosen as substrate. AISI 1020 steel is widely used in different areas of machine constructions. The chemical composition of material used in the experiments was given in Table 1. For boronizing treatment, the samples were shaped in a cylindrical form with a diameter of 12mm and a length of 5mm. Boronizing was performed in a solid medium consisting of Ekabor I powders at 900°C for 2, 4 or 6h. The samples to be boronized were put in a steel box with Ekabor I powders and then placed in an electrical resistance furnace. The borided specimens were polished before the microstructure analysis to determine boride layer thickness. Optic microscope and X-Ray diffraction was used to characterization of boride layer. The thickness of the boride layers was measured through Nikon LV100 optical microscopy with the aid of Clemex Software.

**Table 1**. The chemical compositions of AISI 1040 steel

| Elements (Wt. %) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **C** | **Si** | **Mn** | **P** | **S** | **Cr** | **Ni** | **Mo** | **Al** | **Cu** | **Ti** | **V** | **Pb** | **Fe** |
| 0,41 | 0,22 | 0,82 | 0,018 | 0,04 | 0,02 | 0,07 | 0,02 | <0,01 | 0,06 | <0,01 | <0,01 | <0,001 | 98,3 |

### 3. THE MATHEMATICAL MODEL

An extension of Eq.(1) can be given by

$$\frac{\partial}{\partial t}\rho(x,t) = D\frac{\partial}{\partial x}\left\{\left(\rho(x,t)\right)^m\left|\frac{\partial}{\partial x}\rho(x,t)\right|^n \frac{\partial}{\partial x}\rho(x,t)\right\}, \tag{2}$$

which is based on the nonlinear Fourier law [20]. This equation reduces to Eq.(1) for suitable choices of parameters appeared in Eq.(2). Eq.(2) is called “Gorter-Melling law” and has been employed in nonlinear heat conduction [20], nonlinear flows of non-Newtonian fluids [21], etc. The properties of Eq.(2) have been analyzed in Ref.[19] in the presence of external forces with an absorbent term. Considering these terms, Eq.(2) can be extended

$$\frac{\partial}{\partial t}\rho(x,t) = D\frac{\partial}{\partial x}\left\{|x|^{-\theta}\left|\frac{\partial}{\partial x}\rho(x,t)\right|^n \frac{\partial}{\partial x}\left(\rho(x,t)\right)^{\nu}\right\} - \frac{\partial}{\partial x}\{F(x,t)\rho(x,t)\} - \bar{\alpha}(t)[\rho(x,t)]^{\mu}, \tag{3}$$

where $D(t)$ is diffusion coefficient, $F(x,t)$ stands for an external forces and $\bar{\alpha}(t)$ represents the absorbent term. Eq.(3) can be applied to describe some physical phenomena such as turbulent diffusion [10], percolation of gases through porous media [11], nonlinear diffusion in superconductors [12], etc. It is worth to note that one of aims of this paper is to extent the area of these applications, to the boronizing process.

By using similarity methods, Eq.(3) can be reduced to the ordinary differential equation. Since the explicit form of this ordinary differential equation depends on the boundary conditions, as done in [19], we also restrict the analysis to find a solution, expressed as a scaled function of the type

$$\bar{\rho}(x,t) = \frac{1}{\Phi(t)}\tilde{\rho}\left[\frac{x}{\Phi(t)}\right]. \tag{4}$$

These solutions, given by Eq.(4), satisfy the initial and boundary conditions, and also normalization condition when $\bar{\alpha}(t) = 0$. By following the way described in Ref.[19], the solution of Eq.(3) in the absence of external field can be given by

$$\bar{\rho}(x,t) = \frac{1}{\Phi(t)}\left\{1-(1-q)\left[-\frac{1}{2+\theta+n}\left(\frac{\bar{k}}{\nu}\right)^{1/(n+1)}\left(\frac{|x|}{\Phi(t)}\right)^{\left(\frac{2+\theta+n}{n+1}\right)}\right]^{n+1}\right\}^{1/(1-q)}. \tag{5}$$

In Eq.(5), if we use the $q$-exponential function, $\exp_q(x) = \left(1+(1-q)x\right)^{1/(1-q)}$, then Eq.(5) reduces to

$$\bar{\rho}(x,t) = \frac{1}{\Phi(t)}\exp_q\left\{-\left[-\frac{1}{2+\theta+n}\left(\frac{\bar{k}}{\nu}\right)^{1/(n+1)}\left(\frac{|x|}{\Phi(t)}\right)^{\left(\frac{2+\theta+n}{n+1}\right)}\right]^{n+1}\right\}, \tag{6}$$

where

$$\alpha = \left(\frac{\bar{k}}{\nu}\right)^{1/(n+1)} \left(\frac{\nu + n - 1}{2 + \theta + n}\right), \beta = \left(\frac{n+1}{\nu + n - 1}\right), \lambda = \left(\frac{2 + \theta + n}{n+1}\right).$$

It is interesting to note that the $q$-exponential function appeared in Eq.(6) emerges from the Tsallis formalism. Here the entropic index $q$ is given by $q = 2 - (\nu + n)$. The solution, Eq.(6), has the property that depending on the parameters $\nu$ , $n$ and $\theta$ appeared in Eq.(5), it can show a compact or a long tail character. This property seems to be very important, because our experimental results also exhibit this kind of behavior.

## 4. RESULTS AND DISCUSSION

$Fe_2B$ and FeB compounds are obtained during the boronizing treatment while boron atoms are diffusing to the metal surface. Boride layer thickness formed on boronized AISI 1040 was given in Fig.(1). The highest boride layer thickness was observed for 6h (hours) followed by 4h and 2h respectively. Thickness of borided layer of AISI 1040 steel ranged from 42 µm to 115 µm depending on the boronizing time. According to the results, the longer the boronizing time the thicker the boride layer was. The morphology of borided AISI 1040 at 900oC for different time obtained by optical microscopy are shown in Fig.(2). Boride layer have dentricular morphology. There different region can be identified on the cross section of borided layer. These are i) FeB Phase, ii) $Fe_2B$ phase and iii) diffusion zone formed between the boride layer and substrate.

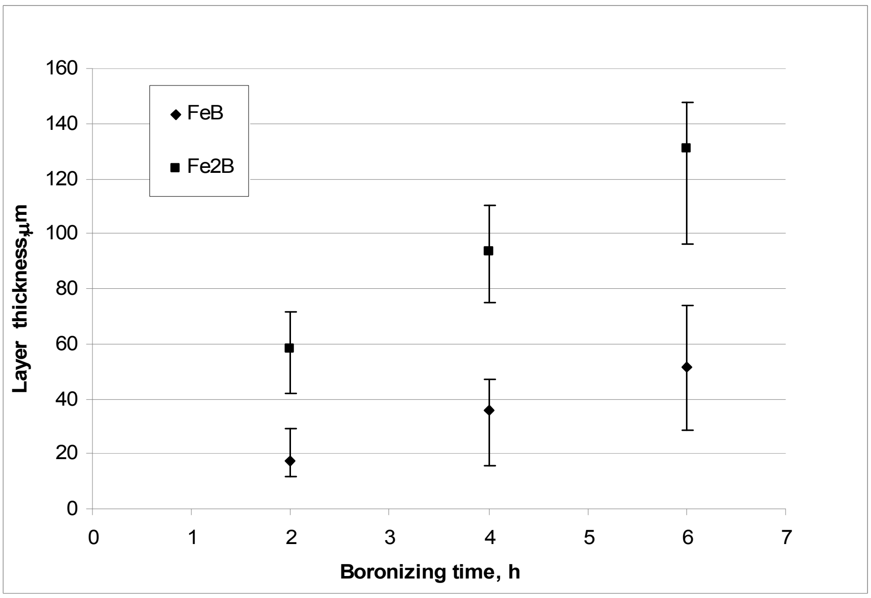


**Figure 1**. Variation of layer thickness as a function of boronizing time for AISI 1040.

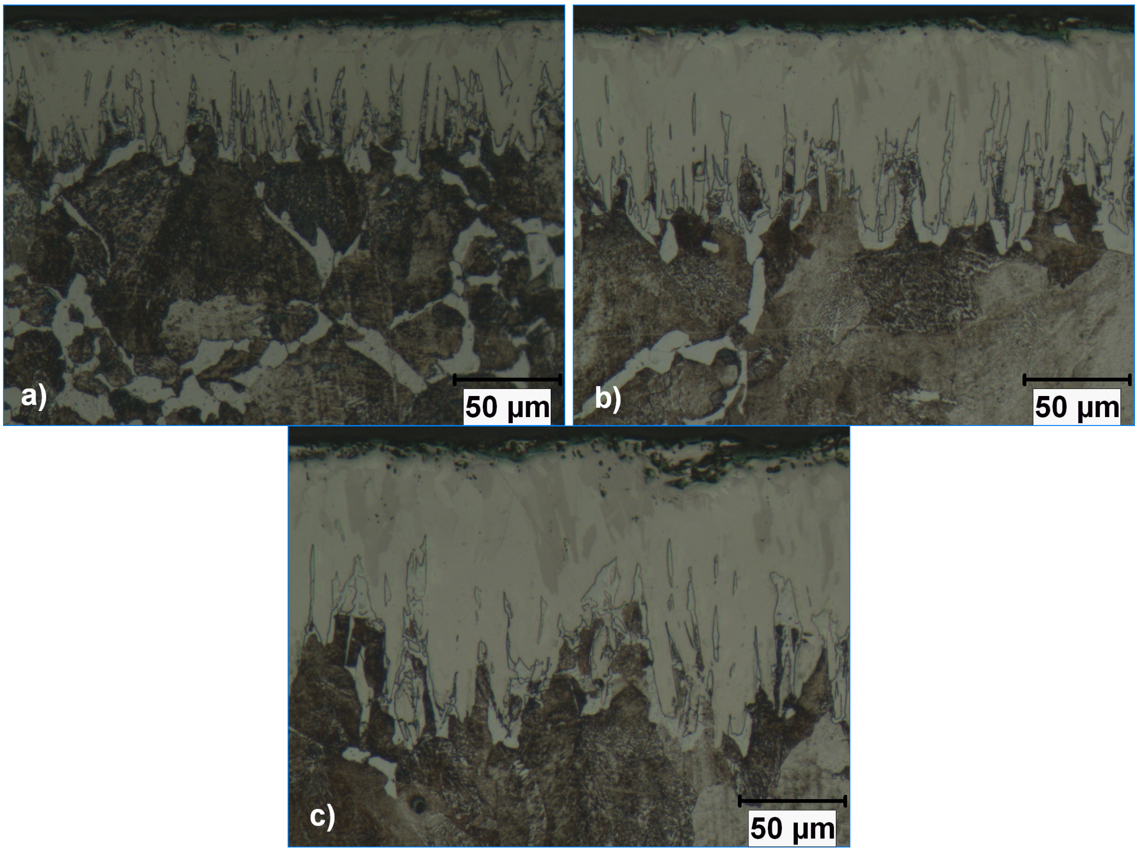


**Figure 2.** Cross-sectional views of borided AISI 1040 at 900 oC a)2h, b)4h, c)6h.

The phases, obtained by boronizing at AISI 1040 surface were determined by XRD analysis for different deepness shown in Fig.(3). From the XRD result, only FeB phase was determined at 20µm depth. However, FeB and $Fe_2B$ phases were determined at 50µm or 80µm depth. $Fe_2B$ phase and Fe Substrate were seen at 120µm depth. These data are in line with results seen in microstructure photo.

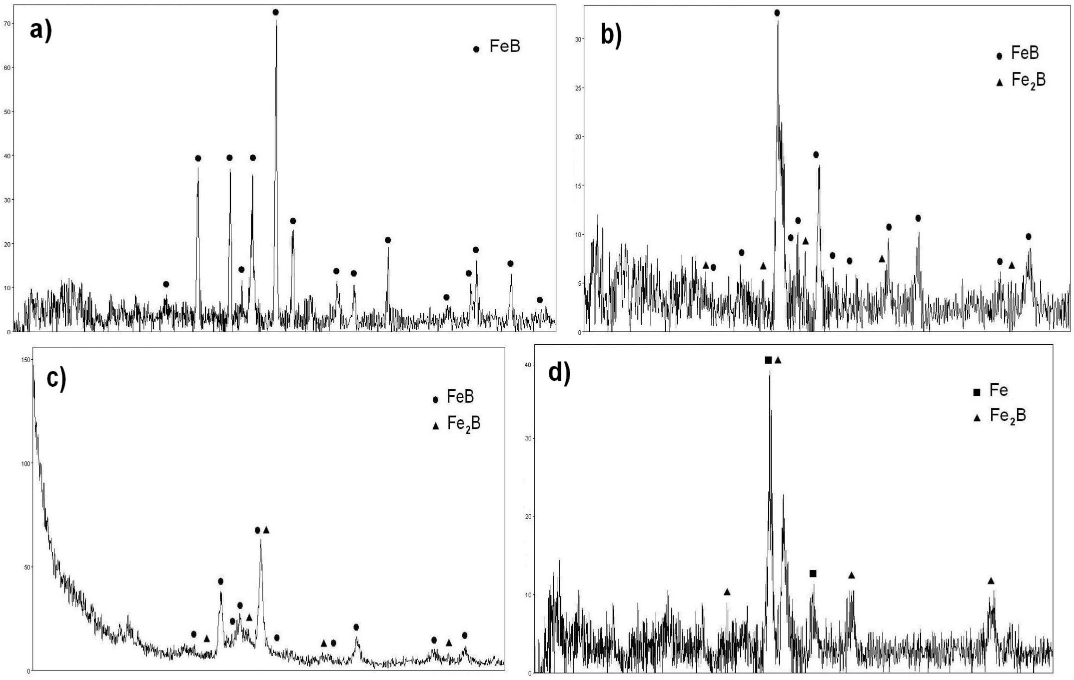


**Figure 3.** XRD Patterns of AISI 1040 at 900 oC, 6 h for different deepness from surface; a) 20µm, b) 50µm, c) 80µm , d) 120µm.

The boron concentration distribution of boronized materials surface is illustrated depending on the formed boron phase (Fig.4). In Fig.(5), the boron concentration depending on the depth from surface for AISI 1040 is shown.

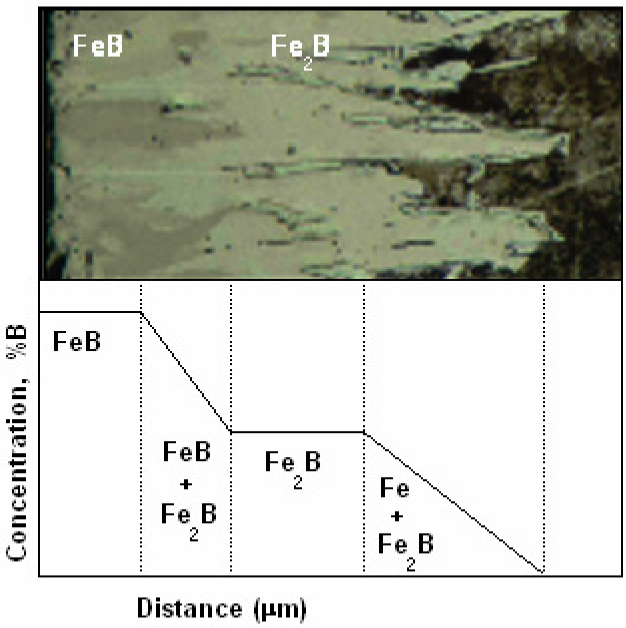


**Figure 4**. Schematic boron concentration profile from experimental data for two phase boride layer.

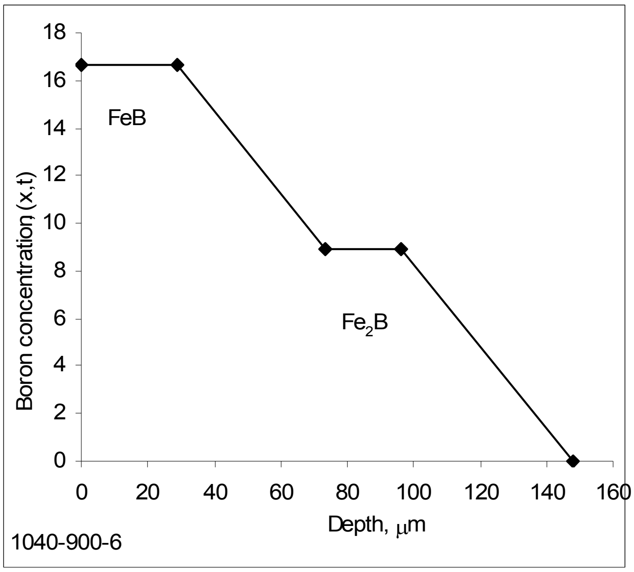


**Figure 5.** The schematic illustration of boron concentration depending on the depth from surface for AISI 1040.

Theoretical results are illustrated in Figs.(6-8) with our experimental data. In all figures, the absence of external force is considered. As the solution Eq.(6) has the *q*-exponential function, it may be concluded that the solution indicates a possible connection between the boronizing process and nonextensive statistics. This could be important and helpful in a deeper connection between the boronizing process and Tsallis formalism. This possible connection was considered elsewhere [19]. It is also interesting to note that the parameter $q$, called "the entropic index", is a function of parameters $\nu$ , $n$ and $\theta$ of the nonlinear differential equation. The solution function has some typical behaviors, depending on the parameters. For example, for $0 < (1+\theta+\nu+2n)/(1-\nu-n) < 2$ the solution of the diffusion equation behaves asymptotically like a Lévy distribution. Another example is that if diffusion coefficient is constant, $D(t)=D$, and the parameters $\nu$, $n$ and $\theta$ is chosen appropriately, we may have an anomalous spreading of the distribution and this situation may describe an anomalous diffusion. These kinds of behaviors have many advantages to study the physical systems and this feature is one of the reasons that we choose here this mathematical model to study on the boronizing process.

In Figs.(6-8), the solution of diffusion equation is fitted to the experimental results, using a subroutine written in Fortran. In these figures, there are two regions, shown as $FeB$ and $Fe_2B$. For these regions, we apply different boundary conditions and then fit the solution to the experimental results. The fitted parameter values are shown on these figures. The diffusion coefficients obtained by fitting process are consistent with the ones obtained elsewhere [22]. As can be seen from Figs.(6-8), there is a good agreement between the theory and the experiment.

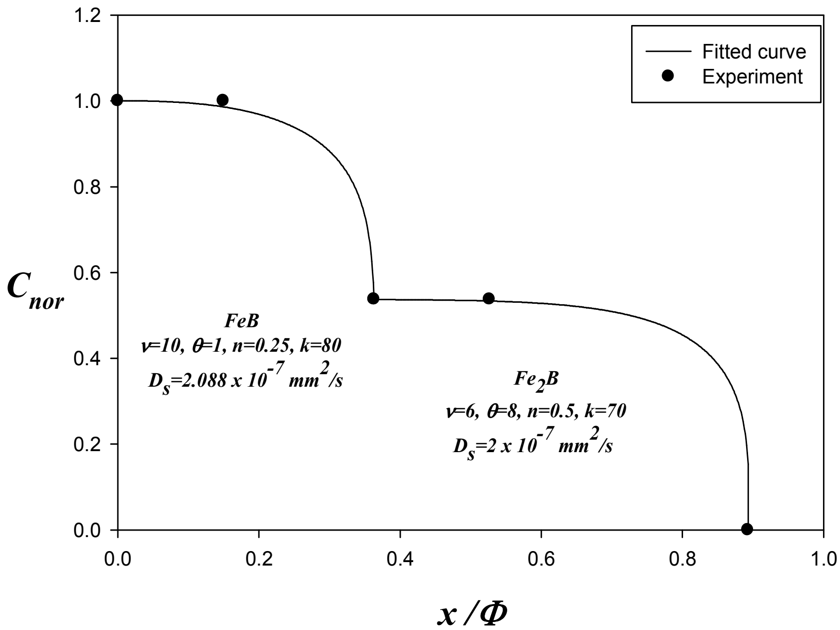


**Figure 6**. The theoretical results of 1040-900-2. The filled circles indicates the experimental results. As seen, the theoretical results and the experiment are in good agreement. The fitted parameter values are shown on the corresponding regions.

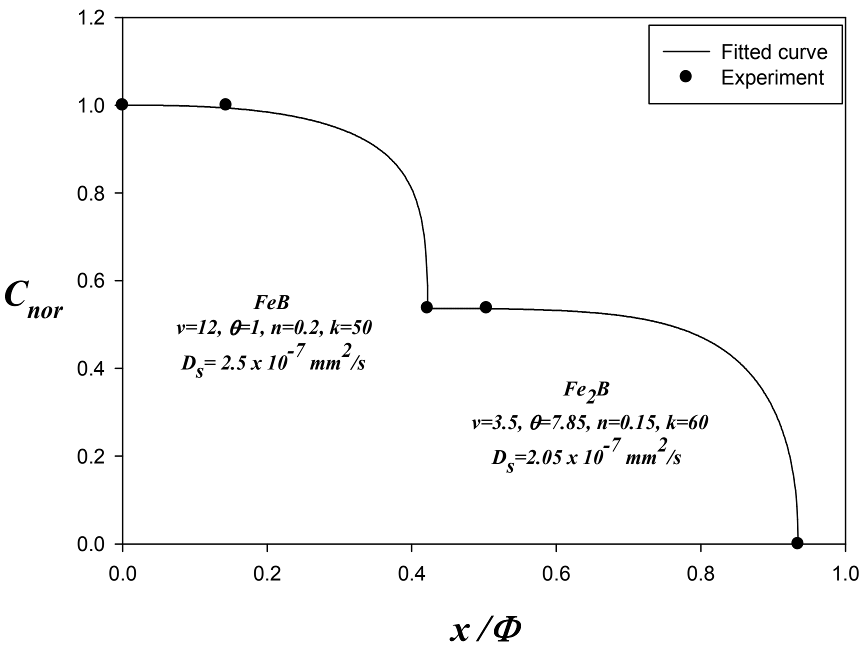


**Figure 7**. The theoretical results of 1040-900-4. The filled circles indicates the experimental results. The theoretical results and the experiment are in good agreement. The fitted parameter values are shown on the corresponding regions.

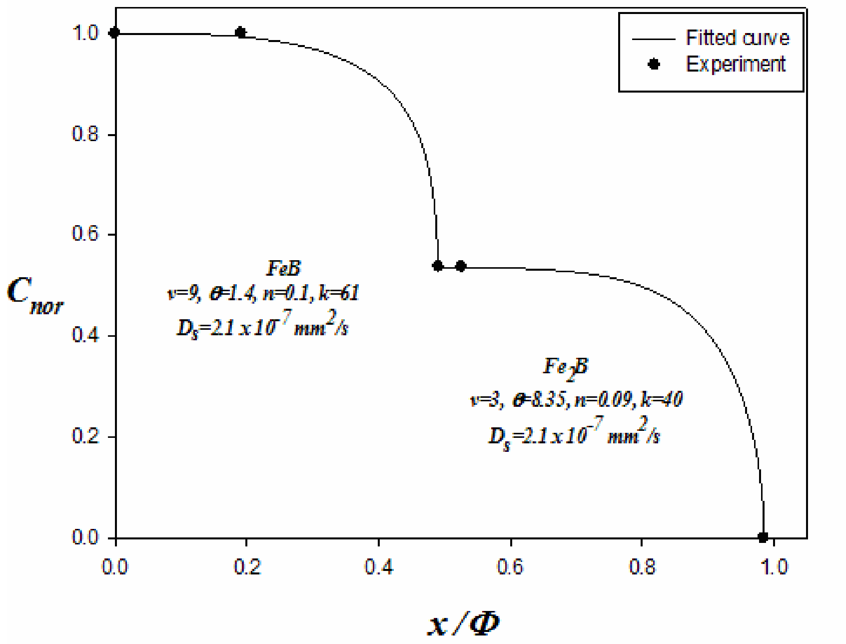


**Figure 8**. The theoretical results of 1040-900-6. The filled circles indicates the experimental results. The theoretical results and the experiment are in good agreement. The fitted parameter values are shown on the corresponding regions.

## 5. CONCLUSION

In this study, we have worked on the boronizing process by using a nonlinear diffusion equation and aimed to study the boronizing process within nonextensive formalism. In this direction, a generalized diffusion equation which was first presented in Ref.[19] has been employed and the obtained results have been compared with the our experimental results. It is interesting to note that the $q$-exponential function of Tsallis statistics appears in the solution of the nonlinear diffusion equation. According to the obtained results, it can be concluded that there is a possible connection between the boronizing process and nonextensive thermostatistics. This connection provides a relation between the entropic index $q$ and the parameters appearing in the nonlinear diffusion equation. This connection could be also useful to understand the boronizing process and the obtained results here could bring some new aspects to the boronizing process. In particular, it can be emphasized that the solution of nonlinear diffusion equation can exhibit a compact or a long tail character. Finally, we hope that the results obtained here may be expanded to the extensional studies of boronizing process in future.

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