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AN APPROXIMATE METHOD FOR FREE VIBRATION ANALYSIS OF MULTI-BAY COUPLED SHEAR WALLS

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Abstract- In this study, an approximate method based on the continuum approach and transfer matrix method for free vibration analysis of multi bay coupled shear walls is presented. In this method the whole structure is idealized as sandwich beam. Initially the differential equation of this equivalent sandwich beam is written then shape function for each storey can be obtained by the solution of differential equations. By using boundary conditions and storey transfer matrices which are obtained by these shape functions**,** system modes and periods can be calculated. Reliability of the study is shown with a few examples. A computer program has been prepared in MATLAB computer algebra system and numerical samples have been solved for demonstration of the reliability of the method. The results of the samples display the convergence of the present method to the other methods used in literature**.**

Keywords- Coupled shear wall, Transfer matrix, Dynamic analysis.

1. INTRODUCTION

Shear walls are commonly used in tall buildings with the aim to increase the resistance to lateral loads. They are formed as coupled shear walls because of the rows of openings constituted for the architectural aspects such as windows, doors etc. Continuous connection method has been used for long time for static and dynamic analysis of coupled shear wall.

Rosman proposed a continuum medium method for a pair of high rise coupled shear walls [1]. The earlier investigation**s** of flexible foundation effects on the stresses and deformations of coupled shear walls were carried out by Coull [2]**.**

Basu gave design of charts for circular frequencies for coupled shear walls [3]. Li and Choo proposed a hybrid approach, based on the analysis of equivalent continuous medium and a discrete lumped mass system for free vibration analysis of stiffened pierced walls on flexible foundations [4].
Aksoğan *et al.* considered the forced vibration analysis of stiffened coupled shear walls with semi –rigid connections having stepwise changes in width [5].

In this study, an approximate method based on continuum system model and transfer matrix approach is suggested for the dynamic analysis of coupled shear walls.

2. ANALYSIS

Under the horizontal loads, coupled shear walls demonstrate neither Timoshenko beam, nor Euler-Bernoulli beam behavior. The behavior of coupled walls is equivalent to that of sandwich beam which denotes the total of these two types of behavior (Fig 1.). Initially the differential equation of this equivalent sandwich beam can be written. The flexural rigidity of sandwich beam consists of the sum of the flexural rigidity of shear walls near the openings. The shear rigidity of the sandwich beam consists of the sum of the connecting beam shear rigidities. The global flexural rigidity of the structural system can be calculated with the help of axial deformation of shear walls near the openings.

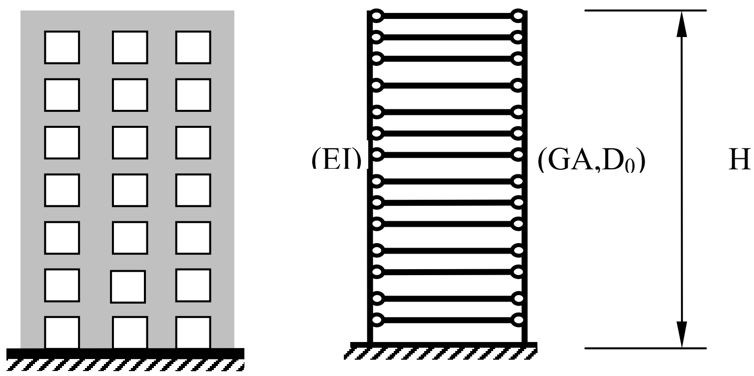

Figure 1. Mathematical model of the equivalent sandwich beam

Obtaining Storey Transfer matrices: Under the horizontal loads, equation of coupled shear wall of i. storey can be written as,

$$EI_i \frac{\partial^4 y_i}{\partial z^4} - GA_i \frac{\partial^2 y_i}{\partial z^2} + GA_i \frac{\partial \psi_i}{\partial z} = 0 \tag{1}$$

$$-GA_i \frac{\partial\ y_i}{\partial z} - D_{0i} \frac{\partial^2 \psi_i}{\partial z^2} + GA_i \psi_i = 0 \tag{2}$$

where y_i is the total shape function, ψ_i is the rotation angle of coupled shear wall because of bending, EI_i is the total bending rigidity of shear wall and D_0 is the bending rigidity which represents the axial deformation and can be calculated as given below,

$$D_0 = \sum EAd^2 \tag{3}$$

GA_i is the equivalent shear rigidity of connecting beams and can be found as follows

$$GA_i = \Sigma \frac{6EI_{bi}[(d_i + s_i)^2 + (d_i + s_{i+1})^2]}{d_i^3 h(1 + \frac{12pEI_{bi}}{Gd_i^2 A_{bi}})} \tag{4}$$

where, d_i is the distance between the i^{th} and $(i+1)^{th}$ walls, s_i is the width of the i^{th} wall, EI_{bi} and GA_{bi} are the flexural rigidity of connecting beam and the shear rigidity of connecting beams and ρ is the Poisson ratio [6].

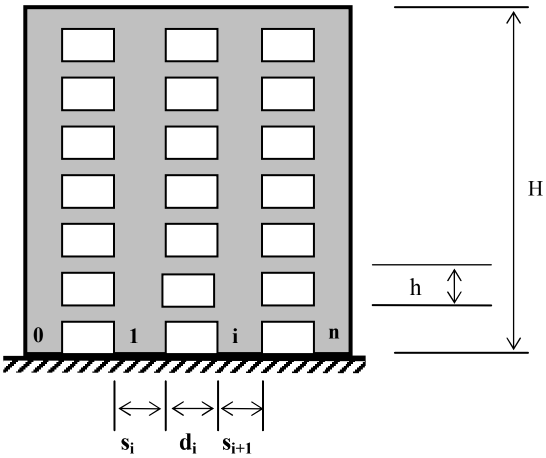

Figure 2. Coupled shear wall.

With the solution of (1) and (2) total shape function and rotation angle due bending can be obtained as

$$y_i(z) = c_1 + c_2 z + c_3 z^2 + c_4 z^3 + c_5 \cosh(a_i z) + c_6 \sinh(a_i z) \tag{5}$$

$$\psi(z) = c_2 + 2c_3 z + \left(3z^2 + 6b_i\right)c_4 + \left(-\frac{EI_i}{GA_i} a_i^2 + a_i\right)c_5 \sinh(a_i z) + \left(-\frac{EI_i}{GA_i} a_i^2 + a_i\right)c_6 \cosh(a_i z) \tag{6}$$

where

$$a_i = \sqrt{\left(1 + \frac{D_{0i}}{EI_i}\right)\frac{GA_i}{D_{oi}}} \tag{7}$$

With the help of equation (5), the total rotation angle, bending moment of shear wall (M_{wi}), bending moment because of the axial deformation (M_{axi}) and the total shear force (V_i) can be obtained as follows.

$$y'(z) = c_2 + 2c_3 z + 3c_4 z^2 + c_5 a_i \sinh(a_i z) + c_6 a_i \cosh(a_i z) \tag{8}$$

$$M_{wi}(z) = EI_i y_i^{11} = EI_i(2c_3 + 6c_4 z + c_5 a_i^2 \cosh(a_i z) + c_6 a_i^2 \sinh(a_i z)) \tag{9}$$

$$M_{axi}(z) = -D_{Oi} \psi_i^{\prime} = -D_{Oi}(2c_3 + 6c_4 z + c_6 f_i a_i \sinh(a_i z) + c_5 f_i a_i \cosh(a_i z)) \tag{10}$$

$$V_i(z) = EI_i \frac{d^3 y_i}{dz^3} - GA_i \frac{dy_i}{dz} + GA_i \psi_i$$
$$= c_4(6EI_i + 6GA_i b_i) + c_5 \sinh(a_i z)(GA_i(f_i - a_i) + EI_i a_i^3) + c_6 \cosh(a_i z)(GA_i(f_i - a_i) + EI_i) \tag{11}$$

where

$$f_i = \frac{-EI_i}{GA_i} a_i^3 + a_i \tag{12}$$

$$b_i = \frac{D_{0i}}{GA_i} \tag{13}$$

When the equations (5), (8),(6), (9), (10) and (11) are written in matrix form,

$$\begin{bmatrix} y_i(z) \\ y'_i(z) \\ \psi_i(z) \\ M_{wi}(z) \\ M_{axi}(z) \\ V(z) \end{bmatrix} = \begin{bmatrix} 1 & z & z^2 & z^3 & \cosh(a_i z) & \sinh(a_i z) \\ 0 & 1 & 2z & 3z^2 & a_i \sinh(a_i z) & a_i \cosh(a_i z) \\ 0 & 1 & 2z & (3z^2 + 6b_i) & f_i \sinh(a_i z) & f_i \cosh(a_i z) \\ 0 & 0 & 2EI_i & 6EI_i z & EI_i a_i^2 \cosh(a_i z) & EI_i a_i^2 \sinh(a_i z) \\ 0 & 0 & -2D_{0i} & -6D_{0i} z & -D_{0i} f_i a_i \cosh(a_i z) & -D_{0i} f_i a_i \sinh(a_i z) \\ 0 & 0 & 0 & (6EI_i + 6GA_i b_i) & \sinh(a_i z)(GA_i(f_i - a_i) + EI_i a_i^3) & \cosh(a_i z)(GA_i(f_i - a_i) + EI_i a_i^3) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{14}$$

is obtained.

At the initial point of the storey for z=0,

$$\begin{bmatrix} y_i(0) \\ y'_i(0) \\ \psi_i(0) \\ M_{wi}(0) \\ M_{axi}(0) \\ V_i(0) \end{bmatrix} = A_i(0) \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{15}$$

equation (15) is written above. When vector c is taken out from formula (15) and substituted into the equation (14), equation (16) is obtained.

$$\begin{bmatrix} y_i(z) \\ y'_i(z) \\ \psi_i(z) \\ M_{wi}(z) \\ M_{axi}(z) \\ V_i(z) \end{bmatrix} = A_i A_i^{-1}(0) \begin{bmatrix} y_i(0) \\ y'_i(0) \\ \psi_i(0) \\ M_{wi}(0) \\ M_{axi}(0) \\ V_i(0) \end{bmatrix} \tag{16}$$

where $T_i = A_i A_i^{-1}(0)$ is the storey transfer matrix for z=h.

3. DYNAMIC ANALYSIS

For free vibration analysis of coupled shear wall the structure is considered as a discrete lumped mass system as shown in Fig.3

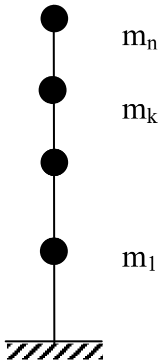

Figure 3.Discrete Model.

The values of m_i can be approximated by

$$m_1 = \frac{M_T}{2n} \qquad m_n = \frac{1.5M_T}{n} \tag{17}$$

$$m_k = \frac{M_T}{2n} \tag{18}$$

The storey transfer matrices which were obtained in equation (16) can be used for dynamic analysis of coupled shear wall. For this, when considering the inertial forces in storey, the relationship of , i. and i+1. storey can be written with this matrix equation.

$$\begin{bmatrix} y_{i+1} \\ y'_{i+1} \\ \psi_{i+1} \\ M_{pi+1} \\ M_{2ei+1} \\ V_{i+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ m_i\omega^2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \underset{-i}{T} \begin{bmatrix} y_i \\ y'_i \\ \psi_i \\ M_{pi} \\ M_{ei} \\ V_i \end{bmatrix} \tag{19}$$

Where, m_i is the mass of i. storey and ω is the natural frequencies. Dynamic transfer matrix can be shown as T_{di} below.

$$\underset{-}{T}_{di} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ m_i\omega^2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \underset{-i}{T} \tag{20}$$

The displacements and internal forces relationship of base and top of structures can be found as follows.

$$\begin{bmatrix} y_{top} \\ y'_{top} \\ \varphi t_{top} \\ M_{ptop} \\ M_{etop} \\ V_{top} \end{bmatrix} = \underset{-}{T}_{dn} \underset{-}{T}_{d(n-1)} \ldots\ldots.. \underset{-}{T}_{d1} \begin{bmatrix} y_{base} \\ y'_{base} \\ \varphi_{base} \\ M_{pbase} \\ M_{ebase} \\ V_{base} \end{bmatrix} \tag{21}$$

When the boundary condition is considered in equation (21), for nontrivial solution in $\underset{-}{t}_{d} = \underset{-}{T}_{dn} \underset{-}{T}_{dn-1} \underset{-}{T}_{dn-2} \ldots\ldots \underset{-}{T}_{d1}$ is obtained in equation (22).

$$\underset{-}{f} = \begin{bmatrix} t_{44} & t_{45} & t_{46} \\ t_{54} & t_{55} & t_{56} \\ t_{64} & t_{65} & t_{66} \end{bmatrix} \tag{22}$$

The value ω which makes the determinant zero is the natural frequencies of coupled shear wall.

4. PROCEDURE OF COMPUTATION

Procedure of computation of transfer matrix method is presented below step by step
1. Calculation of the structural properties of each storey (GA, EI, m,...)
2. Computation of storey transfer matrices for each storey using the structural properties obtained from step 1.
3. Computation of system transfer matrix with the help of storey transfer matrices.
4. Applying the boundary conditions and obtaining the nontrivial equation.
5. Determination of the angular frequencies by using any numerical method.
6. Using the circular frequencies for determination of modes by using the storey transfer matrices.

5. NUMERICAL EXAMPLES

In this section, to verify the present method, three numerical examples have been solved by a program written in MATLAB [7]. The results are compared with the ones which had been solved in literature.

Example 1. Treats the natural vibration analysis of the single bay coupled shear wall [8]. This shear wall rests on rigid foundation having 0.3048 m thickness and the following properties: d_l= 2.438 m, s_1= s_2= 6.096 m, H= 60.96 m, h=3.048 m A_{bi}= 0.2127 m^2, I_{bi}=8.63*10^{-3} m^4, ρ(density)= 24.05 kN/m^3, E=2.876*10^7 kN/m^2. The first and second natural frequencies found for this method and compared with those found in the literature.

Table 1. Comparison of natural frequencies in Example 1 (Hz).

Mode	Li and Choo	Matrix Progression (Aksogan *et al.*)	SAP 2000	Aksogan et *al.*	Present Method
1	2.08	2.05	2.08	2.08	2.01
2	9.34	8.78	9.34	9.34	8.98

Example 2. 12 storey coupled shear wall (Fig. 4) has been solved by using this procedure. This example solved by Ozmen *et al.* [9] and the results are compared in Table 2. The thickness of shear wall and connecting beams are 25 cm, the height of connecting beams are 60 cm and the weight of storey is 350 kN except the top storey has 250 kN.

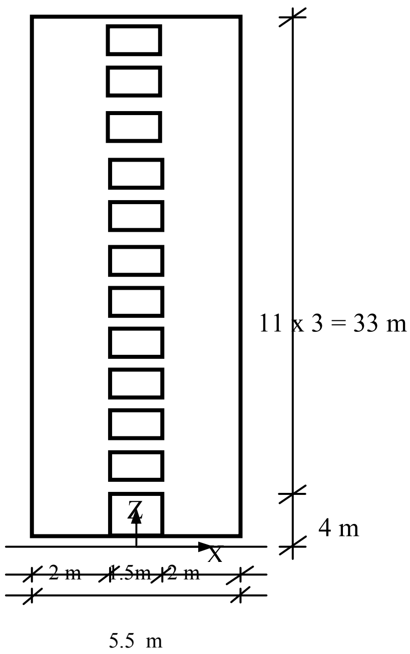

Figure 4. Coupled shear wall in example 2

Table 2.Comparison of natural periods in example 2.

Mode	Natural periods (sn)	
	This Study	Ozmen *et al.*
1.	0.94	0.96
2.	0.20	0.21
3.	0.09	0.09
4.	0.05	0.06
5.	0.04	0.04

Example 3. Treats a coupled shear wall with three bays, for which the geometric properties are seen in Figure 4. The physical properties of the wall are as follows: $E=20*10^{10}$ kN/m^2, ρ=24kN/m^3.The height of connecting beams is 80 cm and the thickness of the wall is 16 cm in everywhere. Free vibration analysis is carried by the present method and is compared with those found in the literature [10].

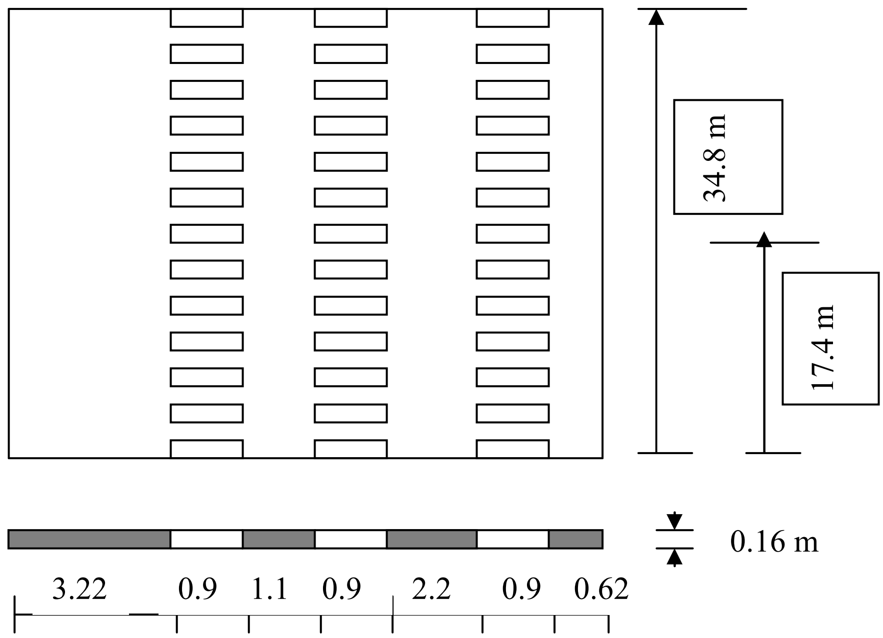

Figure 5. Geometric Properties of Example

Table 3. Comparison of natural frequencies in example 3 (Hz).

Mode	SAP 2000	Bikce	This study
1	3.0030	3.0299	3.0628
2	11.8810	12.0035	12.1803
3	25.8336	25.9693	26.1780
4	42.8116	42.8838	42.7350
5	64.0564	64.0681	63.2911
6	89.2601	89.4336	89.9565
7	118.3703	119.0197	114.9425
8	150.5239	152.1349	144.9275
9	184.3159	187.3833	178.5714
10	217.0551	222.0005	212.7660

6.CONCLUSION

In this study, an approximate method based on continuum system model and transfer matrix approach is suggested for the dynamic analysis of coupled shear walls. In this method the whole structure is idealized as sandwich beam. Examples demonstrate good agreement with the finite element method. The proposed method is simple and accurate enough to be used both at the concept design stage and final analyses. This method is suitable for implementation on any programs.

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