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ELECTROMAGNETIC PROPERTIES OF Z AND γ SELF-COUPLINGS INTERACTIONS

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Abstract-The couplings of three gauge bosons $ZZ\gamma^*$, $Z\gamma\gamma^*$, $Z\gamma Z^*$and ZZZ^* are found in the electroweak standard model at one loop level. Bosonic one loop diagrams do not contribute to these couplings and only fermionic contributions survive. Ward identities and Bose symmetry are checked. Only the anapole dipole moment characterizes the electromagnetic properties of $ZZ\gamma^*$, $Z\gamma\gamma^*$, $Z\gamma Z^*$, and ZZZ^* interactions.
Key Words- Fermion triangle amplitude, Gauge bosons, Ward identities, Bose symmetry.

1. INTRODUCTION

The standard model (SM) of electroweak interactions makes precise predictions for the couplings between gauge bosons. During the last two decades intense activity has taken place about the possible existence of nonstandard contributions (anomalous) to the gauge boson couplings [1,2]. This is because of new experimental results at the fermilab Tevatron [3], which should more improve in the future.

The general form of any three-boson couplings was given, in a model independent way, in terms of a set of seven independent form factors [4,5,6]. This general description has been applied to both charged and neutral gauge bosons [5]. For the neutral three-boson couplings, in a model independent way, its general form was given in terms of a set of two independent form factors only [6]. One of these two form factors is CP conserving and the other is CP violating. They are connected to the anapole dipole moment and anapole quadrupole moment.

In the standard model (SM) for the electroweak interactions, at one loop level, only CP. conserving neutral gauge coupling form factors can arise. Such couplings can only be induced by a fermonic triangle diagram. Therefore, only the terms with anti-symmetric tensor $\varepsilon^{\mu\upsilon\rho\alpha}$ will appear.

Recently, the exact expression of the general form for fermion triangle diagrams was given in ref. [7]. This form possesses six transition form factors, which describe any neutral three-boson couplings off the mass shell. It is seen, in literature [1-7] that the theoretical expectation for these neutral couplings is still open. Therefore, it is important to re-study, using different method, the standard model predictions for these couplings. These couplings are useful in e^+e^- processes: $e^+e^- \to ZZ$ and $e^+e^- \to Z\gamma$. In this paper we use the general form for fermion triangle diagram given in ref [7], to obtain $ZZ\gamma^*$, $Z\gamma\gamma^*$, $Z\gamma Z^*$, and ZZZ^* interactions vertices. Each one of these vertices satisfies canonical Ward identities and the necessary Bose symmetry. The ABJ anomalies [8] (see e.g. [7,9]), for these vertices vanish. In section 2 we rewrite the general amplitude for the fermion triangle diagram of the neutral spin-1 particles and define the neutral electromagnetic current. The $ZZ\gamma^*$, $Z\gamma\gamma^*$, $Z\gamma Z^*$, and ZZZ^*

interactions vertices are calculated in sections 3,4,5 and 6respectively. The transition and on-shell formfactors are given for each vertex. In section 7 we summarize our results.

2. THE GENERAL AMPLITUDE FOR THE FERMION TRIANGLE DIAGRAM OF THE NEUTRAL SPIN-1 PARTICLES

In our previous paper [7] we calculate the general form for the fermion triangle amplitude in the coupling of spin-1 particles and we check Ward identities. At the one loop level, in the standard model (SM), the three-point function for the neutral gauge boson couplings can only be induced by a fermionic triangle diagram. The general form of a triangle fermion diagram with general vertices, Fig. 1, which gives a contribution to the coupling of any chargeless spin-1 particles was obtained in ref. [7] as

$$\Pi_0^{\mu\nu\rho}(p,r)=\left[F_1(r+p)^{\mu}+F_2(r-p)^{\mu}\right]p_{\alpha}\,r_{\beta}\in^{\nu\rho\beta\alpha}+\left[F_3(r+p)^{\nu}+F_4(r-p)^{\nu}\right]p_{\alpha}\,r_{\beta}\in^{\mu\rho\beta\alpha}$$
$$+\left[F_5(r+p)_{\alpha}+F_6(r-p)_{\alpha}\right]\in^{\mu\nu\alpha\rho} \qquad (2.1)$$

Where,

$$F_k=\sigma\int_0^1 dx\int_0^x dy\frac{1}{D_3^0(x,y)}c_k, \qquad k=1,2,\cdots,6$$

and

$$c_1=(\sigma_1+\sigma_2)\,y(x-1), \qquad c_2=(\sigma_1+\sigma_2)\,y(1+x-2y), \qquad c_3=(\sigma_1+\sigma_2)(x-1)(x-y),$$
$$c_4=(\sigma_1+\sigma_2)\left[x(x+3y-1)+y(2y+1)\right],$$
$$c_5=(\sigma_1+B_2)\left[p^2y(1-y)+r^2(x-y)(1-x+y)+2r\cdot p\,y(y-x)\right]+2\sigma_1 m_i^2(x-1)$$
$$c_6=(\sigma_1+\sigma_3)\,p^2y(1-y)+(\sigma_1+\sigma_4)\,r^2(x-y)(x-y-1)$$
$$+\sigma_5(2r\cdot p)\,y(x-y)+2\sigma_1 m_i^2(x-2y),$$

with

A_i , B_i are the vertices constants (i =1,2,3) and

$$\sigma=\frac{1}{2\pi^2}A_1A_2A_3, \quad \sigma_1=B_1B_2B_3, \quad \sigma_2=B_1+B_2+B_3, \quad \sigma_3=B_2+2B_3, \quad \sigma_4=B_2+2B_1,$$
$$\sigma_5=B_1-B_3, \; D_3^0(x,y)=p^2y(1-y)+r^2(x-y)(1-x+y)+2r\cdot p\,y(y-x)-m_i^2.$$

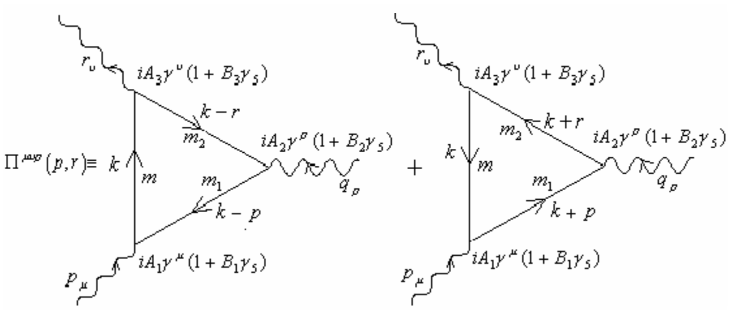

Fig.1: The most general fermion triangle diagram $\Pi^{\mu\nu\rho}(p,r)$ for the coupling of spin-1 particles at one loop approximation

To obtain the electromagnetic formfactors of Z_0 boson we follow the same procedure as in ref. [6] where we define the Z_0 electromagnetic current as

$$j^{\rho} = \varepsilon_{\upsilon}^{*}(\bar{r},\lambda_2)\Gamma^{\mu\upsilon\rho}(P,q)\varepsilon_{\mu}(\bar{p},\lambda_1) \tag{2.2}$$

where

$$\Gamma^{\mu\upsilon\rho}(P,q) = -i\ \Pi^{\mu\upsilon\rho}(on\,shell)$$

with $P = r + p$ and $q = r - p$. Using Shutten identity we get

$$\Gamma^{\mu\upsilon\rho}(P,q) = i\,D\left[q^{\rho}\ \varepsilon^{\mu\upsilon\beta\alpha}\ P_{\alpha}q_{\beta} + q^{2}\ P_{\alpha}\varepsilon^{\mu\upsilon\alpha\rho}\right] + i\,f_5 P_{\alpha}\varepsilon^{\mu\upsilon\rho\alpha} + i\overline{D}q_{\alpha}\varepsilon^{\mu\upsilon\rho\alpha} \tag{2.3}$$

where $D = (f_3 - f_4)/2\,, \overline{D} = \left[2f_6 + q.P\,(f_3 - f_4)\right]/2$

We are going to calculate $ZZ\gamma^{*}$, $Z\gamma\gamma^{*}$, $Z\gamma Z^{*}$, and ZZZ^{*} vertices. We shall write a general form for Ward identities applicable to each vertex. Therefore, from (2.1) we have

$$q_{\rho}\ \Pi^{\mu\upsilon\rho} = 2f_5\ p_{\alpha}r_{\beta}\,\varepsilon^{\mu\upsilon\alpha\beta} \tag{2.4}$$

$$p_{\mu}\ \Pi^{\mu\upsilon\rho} = \left[(f_1 + f_2)\bar{r}.\bar{p} + (f_1 - f_2)\,p^{2} + f_5 + f_6\right]p_{\alpha}r_{\beta}\,\varepsilon^{\upsilon\rho\beta\alpha} \tag{2.5}$$

$$-r_{\upsilon}\ \Pi^{\mu\upsilon\rho} = -\left[(f_3 + f_4)r^{2} + (f_3 - f_4)\bar{r}.\bar{p}\ + f_5 - f_6\right]p_{\alpha}r_{\beta}\,\varepsilon^{\mu\rho\beta\alpha} \tag{2.6}$$

3. THE $ZZ\gamma^{*}$ AMPLITUDE

We have calculated the general form for the fermion triangle amplitude in the coupling of spin-1 particles, in the standard model (SM), the three point function for the neutral gauge boson couplings can only be induced by a fermionic triangle diagram. Then, to calculate $ZZ\gamma$ Green function $\Pi_{ZZ\gamma}^{\mu\nu\rho}(p,r)$ using the general form (2.1) we put

$A_1 = A_3 = A_z$, $A_2 = A_{\gamma}$, $B_1 = B_3 = B_z$ and $B_2 = B_{\gamma} = 0$

with,

$$A_{\gamma} = e\,Q_i\,,\ A_Z = \frac{e}{\sin 2\theta_{\omega}}\left(2Q_i\sin^{2}\theta_{\omega} - \mathrm{T}_{3i}\right),\ B_Z = \frac{\mathrm{T}_{3i}}{\left(2Q_i\sin^{2}\theta_{\omega} - \mathrm{T}_{3i}\right)} \tag{3.1}$$

The obtained amplitude agrees with the one obtained in Ref. [10] using a different method. It is clear that the amplitude $\Pi_{ZZ\gamma}^{\mu\nu\rho}(p,r)$ satisfies Bose symmetry

$$\Pi_{ZZ\gamma}^{\mu\nu\rho}(p,r) = \Pi_{ZZ\gamma}^{\nu\mu\rho}(-r,-p)\,, \tag{3.2}$$

The Ward identities (2.4),(2.5) and (2.6) will be

$$q_{\rho}\,\Pi_{ZZ\gamma}^{\mu\nu\rho}(p,r) = 0\,, \tag{3.3}$$

$$\begin{aligned} p_{\mu}\,\Pi_{ZZ\gamma}^{\mu\nu\rho}(p,r) &= \left[\,F_1^{Z}\left(p^{2} + r\cdot p\right) + F_2^{Z}\left(r\cdot p - p^{2}\right) + F_6^{Z}\,\right]p_{\alpha}\,r_{\beta} \in^{\nu\rho\beta\alpha} \\ &= 2m_i\,\mathrm{T}_{5(II)}^{\nu\rho}(p,r) - \frac{1}{\pi^{2}}A_Z^{2}A_{\gamma}B_Z\left(\frac{1}{2}\right)p_{\alpha}\,r_{\beta} \in^{\nu\rho\beta\alpha}\,, \end{aligned} \tag{3.4}$$

and

$$-r_\nu \, \Pi^{\mu\nu\rho}_{ZZ\gamma}(p,r) = 2m_i \, T^{\mu\rho}_{5(III)}(p,r) + \frac{1}{\pi^2} A_Z^2 A_\gamma B_Z \left(\frac{1}{2}\right) p_\alpha \, r_\beta \in^{\mu\rho\beta\alpha} . \tag{3.5}$$

where,

$$T^{\upsilon\rho}_{5(II)}(p,r) = \frac{1}{\pi^2} A_Z^2 A_\gamma B_Z m_i \int_0^1 dx \int_0^x dy \frac{1}{D_3^0(x,y)} \, p_\alpha \, r_\beta \in^{\upsilon\rho\beta\alpha}$$

and

$$T^{\mu\rho}_{5(III)}(p,r) = \frac{1}{\pi^2} A_Z^2 A_\gamma B_Z m_i \int_0^1 dx \int_0^x dy \frac{1}{D_3^0(x,y)} \, p_\alpha \, r_\beta \in^{\mu\rho\beta\alpha} .$$

The mass independent terms in (2.6) and (2.7) are the ABJ anomaly part which vanishes after the summation over all fermions (for more details see e.g. Ref. [7,9]), where we have

$$\mp A_Z^2 A_\gamma B_Z = \mp \frac{e^3}{\sin^2 2\theta_\omega} \sum_{f_i} Q_i \, T_{3i}\left(2Q_i \sin^2\theta_\omega - T_{3i}\right),$$

but

$$\sum_{f_i} Q_i^2 \, T_{3i} = 0, \text{ and } \sum_{f_i} Q_i \, T_{3i}^2 = 0 .$$

The amplitude $\Pi^{\mu\nu\rho}_{ZZ\gamma}(p,r)$ for the vertex function $ZZ\gamma$ satisfies Bose symmetry and canonical Ward identities. Our results agree with the previous calculations in ref.[10].

To find the formfactors of $ZZ\gamma^*$ Green's function, we have to calculate D, $\overline{D}$ and f_5 in(2.3)

$$\Gamma^{\rho\nu\mu}_{zz\gamma} = D\left[q^2 \, P_\alpha \in^{\alpha\mu\nu\rho} + q^\rho \, P_\alpha \, q_\beta \in^{\beta\alpha\mu\nu}\right],$$

where

$$D = i \sum_{f_i} \Omega_{f_i} \int_0^1 dx \int_0^x dy \frac{1}{D_3^Z(x,y)} y(x-y)$$

$$D_3^Z(x,y) = M_Z^2 \, x(1-x) + q^2 \, y(x-y) - m_{f\,i}^2 + i\varepsilon ,$$

$$\Omega_{f_i} = \frac{1}{\pi^2} A_Z^2 \, A_\gamma \, B_Z = \frac{e^3}{\sin^2 2\theta_\omega} Q_i \, T_{3i}\left(2Q_i \sin^2\theta_\omega - T_{3i}\right).$$

amplitude described by only one formfactor which agree with the results $ZZ\gamma^*$ Thus, the obtained before [1, 10, 12]. The formfactor D describe the anapole dipole moment for Z particle (for more details see the ref. [6]).

4. THE $Z\gamma Z^*$ AMPLITUDE

To calculate $Z\gamma Z^*$ Green function $-\Pi^{\mu\nu\rho}_{Z\gamma Z}(p,r)$ from the general form (2.1) we put

$A_1 = A_2 = A_z$, $A_3 = A_\gamma$, $B_1 = B_2 = B_z$ and $B_3 = B_\gamma = 0$

The amplitude $\Pi^{\mu\nu\rho}_{Z\gamma Z}(p,r)$ for the vertex function $Z\gamma Z^*$ satisfies Bose symmetry and canonical Ward identities.

To find the formfactors of $Z\gamma Z^*$ Green's function, we have to calculate D, $\overline{D}$ and f_5 in (2.3). We find $f_5 - q^2 D = \overline{D}$

$$\Gamma^{\rho\nu\mu}_{z\gamma z} = i D\ q^\rho P_\alpha q_\beta \in^{\mu\nu\beta\alpha} + \overline{D}\left(P_\alpha + q_\alpha\right)\in^{\mu\nu\rho\alpha},$$

where

$$D = \sum_{f_i}\Omega_{f_i}\int_0^1 dx\int_0^x dy\frac{1}{D_3^Z(x,y)}\,y(x-y),$$

$$\overline{D} = \frac{1}{2}\sum_{f_i}\Omega_{f_i}\int_0^1 dx\int_0^x dy\frac{1}{D_3^Z(x,y)}\left[M_z^2(y-xy)-q^2\left(xy-y^2\right)\right]$$

$$\Omega_{f_i} = \frac{1}{\pi^2}A_Z^2 A_\gamma B_Z = \frac{e^3}{\sin^2 2\theta_\omega}Q_i\,\mathrm{T}_{3i}\left(2Q_i\sin^2\theta_\omega - \mathrm{T}_{3i}\right).$$

Thus, the $Z\gamma Z^*$ amplitude is described by two formfactors which describe the anapole dipole moment for the Z particle (for more details see the ref.[6]).

5. THE ZZZ^* AMPLITUDE

As we discussed before, we can obtain the off – mass shell amplitude for ZZZ^* vertex from (2.1) by putting the coupling constants

$A_1 = A_2 = A_3 = A_z$ and $B_1 = B_2 = B_3 = B_z$

The obtained amplitude satisfies Bose symmetry and axial vector Ward identities. Introducing,

$$\Gamma^{\mu\nu\rho}_{ZZZ}(P,q) = -i\,\Pi^{\mu\nu\rho}_{ZZZ}(p^2 = r^2 = M_Z^2)$$

Using schouten identity, and taking into account the hermiticity and CPT invariance we end up with

$$\Gamma^{\rho\nu\mu}_{zzZ} = D\left[q^2 P_\alpha \in^{\mu\nu\alpha\rho} + q^\rho P_\alpha q_\beta \in^{\mu\nu\beta\alpha}\right] + f_5 P_\alpha \in^{\mu\nu\alpha\rho},$$

where

$$D = \sum_{f_i}\Omega_{f_i}\int_0^1 dx\int_0^x dy\frac{1}{D_3^{3Z}(x,y)}\,y(x-y),$$

$$f_5 = \frac{1}{2\pi^2}A_Z^3 B_Z m_i^2\int_0^1 dx\int_0^x dy\frac{1}{D_3^{3Z}(x,y)} + \frac{1}{2\pi^2}A_Z^3 B_Z^3 m_i^2\int_0^1 dx\int_0^x dy\frac{(2x-1)}{D_3^{3Z}(x,y)},$$

with

$$D_3^{3Z}(x,y) = M_Z^2\,x(1-x) + q^2\,y(x-y) - m_{f_i}^2 + i\varepsilon,$$

$$\Omega_{f_i} = \frac{1}{2\pi^2}A_Z^3 B_Z\left(3 + B_Z^2\right)$$

$$=\frac{2e^3}{\pi^2 \sin^3 2\theta_\omega}\left[3Q_i \sin^2\theta_\omega \, T_{3i}\left(Q_i \sin^2\theta_\omega - T_{3i}\right)+T_{3i}\right].$$

As we see, The two formfactors describe the anapole dipole moment for the coupling ZZZ^*.

6. THE $Z\gamma\gamma^*$ AMPLITUDE

To calculate $Z\gamma\gamma^*$ Green function - $\Pi^{\mu\nu\rho}_{Z\gamma\gamma}(p,r)$ from the general form (2.1) we put

$A_1 = A_z$, $A_2 = A_3 = A_\gamma$, $B_1 = B_z$ and $B_2 = B_3 = B_\gamma = 0$

The amplitude $\Pi^{\mu\nu\rho}_{Z\gamma\gamma}(p,r)$ for the vertex function $Z\gamma\gamma^*$ satisfies Bose symmetry and canonical Ward identities.

To find the formfactors of $Z\gamma\gamma^*$ Green's function, we have to calculate D, $\overline{D}$ and f_5 in (2.3) we find

$$f_5 = 0\,,\ \overline{D} = 0 \text{ and } D = \sum_{f_i} \Omega'_{f_i} \int_0^1 dx \int_0^x dy \frac{1}{D_3(x,y)} y(x-y)\,,$$

$$\Omega'_{f_i} = \frac{1}{2\pi^2} A_\gamma^2 A_Z B_Z\ .$$

Thus, the $Z\gamma\gamma^*$ amplitude is described by one formfactor. The formfactor describe the anapole dipole moment for the Z particle (for more details see the ref.[6]).

7. CONCLUSIONS

Electromagnetic formfactors for the neutral gauge boson couplings $ZZ\gamma^*$, $Z\gamma\gamma^*$, $Z\gamma Z^*$ and ZZZ^* are calculated. More formfactors take part in the transition amplitudes. We found that the electric dipole, the magnetic quadrupole and the dipole anapole transition formfactors appear in this case. Only the anapole dipole moment formfactors survive in the on-shell case for incoming and outgoing particles. It was possible to get an anapole moment $T\left(q^2\right)$ different than zero for these neutral gauge boson couplings, because the standard model does not conserve P and C separately though CP is conserved.

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