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A COMPARISON BETWEEN ACTIVE AND PASSIVE VIBRATION CONTROL OF NON-LINEAR SIMPLE PENDULUM PART II: LONGITUDINAL TUNED ABSORBER AND NEGATIVE $G\ddot{\varphi}$ AND $G\varphi^n$ FEEDBACK

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Abstract- In part I [1] we dealt with a tuned absorber, which can move in the transversally direction, where it is added to an externally excited pendulum. Active control is applied to the system via negative velocity feedback or its square or cubic value. The multiple time scale perturbation technique is applied throughout. An approximate solution is derived up to second order approximation. The stability of the system is investigated applying both frequency response equations and phase plane methods. The effects of the absorber on system behavior are studied numerically. Optimum working conditions of the system are obtained applying passive and active control methods. Both control methods are demonstrated numerically. In this paper, a tuned absorber, in the longitudinal direction, is added to an externally excited pendulum. Active control is applied to the system via negative acceleration feedback or via negative angular displacement or its square or cubic value. An approximate solution is derived up to the second order approximation for the system with absorber. The stability of the system is investigated applying both frequency response equations and phase plane methods. The effects of the absorber on system behavior are studied numerically. Optimum working conditions of the system are extracted when applying both passive and active control methods.
Keywords- Spring-pendulum, Absorber, Active and passive control.

1. INTRODUCTION

Vibrations and dynamic chaos are undesired phenomenon in structures. They cause disturbance, discomfort, damage and destruction of the system or the structure. For these reasons, money, time and effort are spent to get rid of both vibrations and noise or chaos or to minimize them. One of the most effective tools for passive vibration control is the dynamic absorber or the damper or the neutralizer [2]. Eissa [3] has shown that a non-linear absorber can be used to control the vibration of a non-linear system. Also, he has shown that the non-linear absorber widens its range of applications, and its damping coefficient should be kept minimum for better performance [4]. Cheng-Tang Lee et al. [5] demonstrated a dynamic vibration absorber system, which can be used to reduce speed fluctuations in rotating machinery. Eissa and El-Ganaini [6,7] studied the control of both vibration and dynamic chaos of both internal combustion engines and mechanical structures having quadratic and cubic non-linearties, subjected to harmonic excitation using single and multi-absorbers. Active constrained layer damping (ACLD) has been successfully utilized as effective means of damping out the vibration of various flexible structures [8-13]. A variable stiffness vibration absorber without damping is used for controlling the principal mode of a vibrating structure. The optimal vibration absorber is also utilized for controlling higher mode [14]. Another approach of active damping of mechanical structures is the hybrid system, which is a combination of semi-active and active treatments, in which the advantages of individual schemes are combined, while eliminating their shortcomings [15]. Active damping of mechanical structures can be utilized using piezoceramic sensors and actuators [16-17]. The vibration of rotating machinery is suppressed by eliminating the root cause of the vibration system imbalance [18].

In the present paper, a tuned absorber, which can move in the longitudinal direction, is added to an externally excited pendulum, which is described by a second order non-linear differential equation having both quadratic and cubic non-linearties, subjected to harmonic excitation. Active control is applied to the system via negative acceleration feedback or via negative feed back of angular displacement or its square or cubic value. The multiple time scale perturbation technique is applied throughout. An approximate solution is derived up to the second order approximation for the system with absorbers. The stability of the system is investigated applying both frequency response equations and phase plane methods. The effects of the absorber on system behavior are studied numerically. Optimum working conditions of the system are obtained applying both passive and active control methods. Both control methods are compared numerically.

2. MATHEMATICAL MODELING

The considered system is shown in Fig. 1. As reported in part I, the kinetic and potential energies are given in the following forms respectively:

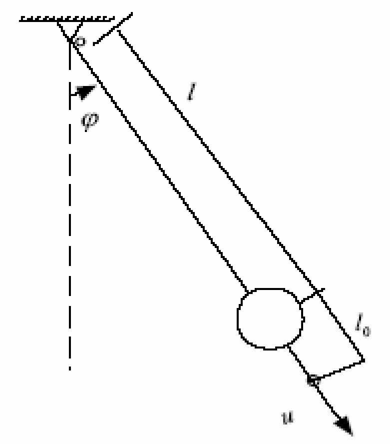


Fig.1. Schematic representation of system

$$T = \frac{1}{2} M\, l^2 \dot{\varphi}^2 + \frac{1}{2} m[\dot{u}^2 + (l + l_0 + u)^2 \dot{\varphi}^2] \tag{1}$$

$$U = Mg\, l(1 - \cos\varphi) + mg(l + l_0 + u)(1 - \cos\varphi) + \frac{1}{2} k_1 u^2 + \frac{1}{4} k_3 u^4 \tag{2}$$

Applying Lagrangian equations and taking into account the effects of linear viscous damping and external excitation on the main system, the following differential equations of motion are obtained:

$$\ddot{\varphi}+c_1\dot{\varphi}+\omega_1^2\sin\varphi+\beta\{[2(l+l_0)+u]u\ddot{\varphi}+2(l+l_0+u)\dot{u}\dot{\varphi}+gu\sin\varphi\}=f\cos\Omega t \quad (3)$$

$$\ddot{u}+c_2\dot{u}+\omega_2^2u+\alpha u^3-(l+l_0+u)\dot{\varphi}^2+g(1-\cos\varphi)=0 \quad (4)$$

where α is the spring stiffness non-linear parameter, ω_1 & ω_2 are the natural frequencies, c_1 & c_2 are the linear damping coefficients of the pendulum and absorber respectively, f is the forcing amplitude and Ω is the forcing frequency of the pendulum. It is assumed that both u and φ are small, and the whole motion is a planer one. Due to these assumptions, both ($\sin\varphi$) and ($\cos\varphi$) can be written in the form:

$$\cos\varphi\cong1-\frac{\varphi^2}{2!}+\frac{\varphi^4}{4!},\qquad \sin\varphi\cong\varphi-\frac{\varphi^3}{3!} \quad (5, 6)$$

The damping coefficients and the forcing amplitudes are assumed to be in the form:

$$c_n=\varepsilon\,\hat{c}_n\;,\; f=\varepsilon^2\hat{f} \qquad n=1, 2. \quad (7)$$

where ε is a small perturbation parameter and $0<\varepsilon<<1$.
Eqns. (3) and (4) can be re-written in the form:

$$\ddot{\varphi}+\varepsilon\hat{c}_1\dot{\varphi}+\omega_1^2\sin\varphi+\beta\{[2(l+l_0)+u]u\ddot{\varphi}+2(l+l_0+u)\dot{u}\dot{\varphi}+gu(\varphi-\frac{\varphi^3}{6})\}=\varepsilon^2\hat{f}\cos\Omega t \quad (8)$$

$$\ddot{u}+\varepsilon\hat{c}_2\dot{u}+\omega_2^2u+\alpha u^3-(l+l_0+u)\dot{\varphi}^2+g(\frac{\varphi^2}{2}-\frac{\varphi^4}{4})=0 \quad (9)$$

Assuming the solution of equations (8) and (9) to be in the form

$$\varphi(t;\varepsilon)=\varepsilon\varphi_1(T_0,T_1)+\varepsilon^2\varphi_2(T_0,T_1)+.... \quad (10)$$

$$u(t;\varepsilon)=\varepsilon u_1(T_0,T_1)+\varepsilon^2u_2(T_0,T_1)+.... \quad (11)$$

where $T_n=\varepsilon^n t$, $(n=0,1)$. The derivatives will be in the forms

$$\frac{d}{dt}=D_0+\varepsilon D_1+....,\qquad \frac{d^2}{dt^2}=D_0^{\;2}+2\varepsilon D_0D_1+\ldots \quad (12, 13)$$

where $Dn=\frac{\partial}{\partial Tn}$, n = 0,1. Equating the similar powers of ε in both side's yields.

$$(D_0^2+\omega_1^2)\varphi_1=0 \quad (14)$$

$$(D_0^2+\omega_2^2)u_1=0 \quad (15)$$

$$(D_0^2+\omega_1^2)\varphi_2=-2D_0D_1\varphi_1-\hat{c}_1D_0\varphi_1-2\beta(l+l_0)u_1D_0^2\varphi_1-2\beta(l+l_0)D_0u_1D_1\varphi_1-\beta gu_1\varphi_1+\frac{\hat{f}}{2}\exp(i\Omega T_0) \quad (16)$$

$$(D_0^2+\omega_2^2)u_2=-2D_0D_1u_1-\hat{c}_2D_0u_1+(l+l_0)(D_0\varphi_1)^2-g\varphi_1/2 \quad (17)$$

The general solutions of Eqns. (14) and (15) can be written in the form

$$\varphi_1=B_1\exp(i\,\omega_1T_0)+cc \quad (18)$$

$$u_1 = B_2 \exp(i\,\omega_2 T_0) + cc \tag{19}$$

where B_1 and B_2 are complex functions in T_1 , which can be determined from eliminating the secular terms at the next approximation, and *cc* represents the complex conjugates. Substituting from Eqns. (18) and (19) into Eqns. (16) and (17) and eliminating the secular terms, then the first-order approximation is obtained as:

$$\varphi_2 = Q_1 \exp(i\omega_1 T_0) + Q_2 \exp(i(\omega_1 + \omega_2)T_0) + Q_3 \exp(i(\omega_1 - \omega_2)T_0) + Q_4 \exp(i\Omega T_0) + cc \tag{20}$$

$$u_2 = R_1 \exp(i\omega_2 T_0) + R_2 \exp(i\omega_1 T_0) + R_3 \exp(2i\omega_1 T_0) + R_4 + cc \tag{21}$$

where $Q_i, R_i (i = 1,2,3,4)$ are complex functions in T_1 and *cc* represents the complex conjugates. From the above-derived solutions, the reported resonance cases are:

a- Primary resonance (1) $\Omega \cong \omega_1$ (2) $\Omega \cong \omega_2$

b- Sub-harmonic and super-harmonic resonance (1) $\Omega \cong 2\omega_1$ (2) $\Omega \cong \omega_2 / 2$

c- Internal or secondary resonance (1) $\omega_1 \cong \omega_2$ (2) $2\omega_1 \cong \omega_2$

d- Simultaneous or incident resonance

Any combination of the above resonance cases is considered as simultaneous or incident resonance.

3. STABILITY OF THE SYSTEM

Using the simultaneous primary resonance conditions $\Omega = \omega_1 + \varepsilon\hat{\sigma}_1$, $\omega_1 = \omega_2 + \varepsilon\hat{\sigma}_2$ (where $\sigma_1 = \varepsilon\hat{\sigma}_1$ and $\sigma_2 = \varepsilon\hat{\sigma}_2$ are called detuning parameters) and eliminating the secular terms leads to solvability conditions.

$$2i\omega_1 D_1 B_1 = -i\hat{c}_1\omega_1 B_1 + \frac{\hat{f}}{2}\exp(i\hat{\sigma}_1 T_1) \tag{22}$$

$$2i\omega_2 D_1 B_2 = -i\hat{c}_2\omega_2 B_2 - \frac{g}{2} B_1 \exp(i\hat{\sigma}_2 T_1) \tag{23}$$

Putting $$B_n = \frac{\hat{b}_n}{2}\exp(i\eta_n) \quad , \quad b_n = \varepsilon\hat{b}_n \quad \text{n} = 1, 2 \tag{24}$$

where $b_n \& \eta_n$ are the steady state amplitudes and the phases of the motions respectively. Substituting from equation (24) into equations (22)-(23) and equating the real and imaginary parts we obtain:

$$\dot{b}_1 = -\frac{c_1 b_1}{2} + \frac{f}{2\omega_1}\sin(\theta_1) \tag{25}$$

$$b_1\dot{\theta}_1 = b_1\sigma_1 + \frac{f}{2\omega_1}\cos(\theta_1) \tag{26}$$

$$\dot{b}_2 = -\frac{c_2 b_2}{2} - \frac{g b_1}{4\omega_2}\sin(\theta_2) \tag{27}$$

$$b_2(\dot{\theta}_1 + \dot{\theta}_2) = b_2(\sigma_1 + \sigma_2) - \frac{g\,b_1}{4\omega_2}\cos(\theta_2) \tag{28}$$

where $\theta_1 = \hat{\sigma}_1 T_1 - \eta_1$ *and* $\theta_2 = \hat{\sigma}_2 T_1 + \eta_1 - \eta_2$

The periodic motions are obtained when $\dot{a}_n = \dot{\theta}_n = 0$. Hence, the fixed points of Eqns. (25)- (28) are given by

$$-\frac{c_1 b_1}{2} + \frac{f}{2\omega_1}\sin(\theta_1) = 0 \quad (29)$$

$$b_1\sigma_1 + \frac{f}{2\omega_1}\cos(\theta_1) = 0 \quad (30)$$

$$-\frac{c_2 b_2}{2} - \frac{g b_1}{4\omega_2}\sin(\theta_2) = 0 \quad (31)$$

$$b_2(\sigma_1 + \sigma_2) - \frac{g b_1}{4\omega_2}\cos(\theta_2) = 0 \quad (32)$$

There are three possibilities in addition to the trivial solution. They are:

(1) $b_1 \neq 0$, $b_2 = 0$ **(2)** $b_2 \neq 0$, $b_1 = 0$ **(3)** $b_1 \neq 0$, $b_2 \neq 0$

Table (1) illustrates the corresponding frequency response equation for each case:

Table (1) Frequency response equation for each case.

No	Case	Frequency response equation (FRE)
1	$b_1 \neq 0$, $b_2 = 0$	$b_1^2\sigma_1^2 + \frac{c_1^2 b_1^2}{4} - \frac{f^2}{4\omega_1^2} = 0$
2	$b_2 \neq 0$, $b_1 = 0$	$b_2^2(\sigma_1^2 + \sigma_2^2) + \frac{c_2^2 b_2^2}{4} = 0$ As $b_2 \neq 0$ then $(\sigma_1^2 + \sigma_2^2) + \frac{c_2^2}{4} = 0$ *
3	$b_1 \neq 0$, $b_2 \neq 0$	$b_1^2\sigma_1^2 + \frac{c_1^2 b_1^2}{4} - \frac{f^2}{4\omega_1^2} = 0$ $b_2^2(\sigma_1^2 + \sigma_2^2) + \frac{c_2^2 b_2^2}{4} - \frac{g^2 b_1^2}{16\omega_2^2} = 0$

* This equation gives nothing, this means that we will never get $b_1 = 0$.

3.1 Stability of the fixed points

To analyze the stability of the fixed points, one lets

$$b_n = b_{n0} + b_{n1}, \theta_n = \theta_{n0} + \theta_{n1} \quad (33)$$

where a_{n0}, θ_{n0} are the solutions of Eqns. (29)-(32). Inserting Eq. (33) into Eqns. (25)-(28) and keeping only the linear terms in a_{n1}, θ_{n1}, we get

$$\dot{b}_{11} = -\frac{c_1 b_{11}}{2} + \frac{f}{2\omega_1}\theta_{11}\cos(\theta_{10}) \quad (34)$$

$$\dot{b}_{21} = -\frac{c_2 b_{21}}{2} - \frac{g}{4\omega_2}\{b_{11}\sin(\theta_{20}) + b_{10}\theta_{21}\cos(\theta_{20})\} \quad (35)$$

$$\dot{\theta}_{11} = \frac{b_{11}\sigma_1}{b_{10}} - \frac{f}{2\omega_1 b_{10}}\theta_{11}\sin(\theta_{10}) \tag{36}$$

$$\dot{\theta}_{21} = \frac{b_{21}\sigma_2}{b_{20}} - \frac{g}{4\omega_2 b_{20}}\{b_{11}\cos(\theta_{20}) - b_{10}\theta_{21}\sin(\theta_{20})\} - \frac{b_{11}\sigma_1}{b_{10}} + \frac{f}{2\omega_1 b_{10}}\theta_{11}\sin(\theta_{10}) \tag{37}$$

The stability of a particular fixed point with respect to perturbations proportional to exp (λT_1) depends on the real parts of the roots of the matrix. Thus a fixed point given in Eqns. (34)-(37) is an asymptotically stable if and only if the real parts of all roots of the matrix are negative.

To study the stability of the fixed points corresponding to case (1), we let $b_{21} = \theta_{21} = 0$ in Eqns. (34)-(37), and obtain the eigenvalues

$$\lambda = (L_1 + L_4) \pm \sqrt{(L_1 + L_4)^2 + 4(L_2 L_3 - L_1 L_4)} \tag{38}$$

where $L_1 = -\frac{c_1}{2}$, $L_2 = \frac{f}{2\omega_1}\cos(\theta_1)$, $L_3 = \frac{\sigma_1}{b_1}$ and $L_4 = -\frac{f}{2\omega_1 b_1}\sin(\theta_1)$ (39)

And hence the fixed points are unstable if and only if

$$L_2 L_3 > L_1 L_4 \tag{40}$$

Otherwise they are stable.

To study the stability of the fixed points corresponding to case (2), we let $b_{11} = \theta_{11} = 0$ in Eqns. (34)-(37), and obtain the eigenvalues

$$\lambda = (L_5 + L_8) \pm \sqrt{(L_5 + L_8)^2 + 4(L_6 L_7 - L_5 L_8)} \tag{41}$$

Where

$L_5 = -\frac{c_2}{2}$, $L_6 = -\frac{g}{4\omega_2} b_1 \cos(\theta_2)$, $L_7 = \frac{\sigma_2}{b_2}$ and $L_8 = \frac{g}{4\omega_2 b_2} b_1 \sin(\theta_2)$ (42)

And hence the fixed points are unstable if and only if

$$L_6 L_7 > L_5 L_8 \tag{43}$$

Otherwise they are stable.

For the stability of the fixed points corresponding to case (3), the eigenvalues are given by the equation

$$\lambda^4 + R_1\lambda^3 + R_2\lambda^2 + R_3\lambda + R_4 = 0 \tag{44}$$

Where R_1, R_2, R_3 and R_4 are functions of the parameters (b_1, b_2, ω_1, ω_2, σ_1, σ_2, f, θ_1, θ_2). According to the Routh-Huriwitz criterion, the necessary and sufficient conditions for all the roots of Eqn. (44) to possess negative real parts is that

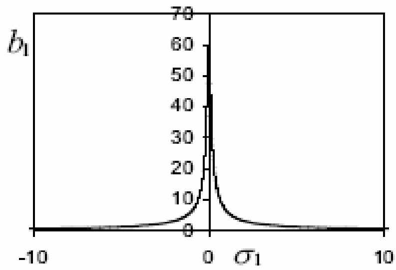
$R_1 > 0$, $R_1R_2 - R_3 > 0$, $R_3(R_1R_2 - R_3) - R_1^2 R_4 > 0$, $R_4 > 0$ (45)

4. RESULTS AND DISCUSSIONS

Results are presented in graphical forms as steady state amplitudes against detuning parameters and as time history or the response for both system and absorber. A good criterion of both stability and dynamic chaos is the phase-plane trajectories, which are shown for some cases. In the following sections, the effects of the different parameters on response and stability will be investigated. Also different primary resonance cases are studied and discussed.

4.1. System stability

Fig. 2a, shows the effects of the detuning parameter σ_1 on the steady state amplitude of the main system b_1 for the stability first case, where $b_1 \neq 0$ and $b_2 = 0$. It can be seen from the figure that the maximum steady state amplitude occurs at primary resonance when $\Omega \cong \omega_1$. Fig. 2b shows that the steady state amplitude of the main system is a monotonic decreasing function to the natural frequency ω_1.



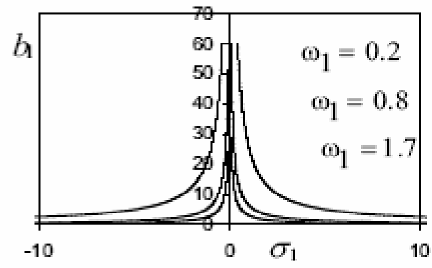
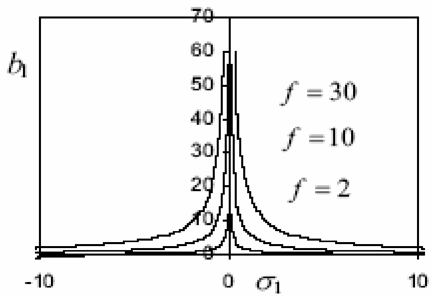


Fig.2a. Effects of the detuning parameter σ_1 *Fig.2b. Effects of the natural frequency ω_1*

$\omega_1 = 0.8$; $f = 10$; $c_1 = 0.2$.



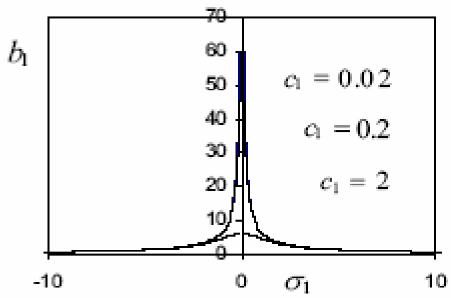
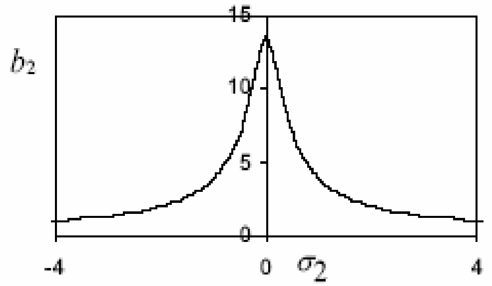


Fig.2c. Effects of excitation amplitude f *Fig.2d. Effects of damping coefficient c_1*

Fig. 2c shows that the steady state amplitude of the main system is a monotonic increasing function to the excitation amplitude f. Fig. 2d shows that the steady state amplitude of the main system is a monotonic decreasing function to the damping coefficient c_1.

Figure 3, shows the effects of the detuning parameter σ_2 on the steady state amplitude of the absorber b_2 for the stability third case, where $b_1 \neq 0$ and $b_2 \neq 0$. It can be seen from the figure that the maximum steady state amplitude occurs at internal resonance when $\omega_1 \cong \omega_2$. Fig. 3b shows that the steady state amplitude of the absorber is a monotonic increasing function to the steady state amplitude of the main system b_1. Figs. (3c-3e) shows that the steady state amplitude of the absorber is a monotonic decreasing function in the natural frequency ω_2, detuning parameter σ_1 and damping coefficient c_2.



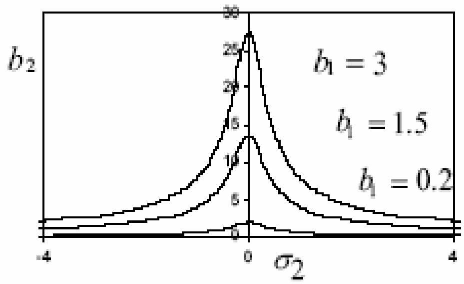
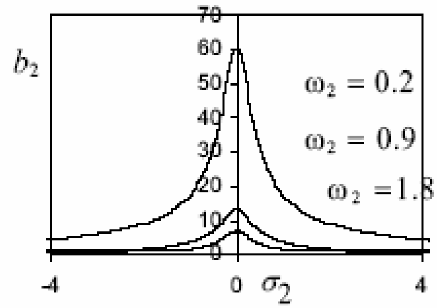


Fig.3a. Effects of the detuning parameter σ_2 *Fig.3b. Effects of the amplitude* b_1

$\omega_2 = 0.9$; $b_1 = 1.5$; $c_2 = 0.02$; $\sigma_1 = 0.3$.



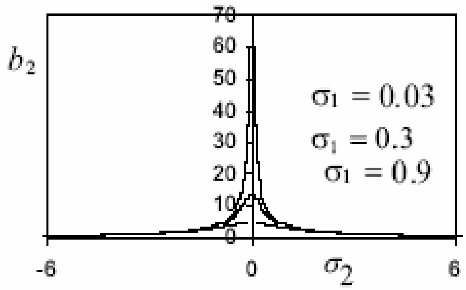


Fig.3c. Effects of the natural frequency ω_2 *Fig.3d. Effects of the detuning parameter* σ_1

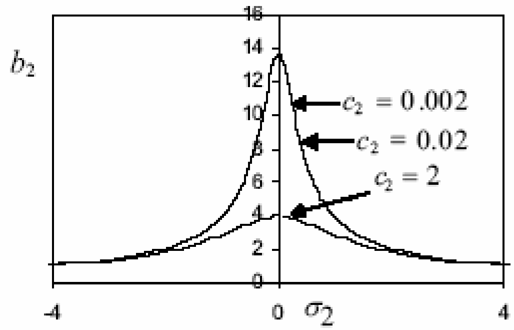


Fig.3e. Effects of damping coefficient c_2

Fig. 4 shows the effect of the non-linear parameter α on the main system and absorber. From the figure, we can see that the steady state amplitude of the main system is a monotonic increasing function for $\alpha \le 25$ and for increasing value we obtain the saturation phenomena. Also the steady state amplitude of the absorber is a monotonic decreasing function in the non-linear parameter α and for increasing value we obtain the saturation phenomena.

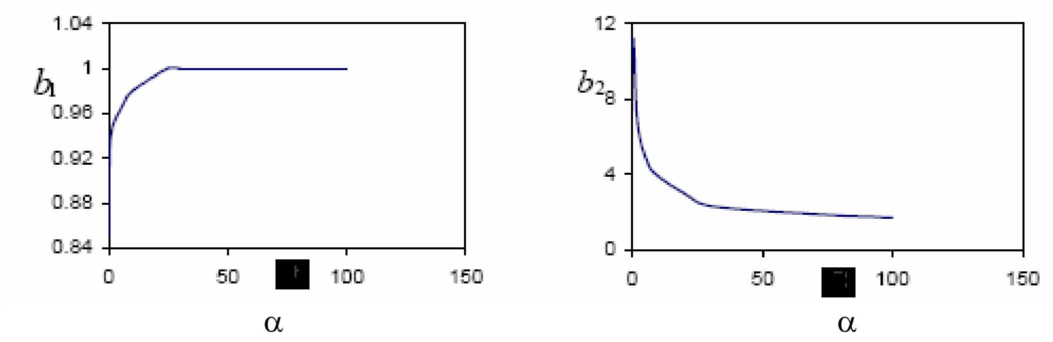


Fig.4. Effects of the non − linear parameter α

4.2. Passive control

In the following section we will discuss the effects of the absorber on pendulum response, stability and dynamic chaos at the worst resonance case. This case is the primary one, where $\Omega \cong \omega_1$. Fig. 5 illustrates both the response and the phase plane for this case. The steady state response without absorber in this case $\Omega \cong \omega$ is about 130%, of the excitation amplitude; the system is stable and free of dynamic chaos.

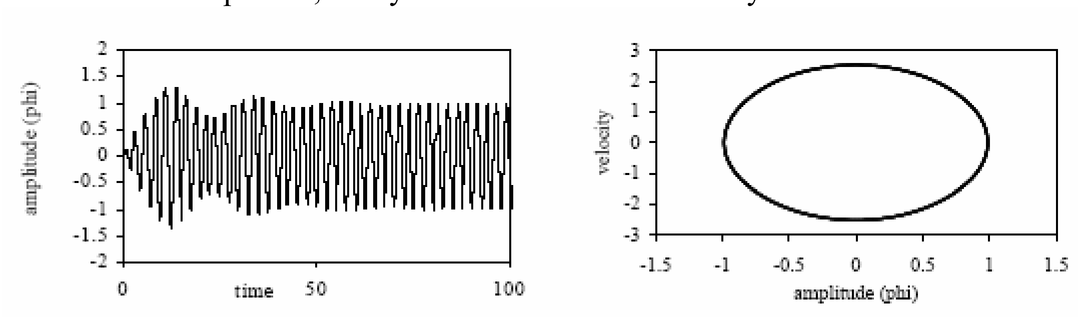
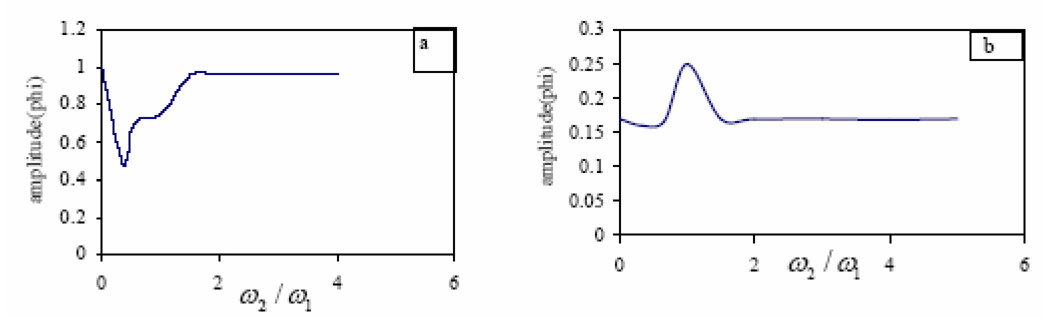


Fig.5. System behavior without absorber at primary resonance $\Omega \cong \omega_1$

Effects of the absorber: Figs. (6a-6b) illustrate the results when the absorber is effective for the different resonance cases. They are $\Omega \cong \omega_1$ and $\Omega \cong 2\omega_1$. Simultaneously the ratio ω_2/ω_1 is varied between zero and 6, i.e., $0 \leq \omega_2/\omega_1 \leq 6$. It can be seen for the first case shown in Fig. 6a that the effectiveness of the absorber E_a is about 2. Best results for the absorber were obtained when $3\omega_2 \cong \omega_1$. Fig. 6b shows that the absorber is ineffective as it may increase the amplitude with its maximum amplitude value occurs when $\omega_2 \cong \omega_1$. A common feature for all cases is the occurrence of saturation phenomenon.



Figs. 6. The effects of variation in $a)\Omega \cong \omega_1$ $b)\Omega \cong 2\omega_1$

4.3. Active control

Active control is applied to improve the behavior of the simple pendulum at the primary resonance case $\Omega \cong \omega_1$. First case**,** we considered negative acceleration feedback. The equation of motion in this case is:

$$\ddot{\varphi} + \varepsilon \hat{c}_1 \dot{\varphi} + \omega_1^2 \sin(\varphi) + G\ddot{\varphi} = f \cos \Omega t \tag{46}$$

Where G is the gain. Here, we are concerned with the effect of the gain G on the pendulum response. From Fig. 7a, we can see that the steady state amplitude is a monotonic decreasing function in the gain and it is decreased to about 2% of the steady state amplitude, and more increase of the gain G leads to saturation phenomena. For second case vibration is controlled via negative angular displacement feedback or its square or cubic value. The equation of motion in this case is:

$$\ddot{\varphi} + \varepsilon \hat{c}_1 \dot{\varphi} + \omega_1^2 \sin(\varphi) + G\varphi^n = f \cos \Omega t \tag{47}$$

Three cases will be considered, when n=1, 2 and 3. For n=1, the amplitude is increasing up to G=0.1. Then for the region $0.1<G<1.2$ the system is unstable. For the region $1.2 \leq G \leq 1.3$ the system is stable with increasing amplitude. When $3.4 \leq G$, the system is stable with decreasing amplitude as shown in Fig. 7b. This means that G should be greater the 10 to control the system where E_a=7, at saturation beginning.

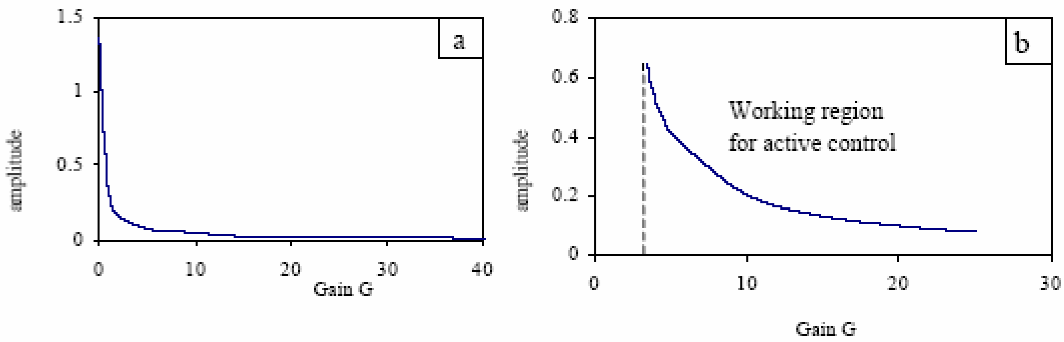


Fig. 7. The effects of the gain G a) negative acceleration feedback.
b) negative angular deflection feedback.

For n=2, the amplitude is increasing up to G=0.1. Then for the region $0.1<G<0.4$ the system is unstable. For the region $0.4 \leq G \leq 0.8$ the system is stable with increasing amplitude. For the region $0.9 \leq G<1.3$ the system is unstable, and for $1.3 \leq G \leq 1.4$ the system is stable with increasing amplitude. When $2.8 \leq G$, the system is stable with decreasing amplitude as shown in Fig. 7c. This means that G should be greater the 10 to control the system where E_a=3, at saturation beginning. It is clear that with active control, care should be taken because the system may be lead to instability instead of reducing the amplitude. For (n=3), Fig. 7d, shows that for $G \leq 1$ the steady state amplitude is a monotonic increasing function in the gain G and it is increased to about 300% of the steady state amplitude. For $G>1$ the steady state amplitude is a monotonic decreasing function and it is decreased to about 30% of the steady state amplitude.

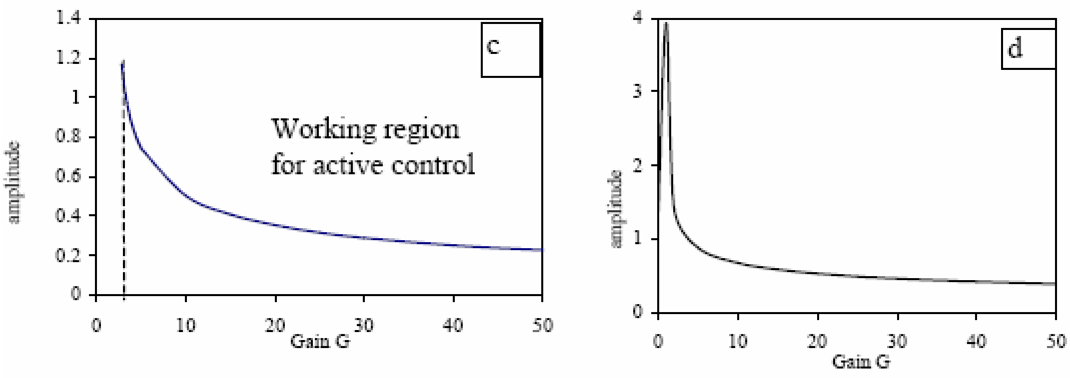


Fig. 7. Effects of the gain G c) negative square angular deflection feedback.
d) negative cubic angular deflection feedback.

5. CONCLUSIONS

From the former results, the following may be concluded.

1- The steady state amplitude of the main system is a monotonic increasing function in the excitation amplitude f.
2- The steady state amplitude of the main system is a monotonic decreasing function in its natural frequency ω_1 and damping coefficient c_1.
3- For passive control the effectiveness of the absorber for the system is about E_a=2 when $\Omega \cong \omega_1$, $\omega_2 \cong \omega_1$ and best results for the absorber is when $3\omega_2 \cong \omega_1$.
4- Non-effective absorber is obtained when $\Omega \cong \omega_1$, $\omega_2 \geq 1.5\omega_1$ or $\Omega \cong 2\omega_1$.
5- The vibration of the system can be controlled actively via negative angular displacement feedback, which can be used to reduce the amplitude of the system to 5% of the original value.
6- For all cases of active control, occurrence of saturation phenomena is noticed.

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