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EXAMINATION OF TEMPERATURE DISTRIBUTION AS A RESULT OF HARD-BRAZING OF TWO DIFFERENT MATERIALS BY FINITE ELEMENTS METHOD

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ABSTRACT

Knowing the temperature effect on machine construction is the first step at material selection and planing machine parts exposed to temperature effect above normal conditions. Since joining processes such as welding and hard-brazing were done at high temperatures, abrupt variations in temperature distributions take place. Because of these temperature variations inner structure and shape changes occur in material. If two different materials are joint, these problem becomes more important due to mismatch of physical properties of materials joint. Because of this reason, temperature distribution must be known at these kind of joinings.

In this study only the temperature distribution calculations around the joining zone were carried out. In the future step the thermal stress and residual stress distributions during and after joining process will be calculated. For this purpose steel-brass material couple were chosen and the materials as thin plates were hard-brazed. The joining parts of this material couple were heated upto hard- brazing temperature and then cooled in air. In this numeric study finite elements method was used and the programming language was chosen Fortran77. In modelling two dimensionel four nodes rectangular elements were selected.

1. INTRODUCTION

It is very important to predict the values of residual stress to obtain the proper dimensioning in production industry. While production proceeses the part can be distorted because of residual stresses.

There can be many different sourcess of residual stresses but the most important one is temperature differences. Especially if a metal is heated and cooled rapidly, temperature differences between surface and center of the material will cause very large residual stresses. Developments in technology bring the need of using different material together. The temperature differences as the result of joining materials having different heat properties are more evident. For this reason the calculation of temperature differences before production is very important. In this study temperature differences at joining zone of two different materials were calculated. As materials brass and steel plates of 4 mm. thickness, 120 mm. in length and 60 mm. in width each were selected. Numerical calculations were carried out by FEM using two dimensional four nodes rectangular elements.

2. HEAT TRANSFER

Heat transfer analysts generally examine temperature distributions for constructions exposed to abnormal temperature levels and for material selection as the first step. Analysts must model the environmental boundary conditions realistically, express complex geometries, and analyze materials besides simple isotropic materials. Known problem solving tecnics are insufficient and for these situations, finite differences method can hardly give desired solutions then finite elements method becomes more important.

Finite elements method is used in many fields of engineering such as stress analyze, temperature and heat transfer analyse, fluid mechanics, electrical and magnetical fields. Problems handled are generally classified as scalar and vectoral field problems[4]. In this study transient heat transfer problem that is a scalar field problem will be handled

The most remarkable feature of scalar field problems is that it can be found in almost every branch of engineering and physics. These are generally expresed as Helmholtz equation.

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial \phi}{\partial z} \right) + \lambda \phi + Q = 0$$
(1)

In this equation $\phi = \phi(x,y,z)$ is the desired variable. Fourier law expressing two dimensional heat flow at termally isotropic environments is:

$$q_x = -k \frac{\partial T}{\partial x}$$
, $q_y = -k \frac{\partial T}{\partial y}$ (2)

here T=T(x,y,t) is the temperature fields in environment, q_x and q_y are heat flows at x and y directions respectly, k is heat transfer coefficient, and $\frac{\partial T}{\partial X}$, $\frac{\partial T}{\partial Y}$ are temperature variations respectly. Heat flow is the vectoral sum of q_x and q_y . If in equation 1. ϕ is taken as $\phi=T$ and fourier law is applied,

$$-\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right) + Q = \rho c \frac{\partial T}{\partial t}$$
(3)

is obtained. Here ρ and c are features of the material and express density and specific heat respectively, t is the time. This is the heat transfer equations and is solved according to an initial condition and to different boundary conditions.

2.1. Finite Elements Formulation

 Ω solution region given in fig.1. is divided into M elements each having are nodes temperature and temperature gradiend in an element is shown as follows.[4]

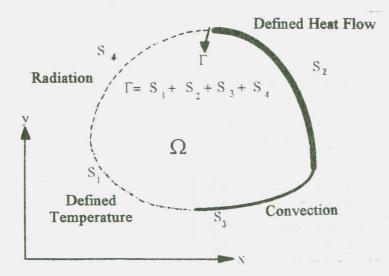


Figure 1. Solution region for two dimensional general heat transfer.

$$T(x,y,t) = \sum_{i=1}^{r} N_{i}(x,y)T_{i}(t)$$
(4)

$$\frac{\partial \mathbf{T}}{\partial \mathbf{x}}(\mathbf{x}, \mathbf{y}, \mathbf{t}) = \sum_{i=1}^{r} \frac{\partial \mathbf{N}_{i}}{\partial \mathbf{x}}(\mathbf{x}, \mathbf{y}) \mathbf{T}_{i}(\mathbf{t})$$
(5)

$$\frac{\partial \mathbf{T}}{\partial \mathbf{y}}(\mathbf{x}, \mathbf{y}, \mathbf{t}) = \sum \frac{\partial \mathbf{N}_{i}}{\partial \mathbf{y}}(\mathbf{x}, \mathbf{y}) \mathbf{T}_{i}(\mathbf{t})$$
(6)

When matrix notation is used then,

$$\mathbf{T}(\mathbf{x},\mathbf{y},\mathbf{t}) = \left[\mathbf{N}(\mathbf{x},\mathbf{y}) \right] \left\{ \mathbf{T}(\mathbf{t}) \right\}$$
(7)

$$\begin{bmatrix} \frac{\partial T}{\partial x}(x, y, t)\\ \frac{\partial T}{\partial y}(x, y, t) \end{bmatrix} = [B(x, y)] \{T(t)\}$$
(8)

Here

$$[N(x,y)] = [N1, N2, ... Nk]$$
(9)

is temperature interpolation matrix,

$$[B(x,y)] = \begin{bmatrix} \frac{\partial N1}{\partial x} & \frac{\partial N2}{\partial x} & \dots & \frac{\partial Nk}{\partial x} \\ \frac{\partial N1}{\partial y} & \frac{\partial N2}{\partial y} & \dots & \frac{\partial Nk}{\partial y} \end{bmatrix}$$
(10)

[B] is temperature variation interpolation matrix. T_i is the temperature of each node and $\{T(t)\}$ is element node temperature vector. Here minimum potential energy principle is applied to equation 3. which is the energy equation.

$$\int \left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial x} - Q + \rho c_p \frac{\partial T}{\partial t}\right) N_i d\Omega = 0$$
(11)

As a result general equation and element equation are found as follows.

$$[C]\left\{\frac{\partial T}{\partial t}\right\} + \left[\left[K_{c}\right] + \left[K_{h}\right]\right] \{T\} = \{R_{q}\} + \{R_{h}\}$$
(12)

$$[C] = \int \rho c_p[N]^{T}[N] dV \qquad (13)$$

$$\left[\mathbf{K}_{c}\right] = \int \mathbf{k} [\mathbf{B}]^{\mathrm{T}} [\mathbf{B}] \mathrm{d} \mathbf{V}$$
(14)

$$\left[\mathbf{K}_{h}\right] = \int_{S1} \mathbf{h}[\mathbf{N}]^{\mathrm{T}}[\mathbf{N}] \,\mathrm{d}\Gamma \tag{15}$$

$$\left\{\mathbf{R}_{q}\right\} = \int_{S1} \mathbf{q}_{S} \left\{\mathbf{N}\right\} d\Gamma$$
(16)

$$\left\{\mathbf{R}_{h}\right\} = \int_{S1} \mathbf{h} \cdot \mathbf{T}\left\{\mathbf{N}\right\} d\Gamma$$
(17)

Each expression is defined as; [C] is element capacitance matrix, $[K_C]$ and $[K_h]$ are element conductance matrices and express conduction and convection respectively. The convection matrix is computed only for elements with surface convection. $\{R_q\}$ and $\{R_h\}$ are surface heat flow and surface convection vector respectively.[1] General equation for two dimensional transient state problem exposed to convection boundary conditions is,

$$[C]\left\{\frac{dT}{dt}\right\} + \left[\left[K_{c}\right] + \left[K_{h}\right]\right]\left\{T(t)\right\} = \left\{R_{q}\right\} + \left\{R_{h}\right\}$$
(18)

Derivative $\frac{dT}{dt}$ is expressed by finite difference and the equation 18. becomes,

$$[\mathbf{C}]\left\{\frac{\mathbf{T}_{n+1} - \mathbf{T}_{n}}{\Delta t}\right\} + \left[\left[\mathbf{K}_{c}\right] + \left[\mathbf{K}_{h}\right]\right]\left\{\mathbf{T}(t)\right\} = \left\{\mathbf{R}_{q}\right\} + \left\{\mathbf{R}_{h}\right\}$$
(19)

then with some regulations,

$$[C] \{T_{n+1}\} = \Delta t [\{R_q\} + \{R_h\}] - \Delta t [[K_c] + [K_h]] \{T_n\} + [C] \{T_n\}$$
(20)

$$\{T_{n+1}\} = [C]^{-1} \{ \Delta t [\{R_q\} + \{R_h\}] + [C] \{T_n\} - \Delta t [[K_c] + [K_h]] \{T_n\} \}$$
(21)

are obtained. [4]

CALCULATIONS AND DISCUSSIONS

Material was modelled by a computer programme and this programme was added to the main programme which was developed in Fortran 77 language. Finite elements model used in this study was given in fig.2 This model was formed of 300 elements and 336 nodes. Hard-brazed region of materials were divided into smaller elements for more precise results. Some physical properties of materials which were needed for calculation, are seen in table 1.

	Steel	Brass
Coefficient of thermal expansion $(\alpha)[1/^{\circ}C]$	11,7.10 ⁻⁶	18,5.10-6
Thermal conductivity (k) [Cal/mm.s.°C]	0,0103	0,0265
Coefficient (p.Cp) [Cal/mm ³ .°C]	1,08.10 ⁻³	7,84.10-4
Heat transfer coefficient (h) [Cal/mm ² .s. °C]	2,22.10-6	2,22.10-6

Table 1. Some physical properties of steel and brass

Firstly, Hard-brazed region was heated upto 610° C which was the working temperature of silver-solder, then it was cooled to the room temperature. Temperatures at all nodes were calculated for $\Delta t=0.001$ sec. time intervals.

Temperature variations on symetric points are shown in fig.3. When cooling values of steel and brass were compared, it was seen that cooling on brass was quicker. Particularly near the joining region important temperature differences accured

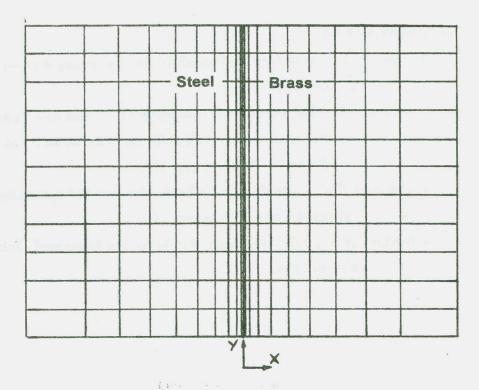


Figure 2. Finite element model of steel - brass joint.

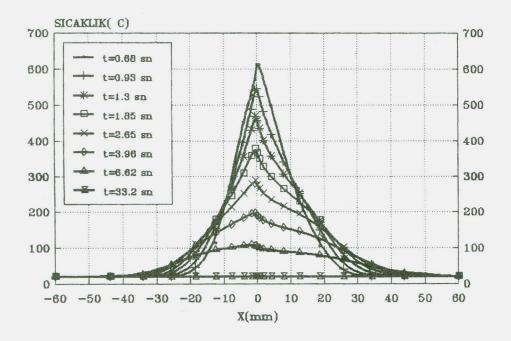


Figure 3. Cooling curves of steel - brass joint.

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