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IBM-1 CALCULATIONS ON THE EVEN-EVEN $^{122-128}$Te ISOTOPES

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Abstract- The $^{122-128}$Te isotopes in O(6)- SU(5) transition region were investigated. For these nuclei, the energy levels, B(E2) transition probabilities, and δ(E2/M1) mixing ratios were calculated within framework of the Interacting Boson Model (IBM-1). The results are compared with the experimental data and the previous calculations. It is shown that there is a good agreement between the results found and especially with the experimental ones.
Keywords- energy levels, transition probabilities, mixing ratios,

1. INTRODUCTION

The neutron-proton interaction is known to play a dominant role in quadrupole correlations in nuclei. As a consequence, the excitation energies of collective quadrupole excitations in nuclei near a closed shell are strongly dependent on the number of nucleons outside the closed shell.

The even-even tellurium isotopes are part of an interesting region beyond the closed proton shell at Z=50, while the number of neutrons in the open shell is much larger, as such these nuclei have been commonly considered to exhibit vibrational-like properties.

The even-mass tellurium isotopes have been extensively investigated both theoretically and experimentally in recent years with special emphasis on interpreting experimental data via collective models. Energy levels, electric quadrupole moments, B(E2) values of $^{122-128}$Te isotopes have been studied within the framework of the semi-microscopic model [1], the two-proton core coupling model [2], dynamic deformation model [3] and the interacting boson model-2 [4-6].

Our aim in this study is to investigate $^{122-128}$Te isotopes in O(6)-SU(5) transition region and calculate the energy levels, B(E2) transition probabilities, and δ(E2/M1) mixing ratios within framework of the Interacting Boson Model (IBM-1).

2. THE INTERACTING BOSON MODEL

The interacting boson model of Arima and Iachello [7-12] has become widely accepted as a tractable theoretical scheme of correlating, describing and predicting low-energy collective properties of complex nuclei. In this model it was assumed that low-lying collective states of even-even nuclei could be described as states of a given (fixed) number N of bosons. Each boson could occupy two levels one with angular momentum

L=0 (s-boson) and another, usually with higher energy, with L=2 (d-boson). In the original form of the model known as IBM-1, proton- and neutron-boson degrees of freedom are not distinguished. The model has an inherent group structure, associated with it. In terms of s- and d-boson operators the most general IBM Hamiltonian can be expressed as [11]

$$H = \varepsilon_s s^\dagger s + \varepsilon_d (d^\dagger . d) + \sum_{L=0,2,4} c_L [(d^\dagger d^\dagger)^{(L)} .(dd)^{(L)}] + 1/2 v_0 [(d^\dagger d^\dagger)_0^{(0)} s^2 + (s^\dagger)^2 (dd)_0^{(0)}]$$
$$+ \sqrt{1/2}\, v_2 [[(d^\dagger d^\dagger)^{(2)} ds]_0^{(0)} + [s^\dagger d^\dagger (dd)^{(2)}]_0^{(0)} + 1/2 u_0 (s^\dagger)^2 s^2 + 1/\sqrt{5} u_2 s^\dagger s (d^\dagger . d). \quad (1)$$

This Hamiltonian contains 2 one-body term, (ε_s and ε_d), and 7 two-body interactions [c_L (L=0,2,4), v_L (L=0,2), u_L (L=0,2)], where ε_s and ε_d are the single-boson energies, and c_L, v_L and u_L describe the two-boson interactions. However, it turn out that for fixed boson number N, only one of the one-body terms and five of the two-body terms are independent, as it can be seen by noting $N = n_s + n_d$. (1) Hamiltonian can be rewritten in terms of the Casimir operators of U(6) group. In that case, one says that the Hamiltonian H has a dynamical symmetry. These symmetries are called SU(5) vibrational, SU(3) rotational and O(6) γ-unstable.

The E2 transition operator must be a hermitian tensor of rank two and therefore the number of bosons must be conserved. Since, with these constraints there are two operators possible in the lowest order, the general E2 operator can be written as [13]

$$T_m(E2) = \alpha_2 [s^\dagger d + ds)]_m^{(2)} + \beta_2 [d^\dagger d]_m^{(2)}, \quad (2)$$

where α_2 plays the role of the effective boson charge and $\beta_2 = \sqrt{7}/2\alpha_2$. The B(E2) strength for the E2 transitions is given by

$$B(E2; L_i \rightarrow L_f) = 1/(2L_i+1)^{1/2} \left|\langle L_f \| T_m(E2) \| L_i \rangle\right|^2 . \quad (3)$$

Similarly, the M1 operator would be just $\beta_1 [d^\dagger d]_1$. To have M1 transitions, the IBM-1 rule must be extended to second-order in the U(6) generators [14]. The most general second-order M1 generator can then be written as

$$T(M1) = (g_b + AN)\mathbf{L} + B_1 [Q_1 \mathbf{L}]_1 + B_2 [Q_2 \mathbf{L}]_1 + C n_d \mathbf{L} . \quad (4)$$

Rather than attempting to evaluate the E2 and M1 matrix elements for $^{122-128}Te$ isotopes essential in theoretical mixing ratio calculations, it is possible to obtain these ratios in an analytic form as the matrix element which has a simple structure in the O(6) and SU(5) limits. The calculated reduced E2/M1 mixing ratio [15]

$$\Delta(E2/M1) = \frac{\langle n_d, K, L \| T(E2) \| n_d + 1, K', L' \rangle}{\langle n_d, K, L \| T(M1) \| n_d + 1, K', L' \rangle} \quad (5)$$

are related to mixing ratios, $\delta(E2/M1)$ by

$$\delta(E2/M1) = 0.835E_{\gamma}\Delta(E2/M1), \quad (6)$$

where E_{γ} is called the transition energy and it is given in MeV and $\Delta(E2/M1)$ is in eb/μ_n.

3. RESULTS AND DISCUSSION

The computer program PHINT [16] was used to make the Hamiltonian diagonal. The best fit values for the Hamiltonian parameters are given in Table 1 and the calculated energy values which are compared with the experimental data [17-21] are given in Figs. 1-4 for $^{122\text{-}128}$Te isotopes. The agreement is good for member of ground state, γ and β bands.

Table 1. Hamiltonian parameters

Isotopes	EPS	PAIR	ELL	QQ	OCT	HEX
^{122}Te	0.500	1.4	-0.01	0.001	0.00	0.0099
^{124}Te	0.570	1.4	-0.01	0.010	-0.0050	0.0131
^{126}Te	0.611	1.4	-0.01	0.020	0.0020	0.0038
^{128}Te	0.611	1.4	-0.01	0.020	-0.0065	0.0209

The calculated $\delta(E2/M1)$ values are given in Table 2 together with experimental data [22-26]. It can be seen from the Table 2 that most of our results are in better agreement with those obtained experimentally. E2/M1 mixing ratios are compared with only experimental data; there exist no previous theoretical value for E2/M1 mixing ratios.

The calculated values in this study show that the transitions connect the levels with the same parity and the E2 transitions are predominant. The later includes transitions originating from, β and γ bands which supports the idea that the β and γ bands may be quadrupole excitations of the perturbed ground state; but the existence of M1 the order of 10% indicates that the β and γ bands can not be pure quadrupole excitations of the ground state band.

Several E2 transition probabilities are experimentally investigated [27-31]. In Table 3, theoretical and experimental data are compared for proton charge e_{π}=2e. It can be seen from the Table 3 that theoretical B(E2) values agree with the experimental data within the indicated errors. The B(E2) values for the so-called cross-over $2_2^+ \rightarrow 0_1^+$ transition are well reproduced and they are very small. This shows that the particle and the collective contributions in the theoretical B(E2) values are out of phase. In the case of $2_2^+ \rightarrow 2_1^+$ transition B(E2) values are much larger when compared to the ones in

$2_2^+ \rightarrow 0_1^+$ transition. This is the inclusion of being both contributions in phase. A satisfactory comparison with the experiments is quite difficult due to the large errors on the experimental values, moreover the theoretical B(E2) values for that the transition seem to be systematically too small. This can be explained by the fact that many small components of the initial and final wave functions contribute coherently to the value of this reduced E2 transition probability. Since these small components are not stable enough against small changes in the model parameters, a quantitative comparison with the experimental data is not possible.

Table 2. E2/M1 mixing ratios for $^{122\text{-}128}$Te isotopes

Isotopes	Transition Energy E_γ (keV)	Spin Parity $I_i^\pi \rightarrow I_f^\pi$	This Work $\lvert\delta(E2/M1)\rvert$	Experiment* $\delta(E2/M1)$
^{122}Te	692.6	$2_2^+ \rightarrow 2_1^+$	1.04	-3.48
	728.3	$4_2^+ \rightarrow 4_1^+$	0.57	-0.57
	694.3	$2_3^+ \rightarrow 2_1^+$	1.04	----
	1386.9	$2_3^+ \rightarrow 2_2^+$	2.08	$-0.3<\delta<0.0$
	860.7	$4_3^+ \rightarrow 4_1^+$	1.20	$1.3^{+0.3}_{-0.4}$
^{124}Te	722.8	$2_2^+ \rightarrow 2_1^+$	3.35	-3.55, -3.40
	709.3	$4_2^+ \rightarrow 4_1^+$	0.18	-0.18 -0.26
	1436.5	$2_3^+ \rightarrow 2_1^+$	1.27	$1.5^{+0.6}_{-0.3}$
	713.7	$2_3^+ \rightarrow 2_2^+$	1.54	0.52 0.10 0.23
^{126}Te	753.9	$2_2^+ \rightarrow 2_1^+$	4.80	$-4.8,\ -4.25^{+0.15}_{-0.01}$
	1378.8	$4_2^+ \rightarrow 4_1^+$	1.74	$0.09<\delta<1.8^{+0.7}_{-0.4}$
^{128}Te	776.8	$2_2^+ \rightarrow 2_1^+$	2.66	$4.6^{+1.6}_{-1.0}$
	1225.3	$2_3^+ \rightarrow 2_1^+$	4.20	$4.2^{+2.0}_{-1.0}$
	531.0	$3_3^+ \rightarrow 4_1^+$	1.24	1.4
	643.6	$3_3^+ \rightarrow 2_2^+$	1.84	$0.45,\ 4.2^{+2.5}_{-1.2}$

* Experimental values from Teixeira and Goldman (1993), Samuel *et al.* (1977), Hashizame *et al.* (1987), Kitao *et al.* (1986), Warr *et al.* (1998)

Table 3. B(E2) values for $^{122\text{-}128}$Te isotopes in e^2b^2

Isotopes	$I_i^\pi \to I_f^\pi$	This Work	Experiment*	Previous Work*
^{122}Te	$2_1^+ \to 0_1^+$	0.13	0.132±0.012	0.0858
			0.130±0.020	0.1454
			0.130	0.129
	$2_2^+ \to 2_1^+$	0.23	0.35±0.16	0.146
			0.350	0.1128
	$2_2^+ \to 0_1^+$	0.01	0.0035±0.0017	0.0050
			0.0033	0.0038
				0.003
	$4_1^+ \to 2_1^+$	0.20	0.196	0.198
				0.1612
^{124}Te	$2_1^+ \to 0_1^+$	0.11	0.1138±0.0015	0.1145
			0.1140±0.0018	0.1150
			0.1150±0.0040	0.112
	$2_2^+ \to 2_1^+$	0.16	0.5686±0.2710	0.1598
			0.6220±0.3230	0.57
	$2_2^+ \to 0_1^+$	0.004	0.0045±0.0022	0.0041
			0.0050±0.0030	0.0044
	$4_1^+ \to 2_1^+$	0.16	0.1630	0.1874
			0.144	0.1581
			0.2280	0.164
^{126}Te	$2_1^+ \to 0_1^+$	0.09	0.094±0.004	0.091
			0.1064±0.0010	0.102
				0.139
	$2_2^+ \to 2_1^+$	0.14	0.126	0.117
			0.170	0.118
				0.178
	$2_2^+ \to 0_1^+$	0.00	0.008	0.001
				0.0013
				0.0006
	$4_1^+ \to 2_1^+$	0.16	0.159	0.1
				0.131
				0.1685
^{128}Te	$2_1^+ \to 0_1^+$	0.07	0.076±0.006	---
			0.078	
	$2_2^+ \to 2_1^+$	0.04	0.049	0.130
	$2_2^+ \to 0_1^+$	0.00	---	0.09
	$4_1^+ \to 2_1^+$	0.12	---	---

* Experimental and previous theoretical values are taken from Lombard (1969), Lopac (1970), Degrieck and Berghe (1974), Samuel *et al.* (1977), Nagib *et al.* (1977), Robinson *et al.* (1983), Mardirosian and Stewart (1984), Subber *et al.* (1986), Rikovska *et al.* (1989), Subrahmanyeswara Rao, Bhaskara Rao (1990), Küçükbursa and Yörük (1999)

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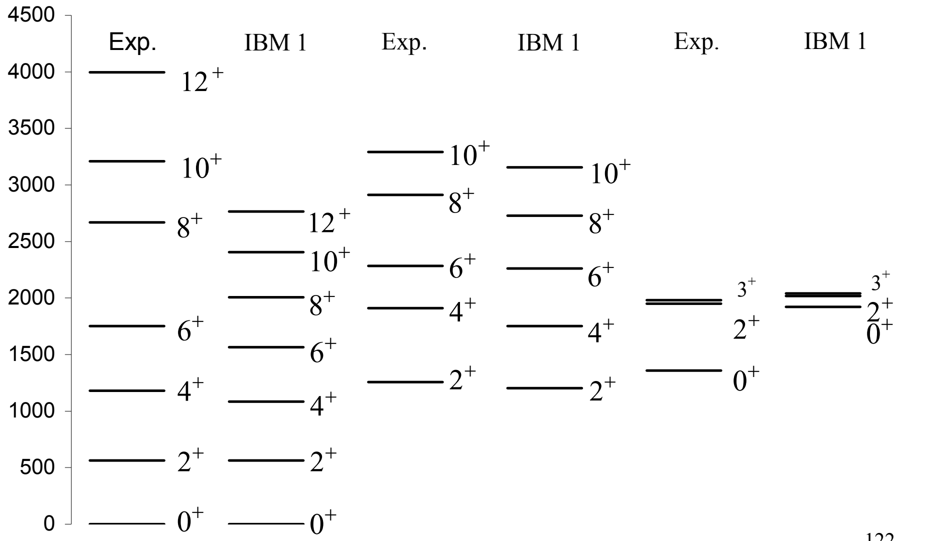
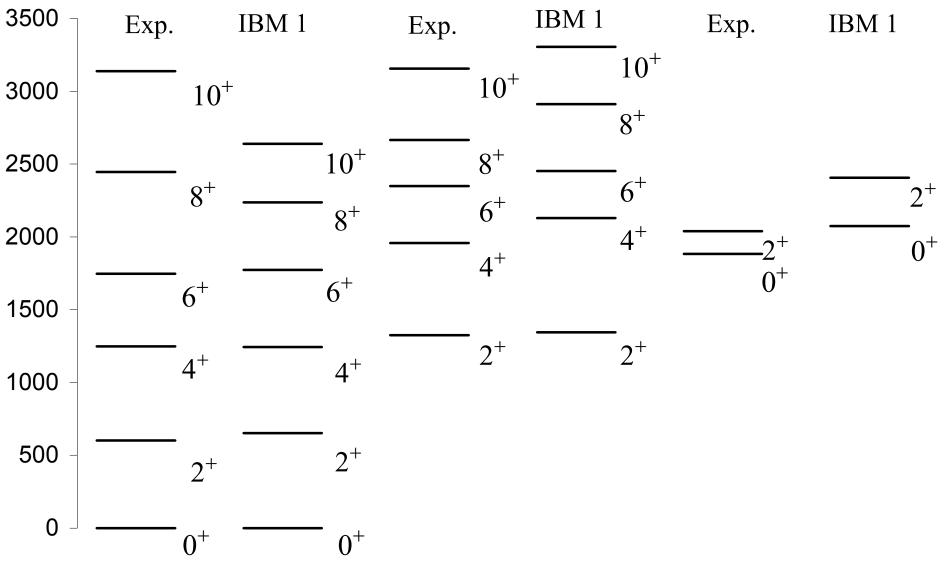


Fig.1. The experimental and theoretical energy levels for ^{122}Te



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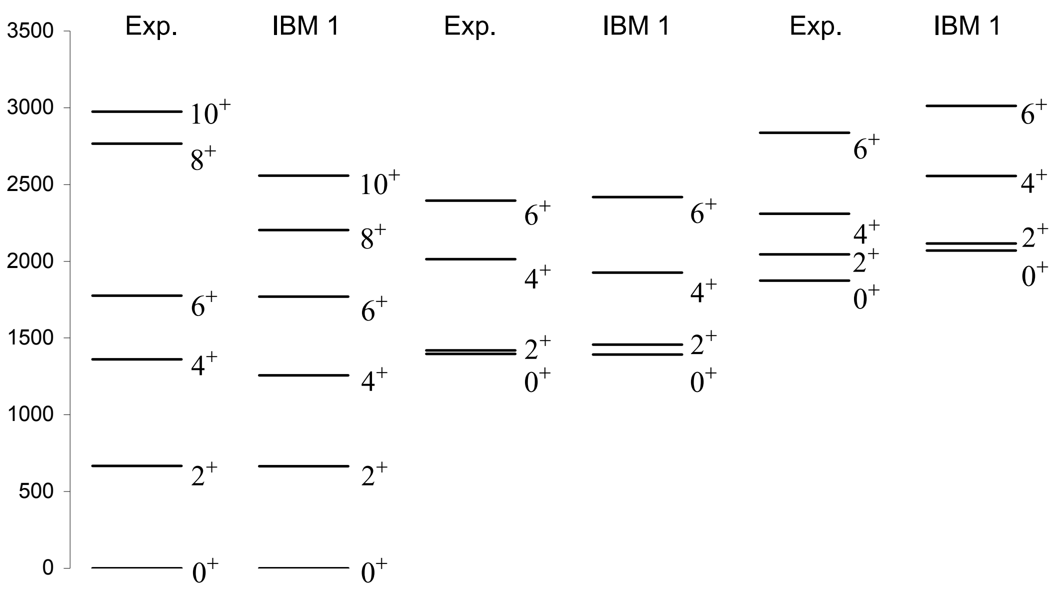

Fig.3. The experimental and theoretical energy levels for ^{126}Te

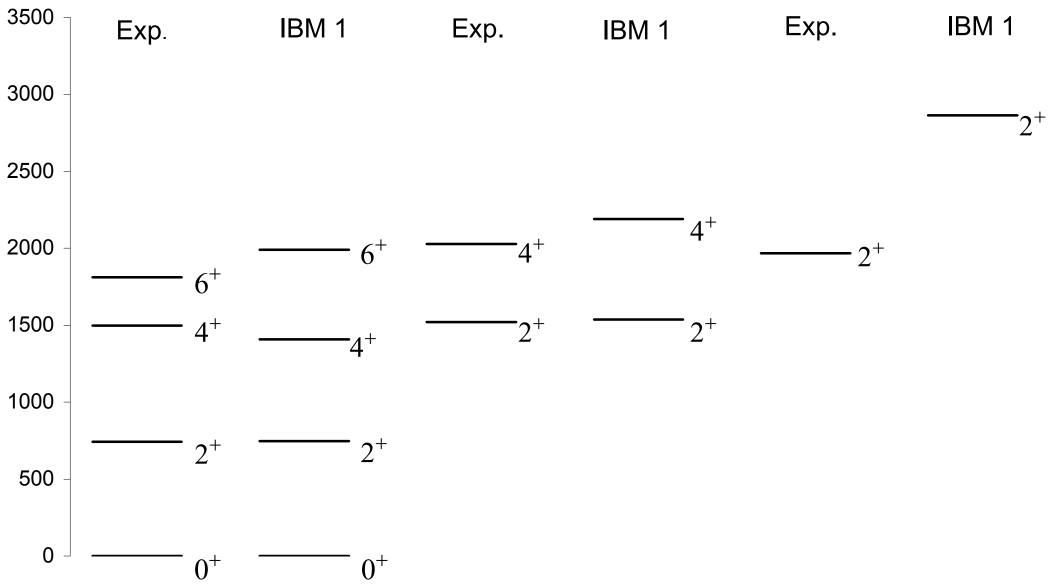

Fig.4. The experimental and theoretical energy levels for ^{128} Te