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DETERMINATION OF THE STEADY STATE RESPONSE OF EFEF / VFVF SUPPORTED RECTANGULAR SPECIALLY ORTHOTROPIC PLATES

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Abstract-Determination of The Steady State Response of EFEF / VFVF Supported Rectangular Specially Orthotropic Plates is presented. EFEF and VFVF plates, these are rectangular plates with two opposite edges elastically or viscoelastically supported and remaining others free. Using the energy based finite difference method; the problem is modeled by a kind of finite difference element. Due to the significance of the fundamental frequency of the plate, its variation was investigated with respect to mechanical properties of plate material and translational spring coefficient of supports. The steady state response of viscoelastically supported plates was also investigated numerically for various damping coefficients. In the numerical examples, the natural frequency parameters and steady state responses to a sinusoidally varying force are assessed for the fundamental mode. Convergence studies are made. Many new results have been presented. Considered problems are solved within the frame work of Kirchhoff-Love hypothesis.
Keywords- SFSF, EFEF, VFVF, viscoelastic, steady state.

1. INTRODUCTION

It is generally accepted that classical support conditions employed in the analysis of rectangular plate behavior represent only limiting mathematical conditions. The actual boundary conditions of a real system are mostly not classical, for example in ship plating, machine tables, circuit boards, solar panels, bridge decks, aircraft and marine structures supports are generally accepted are elastic. In addition, rectangular plates, with two opposite edges supported and remaining others free, extensively use in many branches of modern industry, these panels and plates are fabricated from composite materials. Therefore, the present investigation may be considered to be a problem of the mechanics of elements fabricated from composite materials.
There are lots of work has been undertaken for the analysis of a rectangular plate in the case of free and forced vibrations in literature. Extensive investigation has been carried out on the analysis of the free vibration of rectangular plates having classical boundary conditions, [1-6] and elastically restrained edges [7-31] has been widely analyzed. Viscoelastically supported plates studied by several researchers for point supported plate systems. Yamada and co-workers [32] studied free vibrations of elastically point-supported plates and forced vibrations of viscoelastically point-supported isotropic plates. Kocatürk and Altıntaş [33, 34] extended Yamada's [32] problem in case of anisotropic plates by using finite difference technique.

In this paper, plate problems are studied particularly for the case of boundary conditions elastically and viscoelastically restrained against translation.

A review of the related literature reveals that this problem has not heretofore been properly addressed. Prompted by lack of research work in this area, this paper aims to provide some vibration solutions for plates systems. The accuracy of the results was partially shown by comparing results available from other sources wherever possible.

2. ANALYSIS

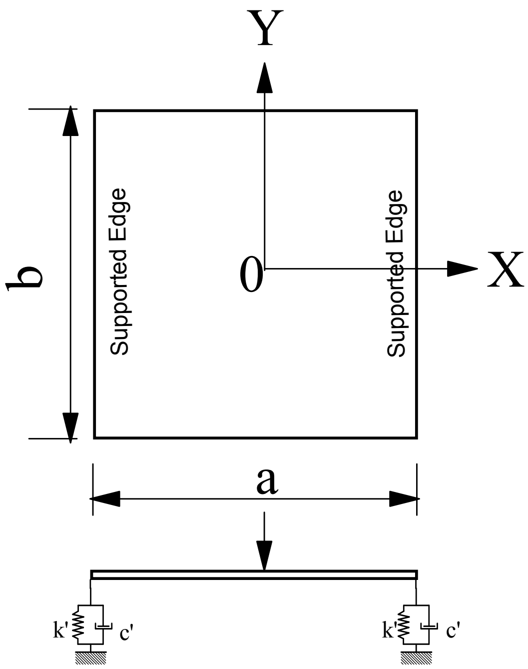

Figure 1. Viscoelastically supported plate subjected to concentrated force.

Consider a viscoelastically supported plate with side lengths a, b and thickness h subjected to a concentrated force as shown in Fig 1. Translational stiffness and damping coefficients were assigned equally along supported edges. The elastic symmetry axis of the plate material coincide with the OX and OY axes. Therefore the plate is specially orthotropic. Given W is the lateral displacement of the mid-surface of the plate corresponding coordinate Z, maximum strain energy of the plate is

$$U=\frac{D_{XX}}{2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\left[\left(\frac{\partial^2 W}{\partial X^2}\right)^2+2\nu_{YX}\frac{\partial^2 W}{\partial X^2}\frac{\partial^2 W}{\partial Y^2}+e\left(\frac{\partial^2 W}{\partial Y^2}\right)^2+4\frac{D_{66}}{D_{XX}}\left(\frac{\partial^2 W}{\partial X\,\partial Y}\right)^2\right]dXdY \tag{1}$$

and maximum kinetic energy of the plate is

$$T=\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\left[\frac{\rho h\omega^2}{2}\right]WdXdY \tag{2}$$

where D_{XX}, D_{YY} and D_{66} are expressed as follows:

$$D_{XX} = \left(E'_X \quad h^3\right)\Big/12 \ , \ D_{YY} = \left(E'_Y \quad h^3\right)\Big/12 \ , \ D_{66} = \left(G_{XY} \quad h^3\right)\Big/12 \tag{3}$$

where G_{XY} is shear modulus. E'_X , E'_Y are derived using:

$$\nu_{XY}E_Y = \nu_{YX}E_X \ , \ e = E_Y/E_X \ , \ E'_X = E_X\Big/\left(1-\nu_{YX}^2/e\right), \ E'_Y = E_X e\Big/\left(1-\nu_{YX}^2/e\right) \tag{4}$$

The additive strain energy and dissapation function of per viscoelastic support is

$$F_s = \frac{1}{2}k^{'}\,W_{Si}^2, \quad D = \frac{1}{2}c^{'}\,(\dot{W}_{Si})^2 \tag{5}$$

where k' and c' is spring coefficient and damping coefficient of per viscoelastic support, E_X , E_Y are Young's moduli in the OX and OY directions, respectively, and ν_{YX} is the Poisson's ratio for the strain response in the X direction due to an applied stress in the Y direction. The total energy of whole plate can be found by summing of entire area of plate with supports and external force. The potential energy from external force is

$$F_e = -F_{EXT}W_E \tag{6}$$

where F_{EXT} and W_E are external force and corresponding displacement.

Introducing the following non-dimensional parameters

$$x = \frac{X}{a}, \quad y = \frac{Y}{b}, \ \alpha = \frac{a}{b}, \ \ \overline{w}(x,y,t) = w\left(x\ ,y\right)e^{i\,\omega\,t} = W/a, \qquad i = \sqrt{-1} \tag{7}$$

the above energy expressions can be written as

$$\begin{aligned} U_{m,n} = \frac{D_{XX}}{2}\Bigg[& \frac{1}{\alpha\Delta x^4}\left(\overline{w}_{m-1,n} - 2\overline{w}_{m,n} + \overline{w}_{m+1,n}\right)^2 \\ & + \frac{2\alpha\nu_{YX}}{\Delta x^2\Delta y^2}\left(\overline{w}_{m-1,n} - 2\overline{w}_{m,n} + \overline{w}_{m+1,n}\right)\left(\overline{w}_{m,n-1} - 2\overline{w}_{m,n} + \overline{w}_{m,n+1}\right) \\ & + \frac{4\alpha D_{66}}{D_{XX}\left(4\Delta x^2\Delta y^2\right)^2}\left(\overline{w}_{m-1,n-1} - \overline{w}_{m+1,n-1} - \overline{w}_{m-1,n+1} + \overline{w}_{m+1,n+1}\right) + \frac{e\alpha^3}{\Delta y^4}\left(\overline{w}_{m,n-1} - 2\overline{w}_{m,n} + \overline{w}_{m,n+1}\right)^2 \Bigg]\Delta x\ \Delta y \end{aligned}$$

$$T_{m,n} = \frac{\rho h a^3 b}{2}\dot{\overline{w}}_{m,n}^2\Delta x\ \Delta y$$

$$F_s = \frac{1}{2}a^2 k^{'}\overline{w}_{m,n}^2 \tag{8a-e}$$

$$D = \frac{1}{2} a^2 c' \left(\dot{\bar{w}}_{m,n} \right)^2$$
$$F_e = -a Q \, \bar{w}_{m,n}$$

The derivative terms was approximated in terms of discrete displacements at grid points by using the following finite difference operators:

$$\left(\frac{\partial^2 \Theta}{\partial x^2} \right)_{m,n} = \frac{1}{\Delta x^2} \left(\Theta_{m-1,n} - 2\Theta_{m,n} + \Theta_{m+1,n} \right)$$

$$\left(\frac{\partial^2 \Theta}{\partial y^2} \right)_{m,n} = \frac{1}{\Delta y^2} \left(\Theta_{m,n-1} - 2\Theta_{m,n} + \Theta_{m,n+1} \right) \tag{9}$$

$$\left(\frac{\partial^2 \Theta}{\partial x \, \partial y} \right)_{m,n} = \frac{1}{4\Delta x \, \Delta y} \left(\Theta_{m-1,n-1} - \Theta_{m+1,n-1} - \Theta_{m-1,n+1} + \Theta_{m+1,n+1} \right)$$

The energy for the whole plate can be found by summing over the entire area of the plate. Thus

$$U = \sum_{m=1}^{N} \sum_{n=1}^{N} U_{m,n} \, , \quad T = \sum_{m=1}^{N} \sum_{n=1}^{N} T_{m,n} \, , \qquad F_s = \sum F_{si}$$

$$D = \sum D_{ci} \, , \quad F_e = \sum F_{ei} \tag{10}$$

where N is taken as the number of the mesh points in each of the two directions in the plate region, $N \times N$ is the total number of the area elements on the plate.

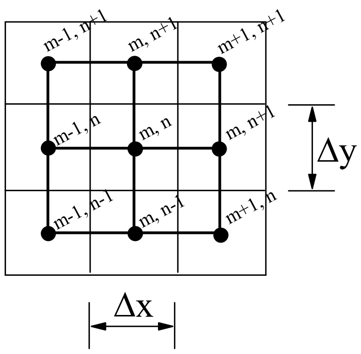

Figure 2 Typical finite difference mesh on part of a plate

The governing differential equation as obtained from the Lagrange's equation is given as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\bar{w}}_{m,n}}\right)-\frac{\partial\left(T-U\right)}{\partial \bar{w}_{m,n}}+\frac{\partial D}{\partial \dot{\bar{w}}_{m,n}}+\frac{\partial F_s}{\partial \bar{w}_{m,n}}+\frac{\partial F_e}{\partial \bar{w}_{m,n}}=0 \tag{11}$$

where $\bar{w}_{m,n}$ is the m,n th discrete displacement and the overdot stands for the partial derivative with respect to time. Introducing the following non-dimensional parameters,

$$\kappa_j=\frac{k_j\ a^3}{b\ D_{xx}}\ ,\ \gamma_j=\frac{c_j}{\sqrt{\rho\ h\ D_{xx}}}\ ,\ \lambda^2=\frac{\rho\ h\ \omega^2 a^4}{D_{xx}}\ ,\ q=\frac{Q\ a}{D_{xx}} \tag{12}$$

and remembering that $\bar{w}(x_1,x_2,t)=w\left(x_1,x_2\right)e^{i\,\omega\,t}$, which was given in equation (7), by using equation (11) for the mesh point m, n with equation (8a-e) results in the following expression:

$$\begin{aligned}
&\left[-\frac{4}{\alpha\Delta x^4}(w_{m+1,n}-2w_{m,n}+w_{m-1,n})-\frac{4\nu_{YX}\alpha}{\Delta x^2\Delta y^2}(w_{m,n-1}-2w_{m,n}+w_{m,n+1})\right.\\
&\left.-\frac{4\nu_{YX}\alpha}{\Delta x^2\Delta y^2}(w_{m-1,n}-2w_{m,n}+w_{m+1,n})-\frac{4\alpha^3}{\Delta y^4}\frac{D_{YY}}{D_{XX}}(w_{m,n-1}-2w_{m,n}+w_{m,n+1})\quad-\frac{2\lambda^2}{\alpha}w_{m,n}\right]\Delta x\Delta y\\
&+\left[\frac{2}{\alpha\Delta x^4}(w_{m+2,n}-2w_{m+1,n}+w_{m,n})+\frac{2\nu_{YX}\alpha}{\Delta x^2\Delta y^2}(w_{m+1,n-1}-2w_{m+1,n}+w_{m+1,n+1})\right]\Delta x\Delta y\\
&+\left[\frac{2}{\alpha\Delta x^4}(w_{m,n}-2w_{m-1,n}+w_{m-2,n})+\frac{2\nu_{YX}\alpha}{\Delta x^2\Delta y^2}(w_{m-1,n-1}-2w_{m-1,n}+w_{m-1,n+1})\right]\Delta x\Delta y\\
&+\left[\frac{2\nu_{YX}\alpha}{\Delta x^2\Delta y^2}(w_{m+1,n-1}-2w_{m,n-1}+w_{m-1,n-1})+\frac{2\alpha^3}{\Delta y^4}\frac{D_{YY}}{D_{XX}}(w_{m,n-2}-2w_{m,n-1}+w_{m,n})\right]\Delta x\ \Delta y\\
&+\left[\frac{2\nu_{YX}\alpha}{\Delta x^2\Delta y^2}(w_{m+1,n+1}-2w_{m,n+1}+w_{m-1,n+1})+\frac{2\alpha^3}{\Delta y^4}\frac{D_{YY}}{D_{XX}}(w_{m,n}-2w_{m,n+1}+w_{m,n+2})\right]\Delta x\ \Delta y\\
&+\left[\frac{\alpha}{\Delta x^2\Delta y^2}\frac{D_{66}}{2D_{XX}}(w_{m-2,n-2}-w_{m-2,n}-w_{m,n-2}+w_{m,n})\right]\Delta x\ \Delta y\\
&+\left[-\frac{\alpha}{\Delta x^2\Delta y^2}\frac{D_{66}}{2D_{XX}}(w_{m-2,n}-w_{m-2,n+2}-w_{m,n}+w_{m,n+2})\right]\Delta x\ \Delta y\\
&+\left[-\frac{\alpha}{\Delta x^2\Delta y^2}\frac{D_{66}}{2D_{XX}}(w_{m,n-2}-w_{m,n}-w_{m+2,n-2}+w_{m+2,n})\right]\Delta x\ \Delta y\\
&+\left[\frac{\alpha}{\Delta x^2\Delta y^2}\frac{D_{66}}{2D_{XX}}(w_{m,n}-w_{m,n+2}-w_{m+2,n}+w_{m+2,n+2})\right]\Delta x\Delta y
\end{aligned}$$

$$+\alpha_S \frac{2(\kappa + i\lambda\gamma)}{\Delta x\ \Delta y} w_{m,n} = \alpha_Q \frac{q}{\Delta x\ \Delta y}, \qquad i = \sqrt{-1} \tag{13}$$

In equation (13) α_S and α_Q is taken values 0, 1 depending of existence of support and load respectively on pivotal point m, n.

For the whole mesh points, by using equation (13), the following set of linear algebraic equations is obtained which can be expressed in the following matrix form

$$[A]\{w\} + i\lambda\gamma[B]\{w\} - \lambda^2[C]\{w\} = \{q\} \tag{14}$$

where $[A]$, $[B]$ and $[C]$ are coefficient matrices obtained by using equation (13) for all mesh points. For free vibration analysis, when the external force and damping of the supports are zero in (14), this situation results in a set of linear homogeneous equations that can be expressed in the following matrix form:

$$[A]\{w\} - \lambda^2[C]\{w\} = \{0\} \tag{15}$$

Numbering of the mesh points is shown in Figure 2. By decreasing the dimensionless mesh widths, the accuracy can be increased.

The total magnitude of the reaction forces of the supports is given by

$$\sum P_i = \sum\left(\kappa_j + i\gamma_j\right)w \tag{16}$$

and therefore the force transmissibility at the supports is determined by

$$T_R = \sum P_i / \sum F_{EXT} = \sum\left(\kappa_j + i\gamma_j\lambda\right)w/q \tag{17}$$

where $\sum F_{EXT}$ is total amount of external force.

The number of unknown displacements is $(N+2)^2$, where N^2 is the mesh size in the plate region.

3. NUMERICAL RESULTS

Because of the lackness of the comparable results for different E_2/E_1 ratios, only fundamental frequency of simple supported isotropic plate was compared in table 1. Gorman also studied elastically supported plate but results are given in reference [11] graphically and the results are in good agreement not shown here.

Table 1. Fundamental frequency parameter, λ, for a square plate for different boundary conditions.
(ν = 0.333, κ = 1e10)

BC (Boundary Conditions)	**Actual value** [5]	**Present method** Mesh Size (151x151)
SSSS	19,74	19,737
SFSS	11,58	11,582
SFSF	9,568	9,568
SFFF	14,77	14,767
FFFF	19,22	19,220
SFFS	3,292	3,292

The steady state response to a concentrated force acting on an orthotropic square plate, which of two edges are viscoelastically supported oppositely others are free, is calculated numerically.

Viscoelastic supports are given equally per supported area. A brief investigation of the free vibration of an elastically supported plate is necessary for a better understanding of the responses presented in this study. The natural frequencies of the elastically supported plate are determined by calculating the eigenvalues, assuming that the damping parameter of the supports and external force are zero.

In Table 1. the convergence of the fundamental mode is presented for the for κ=50, κ=1e20 (almost rigid for translational deflection) and υ=0.3, υ=0.333 respectively.

It is shown that the convergence with respect to mesh size is quite rapid in the considered cases. As it is observed from Table 2, the frequency parameter monotonically convergences while the mesh size increase. Convergence can be below or above depending of the value of translational spring coefficient. Convergency properties are not effected from E_2/E_1 ratios and not shown here.

Table 2. The effect of mesh size on the fundamental frequency.
$\left(E_2 / E_1 = 1\right)$

Mesh Size	**υ=0,3-κ=50**	**υ=0,3-κ=1^e20**	**υ=0,333-κ=1^e20**	**υ=0,333-κ=50**
9X9	5,8180	9,4898	5,8039	9,4242
11X11	5,8195	9,5398	5,8039	9,4758
15 X 15	5,8180	9,5852	5,8008	9,5211
17 X 17	5,8180	9,5961	5,8008	9,5320
21 X 21	5,8154	9,6085	5,7974	9,5447
51 X 51	5,8076	9,6277	5,7883	9,5645
81 X 81	5,8050	9,6299	5,7853	9,5668
101 X 101	5,8041	9,6304	5,7844	9,5673
131 X 131	5,8033	9,6308	5,7834	9,5677
151 X 151	5,8029	9,6309	5,7829	9,5678

The effect of mesh size and translational spring coefficient on the fundamental frequency can be seen clearly in Figure 3.

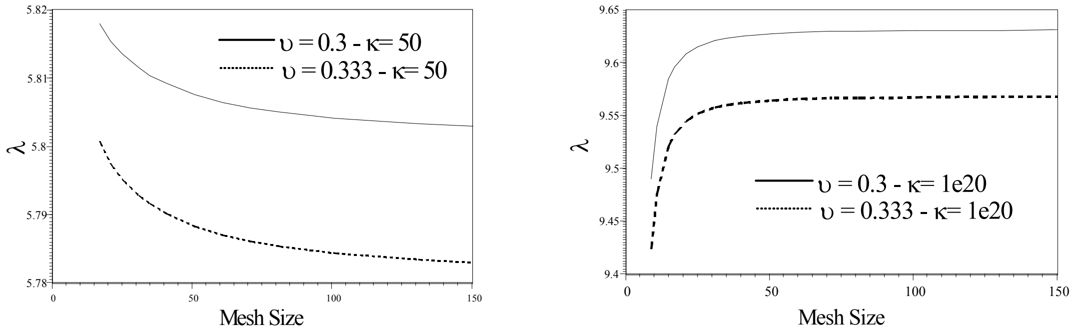

Figure 3. The effect of mesh size on the fundamental frequency.
$\left(E_2 / E_1 = 1\right)$

Figure 4 shows the frequency parameters λ versus the stiffness parameter κ. The translational stiffness coefficient is given equal values along supported edges in the case $\gamma = 0, F_{EXT} = 0$. In Figure 4, the values of ordinates at $\kappa = 0$ and $\kappa = \infty$ represent the frequency parameters of a free plate and a simply supported plate, respectively. As the value of the translational stiffness parameter increases, the frequency parameter also increases and ultimately becomes the value of a SFSF supported plate. All eigen values approach zero as their lover limit, as expected.

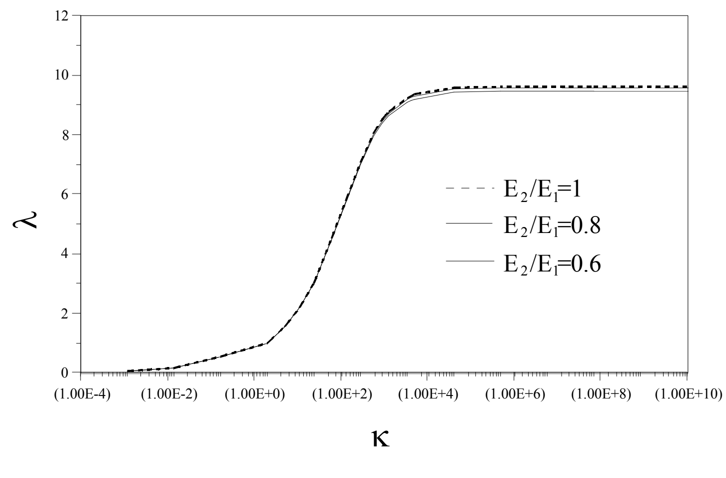

Figure 4. The effect of and spring stiffness on the non dimensional fundamental frequency.
(ν = 0.3, Mesh size (151X151))

Table 3. depicts eigenvalues of frequency parameter numerically for anisotropic plate. Last three line of Table 3. corresponds to the case of SFSF supported plate, while others EFEF relatively. It can be seen that effect of E_2/E_1 ratio on fundamental frequency of plate is getting significant, while the value of **κ** increases.

Table 3. Fundamental frequency parameters of elastically supported plate
(ν = 0.3, Mesh size (151X151))

κ	E_2/E_1=1	E_2/E_1=0,8	E_2/E_1=0,6
50	5,803	5,782	
100	7,064	7,028	6,966
500	8,897	8,839	8,742
1000	9,233	9,173	9,074
1e4	9,581	9,523	9,425
1e6	9,630	9,574	9,479
1e8	9,631	9,575	9,480
1e10	9,631	9,575	9,480
1e20	9,631	9,575	9,480

Table 4 depicts frequency parameters and transmissibility values where the peak values occur for set of κ, γ, E_2/E_1. When comparing Table 3 with Table 4, one can see clearly, frequency parameter tend to be close natural fundamental frequency of the plate for lower values of γ.

Table 4. The frequencies at which the peak values of the force transmissibilities occur,
(ν = 0.3, Mesh Size (151X151))

E_2/E_1=1				**E_2/E_1=0.8**				**E_2/E_1=0.6**			
κ	γ	λ	T_R	κ	γ	λ	T_R	κ	γ	λ	T_R
5	1	2,068	3,515	5	1	2,189	2,800	5	1	2,114	2,691
10	1	2,986	4,258	10	1	2,969	4,202	10	1	2,979	3,929
50	1	5,810	14,794	50	1	5,785	15,101	50	1	5,750	14,745
100	1	7,078	35,435	100	1	7,047	36,338	100	1	6,973	34,565
500	1	8,895	625,255	500	1	8,875	872,252	500	1	8,744	510,529
1e10	1	9,635	6038,343	1e10	1	9,500	983,483	1e10	1	9,438	289,486
1	5	0,609	1,052	1	5	0,678	1,061	1	5	0,647	1,036
5	5	1,938	1,178	5	5	1,945	1,177	5	5	1,918	1,174
10	5	2,902	1,358	10	5	2,914	1,358	10	5	2,892	1,358
50	5	5,934	3,393	50	5	5,914	3,396	50	5	5,869	3,399
100	5	7,152	7,370	100	5	7,125	7,371	100	5	7,047	7,366
500	5	8,900	95,097	500	5	8,875	126,926	500	5	8,750	92,930
1e10	5	9,635	6038,343	1e10	5	9,500	983,483	1e10	5	9,438	289,486
1	10	8,752	1,500	1	10	8,521	1,472	1	10	8,119	1,424
5	10	8,502	1,518	5	10	8,273	1,493	5	10	7,896	1,451
10	10	8,172	1,554	10	10	7,954	1,533	10	10	7,596	1,497
50	10	6,876	2,437	50	10	6,828	2,430	50	10	6,704	2,416
100	10	7,464	4,458	100	10	7,375	4,450	100	10	7,329	4,425
500	10	8,915	48,398	500	10	8,867	51,486	500	10	8,761	47,333
1e10	10	9,629	14156,873	1e10	10	9,594	13175,940	1e10	10	9,488	65452,418
1	50	9.,541	7,004	1	50	9,484	6,872	1	50	9,372	6,652
5	50	9,535	7,011	5	50	9,484	6,876	5	50	9,365	6,657
10	50	9,526	7,020	10	50	9,484	6,885	10	50	9,356	6,665
50	50	9,460	7,146	50	50	9,406	7,010	50	50	9,288	6,783
100	50	9,380	7,444	100	50	9,344	7,304	100	50	9,206	7,073
500	50	9,175	15,798	500	50	9,121	15,618	500	50	9,012	15,277

1e10	50	9,630	22358,833	1e10	50	9,500	1449,644	1e10	50	9,468	9883,807

Figure 5 shows that existence of a suitable value of damping parameter γ, for possible to reduce the peak values of the force transmissibilities to certain minimum value for any κ value. Existence of such points is useful for an optimum design of a system by choosing appropriate damping parameter. Within certain range of the frequencies the force transmissibilities are less than unity, which indicates the possibility of vibration isolation.

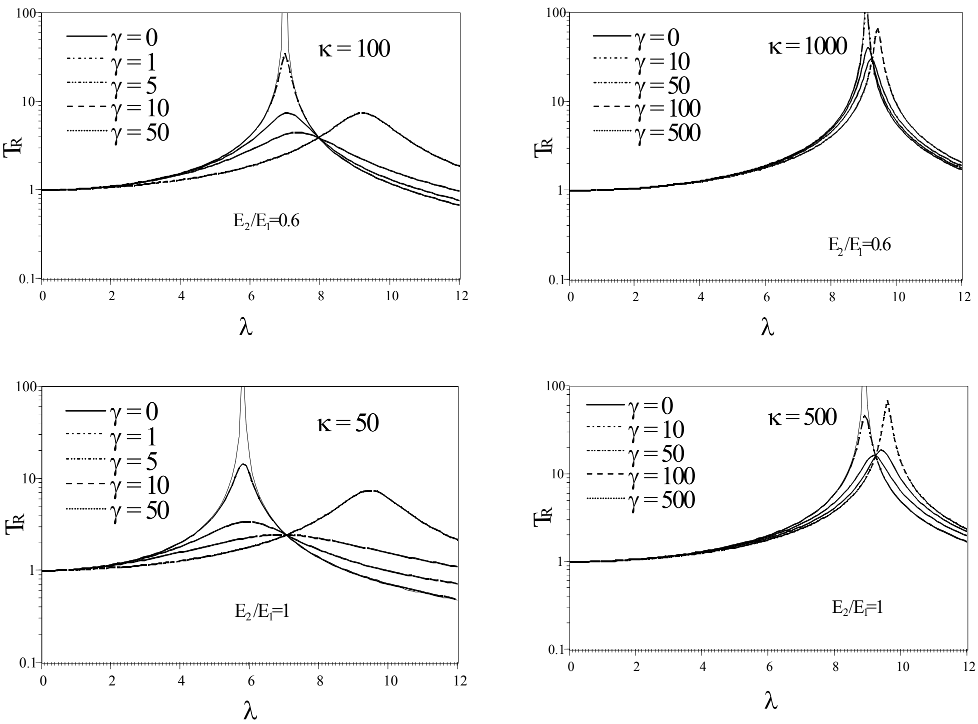

Figure 5. The Effect of damping coefficient on force transmissibilities for different κ values. ($\nu = 0.3$, Mesh Size (151X151))

The effect of anisotropy on force transmissibilities are shown in Figure 6. It is apparently seen that the value of force transmissibility is not affected from anisotropy significantly. But the values of frequencies where the peak values occur are affected from anisotropy as suitable for natural frequency occurrence range for different E_2/E_1 ratios.

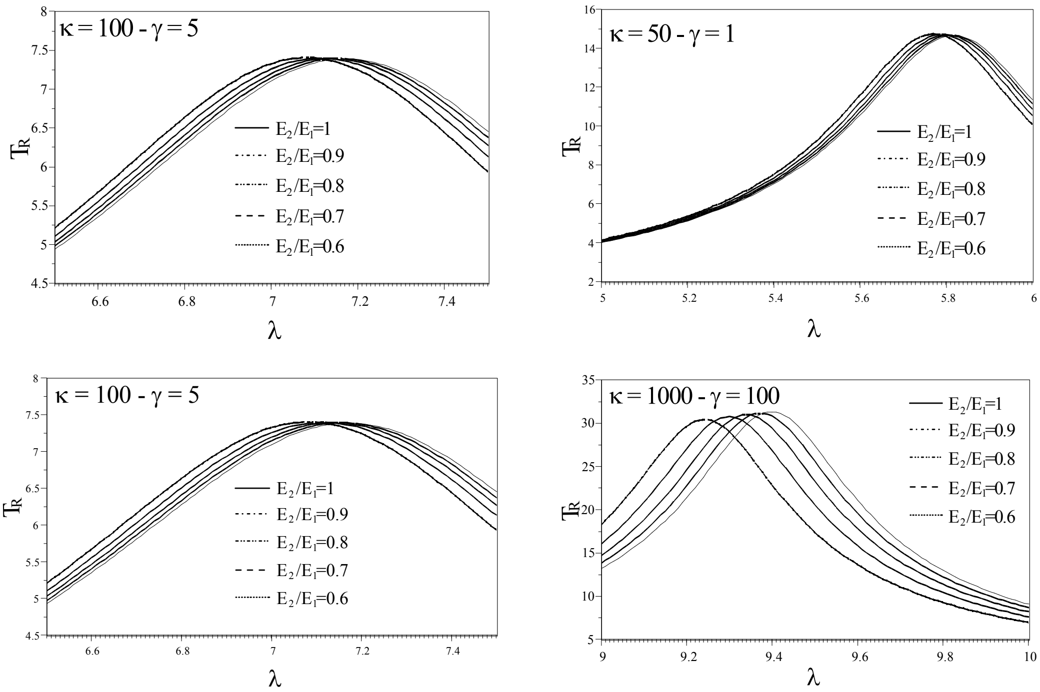

Figure 6. The effect of anisotropy on force transmissibilities for different κ and γ values. (ν = 0.3, Mesh Size (151X151))

4. CONCLUSIONS

This present paper, to the authors' knowledge, the first known vibration analysis of EFEF / VFVF supported anisotropic plate. Model can be used to simulate the actual boundary conditions of the plates. A simple numerical method has been presented for determining natural frequencies of plates. The convergence studies are made. Fundamental frequency was determined depending on spring coefficient and anisotropy. The frequencies at which the peak values of the force transmissibilities occur were obtained. It was shown that existence of a suitable value of damping parameter γ for possible to reduce the peak values of the force transmisibilities to minimum certain value for any κ. The effect of anisotropy on the force transmissibilities was also investigated.

It is hoped that these novel results will be useful to designers in the various types of practical applications.

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