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DYNAMIC ANALYSIS OF SEMI-RIGID FRAMES

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Abstract-The dynamic response of semi-rigid frames is studied by using a computer program. The connection flexibility is modeled by linear elastic rotational springs. Having the same geometry and cross-section; semi-rigid frames, with different spring coefficients, are examined. The reducing coefficients and lateral rigidity values, representing the real behavior of frames, are determined for each frame. To represent the real behavior, all deformations of a frame are accounted for a dynamic analysis. Response characteristics of five different multistory frames are compared with reference to their modal attributes. The study indicates that connection flexibility tends to increase vibration periods, especially in lower modes, while it causes vibration frequencies decrease.
Keywords- semi-rigid, reducing coefficient, rotational spring

1. INTRODUCTION

Defining behaviors of frames under dynamic loads exactly takes a important place in earthquake engineering. In engineering design, to know the real behavior of a structure is provided by determining geometrical, damping, mass and connection model well. In design purposes; structures are designed as having rigid connections. However, the behavior of connections is not rigid. Structures having such flexible connections in which connection flexibility becomes important are called semi-rigid frames.

Semi-rigid frames are frames for which the beam-to-column joints are neither pinned nor rigid [1]. In reality all frames are semi-rigid , because there is not a frame which has truly pinned and perfectly rigid connections. For practical design; two classification systems were developed. The classification system by Bjorhovde et al [2] is based on the response of a frame element, while the other classification system by Eurocode 3 [3] is based on the response of a frame. These classification systems were developed by using the results of many studies performed in last decades.

2. SEMI-RIGID FRAME MODEL AND REDUCING COEFFICIENT

The semi-rigid frame model used for the present study is shown in Figure 1. This model includes a beam with moment of inertia I_b and length L, and two columns with moment of inertia I_c , length h and cross-section A_c. The modulus of elasticity E is the same in all frame elements.

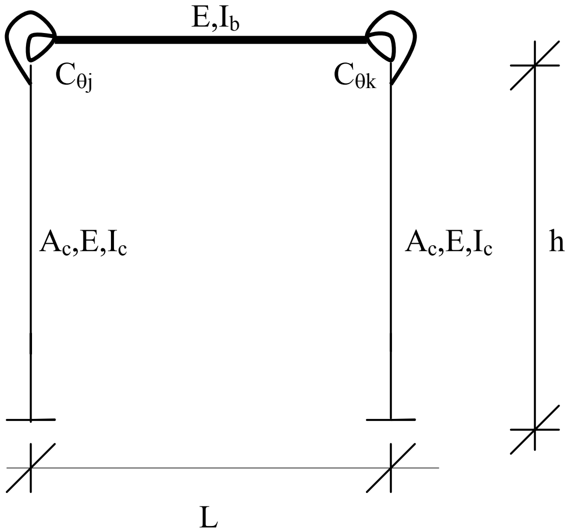

Figure 1 Semi-rigid frame model

The connections are modeled as rotational springs at beam-to-column joints. All deformations are incorporated in this study. One can define rigidity at the ends of frame element by the term of rigidity index. For a connection with hinge, rigidity index is zero , and flexural moments do not occur at the ends of a frame element. For a rigid connection , this value is infinite , and flexural moments occur at the ends of a frame element [4]. Flexural moments at the two ends for a frame element, with spring coefficients represented by $C_{\theta j}$ and $C_{\theta k}$, can be given by

$$M_{jf} = C_{\theta j} x \overline{\Phi}_j \ ; \ M_{kf} = C_{\theta k} x \overline{\Phi}_k \tag{1}$$

where M_{jf} and M_{kf} are flexural moments, respectively, at j and k ends of a frame element , $\overline{\Phi}_j$ and $\overline{\Phi}_k$ are rotations, occurred by rotational springs.

The relationship between spring coefficients and rigidity index can be written by

$$R_j = \frac{C_{\theta j} L_i}{EI_x} \quad ; \quad R_k = \frac{C_{\theta k} L_i}{EI_x} \tag{2}$$

where R_j and R_k are rigidity index at two ends of a frame element, respectively.

Rotations at two ends and axial displacement of a semi-rigid frame element given by Figure 2 are element displacements.

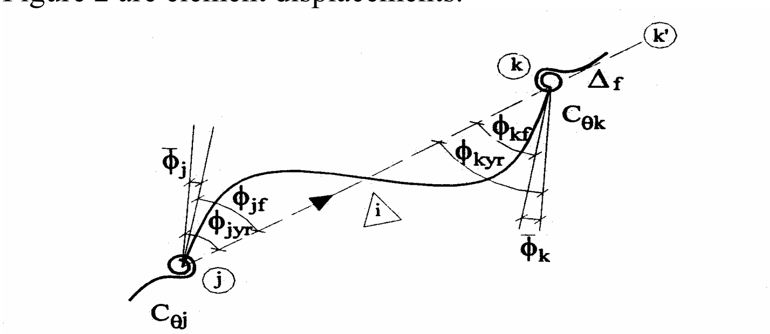

Figure 2 Displacements of a semi-rigid frame element

ϕ_{jyr} and ϕ_{kyr} are total rotations at two ends of a semi-rigid element, ϕ_{jf} ve ϕ_{kf} are rotations occurred without rotational springs at two ends of a semi-rigid element, respectively. $\bar{\phi}_j$ and $\bar{\phi}_k$ can be written by using equation (1) and equation (2).

$$\bar{\phi}_j = \frac{M_{jf} L_i}{R_j E I_x} \quad ; \quad \bar{\phi}_k = \frac{M_{kf} L_i}{R_k E I_x} \tag{3}$$

Using rotational springs, the stiffness matrix relating rigidity index at the ends is given by equation (4) [4].

$$[K_{yr}^t] = \begin{bmatrix} \frac{4EI_x}{L_i}\beta_1 & \frac{2EI_x}{L_i}\beta_2 & 0 \\ \frac{2EI_x}{L_i}\beta_2 & \frac{4EI_x}{L_i}\beta_3 & 0 \\ 0 & 0 & \frac{AE}{L_i} \end{bmatrix} \tag{4}$$

where ;

$$\beta_1 = \frac{3\lambda_1\lambda_2}{\left(4\lambda_1^2\lambda_2 - \lambda_1\right)} \quad ; \quad \beta_2 = \frac{3}{\left(4\lambda_1\lambda_2 - 1\right)} \quad ; \quad \beta_3 = \frac{3\lambda_1}{\left(4\lambda_1\lambda_2 - \lambda_1\right)} \tag{5}$$

$$\lambda_1 = \left(1 + \frac{3}{R_j}\right) \quad ; \quad \lambda_2 = \left(1 + \frac{3}{R_k}\right) \tag{6}$$

The stiffness matrix of a semi-rigid column element in Figure 1 can be written by

$$[K_{cf}] = \begin{bmatrix} \frac{12EI_c}{h^3}\gamma_1 & 0 & -\frac{6EI_c}{h^2}\gamma_2 & -\frac{12EI_c}{h^3}\gamma_1 & 0 & -\frac{6EI_c}{h^2}\gamma_3 \\ 0 & \frac{A_cE}{h} & 0 & 0 & -\frac{A_cE}{h} & 0 \\ -\frac{6EI_c}{h^2}\gamma_2 & 0 & \frac{4EI_c}{h}\beta_1 & \frac{6EI_c}{h^2}\gamma_2 & 0 & \frac{2EI_c}{h}\beta_2 \\ -\frac{12EI_c}{h^3}\gamma_1 & 0 & \frac{6EI_c}{h^2}\gamma_2 & \frac{12EI_c}{h^3}\gamma_1 & 0 & \frac{6EI_c}{h^2}\gamma_3 \\ 0 & -\frac{A_cE}{h} & 0 & 0 & \frac{A_cE}{h} & 0 \\ -\frac{6EI_c}{h^2}\gamma_3 & 0 & \frac{2EI_c}{h}\beta_2 & \frac{6EI_c}{h^2}\gamma_3 & 0 & \frac{4EI_c}{h}\beta_3 \end{bmatrix} \tag{7}$$

where ;

$$\gamma_1 = \frac{\beta_1 + \beta_2 + \beta_3}{3} \quad ; \quad \gamma_2 = \frac{2\beta_1 + \beta_2}{3} \quad ; \quad \gamma_3 = \frac{2\beta_3 + \beta_2}{3} \tag{8}$$

The stiffness matrix of a semi-rigid beam element in Figure 1 can be written by

$$[K_{bf}]=\begin{bmatrix} \frac{A_bE}{L} & 0 & 0 & -\frac{A_bE}{L} & 0 & 0 \\ 0 & -\frac{12EI_b}{L^3}\gamma_1 & \frac{6EI_b}{L^2}\gamma_3 & 0 & -\frac{12EI_b}{L^3}\gamma_1 & \frac{6EI_b}{L^2}\gamma_3 \\ 0 & \frac{6EI_b}{L^2}\gamma_2 & \frac{4EI_b}{L}\beta_1 & 0 & -\frac{6EI_b}{L^2}\gamma_1 & \frac{2EI_b}{L}\beta_2 \\ -\frac{A_bE}{L} & 0 & 0 & \frac{A_bE}{L} & 0 & 0 \\ 0 & -\frac{12EI_b}{L^3}\gamma_1 & -\frac{6EI_b}{L^2}\gamma_1 & 0 & \frac{12EI_b}{L^3}\gamma_1 & -\frac{6EI_b}{L^2}\gamma_3 \\ 0 & \frac{6EI_b}{L^2}\gamma_3 & \frac{2EI_b}{L}\beta_2 & 0 & -\frac{6EI_b}{L^2}\gamma_3 & \frac{4EI_b}{h}\beta_3 \end{bmatrix} \tag{9}$$

The structure stiffness matrix is obtained by assembling the column and beam stiffness matrices described above according to conventional stiffness matrix analysis procedure. One obtains a 6x6 stiffness matrix for the frame of Figure 3.

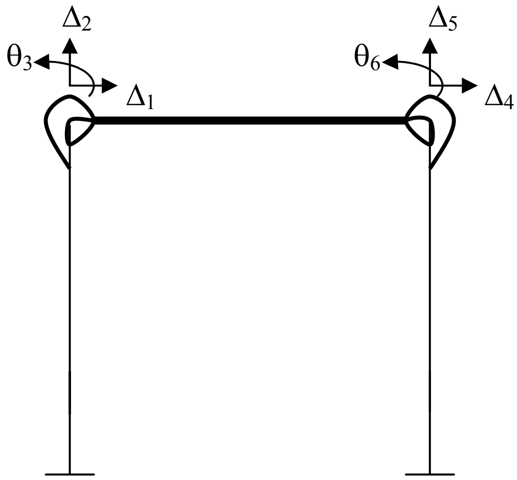

Figure 3 Degrees-of-freedom

By assuming that Δ_1 and Δ_4 are equal, one can eliminate Δ_4 from the frame of Figure 4.The reduced displacements are given by Figure 4. The remaining stiffness matrix is a 5x5 matrix.

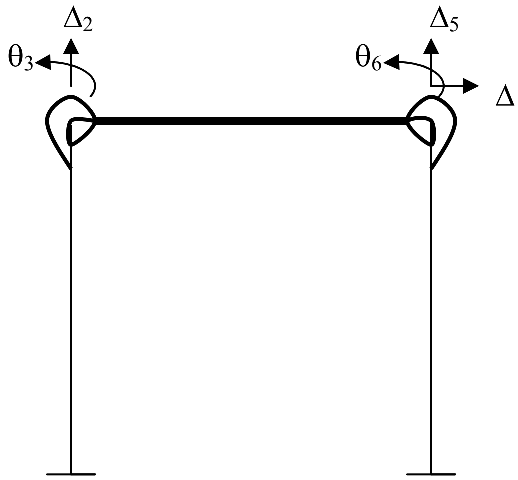

Figure 4 The reduced displacements

$$\{F\} = [Ksf] \times \{ \delta \} \tag{10}$$

The relationship between deformations and forces are given by equation 10.Solving the above matrix equation for displacements except Δ and back substituting the result into the first row, the one-degree-of –freedom system stiffness relationship can be written as

$$F=\frac{24EI_x}{h^3}\alpha_r \times \Delta \tag{11}$$

where Δ is the lateral displacement ,and F and α_r are the lateral force and reducing coefficient respectively.

3. DYNAMIC ANALYSIS AND NUMERICAL STUDIES

The primary objective of the present study is to investigate the dynamic characteristics of semi-rigid frames and how connection flexibility influences them. For a given frame in Figure 4, the equation of motion for a semi-rigid frame in free vibration is given by

$$[\,M\,]\,\{\ddot{v}\}+[k]\{\,v\,\}=\{0\} \tag{12}$$

where $\ddot{v}$ and v are, respectively, acceleration and displacement of a structure.

The dynamic characteristics of semi-rigid frame are determined by modal analysis. The frequency and period of a vibration will be investigated. The influence of connection flexibility will be studied.

In the present study, 3-story semi-rigid frames having four different spring coefficients and a rigid connected frame were studied. The semi-rigid model for the present analysis is given in Figure 5. All frames have the same geometry, cross-section and material property to compare the influence of connection flexibility on dynamic characteristics. First, the reducing coefficients were determined by using a computer program. Then, lateral rigidity values were calculated for each frame. The reducing coefficients and periods are given in Table 1 below.

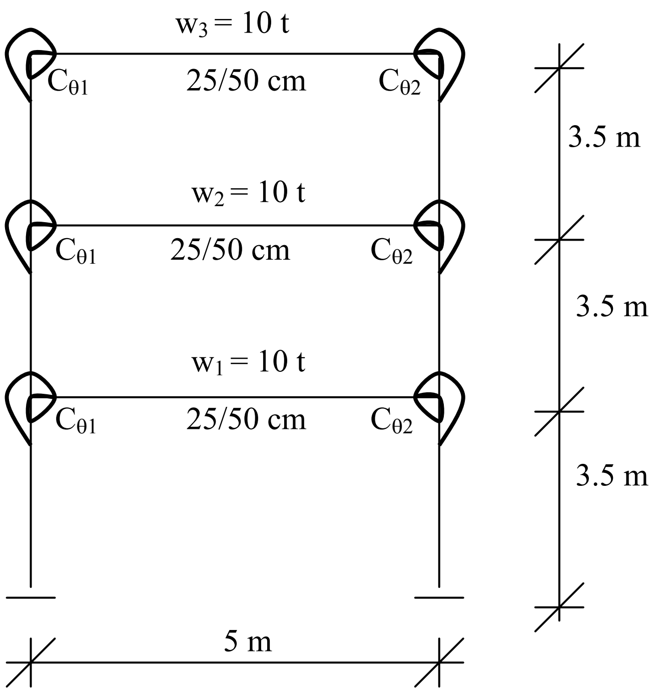

Figure 5 Semi-rigid model for the present analysis

Table 1 Reducing coefficients

Connection model	Reducing coefficient (α_r)	Lateral rigidity (t / m)
Semi-rigid (C_θ=2000 tm/rd)	0.1217	596.28
Semi-rigid (C_θ=5000 tm/rd)	0.2309	1131.23
Semi-rigid (C_θ=20000 tm/rd)	0.4255	2084.29
Semi-rigid (C_θ=10^{20} tm/rd)	0.5978	2928.11
Rigid	0.5978	2928.11

The results of the conducted analysis are given for each mod of vibration below.

Table 2 Dynamic results of 1st mod

Connection model	Frequency ω_i (rd / sec)	Period T_i (sec)
Semi-rigid (2000 tm/rd)	10.7603	0.5839
Semi-rigid (5000 tm/rd)	14.8210	0.4239
Semi-rigid (20000 tm/rd)	20.1178	0.3123
Semi-rigid (10^{20} tm/rd)	23.8448	0.2635
Rigid	23.8448	0.2635

Table 3 Dynamic results of 2nd mod

Connection model	Frequency ω_i (rd / sec)	Period T_i (sec)
Semi-rigid (2000 tm/rd)	30.1497	0.2084
Semi-rigid (5000 tm/rd)	41.5274	0.1513
Semi-rigid (20000 tm/rd)	57.7350	0.1088
Semi-rigid (10^{20} tm/rd)	66.8117	0.0940
Rigid	66.8117	0.0940

Table 4 Dynamic results of 3rd mod

Connection model	Frequency ω_i (rd / sec)	Period T_i (sec)
Semi-rigid (2000 tm/rd)	30.1497	0.2084
Semi-rigid (5000 tm/rd)	43.5677	0.1442
Semi-rigid (20000 tm/rd)	81.4552	0.0771
Semi-rigid (10^{20} tm/rd)	96.5480	0.00651
Rigid	96.5480	0.00651

4. CONCLUSIONS

A semi-rigid frame was modeled by rotational springs. The stiffness matrix was obtained by using rigidity index at the ends of a semi-rigid frame element. A computer program was written to obtain the reducing coefficients from this 5x5 stiffness matrix. Dynamic analysis was performed for five different types of connection. The effects of connection flexibility were investigated.

In a semi-rigid frame, an increase in the rate between length of bay and height of story (L/h) causes reducing coefficient and lateral rigidity decrease, and in the same rate between length of bay and height of story (L/h) , the reducing coefficients for frames with lower spring coefficients are lower than the reducing coefficients for frames with higher spring coefficients.

The dynamic behavior of a semi-rigid frame is different from the dynamic behavior of a rigid connected frame. Since the connection flexibility influences the dynamic characteristics of frames. The study indicates that connection flexibility tends to increase periods, especially in lower modes, while it tends to decrease the frequency.

REFERENCES

1. J.C. Awkar and E.M. Lui, Seismic analysis and response of multistory semi-rigid frames, Journal of Engineering Structures, 30, 425-441, 1997.
2. R .Bjorhovde, A .Colson and J. Brozetti, A classification system for beam to column connections, ASCE Journal of Structural Engineering,116,3059-3076,1990
3. European Community, Design of steel structure; part1 general rules and rules for buildings, Brussels,Belgium,1990.
4. H.H. Catal, Yapı ve deprem mühendisliğinde matris yöntemler, İzmir, 2002.
5. W.F .Chen, Joint flexibility in steel frames, New York, Elsevier, 987.
6. W.F .Chen, Steel beam to column connections, New York, Elsevier, 1987.
7. W.F .Chen, Y. Goto and R. Liew, Stability design of semi rigid frames, New York, Wiley, 1993.
8. A.K. Chopra, Dynamics of structures: Theory and applications to earthquake engineering, Englewood Cliffs, NJ, Prentice Hall, 1995.
9. R.W. Clough and J. Penzien, Dynamics of structures 2nd ed, New York, NY, McGraw-Hill,1993.
10. R.F. Lorenz, B. Kato and W.F. Chen, Council on tall buildings and urban habitat, Semi rigid connections in steel frames, New York, 1993.
11. E.M. Lui, A. Lopes, Dynamic analysis and response of semi-rigid Frames. Journal of Engineering Structures,19(8),644-654,1997.