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FINITE ELEMENT MODELING OF DRILLSTRINGS

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Abstract

A finite element model is presented for investigating dynamic behavior of drillstrings used in oil and gas wells. A Timoshenko beam element is used to model axial and transverse vibrations. The effect of axial load, gravity, the constraints due to stabilizers are considered. The results show that a small amount of disorder in the configuration as well as the gravity cause strong mode localization which may be catastrophic. The implications for design and operation in the presence of mode localization is also discussed.

1 Introduction

The Bottom Hole Assemblies (BHA), the lower part of drillstrings used for the drilling of oil and gas wells are subject to severe vibrations that are often blamed for failures. There is now sufficient evidence that most drillstring failures are primarily caused by fatigue induced by vibrations [1]. In addition to the component failures, in most cases drillstring vibrations represent a loss or waste of drilling energy. Therefore, severe vibrations may result in deviations from optimal drilling conditions [2].

Though there has been considerable research in the modeling and analysis of drillstring dynamics, a comprehensive understanding of all the vibration phenomena involved is still lacking. Furthermore, the complex and varying nature of the boundary conditions, operational characteristics and parameters undermine the utility of available models with respect to their predictive capabilities. For this reason the use of experimental drillstring measurement tools are currently the only reliable methods for improving performance and solving real-time drilling problems [3]. Theoretical studies on drillstring dynamics are still important however, to improve the understanding of the various phenomena and thus provide better interpretation of experimental data.

Finite Element Method (FEM) has long been used for both static and dynamic analysis of drillstrings. Of note is the GEODYN program developed in Sandia National Laboratories [4]. This program is designed to simulate the three-dimensional, transient dynamic response of a bit-drillstring system interacting with a geological formation. Kalsi et al., [5] used a one dimensional FEM model for transient dynamic analysis of the drillstring under jarring operations. Mitchell and Allen [6] used MARC general purpose FEM code to investigate axial, lateral and torsional modes of vibration. They concluded that lateral vibration was the dominant cause of reported failures.

The present work is an attempt to examine the drillstring dynamics as a whole structure. An FEM model of the entire BHA is developed. The natural frequencies as well as the mode shapes are investigated through modal analysis. It is found that a phenomenon known as "mode localization" may occur in certain drillstring configurations, which may explain some drillstring failures.

2 FEM Model Description

In this study only planar vibrations are considered. A two-dimensional Timoshenko beam element, including shear and rotary inertia effects is used to discretize the drillstring. The effect of axial load, as well as parametric coupling between the axial and transverse vibrations is included by considering geometric stiffening effect. The resulting element stiffness and mass matrices are given in the Appendix.

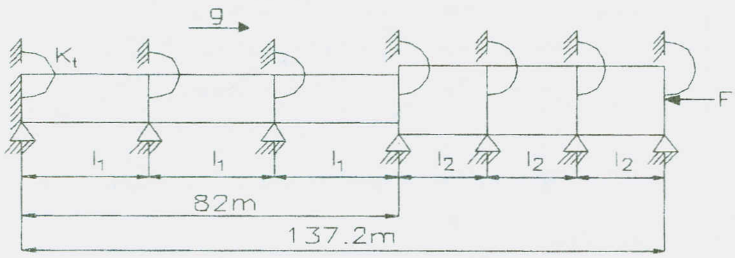

Fig. 1 A Sketch of the Bottom Hole Assembly.

A field case reported in Reference [6] has been selected and the whole bottomhole assembly shown in Fig. 1 is modeled. There are six steel drill collars of two different cross sections, supported by seven stabilizers. Stabilizers are modeled as permanent point contacts which constrain the transverse motion, but not the axial motion. The effect of finite stabilizer contact is modeled as effective torsional springs. Certainly, this is an approximation and can only be justified for certain cases (i.e., fully gauged stabilizers). The inner and outer diameters for one set of drill collars are 0.2m and 0.073m respectively whereas for the other set, they are 0.23m and 0.073m respectively. The torsional springs located at the stabilizers have a nominal value of $1 \times 10^{10}\ Nm/rad$. For the axial vibrations, the BHA is assumed to be fixed at the bit and free at the drillpipe interface. This assumption can be justified by noting that the drillpipe has relatively smaller cross sectional area and has very little effect on the dynamics of BHA. The well is assumed to be perfectly vertical and gravity load is taken into account as a body force. The amount of Weight-on-Bit (WOB), the axial force at the bit, is adjusted by adding a constant force at the bit (see Fig. 1).

A standard assembly procedure by applying the appropriate compatibility conditions and boundary conditions yields the following discrete equations of motion:

$$M\ddot{x} + Kx = F \tag{1}$$

where M and K are the global mass and stiffness matrices, respectively; x is the vector of nodal degrees of freedoms, and F is the global load vector. The corresponding eigenvalue problem can be written as

$$Ku = \omega^2 Mu \tag{2}$$

where u is the mode shape associated with natural frequency ω.

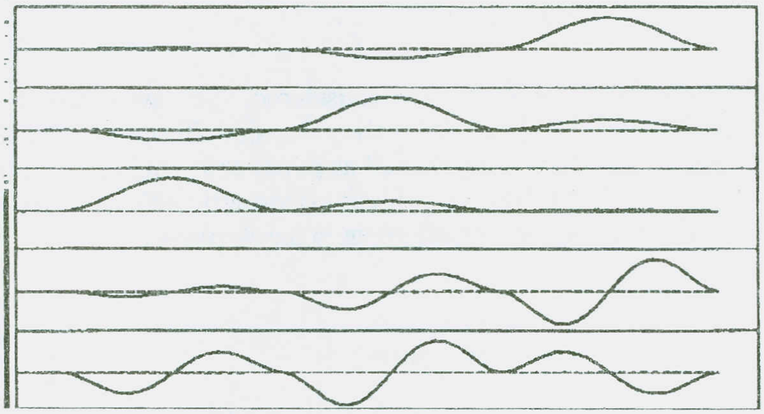

Fig. 2 Normal Mode Shapes With Gravity Force and without disorder, (modes 4,5,6,14,15).

3 Results and Discussion

Modal analysis is carried out for the model described above. A constant axial force of 170 kN is applied at the bit in addition to the distributed gravity force. The first 25 modes are extracted and classified as either axial or transverse mode. Table 1 gives the first ten natural frequencies. Some of the transverse mode shapes relevant for the BHA are given in Fig 2.

Mode	1	2	3	4	5	6	7	8	9	10
Case 1	1.26	1.30	1.35	2.99	3.02	3.05	3.44	3.50	3.56	6.72
Case 2	1.37	1.37	1.37	3.00	3.00	3.00	3.60	3.60	3.60	6.90
Case 3	1.36	1.37	1.37	2.95	2.98	3.02	3.59	3.59	3.60	6.89

Table 1. First Ten Natural Frequencies (*Hz*).

From the modal analysis it can be concluded that the whole BHA has to be modeled for an accurate prediction of transverse mode shapes and frequencies since the mode shapes are distributed over few segments. This is in contrast with some of the earlier studies in bending vibrations where only the segment between the bit and the first stabilizer is considered (this can only be justified if the stabilizer is assumed to constrain rotation of the section completely).

This drillstring configuration is demonstrating a strong mode localization. This can be verified by observing the mode shapes which show typical localized behavior (see Fig. 2). The effect of disorder and imperfect boundary conditions should also be examined from this viewpoint. It is believed that the primary reason for this mode localization is the gravity force which acts as a disorder. In order to test this hypothesis, the same system is analyzed without gravity. For a fair comparison the drillstring is loaded by constant axial loads to result in the same amount of WOB. The resulting mode shapes are not localized as shown in Fig. 3. It is therefore concluded that the gravity is the main culprit for mode localization. Fig. 4 shows the mode shapes obtained by introducing a small disorder by perturbing the span lengths by only 0.6 %. In this case the gravity is not included. As is seen the effect of this disorder is to cause a very strong mode localization.

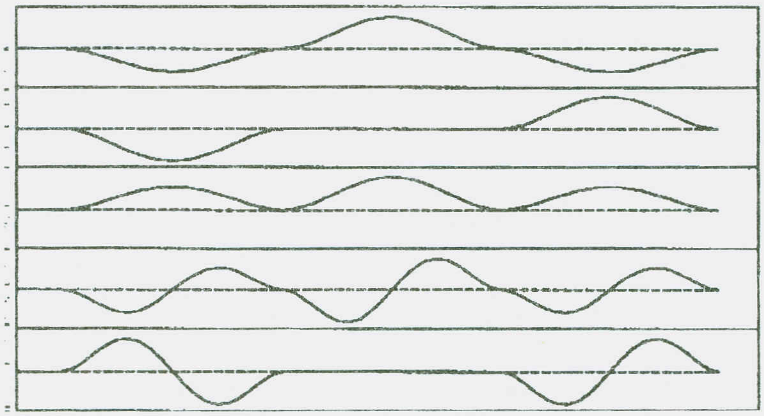

Fig. 3 Normal Mode Shapes without Gravity Force and without disorder, (Modes 4,5,6,14,15).

The mode shapes shown are also useful in identifying the critical locations for harmonic analysis. Clearly, some of the modes will not be observed well at certain locations due to the low amplitudes of the mode shapes at these locations. These mode shapes should also be considered for the placement of downhole measurement equipment.

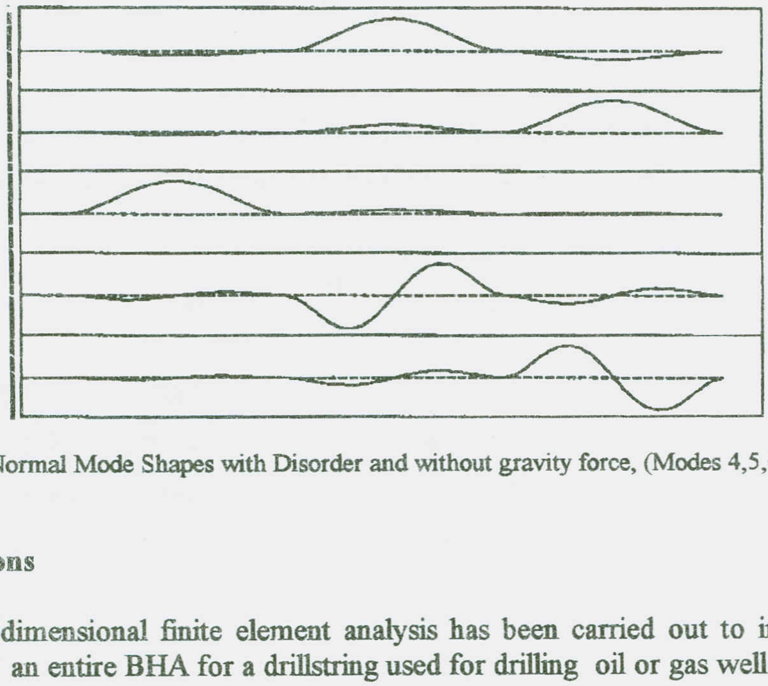

Fig. 4 Normal Mode Shapes with Disorder and without gravity force, (Modes 4,5,6,14,15).

4 Conclusions

A two dimensional finite element analysis has been carried out to investigate the dynamics of an entire BHA for a drillstring used for drilling oil or gas wells. The model includes the effect of axial load as well as gravity. The stabilizers are modeled as transverse and torsional springs. It has been found that most of the mode shapes are strongly localized in small geometric regions. This may explain some of the failures in these systems. Furthermore, this phenomenon should be considered and special attention should be paid for the placement of transducers used to measure vibrations.

Acknowledgment

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Appendix

Element stiffness matrix is given as $K = K_l + K_g$, where K_l is the usual elastic stiffness matrix and K_g is the geometric stiffness matrix associated with the axial load.

$$
[K_l] = \begin{bmatrix}
\frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\
 & \frac{12EI}{L^3(1+\Phi)} & \frac{6EI}{L^2(1+\Phi)} & 0 & -\frac{12EI}{L^3(1+\Phi)} & \frac{6EI}{L^2(1+\Phi)} \\
 & & \frac{EI(4+\Phi)}{L(1+\Phi)} & 0 & -\frac{6EI}{L^2(1+\Phi)} & \frac{EI(2-\Phi)}{L(1+\Phi)} \\
 & & & \frac{AE}{L} & 0 & 0 \\
 & & & & \frac{12EI}{L^3(1+\Phi)} & -\frac{6EI}{L^2(1+\Phi)} \\
 & & & & & \frac{EI(4+\Phi)}{L(1+\Phi)}
\end{bmatrix}
$$

where A is the cross-sectional area, E is Young's Modulus, L is the element length, I is the area moment of inertia. The shear factor is given by $\Phi = \dfrac{12EI}{GA_S L^2} = 24(1+\gamma)\dfrac{A}{A_S}\left(\dfrac{r}{L}\right)^2$ where G is the shear modulus, γ is the Poission's ratio. $A_S = \dfrac{A}{F_S}$ is the shear area with F_s being the shear deflection constant and r is the radius of gyration.

For an axial load of F, the geometric stiffness matrix is given as

$$[K_g] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 6F/5L & F/10 & 0 & -6F/5L & F/10 \\ & & 2FL/15 & 0 & -F/10 & -FL/30 \\ & & & 0 & 0 & 0 \\ & & & & 6F/5L & -F/10 \\ & & & & & 2FL/15 \end{bmatrix}$$

The element mass matrix is given by :

$$[M_l] = \rho AL \begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ & \frac{13}{35}+\frac{6I}{5AL^2} & \frac{11L}{210}+\frac{I}{10AL} & 0 & \frac{9}{70}-\frac{6I}{5AL^2} & -\frac{13L}{420}+\frac{I}{10AL} \\ & & \frac{L^2}{105}+\frac{2I}{15A} & 0 & \frac{13L}{420}-\frac{I}{10AL} & -\frac{L^2}{140}-\frac{I}{30A} \\ & & & \frac{1}{3} & 0 & 0 \\ & & & & \frac{13}{35}+\frac{6I}{5AL^2} & -\frac{11L}{210}-\frac{I}{10AL} \\ & & & & & \frac{L^2}{105}+\frac{2I}{15A} \end{bmatrix}$$

where ρ is the material density.