Mathematical and Computational Applications, Vol. 14, No. 2, pp. 85-96, 2009.

HOMOTOPY PERTURBATION METHOD FOR SOLVING HUMAN T-CELL LYMPHOTROPIC VIRUS I(HTLV-I) INFECTION OF CD4$^+$ T-CELLS MODEL

Mehmet Merdan
Karadeniz Technical University, Engineering Faculty of Gümüshane
Civil Engineering, 29000, Gümüshane, Turkey. merdan@ktu.edu.tr

Abstract-In this article, homotopy perturbation method is implemented to give approximate and analytical solutions of nonlinear ordinary differential equation systems such as human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells model. The proposed scheme is based on homotopy perturbation method (HPM), Laplace transform and Padé approximants. The results to get the homotopy perturbation method (HPM) is applied Padé approximants. Our proposed approach showed results to analytical solutions of nonlinear ordinary differential equation systems. Some plots are presented to show the reliability and simplicity of the methods.

Keywords- Padé approximants; Homotopy perturbation method; human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells model

1. INTRODUCTION

Dynamics of human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells is examined [3] at the study. The components of the basic four-component model are the concentration of healthy CD4+ T-cells at time t, the concentration of latently infected CD4+ T-cells, the concentration of actively infected CD4+ T-cells and the concentration of leukemic cells at time t are denoted respectively by T(t), $T_L(t)$, $T_A(t)$ and $T_M(t)$ .These quantities satisfy

$$\begin{cases} \dfrac{dT}{dt} = \lambda - \mu_T T - kT_A T \\ \dfrac{dT_L}{dt} = kT_A T - (\mu_L + \alpha)T_L \\ \dfrac{dT_A}{dt} = \alpha T_L - (\mu_A + \rho)T_A \\ \dfrac{dT_M}{dt} = \rho T_A + \beta T_M \left(1 - \dfrac{T_M}{T_{Max}}\right) - \mu_M T_M \end{cases} \tag{1}$$

with the initial conditions:
$T(0) = P_1$, $T_L(0) = P_2$, $T_A(0) = P_3$, $T_M(0) = P_4$. Throughout this paper, we set $\mu_T = 0.66, \mu_L = 0.06, \mu_A = 0.05, \mu_M = 0.005$, $k = 0.5, \alpha = 0.004, \beta = 0.0003, \rho = 0.00004$ and $T_{\max} = 2200$

A technique for calculating the analytical solutions of nonlinear ordinary differential equation systems is developed in this paper. The developed technique depends only on the fundamental operation properties of Laplace transform and Padé approximants. The calculated results are exactly the same as those obtained by other analytical or approximate methods and demonstrate the reliability and efficiency of the

technique. We will use Laplace transform and Pade´ approximant to deal with the truncated series. Pade´ approximant [2] approximates a function by the ratio of two polynomials. The coefficients of the powers occurring in the polynomials are determined by the coefficients in the Taylor series expansion of the function. Generally, the Pade´ approximant can enlarge the convergence domain of the truncated Taylor series and can improve greatly the convergence rate of the truncated Maclaurin series.

The motivation of this paper is to extend the application of the analytic homotopy-perturbation method (HPM) [8-9] and Padé approximants [1] to solve of human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells (1). The homotopy perturbation method (HPM) and the variational iteration method was first proposed by Chinese mathematician He [8-9,12-15]. The first connection between series solution methods such as an Adomian decomposition method and Padé approximants was established in [6]. The transmission and dynamics of HTLV-I feature several biological characteristics that are of interest to epidemiologists, mathematicians, and biologists, see, for example, [10-11,16], etc. Like HIV, HTLV-I targets CD4+ T-cells, the most abundant white cells in the immune system, decreasing the body's ability to fight infection.

2 PADÉ APPROXIMATON

A rational approximation to $f(x)$ on $[a,b]$ is the quotient of two polynomials $P_N(x)$ and $Q_M(x)$ of degrees N and M, respectively. We use the notation $R_{N,M}(x)$ to denote this quotient. The $R_{N,M}(x)$ Padé approximations to a function $f(x)$ are given by [1]

$$R_{N,M}(x)=\frac{P_N(x)}{Q_M(x)} \quad \text{for } a \le x \le b. \tag{2}$$

The method of Padé requires that $f(x)$ and its derivative be continuous at $x=0$. The polynomials used in (2.1) are

$$P_N(x)=p_0+p_1x+p_2x^2+...+p_Nx^N \tag{3}$$

$$Q_M(x)=1+q_1x+q_2x^2+...+q_Mx^M \tag{4}$$

The polynomials in (3) and (4) are constructed so that $f(x)$ and $R_{N,M}(x)$ agree at $x=0$ and their derivatives up to $N+M$ agree at $x=0$. In the case $Q_0(x)=1$, the approximation is just the Maclaurin expansion for $f(x)$. For a fixed value of $N+M$ the error is smallest when $P_N(x)$ and $Q_M(x)$ have the same degree or when $P_N(x)$ has degree one higher then $Q_M(x)$.

Notice that the constant coefficient of Q_M is $q_0=1$. This is permissible, because it notice be 0 and $R_{N,M}(x)$ is not changed when both $P_N(x)$ and $Q_M(x)$ are divided by the same constant. Hence the rational function $R_{N,M}(x)$ has $N+M+1$ unknown coefficients. Assume that $f(x)$ is analytic and has the Maclaurin expansion

$$f(x)=a_0+a_1x+a_2x^2+...+a_kx^k+..., \tag{5}$$

And from the difference $f(x)Q_M(x)-P_N(x)=Z(x)$:

$$\left[\sum_{i=0}^{\infty} a_i x^i\right]\left[\sum_{i=0}^{M} q_i x^i\right]-\left[\sum_{i=0}^{N} p_i x^i\right]=\left[\sum_{i=N+M+1}^{\infty} c_i x^i\right], \tag{6}$$

The lower index $j = N + M + 1$ in the summation on the right side of (6) is chosen because the first $N + M$ derivatives of $f(x)$ and $R_{N,M}(x)$ are to agree at $x = 0$.

When the left side of (6) is multiplied out and the coefficients of the powers of x^i are set equal to zero for $k = 0,1,2,..., N + M$, the result is a system of $N + M + 1$ linear equations:

$$\begin{aligned}
&a_0 - p_0 = 0\\
&q_1 a_0 + a_1 - p_1 = 0\\
&q_2 a_0 + q_1 a_1 + a_2 - p_2 = 0\\
&q_3 a_0 + q_2 a_1 + q_1 a_2 + a_3 - p_3 = 0\\
&q_M a_{N-M} + q_{M-1} a_{N-M+1} + a_N - p_N = 0
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
&q_M a_{N-M+1} + q_{M-1} a_{N-M+2} + ... + q_1 a_N + a_{N+2} = 0\\
&q_M a_{N-M+2} + q_{M-1} a_{N-M+3} + ... + q_1 a_{N+1} + a_{N+2} = 0\\
&\cdot \qquad\qquad\qquad\qquad\qquad\qquad \cdot\\
&\cdot \qquad\qquad\qquad\qquad\qquad\qquad \cdot\\
&\cdot \qquad\qquad\qquad\qquad\qquad\qquad \cdot\\
&q_M a_N + q_{M-1} a_{N+1} + ... + q_1 a_{N+M+1} + a_{N+M} = 0
\end{aligned} \tag{8}$$

Notice that in each equation the sum of the subscripts on the factors of each product is the same, and this sum increases consecutively from 0 to $N + M$. The M equations in (8) involve only the unknowns $q_1, q_2, q_3, ..., q_M$ and must be solved first. Then the equations in (7) are used successively to find $p_1, p_2, p_3, ..., p_N$ [1].

3. HOMOTOPY PERTURBATION METHOD

To illustrate the homotopy perturbation method (HPM) for solving non-linear differential equations, He [8, 9] considered the following non-linear differential equation:

$$A(u) = f(r), \quad r \in \Omega \tag{9}$$

subject to the boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma \tag{10}$$

where A is a general differential operator, B is a boundary operator, f(r) is a known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along the normal vector drawn outwards from Ω. The operator A can generally be divided into two parts M and N. Therefore, (9) can be rewritten as follows:

$$M(u) + N(u) = f(r), \quad r \in \Omega \tag{11}$$

He [8,9] constructed a homotopy $v(r,p):\Omega x[0,1]\to\Re$ which satisfies

$$H(v,p)=(1-p)[M(v)-M(u_0)]+p[A(v)-f(r)]=0, \tag{12}$$

which is equivalent to

$$H(v,p)=M(v)-M(u_0)+pM(v_0)+p[N(v)-f(r)]=0, \tag{13}$$

where $p\in[0,1]$ is an embedding parameter, and u_0 is an initial approximation of (9). Obviously, we have

$$H(v,0)=M(v)-M(u_0)=0,\quad H(v,1)=A(v)-f(r)=0. \tag{14}$$

The changing process of p from zero to unity is just that of H(v,p) from $M(v)-M(v_0)$ to $A(v)-f(r)$. In topology, this is called deformation and $M(v)-M(v_0)$ and $A(v)-f(r)$ are called homotopic. According to the homotopy perturbation method, the parameter p is used as a small parameter, and the solution of Eq. (12) can be expressed as a series in p in the form

$$v=v_0+pv_1+p^2v_2+p^3v_3+... \tag{15}$$

When $p\to1$, Eq. (12) corresponds to the original one, Eqs. (11) and (15) become the approximate solution of Eq. (11), i.e.,

$$u=\lim_{p\to1}v=v_0+v_1+v_2+v_3+... \tag{16}$$

The convergence of the series in Eq. (16) is discussed by He in [8, 9].

4. APPLICATIONS

In this section, we will apply the homotopy perturbation method to nonlinear ordinary differential systems (1).

4.1 Homotopy perturbation method to a human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells

According to homotopy perturbation method, we derive a correct functional as follows:

$$\begin{aligned}
&(1-p)(\dot v_1-\dot x_0)+p(\dot v_1-\lambda+\mu_T v_1+kv_1v_3)=0,\\
&(1-p)(\dot v_2-\dot y_0)+p(\dot v_2-kv_1v_3+(\mu_L+\alpha)v_2)=0,\\
&(1-p)(\dot v_3-\dot z_0)+p(\dot v_3-\alpha v_2+(\mu_A+\rho)v_3)=0,\\
&(1-p)(\dot v_4-\dot u_0)+p\left(\dot v_4-\rho v_3-\beta v_4\left(1-\frac{v_4}{T_{Max}}\right)+\mu_M v_4\right)=0,
\end{aligned} \tag{17}$$

where “dot” denotes differentiation with respect to t, and the initial approximations are as follows:

$$
\begin{aligned}
&v_{1,0}(t) = x_0(t) = T(0) = P_1,\\
&v_{2,0}(t) = y_0(t) = T_L(0) = P_2,\\
&v_{3,0}(t) = z_0(t) = T_A(0) = P_3,\\
&v_{4,0}(t) = u_0(t) = T_M(0) = P_4.
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
&v_1 = v_{1,0} + pv_{1,1} + p^2v_{1,2} + p^3v_{1,3} + ...,\\
&v_2 = v_{2,0} + pv_{2,1} + p^2v_{2,2} + p^3v_{2,3} + ...,\\
&v_3 = v_{3,0} + pv_{3,1} + p^2v_{3,2} + p^3v_{3,3} + ...,\\
&v_4 = v_{4,0} + pv_{4,1} + p^2v_{4,2} + p^3v_{4,3} + ...,
\end{aligned}
\tag{19}
$$

where $v_{i,j}, i, j = 1,2,3,...$ are functions yet to be determined. Substituting Eqs.(18) and (19) into Eq. (17) and arranging the coefficients of "p" powers, we have

$$
\begin{aligned}
&\left(\dot{v}_{1,1} - \lambda + \mu_T P_1 + kP_1P_3\right)p + \left(\dot{v}_{1,2} + \mu_T v_{1,1} + k\left(P_1v_{3,1} + P_3v_{1,1}\right)\right)p^2\\
&\quad + \left(\dot{v}_{1,3} + \mu_T v_{1,2} + k\left(P_1v_{3,2} + P_3v_{1,2} + v_{1,1}v_{3,1}\right)\right)p^3 + ... = 0,\\
&\left(\dot{v}_{2,1} - kP_1P_3 + (\mu_L + \alpha)P_2\right)p\\
&\quad + \left(\dot{v}_{2,2} - k\left(P_1v_{3,1} + P_3v_{1,1}\right) + (\mu_L + \alpha)v_{2,1}\right)p^2\\
&\quad + \left(\dot{v}_{2,3} - k\left(P_1v_{3,2} + P_3v_{1,2} + v_{1,1}v_{3,1}\right) + (\mu_L + \alpha)v_{2,2}\right)p^3 + ... = 0,\\
&\left(\dot{v}_{3,1} - \alpha P_2 + (\mu_A + \rho)P_3\right)p + \left(\dot{v}_{3,2} - \alpha v_{2,1} + (\mu_A + \rho)v_{3,1}\right)p^2\\
&\quad + \left(\dot{v}_{3,3} - \alpha v_{2,2} + (\mu_A + \rho)v_{3,2}\right)p^3 + ... = 0,\\
&\left(\dot{v}_{4,1} - \rho P_3 + (\mu_M - \beta)P_4 + \frac{\beta}{T_{Max}}P_4^2\right)p + \left(\dot{v}_{4,2} - \rho v_{3,1} + (\mu_M - \beta)v_{4,1} + \frac{\beta}{T_{Max}}2P_4v_{4,1}\right)p^2\\
&\quad + \left(\dot{v}_{4,3} - \rho v_{3,2} + (\mu_M - \beta)v_{4,2} + \frac{\beta}{T_{Max}}\left(v_{4,1}^2 + 2P_4v_{4,2}\right)\right)p^3 + ... = 0,
\end{aligned}
\tag{20}
$$

In order to obtain the unknowns $v_{i,j}(t), i, j = 1,2,3,$ we must construct and solve the following system which includes nine equations with nine unknowns, considering the initial conditions

$$v_{i,j}(0) = 0, i, j = 1,2,3,$$

$$
\begin{aligned}
&\dot{v}_{1,1} - \lambda + \mu_T P_1 + kP_1P_3 = 0,\\
&\dot{v}_{1,2} + \mu_T v_{1,1} + k\left(P_1v_{3,1} + P_3v_{1,1}\right) = 0,\\
&\dot{v}_{1,3} + \mu_T v_{1,2} + k\left(P_1v_{3,2} + P_3v_{1,2} + v_{1,1}v_{3,1}\right) = 0,\\
&\dot{v}_{2,1} - kP_1P_3 + (\mu_L + \alpha)P_2 = 0,\\
&\dot{v}_{2,2} - k\left(P_1v_{3,1} + P_3v_{1,1}\right) + (\mu_L + \alpha)v_{2,1} = 0,\\
&\dot{v}_{2,3} - k\left(P_1v_{3,2} + P_3v_{1,2} + v_{1,1}v_{3,1}\right) + (\mu_L + \alpha)v_{2,2} = 0,\\
&\dot{v}_{3,1} - \alpha P_2 + (\mu_A + \rho)P_3 = 0,\\
&\dot{v}_{3,2} - \alpha v_{2,1} + (\mu_A + \rho)v_{3,1} = 0,\\
&\dot{v}_{3,3} - \alpha v_{2,2} + (\mu_A + \rho)v_{3,2} = 0,\\
&\dot{v}_{4,1} - \rho P_3 + (\mu_M - \beta)P_4 + \frac{\beta}{T_{Max}}P_4^2 = 0,\\
&\dot{v}_{4,2} - \rho v_{3,1} + (\mu_M - \beta)v_{4,1} + \frac{\beta}{T_{Max}}2P_4v_{4,1} = 0,\\
&\dot{v}_{4,3} - \rho v_{3,2} + (\mu_M - \beta)v_{4,2} + \frac{\beta}{T_{Max}}\left(v_{4,1}^2 + 2P_4v_{4,2}\right) = 0.
\end{aligned}
\tag{21}
$$

From Eq. (16), if the three terms approximations are sufficient, we will obtain:

$$
\begin{aligned}
&T(t) = \lim_{p\to 1} v_1(t) = \sum_{k=0}^{3} v_{1,k}(t),\\
&T_L(t) = \lim_{p\to 1} v_2(t) = \sum_{k=0}^{3} v_{2,k}(t),\\
&T_A(t) = \lim_{p\to 1} v_3(t) = \sum_{k=0}^{3} v_{3,k}(t),\\
&T_M(t) = \lim_{p\to 1} v_4(t) = \sum_{k=0}^{3} v_{4,k}(t),
\end{aligned}
\tag{22}
$$

therefore

$$
\begin{aligned}
T(t) &= P_1 + (\lambda - \mu_T P_1 + kP_1P_3)t\\
&+ \frac{1}{2}\left[\left(-\mu_T - kP_3\right)(\lambda - \mu_T P_1 + kP_1P_3) - kP_1\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right)\right]t^2\\
&+ \frac{1}{6}\begin{bmatrix}\left(\mu_T + kP_3\right)^2(\lambda - \mu_T P_1 + kP_1P_3) - k\alpha P_1\left(kP_1P_3 - \left(\mu_L + \alpha\right)P_2\right)\\ +\left(kP_1\mu_T + k^2P_1P_3 + kP_1\left(\mu_A + \rho\right)\right)\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right)\\ -2k(\lambda - \mu_T P_1 + kP_1P_3)\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right)\end{bmatrix}t^3
\end{aligned}
$$

$$
\begin{aligned}
T_L(t) &= P_2 + \left(kP_1P_3 - \left(\mu_L + \alpha\right)P_2\right)t \\
&\quad + \frac{1}{2}\begin{bmatrix} kP_3(\lambda - \mu_T P_1 + kP_1P_3) + kP_1\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right) \\ -k\left(\mu_L + \alpha\right)\left(kP_1P_3 - \left(\mu_L + \alpha\right)P_2\right) \end{bmatrix} t^2 \\
&\quad + \frac{1}{6}\begin{bmatrix} \left(-k^2P_3^2 - kP_3\left(\mu_L + \alpha\right) - kP_3\mu_T\right)^2(\lambda - \mu_T P_1 + kP_1P_3) \\ \left(k\alpha P_1 + \left(\mu_L + \alpha\right)^2\right)\left(kP_1P_3 - \left(\mu_L + \alpha\right)P_2\right) \\ +\left(-kP_1\left(\mu_A + \rho\right) - k^2P_1P_3 - kP_1\left(\mu_L + \alpha\right)\right)\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right) \\ +2k(\lambda - \mu_T P_1 + kP_1P_3)\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right) \end{bmatrix} t^3 \\
T_A(t) &= P_3 + \left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right)t \\
&\quad + \frac{1}{2}\begin{bmatrix} \alpha\left(kP_1P_3 - \left(\mu_L + \alpha\right)P_2\right) \\ -\left(\mu_A + \rho\right)\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right) \end{bmatrix} t^2 \\
&\quad + \frac{1}{6}\begin{bmatrix} k\alpha P_3(\lambda - \mu_T P_1 + kP_1P_3) + \left(k\alpha P_1 + \left(\mu_A + \rho\right)^2\right)\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right) \\ -\alpha\left(\mu_A + \rho + \mu_L + \alpha\right)\left(kP_1P_3 - \left(\mu_L + \alpha\right)P_2\right) \end{bmatrix} t^3 \\
T_M(t) &= P_4 + \left(\rho P_3 - \left(\mu_M - \beta\right)P_4 - \frac{\beta P_4^2}{T_{\max}}\right)t \\
&\quad + \frac{1}{2}\begin{bmatrix} -\left(\mu_M - \beta + \frac{2\beta P_4}{T_{\max}}\right)\left(\rho P_3 - \left(\mu_M - \beta\right)P_4 - \frac{\beta P_4^2}{T_{\max}}\right) \\ \rho\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right) \end{bmatrix} t^2 \\
&\quad + \frac{1}{6}\begin{bmatrix} \rho\alpha\left(kP_1P_3 - \left(\mu_L + \alpha\right)P_2\right) \\ +\left(-\rho\left(\mu_A + \rho\right) - \rho\left(\mu_M - \beta\right) - \frac{2\beta\rho P_4}{T_{\max}}\right)\left(\alpha P_2 - \left(\mu_A + \rho\right)P_3\right) \\ +\left(\left(\mu_M - \beta\right) + \frac{2\beta P_4}{T_{\max}}\right)^2\left(\rho P_3 - \left(\mu_M - \beta\right)P_4 - \frac{\beta P_4^2}{T_{\max}}\right) \end{bmatrix} t^3 \qquad (23) \\
&\quad \vdots
\end{aligned}
$$

Here
$T(0) = 1000/mm^3, T_L(0) = 250/mm^3, T_A(0) = 1.5/mm^3$ and $T_M(0) = 0$ for the four-component model.

A few first approximations for $T(t), T_L(t), T_A(t)$ and $T_M(t)$ are calculated and presented below:

Three terms approximations:

$$\begin{aligned} T(t) &= 1000 + 90.6t - 70.108t^2 + 7.731499133t^3, \\ T_L(t) &= 250 + 748.4t + 37.81512t^2 + 7.61158861t^3, \\ T_A(t) &= 01.5 + .02494t + .1490560012t^2 + .0025557619t^3, \\ T_M(t) &= .00006t + .0000003578t^2 + .000001986852796t^3. \end{aligned} \tag{24}$$

Four terms approximations:

$$\begin{aligned} T(t) &= 1000 + 90.6t - 70.108t^2 + 7.731499133t^3 - 4.514321208t^4, \\ T_L(t) &= 250 + 748.4t + 37.81512t^2 + 7.61158861t^3 + 3.226445308t^4, \\ T_A(t) &= 01.5 + .02494t + .1490560012t^2 + .0025557619t^3 + .0007291862795t^4, \\ T_M(t) &= .00006t + .0000003578t^2 + .000001986852796t^3 - .5811660548\text{e-}8t^4. \end{aligned} \tag{25}$$

Five terms approximations:

$$\begin{aligned} T(t) &= 1000 + 90.6t - 70.108t^2 + 7.731499133t^3 - 4.514321208t^4 \\ &\quad + 2.202684204t^5, \\ T_L(t) &= 250 + 748.4t + 37.81512t^2 + 7.61158861t^3 + 3.226445308t^4 \\ &\quad - 1.610923654t^5, \\ T_A(t) &= 01.5 + .02494t + .1490560012t^2 + .0025557619t^3 + .0007291862795t^4 \\ &\quad + .0002508179284t^5, \\ T_M(t) &= .00006t + .0000003578t^2 + .000001986852796t^3 - .5811660548\text{e-}8t^4 \\ &\quad + .5811660548\text{e-}8t^5. \end{aligned} \tag{26}$$

Six terms approximations:

$$\begin{aligned} T(t) &= 1000 + 90.6t - 70.108t^2 + 7.731499133t^3 - 4.514321208t^4 \\ &\quad + 2.202684204t^5 - .6157592898t^6, \\ T_L(t) &= 250 + 748.4t + 37.81512t^2 + 7.61158861t^3 + 3.226445308t^4 \\ &\quad - 1.610923654t^5 + .3751823458t^6, \\ T_A(t) &= 01.5 + .02494t + .1490560012t^2 + .0025557619t^3 + .0007291862795t^4 \\ &\quad + .0002508179284t^5 - .1094867318\text{e-}3t^6, \\ T_M(t) &= .00006t + .0000003578t^2 + .000001986852796t^3 - .5811660548\text{e-}8t^4 \\ &\quad + .5811660548\text{e-}8t^5 + .1667567057\text{e-}8t^6. \end{aligned} \tag{27}$$

In this section, we apply Laplace transformation to (27), which yields

$$
\begin{aligned}
L\left(T(s)\right) &= \frac{100}{s} + \frac{90.6}{s^2} - \frac{140.216}{s^3} + \frac{46.3889948}{s^4} \\
&\quad - \frac{108.343709}{s^5} + \frac{264.3221045}{s^6} - \frac{443.3466887}{s^7} \\
L\left(T_L(s)\right) &= \frac{250}{s} + \frac{748.4}{s^2} + \frac{75.63024}{s^3} + \frac{45.66953166}{s^4} \\
&\quad + \frac{77.43468739}{s^5} - \frac{193.3108385}{s^6} + \frac{270.131289}{s^7} \\
L\left(T_A(s)\right) &= \frac{1.5}{s} + \frac{.02494}{s^2} + \frac{.2981120024}{s^3} + \frac{.0153345714}{s^4} \\
&\quad + \frac{.01750047071}{s^5} + \frac{.03009815141}{s^6} - \frac{.0788304469}{s^7} \\
L\left(T_M(s)\right) &= \frac{0.00006}{s^2} + \frac{.0000007156}{s^3} + \frac{.00001192111678}{s^4} \\
&\quad + \frac{.000000557353607}{s^5} + \frac{.000001200648281}{s^7}
\end{aligned}
\tag{28}
$$

For simplicity, let $s = \frac{1}{t}$; then

$$
\begin{aligned}
L(T(t)) &= 100t + 90.6t^2 - 140.216t^3 + 46.3889948t^4 - 108.343709t^5 \\
&\quad + 264.3221045t^6 - 443.3466887t^7 \\
L(T_L(t)) &= 250t + 748.4t^2 + 75.63024t^3 + 45.66953166t^4 + 77.43468739t^5 \\
&\quad - 193.3108385t^6 + 270.131289t^7 \\
L(T_A(t)) &= 1.5t + .02494t^2 + .2981120024t^3 + .0153345714t^4 + .01750047071t^5 \\
&\quad + .03009815141t^6 - .788304469t^7 \\
L(T_M(t)) &= 0.00006t^2 + .0000007156t^3 + .00001192111678t^4 + .000000557353607t^5 \\
&\quad + .000001200648281t^7
\end{aligned}
\tag{29}
$$

Padé approximant $[4/4]$ of (29) and substituting $t = \frac{1}{s}$, we obtain $[4/4]$ in terms of s. By using the inverse Laplace transformation, we obtain

$$
\begin{aligned}
T(t) &= -2029.558117e^{-.449004857t} + 3049.429417e^{-.2743271492t} \\
&\quad + e^{-.4112696069t}\left[-19.87130089\cos(1.572742785t) + 4.887871691\sin(1.572742785t)\right] \\
T_L(t) &= e^{-.7972283788t}\left[5.705934771\cos(1.211231026t) + 29.58485206\sin(1.211231026t)\right] \\
&\quad + e^{.09932940685t}\left[244.2940654\cos(.1019716214t) + 6794.530172\sin(.1019716214t)\right] \\
T_A(t) &= -.03754095721e^{-1.095137053t} + .7224835299e^{-.5130323164t} \\
&\quad + .8155286715e^{.4358409947t} - .0004712445489e^{2.028504412t} \\
T_M(t) &= -.00006732169426e^{-.4396582501t} + .00006732169426e^{.4515849168t}
\end{aligned}
\tag{30}
$$

These results obtained by Padé approximations for $T(t), T_L(t), T_A(t)$ and $T_M(t)$ are calculated and presented follow.

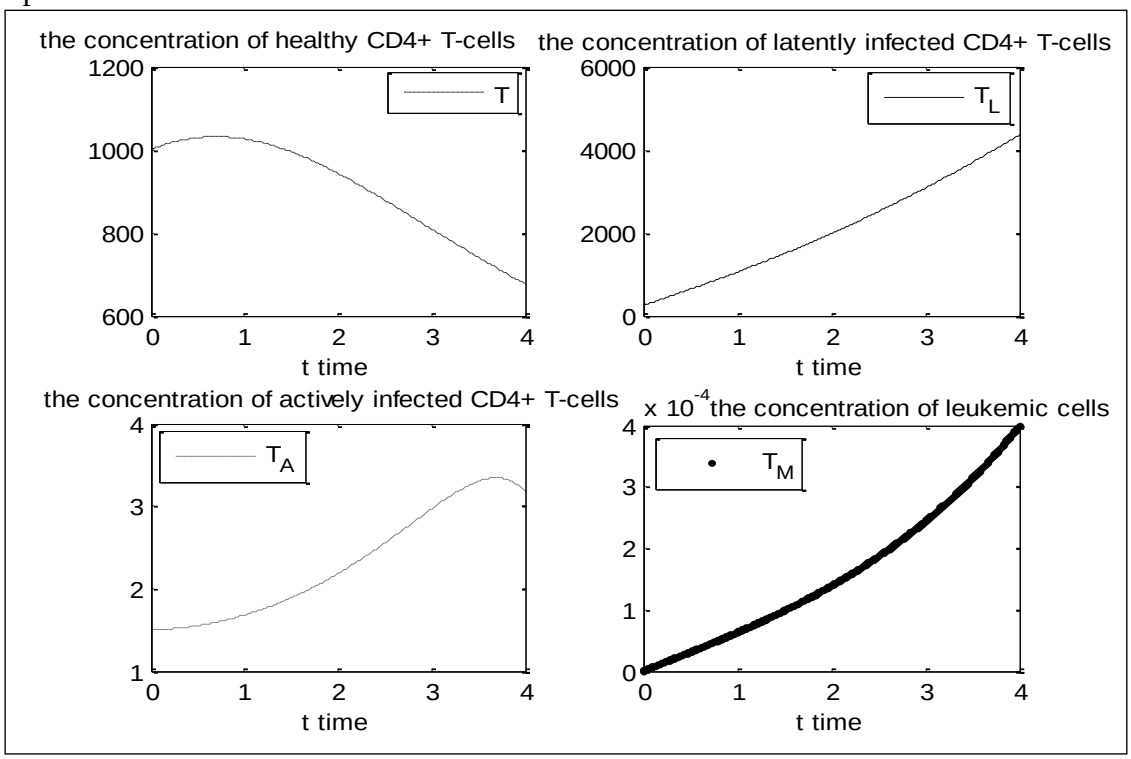

Figure. 1. Plots of Padé approximations for human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells model.

These results obtained by homotopy perturbation method, three, four, five and six terms approximations for $T(t), T_L(t), T_A(t)$ and $T_M(t)$ are calculated and presented follow.

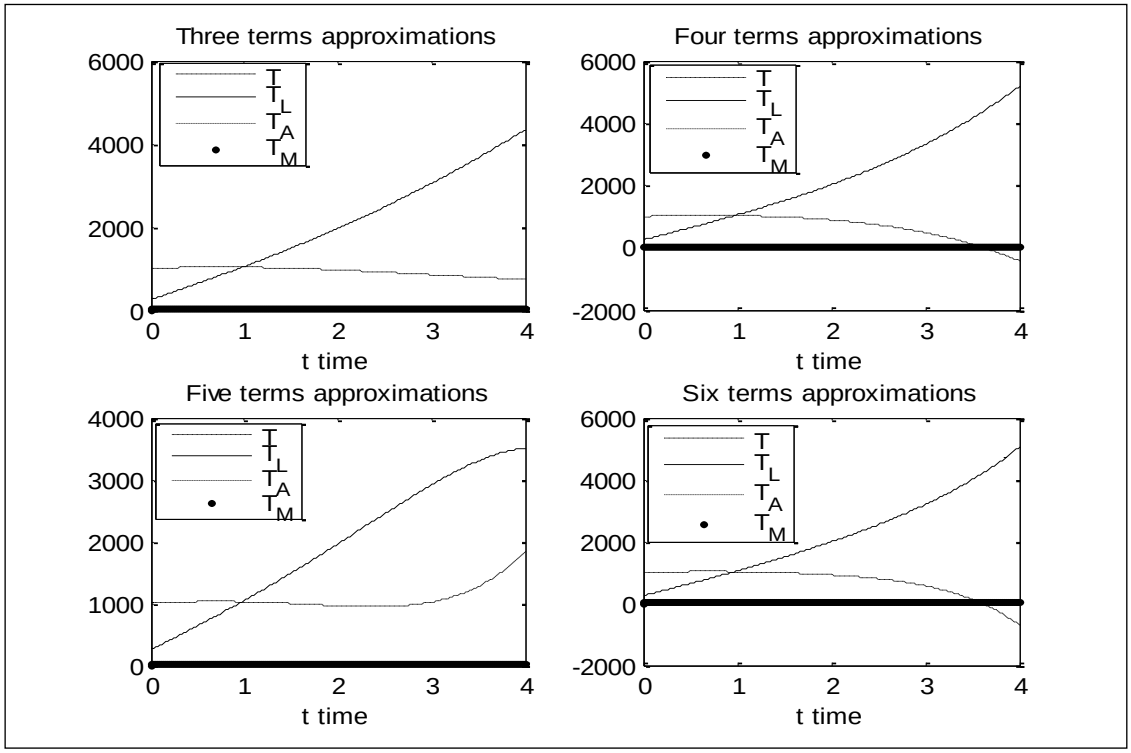

Figure. 2. Plots of three, four, five and six terms approximations for human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells Model

5. CONCLUSIONS

In this paper, we have presented an after treatment technique for the homotopy perturbation method. Because the Pade´ approximant usually improves greatly the Maclaurin series in the convergence region and the convergence rate, the at leads to a better analytic approximate solution from homotopy perturbation method truncated series The homotopy perturbation method was used for finding the solutions of nonlinear ordinary differential equation systems such as human T-cell lymphotropic virus I (HTLV-I) infection of CD4+ T-cells model. We demonstrated the accuracy and efficiency of these methods by solving some ordinary differential equation systems. We use Laplace transformation and Padé approximant to obtain an analytic solution and to improve the accuracy of homotopy perturbation method. The reliability of the method and reduction in the size of computational domain give this method a wider applicability. It is observed that The results to get the homotopy perturbation method (HPM) applied Padé approximants is an effective and reliable tool for the solution of the nonlinear ordinary differential equation systems considered in the present paper.

The computations associated with the examples in this paper were performed using Maple 7 and Matlab 7

6. REFERENCES

1. G.A. Baker, *Essentials of Pade´ Approximants*, Academic Press, London, 1975.
2. G.A. Baker, P. Graves-Morris, *Pade´ approdmants*, Cambridge: Cambridge University Press; 1996.
3. P. Katri, S. Ruan,Dynamics of Human T-cell lymphotropic virus I(HTLV-I) infection of CD4+ T-cells, *C.R. Biologies* **327**, 1009-1016, 2004.

4. S Iwami, Y Takeuchi, X Liu, Avian–human influenza epidemic model, *Mathematical Biosciences* **207** ,1–25, 2007.
5. D.W. Jordan, P. Smith, *Nonlinear Ordinary Differential Equations*, third ed., Oxford University Press, 1999.
6. A.M. Wazwaz, The modified decomposition method and Padé approximants for solving the Thomas-Fermi equation, *Applied Mathematics and Computation* **105**(1), 11-19 1999.
7. G.F. Simmons, *Differential Equations with Applications and Historical Notes*, McGraw-Hill (1972).
8. J.H. He. Homotopy perturbation technique, *Comput Methods Appl Mech Engrg* **178**, 257–62 1999.
9. J.H. He. A coupling method of a homotopy technique and a perturbation technique for non-linear problems, *Int J Non-linear Mech* **35**(1), 37–43, 2000.
10. B. Asquith, C.R.M. Bangham, The dynamics of T-cell fratricide:application of a robust approach to mathematical modeling in immunology, *J. Theor. Biol*. **222**, 53–69, 2003.
11. B.A. Finlayson, *The Method of Weighted Residuals and Variational Principles*, Academic press, New York, 1972.
12. J.H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media, *Computer Methods in Applied Mechanics and Engineering* **167** (1–2), 57–68, 1998.
13. J.H. He, Approximate solution of nonlinear differential equations with convolution product nonlinearities, *Computer Methods in Applied Mechanics and Engineering* **167** (1–2), 69–73, 1998.
14. J.H. He,Variational iteration method-a kind of nonlinear analytical technique: some examples, *International Journal of Nonlinear Mechanics* **34** (4) (1999) 699–708.
15. J.H. He, Some asymptotic methods for strongly nonlinear equations, *International Journal of Modern Physics B* **20** (10), 1141–1199, 2006.
16. M.A. Abdou, A.A. Soliman, Variational-iteration method for solving Burger's and coupled Burger's equations, *Journal of Computational and Applied Mathematics* **181** (2), 245–251, 2005.