

GENERALIZED STATISTICAL MECHANICS FOR NUCLEUS

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Abstract- In this study, we derived the grand partition function and some thermodynamical quantities of the nucleus in the high temperature limit by adopting the ideal Fermi gas partition function of generalized Tsallis thermostatics. The behaviour of number of neutron and internal energy are depicted as a function of temperature. Sensitivity of q entropy index to the number neutron are also analyzed.

Key Words- Tsallis thermostatics, Fermi gas model, nucleus

1. INTRODUCTION

A nonextensive entropy definition was proposed in 1988 [1] by Tsallis,

$$S_q = -k \frac{1 - \sum_{i=1}^W p_i^q}{1 - q}, \quad q \in R \quad (1)$$

where k is a positive constant, p_i is the probability of the system in the i th microstate, W is the total number of the configurations of the system and q is any real number. Since then, Tsallis thermostatics (TT) has commonly been used for the investigation of the physical systems. As well-known, Boltzmann-Gibbs (BG) statistics is a powerful one to study a variety of the physical systems. However it falls for the systems which 1) have long-ranged interactions, 2) have long-ranged memory effects, and 3) evolve in multifractal space-time. In this manner, it has been understood that extensive BG statistics fails to study nonextensive physical systems not having these conditions. Consequently the standard BG thermostatics is not universal and is appropriate for extensive systems. The parameter, q , in Eq.(1) is called the entropy index and measures the degree of the nonextensivity of the system under consideration. After a few years from 1988, the formalism was revised [2] introducing the unnormalized constraint to the internal energy. This generalization included the celebrated BG thermostatics, which could be recovered when $q = 1$. After this work, this formalism has been commonly employed to study various physical systems. Some of them can be given such as; self gravitating systems [3,4], turbulence [5-8], anomalous diffusion [9-12], solar neutrinos [13], liquid crystals [14-22] etc.

The constraint to the internal energy of the system is given by

$$\sum_{i=1}^W p_i^q \varepsilon_i = U_q. \quad (2)$$

The optimization of Tsallis entropy given by Eq.(1) according to this internal energy constraint results in

$$p_i = \frac{[1 - (1 - q)\beta\varepsilon_i]^{-\frac{1}{q}}}{Z_q} \quad (3)$$

with

$$Z_q = \sum_{i=1}^W [1 - (1 - q)\beta\varepsilon_i]^{-\frac{1}{q}} \quad (4)$$

Therefore the q -expectation value of any observable is defined as

$$\langle A_i \rangle_q = \sum_{i=1}^W p_i^q A_i, \quad (5)$$

where A represents any observable quantity which commutes with Hamiltonian. The q -expectation value of the observable reduces to the conventional one when $q = 1$.

In literature, there exists some applications of nonextensive statistics to; the classical ideal gas [23], the quantum ideal gas [24], Bose-Einstein and Fermi-Dirac distribution functions [25], the ideal Fermi gas [26]. Prato [23] derived an partition function of the classical ideal gas for $q < 1$ by using integral representation of Gamma function in the complex plane. Using this partition function specific heat also obtained and recovered the BG expression for $q = 1$. In another study related with the ideal gases [24] the ideal gases partition function, internal energy and number of particles for both $q < 1$ and $q > 1$ are derived using the transformations obtained from the gamma function representations at high and low temperature limit. Lenzi et al. [23] analyzed the generalized Bose-Einstein and Fermi-Dirac distributions within nonextensive statistics by considering the normalized constraints in the effective temperature approach. Martinez et al. [26] discussed the relevant aspects of the exact q -thermostatistical treatment for an ideal Fermi system. The grand canonical exact generalized partition function was given for arbitrary values of the entropic index q , and ensuing statistics was derived.

In the present study, we obtained the grand partition function and related thermodynamical quantities at high temperature for the nucleus which is considered as a fermi gas. This is crude but useful model to explain the statistical features of nuclear spectra. In doing so, we use the expressions obtained in Ref. [24]. Details of this calculations are given in Section 2. In section 3, generalized internal energy, number of particles (neutrons and protons), spin for the nucleus is obtained. In last section, our results are compared to those in the literature and we present a summary and our main conclusions.

2. GENERALIZED GRAND PARTITION FUNCTION

Consider a system of N neutrons and Z protons. The neutrons have single-particle levels with energy a_s , occupation number n_s , magnetic quantum number m_{1s} ; proton single-particle levels are similarly designated by b_s, z_s, m_{2s} . A single state of whole system is defined by four constants of the motion:

$$\begin{aligned} N &= \sum n_s, & Z &= \sum z_s, \\ E &= \sum n_s a_s + \sum z_s b_s, & M &= \sum n_s m_{1s} + \sum z_s m_{2s} \end{aligned} \quad (6)$$

For this system, the generalized grand partition function is given by

$$\Xi_q = e_q^\Omega = \sum_{N'} \sum_{Z'} \sum_{M'} \sum_{E'} [1 - (1-q)\beta(E' - \mu_1 N' - \mu_1 Z' - \mu_1 M')]^{\frac{1}{1-q}} \quad (7)$$

The Hilhorst integral transformations [27] and the extension for $q < 1$ shown by Prato [23] are useful because it provides a connection between a thermodynamic or statistical generalized quantity and its respective standard quantity. Thus, generalized thermodynamics could easily be established. Therefore, the generalized statistical mechanics can be regarded as a Hilhorst integral transformation of respective standard quantities. From the integral representation of Gamma function

$$\eta^{-\nu} = \frac{1}{\Gamma(\nu)} \int_0^\infty d\xi \xi^{\nu-1} e^{-\eta\xi}, \quad (8)$$

the grand partition function (7) can be written, with the identifications $\nu=1/(q-1)$ and $\eta=1+(q-1)\beta(E' - \mu_1 N' - \mu_1 Z' - \mu_1 M')$, in the following form:

$$\begin{aligned} \Xi_q(\beta, \mu_1, \mu_2, \mu_3) &= \frac{1}{\Gamma\left(\frac{1}{q-1}\right)} \sum_{N'} \sum_{Z'} \sum_{M'} \sum_{E'} \int_0^\infty d\xi \xi^{\frac{1}{q-1}-1} \\ &\quad \times e^{[(1+(1-q)\beta(E' - \mu_1 N' - \mu_1 Z' - \mu_1 M'))\xi]} \end{aligned} \quad (9)$$

When the summations and integral are interchanged, Eq.(9) becomes

$$\Xi_q(\beta, \mu_1, \mu_2, \mu_3) = \frac{1}{\Gamma\left(\frac{1}{q-1}\right)} \int_0^\infty d\xi \xi^{\frac{1}{q-1}-1} e^\xi \Xi_q(\beta(q-1)\xi, \mu_1, \mu_2, \mu_3) \quad (10)$$

for $q > 1$, and using the integral representation of Gamma function in the complex plane we have [23]

$$\Xi_q(\beta, \mu_1, \mu_2, \mu_3) = \Gamma\left(\frac{2-q}{1-q}\right) \frac{i}{2\pi} \oint_C d\xi (-\xi)^{\frac{1}{1-q}-1} e^\xi \Xi_q(\beta(q-1)\xi, \mu_1, \mu_2, \mu_3) \quad (11)$$

for $q < 1$, where

$$\begin{aligned} \Xi_q(\beta, \mu_1, \mu_2, \mu_3) &= \sum_{N'} \sum_{Z'} \sum_{M'} \sum_{E'} e^{-(q-1)\beta(E' - \mu_1 N' - \mu_1 Z' - \mu_1 M')\xi} \\ &= e^{\Omega_1(\beta(q-1)\xi, \mu_1, \mu_2, \mu_3)} \end{aligned} \quad (12)$$

The contour C in the complex plane is Hankel contour. For the system that contains N neutrons and Z protons, standart number of states is defined as

$$\begin{aligned} \Omega_1(\beta(q-1)\xi, \mu_1, \mu_2, \mu_3) &= \sum_s \ln\{1 + \exp(\beta(q-1)(\mu_1 + \mu_3 m_{1s} - a_s)\xi)\} + \\ &\quad \sum_s \ln\{1 + \exp(\beta(q-1)(\mu_2 + \mu_3 m_{2s} - b_s)\xi)\}. \end{aligned} \quad (13)$$

In order to calculate Ω_1 , we shall replace the sums in Eq.(13) by integrals over the energy ε , introducing for this purpose the single-particle level densities $g_1(\varepsilon, m_1)$ and $g_2(\varepsilon, m_2)$ for neutrons and protons respectively:

$$\begin{aligned}\Omega_1 &= \sum_{m_1} \int_0^\infty d\varepsilon g_1(\varepsilon, m_1) \ln[1 + \exp((q-1)(\alpha_1 + \alpha_3 m_1 - \beta\varepsilon)\xi)] \\ &= \sum_{m_2} \int_0^\infty d\varepsilon g_2(\varepsilon, m_2) \ln[1 + \exp((q-1)(\alpha_2 + \alpha_3 m_2 - \beta\varepsilon)\xi)] \\ &= \Omega_1' + \Omega_1''\end{aligned}\quad (14)$$

where $\alpha_i = \beta\mu_i$. Replacement of single-particle level densities by a smooth function is valid, provided $g/\beta \gg 1$ which is equivalent to $g/E \gg 1$. Evaluating integral over the energy ε results with

$$\begin{aligned}\Omega_1 &= \beta(q-1)\xi \left[\frac{\mu_1^2}{2} \sum_{m_1} g_1 + \frac{\mu_2^2}{2} \sum_{m_2} g_2 + \frac{\mu_3^2}{2} \sum_{m_1} g_1 m_1^2 + \frac{\mu_3^2}{2} \sum_{m_2} g_2 m_2^2 \right] \\ &\quad + \frac{\pi^2}{6\beta(q-1)\xi} \left(\sum_{m_1} g_1 + \sum_{m_2} g_2 \right).\end{aligned}\quad (15)$$

In the evaluation of integrals the terms depending on derivatives of g are neglected since they are small compared with the terms containing g . For the details of this calculations see [28,29]. Using the following definitions

$$\begin{aligned}\sum_{m_1} g_1(\mu_1, m_1) &\equiv g_1, \quad \sum_{m_2} g_2(\mu_2, m_2) \equiv g_2, \quad \sum_{m_1} g_1(\mu_1, m_1) + \sum_{m_2} g_2(\mu_2, m_2) = g \\ \sum_{m_1} g_1(\mu_1, m_1) m_1^2 + \sum_{m_2} g_2(\mu_2, m_2) m_2^2 &= g \langle m^2 \rangle,\end{aligned}\quad (16)$$

the number of states Ω_1 can be written in the final form

$$\Omega_1 = \beta(q-1)\xi \left[\frac{\mu_1^2}{2} g_1 + \frac{\mu_2^2}{2} g_2 + \frac{\mu_3^2}{2} g \langle m^2 \rangle \right] + \frac{\pi^2 g}{6\beta(q-1)\xi}.\quad (17)$$

Introducing the number of states into standart partition function Ξ_1 in Eq.(12) and replacing it into the integral in Eq.(11) we obtain

$$\Xi_q(\beta, \mu_1, \mu_2, \mu_3) = \Gamma(1-\nu) \frac{i}{2\pi} \oint_C d\xi (-\xi)^{-\alpha-1} e^{-\xi \left[1 + \beta(1-q) \left\{ \frac{\mu_1^2}{2} g_1 + \frac{\mu_2^2}{2} g_2 + \frac{\mu_3^2}{2} g \langle m^2 \rangle \right\} \right]} e^{-\frac{\alpha}{\beta(1-q)\xi}}.\quad (18)$$

Where $\alpha = \frac{1}{1-q}$. Substituting $z = \lambda_q \xi$ and taking $\gamma_q = \lambda_q a_q$ with

$$\lambda_q = 1 + \beta(1-q) \left[\frac{\mu_1^2}{2} g_1 + \frac{\mu_2^2}{2} g_2 + \frac{\mu_3^2}{2} g \langle m^2 \rangle \right] \quad \text{and} \quad a_q = \frac{\pi^2 g}{6\beta(1-q)} = \frac{a}{\beta(1-q)},$$

generalized partition function becomes

$$\Xi_q(\beta, \mu_1, \mu_2, \mu_3) = \Gamma(1 + \alpha) \frac{i}{2\pi} \oint_C dz (-z)^{-\alpha-1} e^{-\left(z + \frac{\gamma_q}{z}\right)}. \quad (19)$$

Above integral with $-z = t$ is integral representation of the Wright function for $\rho=1$ which is in the form of

$$\Phi(1, 1 - \nu, ; \gamma_q) = \frac{1}{2\pi i} \int_{H\alpha} dt t^{-(1+\alpha)} e^{t + \gamma_q t^{-1}}. \quad (20)$$

$H\alpha$ is the Hankel contour chosen on negative real axis. Therefore, generalized partition function becomes

$$\Xi_q(\beta, \mu_1, \mu_2, \mu_3) = \Gamma(1 + \alpha) \lambda_q^\alpha \Phi(1, 1 + \alpha; \gamma_q) \quad (21)$$

where $\Phi(1, 1 + \alpha; \gamma_q)$ is Wright function introduced by E. M. Wright in the asymptotic theory of partitions. For analytical properties and some generalizations of this function see the [30]. For large γ_q Wright function can be written in the following asymptotic expression [30]

$$\Phi(1, 1 + \alpha; \gamma_q) = \sqrt{\pi} (\gamma_q)^{\frac{1+2\alpha}{4}} \exp(2\sqrt{\gamma_q}) \left[1 + \sum_{m=1}^M \frac{(-1)^m a_m(1, 1 + \alpha)}{(\gamma_q)^{m/2}} + O\left[(\gamma_q)^{\frac{M+1}{2}} \right] \right]. \quad (22)$$

This form of this function allows to obtain the thermodynamical quantities in more simple form and in high temperature limit since $\gamma_q \gg 1$. The constants $a_m(1, 1 + \alpha)$ is exactly evaluated in [30]. For our case we need

$$a_1(1, 1 + \alpha) = \frac{\alpha^2 - 1}{4} + \frac{3}{16} \quad \text{and} \quad a_2(1, 1 + \alpha) = \frac{(\alpha^2 - 1)(2\alpha^2 - 3)}{64} + \frac{945}{512}.$$

Using this asymptotic expression final form of generalized partition function for $q < 1$ can be written in the following form

$$\Xi_q(\beta, \mu_1, \mu_2, \mu_3) = A_1 \left(\frac{\beta}{\alpha} \right)^\alpha \tau^{\frac{2\alpha-1}{4}} \exp(2\sqrt{a\tau}) \left(1 + \frac{a_2}{a\tau} - \frac{a_1}{\sqrt{a}\sqrt{\tau}} \right),$$

where

$$A_1 = \frac{\sqrt{\pi} \Gamma(1 + \alpha)}{\alpha^{\frac{2\alpha+1}{4}}} \quad \text{and} \quad \tau = \frac{\alpha}{\beta} + g_n.$$

Using connection between Eq.(10) and Eq.(11)

$$\begin{aligned} & \oint_C d\xi (-\xi)^{\frac{1}{1-q}-1} e^\xi \Xi_1(\beta(q-1)\xi, \mu_1, \mu_2, \mu_3) = -2i \sin(\pi\nu) \\ & \times \int_0^\infty d\xi \xi^{\frac{1}{1-q}-1} e^{-\xi} \Xi_1(\beta(q-1)\xi, \mu_1, \mu_2, \mu_3) \end{aligned} \quad (24)$$

the generalization partition function for $q > 1$ can be obtained as

$$\Xi_q(\beta, \mu_1, \mu_2, \mu_3) = A_2 \left(\frac{\beta}{\alpha} \right)^\alpha \tau^{\frac{2\alpha-1}{4}} \exp(2\sqrt{a\tau}) \left(1 + \frac{a_2}{a\tau} - \frac{a_1}{\sqrt{a}\sqrt{\tau}} \right), \quad (25)$$

where

$$A_2 = \frac{\pi^{3/2}}{\Gamma(-\alpha) \sin(-\pi\alpha) \alpha^{\frac{2\alpha+1}{4}}}.$$

Since α is negative for $q > 1$, $\Xi_q(\beta, \mu_1, \mu_2, \mu_3)$ has valid only if $g_N > \frac{\alpha}{\beta}$.

3. q -EXPECTATION OF INTERNAL ENERGY, THE NUMBER OF PROTONS, NEUTRONS AND SPIN

Now we can calculate the internal energy of the physical system under consideration, using the partition functions obtained above. After similar transformations for the q -expectation value of the energy are written, the internal energy is obtained

$$U_q = \frac{1}{[\Xi_q(\beta)]^q \Gamma(-q\alpha)} \int_0^\infty d\xi \xi^{-\alpha} e^{-\xi} \Xi_1(\beta(q-1), \xi, \mu) U_1(\beta(q-1)\xi) \quad (26)$$

for $q > 1$, and

$$U_q = \frac{\Gamma(\alpha)}{[\Xi_q(\beta)]^q} \frac{i}{2\pi_c} \oint d\xi (-\xi)^{-\alpha} e^{-\xi} \Xi_1(\beta(q-1), \xi, \mu) U_1(\beta(q-1)\xi) \quad (27)$$

for $q < 1$.

Similar expressions are obtained for the q -expectation values of the number of neutrons, protons and spin

$$N_q = \frac{1}{[\Xi_q(\beta)]^q \Gamma(-q\alpha)} \int_0^\infty d\xi \xi^{-\alpha} e^{-\xi} \Xi_1(-\beta(1-q), \xi, \mu) N_1(\beta(q-1)\xi) \quad (28)$$

for $q > 1$, and

$$N_q = \frac{\Gamma(\alpha)}{[\Xi_q(\beta)]^q} \frac{i}{2\pi_c} \oint d\xi (-\xi)^{-\alpha} e^{-\xi} \Xi_1(\beta(q-1), \xi, \mu) N_1(\beta(q-1)\xi) \quad (29)$$

for $q < 1$. By using the integral representation of Wright function, i.e. Eq.(20), and after some algebra, the q -expectations of internal energy, the particle number (the number of protons and neutrons) and spin are obtained

$$U_q = \frac{\Gamma(\alpha)}{[\Gamma(1+\alpha)]^q} \left[\left(U_0 + \frac{M_1^2 \eta^{\frac{1}{1-q}}}{2g \langle m^2 \rangle} \right) \frac{\Phi(1, \alpha; \gamma_q)}{\Phi_q(1, 1+\alpha; \gamma_q)} + \frac{\gamma_q^2 \Phi(1, 2+\alpha; \gamma_q)}{\pi^2 g \Phi_q(1, 1+\alpha; \gamma_q)} \right]$$

$$N_q = \frac{\Gamma(\alpha) \mu_1 g_1}{[\Gamma(1+\alpha)]^q} \eta^2 \frac{\Phi(1, \alpha; \gamma_q)}{\Phi_q(1, 1+\alpha; \gamma_q)}, \quad z_q = \frac{\Gamma(\alpha) \mu_2 g_2}{[\Gamma(1+\alpha)]^q} \eta^2 \frac{\Phi(1, \alpha; \gamma_q)}{\Phi_q(1, 1+\alpha; \gamma_q)},$$

$$M_q = \frac{\Gamma(\alpha) \mu_3 g \langle m^2 \rangle}{[\Gamma(1+\alpha)]^q} \eta^2 \frac{\Phi(1, \alpha; \gamma_q)}{\Phi_q(1, 1+\alpha; \gamma_q)},$$

respectively, where $M_1 = \mu_3 g \langle m^2 \rangle$.

4. RESULTS AND DISCUSSION

In this study, we derive the grand partition functions for $q < 1$ (eq. (23)) and $q > 1$ (eq. (25)), and internal energy, number of neutrons, number of protons and spin (eq. (30)), probably first time, using the Tsallis second choice of constraints. We focus only on the generalized number of neutron and internal energy of the nucleus because another thermodynamical quantities presents the similar behaviour to the number of neutrons and it is adequate for our present purposes. Our results are obtained by using the eqs. (30) and (31) for ^{24}Na . In Figs.(1) and (2), the variation of the q -expectation value of the number of neutrons on the entropic index q is plotted for $\beta = 0.1$ and 0.5 respectively. We restrict the q values between 0.9 and 0.99. The reason for this is simply that the q -expectation value of number of neutrons becomes very large out of this range of q and there is a convergence problem at $q = 1$; very similar problem was also reported elsewhere [31]. In this region, q -expectation value of number of neutrons is sensitive to the q entropy index.

In Fig.(3), the dependence of the q -expectation value of the number of neutrons on the inverse temperature is illustrated for some q values. Number of neutrons increases with the inverse temperature. Our results exhibit similar behaviour to that of Fermi systems obtained in Refs. [32,33].

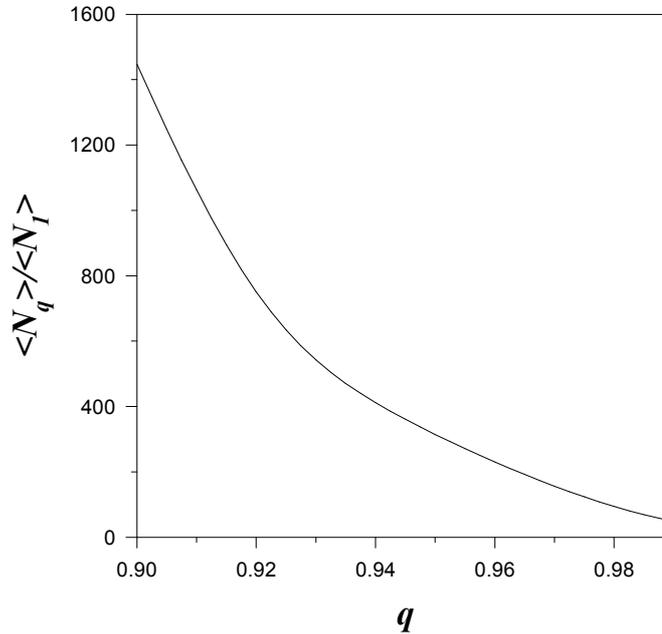


Figure 1. Plot of $\langle N \rangle_q / \langle N \rangle_1$ as a function of q for ^{24}Na and for q near 1 at $\beta = 1 \text{ (MeV)}^{-1}$.

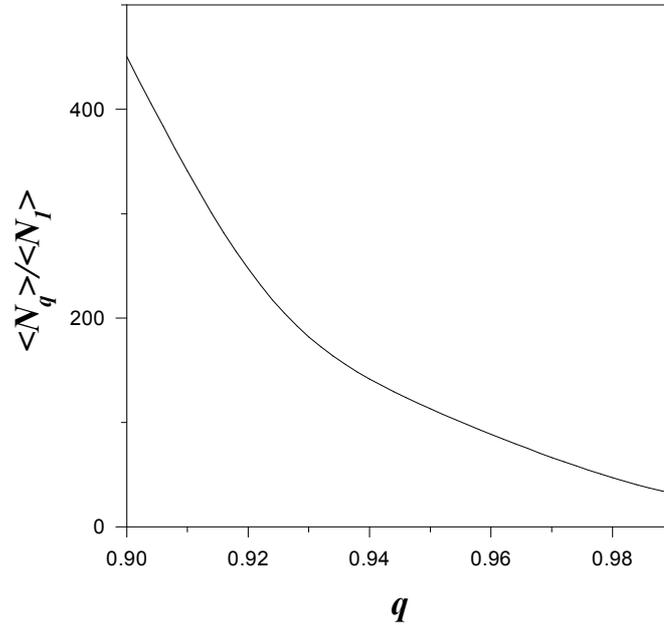


Figure 2. Plot of $\langle N \rangle_q / \langle N \rangle_1$ as a function of q for ^{24}Na and for q near 1 at $\beta = 0.5 \text{ (MeV)}^{-1}$.

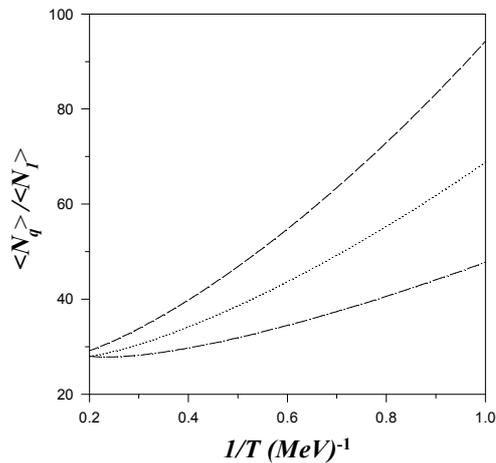


Figure 3. Plot of $\langle N \rangle_q / \langle N \rangle_1$ as a function of inverse temperature for some q values.

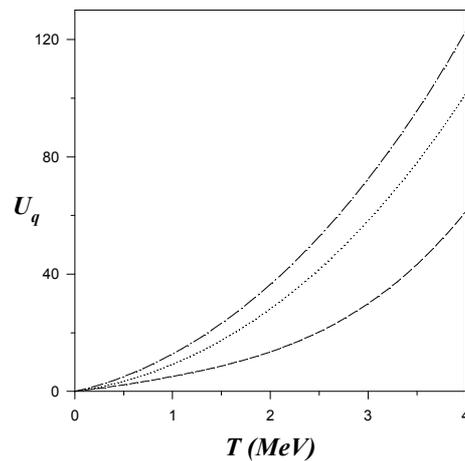


Figure 4. The variation of the mean internal energy of ^{24}Na with temperature for $q = 0.85, 0.9$ and 0.95 .

Finally, we plot the generalized mean energy vs. temperature for $q = 0.85, 0.9$ and 0.95 in Fig.(4). The behaviour of the generalized mean energy vs. temperature is typically in similar to that obtained from Fermi-Gas model proportional to T^2 . It is

worth to note that whereas there is a convergence problem at $q = 1$, the relevant region could be $q < 1$ because for $q > 1$ number of neutron and internal energy have discontinuities at different q values.

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