



Article Computational Investigation of the Combined Impact of Nonlinear Radiation and Magnetic Field on Three-Dimensional Rotational Nanofluid Flow across a Stretchy Surface

Azad Hussain ^{1,*}, Mohamed Abdelghany Elkotb ^{2,3}, Mubashar Arshad ¹, Aysha Rehman ^{1,*}, Kottakkaran Sooppy Nisar ⁴, Ali Hassan ¹, and C. Ahamed Saleel ²

- ¹ Department of Mathematics, University of Gujrat, Gujrat 50700, Pakistan; imbashrii@gmail.com (M.A.); muhammadali0544@gmail.com (A.H.)
- ² Mechanical Engineering Department, College of Engineering, King Khalid University, P.O. Box 394, Abha 61421, Saudi Arabia; melkotb@kku.edu.sa (M.A.E.); ahamedsaleel@gmail.com (C.A.S.)
- ³ Mechanical Engineering Department, Faculty of Engineering, Kafrelsheikh University, Kafr Elsheikh 8655, Egypt
- ⁴ Department of Mathematics, College of Arts and Sciences, Prince Sattam bin Abdulaziz University, Wadi Aldawaser 11991, Saudi Arabia; n.sooppy@psau.edu.sa
- * Correspondence: azad.hussain@uog.edu.pk (A.H.); aysharehman1986@gmail.com (A.R.)

Abstract: This comparative study inspects the MHD three-dimensional revolving flow and temperature transmission of a radiative stretching surface. The flow of nanofluid is modeled using the Tiwari and Das model. Water is the base fluid, and the nanoparticles are composed of two different types of nanoparticle, i.e., gold and silver (Au and Ag). The non-radiative heat flow notion is examined in a temperature field that results in a nonlinear energy equation. Conformist transformations are used to generate a self-similar arrangement of the leading differential system. The resulting system has an intriguing temperature ratio constraint, which shows whether the flow has a little or significant temperature differential. By using a powerful mathematical technique, numerical results are obtained. The solutions are influenced by both stretching and rotation. The difference in velocity constituents with the elements' volume fraction is non-monotonic. Results for the rotating nanofluid flow and heat transfer properties for both types of nanoparticles are highlighted with graphs. The impact of physical concentrations, such as heat flux rates and skin friction constants, are examined at the linear extending surface and clarified graphically. *Ag*-water nanofluid has a high-temperature transfer constant compared to *Au*-water nanofluid. The velocity profile was also discovered to have a parabolic distribution shape.

Keywords: MHD; nonlinear solar radiation; rotating three-dimensional flow; linear stretching surface; *Au* and *Ag* as nanoparticles

1. Introduction

Problems associated with pouring in a spinning setting occur in many fields, such as engineering and geophysics. The movement of volcanic emissions near the Earth's crust's mantle, as well as its rotation, is a prominent geophysical application. The engineering applications of pouring in a spinning setting occur in a large range of chemical industries, the processing of food, centrifugal filtration processes, the design of the multi-pore supplier in the gas-solid liquidized bed, rotating machinery, and in manufacturing processes. Wang [1]. Conducted innovative research on three-dimensional revolving viscid flow, created by a widening surface. First, the problem was addressed with a constraint that denotes the ratio of the revolution to the surface broadening rate. Then, by using a regular perturbation approach, he found a series of solutions. He concluded that the velocity of fluid decays as the parameter increases. Rajeswari and Nath [2], and Nazar et al. [3] furthered Wang's efforts concerning unstable flow problems. Their outcomes specify a seamless alteration



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Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). from the initial unstable flow to the end smooth state flow. Vajravelu and Kumar [4] conducted a noteworthy exploration of the arithmetical and consistent clarifications of a nonlinear scheme caused by a three-dimensional rotational flow. Hayat et al. [5] provided the homotopic solution of succeeding-grade non-Newtonian rotating flow. They found that velocity has a direct relationship with material parameters. Zaimi et al. [6] and Hayat et al. [7] calculated the rotating flow of viscid fluid and proved that frontier layer viscosity is an accumulative function of the viscid fluid parameter. Entropy generation in steady fluid, due to a rotating disk, was investigated by Rashidi et al. [8].

The poor temperature conductivity of traditional coolants such as water, crude oil, and mineral oil limits their ability to cool heat transfer devices. As it is widely known, metals have a relatively high heat conductivity as compared to liquids. Thus, scattering nanometer-sized metallic particles in liquids is one of the most effective ways to improve heat transfer performance. Choi and Eastman [9] conceived the concept of nanofluid. During the last few decades, nanofluid convective heat transfer has been a popular research topic [10–15]. Turkyilmazoglu [16] studied the different types of water-based nanofluids. He used the spectral Chebyshev collocation technique to represent numerical results.

Magnetic nanofluids are colloidal solutions of magnetizable nanoparticles with fluid, magnetic, and thermal characteristics. They are beneficial in a variety of applications. High-temperature plasmas, nuclear reactor cooling, MHD accelerators, power-generating systems, and liquid-metal fluids are the major industrial and geothermal uses of the magnetohydrodynamic (MHD) flow. Cobalt, magnetite, hematite, and other iron-based nanoparticles can be found in them. Such types of nanoparticles can be used for MRI, hyperthermia, and magnetic cell separation. Sheikholeslami and Ganji [17] considered ferroand magnetohydrodynamic fluid. They estimated the electrical and thermal conductivity using Maxwell's model. Moraveji and Hejazian [18] considered the natural convection of magnetite–water nanofluid via a rectangular container. Aydin and Kaya [19] examined mixed convection MHD flow over an inclined plate. They found that increasing the angle of inclination is a decreasing function of skin friction and heat flux. Ellahi [20] provided the analytical solutions of MHD non-Newtonian nanofluids passing through a pipe by considering the temperature-dependent viscosity. Sheikholeslami et al. [21] presented the numerical outcomes of nanofluid flow and temperature transmission in the presence of a magnetic field. Takhar and Nath [22] conducted a statistical study of MHD spinning flow above an unsteady enlarging pane. The combined impact of slip circumstances and Joule heating on the MHD peristaltic flows of a hyperbolic curvature nanofluid in a tending planar channel was investigated by Hayat et al. [23]. Rizwana et al. [24] studied the MHD oblique immobility point flow of a nanofluid. Ahmed et al. [25] described the carbon nanotube magneto-hydrodynamics of nanofluid flow on a stretching surface. Hybrid nanofluid entropy generation and temperature-dependent viscosity were discussed by Ahmad et al. [26]. Shahid et al. [27] conducted a numerical examination of stimulation energy on MHD nanofluid flow. Bahireal et al. [28] examined a manganese-zinc ferrite water nanofluid under a nonuniform magnetic field. Recently, many researchers have determined the effects of heat transfer and magnetism on nanofluids [29–31]. Mahian et al. [32] contributed to this research area by reviewing recent advances in the modeling of nanofluid flow in three dimensions for single-phase and two-phase nanofluids. Waqas et al. [33] examined the MHD flow of a hybrid nanofluid over a spinning disc with nonlinear thermal radiation and found favorable agreement in their results. Yasin et al. [34] investigated the impact of several factors on the 2D MHD frontier layer movement of an electoral conducting fluid above a stretchy surface, including the heat sink and suction parameter.

Thermal radiation has a critical part of a variety of industrial uses, including glass manufacturing and heating system design, as well as cosmos technology uses such as aeronautical aerodynamic rockets. In recent years, some research on nonlinear radiative heat transport has been published [35–37]. Imitaz et al. [38] considered radiation effects on the movement of carbon nanotubes amongst stretchy rotational disks. Over a nonlinear stretched sheet, Cortell [39] investigated the properties of viscous degeneration and heat

radiation. In the presence of thermal radiation from a nanofluid, Hady et al. [40] examined the impact of heat transmission across a nonlinear expanding sheet. Hussain et al. [41] examined the Carreau–Yasuda temperature convention near a stagnation point. Benos et al. [42] conducted analytical research in the presence of radiation, the Lorentz force, the Navier slip boundary condition, and the heat source. Alsani et al. [43] deliberated the joint effects of the inclined applied magnetic field, the permeable enlarging surface, mass transportation, and radiative warming on micropolar flow. Hussain et al. [44] discussed the MHD for viscid nanofluid with radiative effects. Mousavi et al. [45] presented a dual solution for Casson hybrid nanofluid flow above a widening surface. Mikhailenko et al. [46,47] computationally examined the convective nanofluid circulation and heat transmission within a rotating square chamber with a local isothermal element using a wide range of Rayleigh numbers, nanoparticle concentrations, and Taylor numbers. Mustafa et al. [48] considered boundary layer flow with the existence of nonlinear radiative temperature transmission. They concluded that the heat and temperature frontier layer width increase as the thermophoretic effects and Brownian motion intensify.

To the best of our knowledge, no one has ever described the rotating flow of a nanofluid generated by a broadening surface. Therefore, this comparative study addresses this gap in the current research. Interesting aspects of nonlinear radiative heat transmission for Ag- and Au-water-based nanofluid are also discussed. This research enables the reader to investigate the issue of both large and small temperature variations between the enlarging surface and the fluid. The leading nonlinear differential coordination is dealt with by using the powerful bvp4c technique. The Tiwari and Das [49] model is employed to discuss the problem. The effects of different constraints on velocity and temperature are emphasized. The impact of the surface radiative parameter Rd, the magnetic parameter M, and the rotation parameter λ on temperature, velocity, and skin friction coefficients are calculated, visually confirmed, and discussed for Ag0 and Au-water-based nanofluids.

2. Materials and Methods

Consider the rotating flow of *Ag*-water and *Au*-water ferrofluid that is generated by a radiative surface parallel to the *xy* plane. The surface is stretched uniformly in the *x* direction at constant rate *a*. The temperature beyond the thermal boundary layer is T_{∞} , whereas T_w denotes the temperature on the surface of the sheet. Geometry of the problem is shown in Figure 1. The Tiwari and Das model [49] equations symbolizing the preservation of mass, energy, and momentum are expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = 2\Omega v + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial z^2} - \frac{\sigma_{nf} B_o^2}{\rho_{nf}} u,$$
(2)

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -2\Omega u + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 v}{\partial z^2} - \frac{\sigma_{nf} B_o^2}{\rho_{nf}} v, \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_{nf} \frac{\partial^2 T}{\partial z^2} - \frac{1}{\left(\rho C_p\right)_{nf}} \frac{\partial q_r}{\partial z}.$$
 (4)



Figure 1. Geometry of the problem.

With the following boundary conditions:

$$u = u_w = ax, \ v = v_w = 0, \ w = 0, \ T = T_w \quad at \ z = 0.$$

$$u = 0, \ v = 0, \ T = T_\infty \qquad as \ z \to \infty.$$
(5)

where *u* is the *x* component, *v* is the *y* component, and *w* is *z* the component of angular velocity Ω ; μ_{nf} is the fluid's dynamic viscosity; ρ_{nf} is the density of the nanofluid; α_{nf} is the thermal diffusivity of the nanofluid; *T* is the nanofluid's temperature; T_w is the wall temperature; T_{∞} is the ambient fluid temperature; $(\rho C_p)_{nf}$ is the volumetric heat capacity of the nanofluid; $(\rho C_p)_f$ is the volumetric heat capability of the base fluid; $(\rho C_p)_s$ is the volumetric heat capacity of the solid nanoparticles; and B_0 is a consistent magnetic field applied normally to the sheet. $q_r = -\left(\frac{4\sigma^*}{3k^*}\right)\left(\frac{\partial T^4}{\partial z}\right)$ is the Rosseland radiative temperature fluidity, where σ^* is the Stefan–Boltzmann coefficient and k^* is the mean immersion coefficient. As shown below, all these are connected to the particle volume fraction φ of the nanoparticles [50].

$$\mu_{nf} = \frac{\mu_n}{\left(1 - \varphi\right)^{5/2}} \tag{6}$$

$$\rho_{nf} = \rho_f (1 - \varphi) + \varphi \rho_s \tag{7}$$

$$\left(\rho C_{p}\right)_{nf} = \left(\rho C_{p}\right)_{f} \left(1 - \varphi\right) + \varphi \left(\rho C_{p}\right)_{s}$$

$$\tag{8}$$

$$\frac{k_f}{k_{nf}} = \frac{k_s + 2k_f + 2\varphi\left(k_f - k_s\right)}{k_s + 2k_f - 2\varphi\left(k_f - k_s\right)} \tag{9}$$

Furthermore, Maxwell [51] describes the electrical conductivity of nanofluid σ_{nf} as:

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(\sigma_s - \sigma_f)}{(\sigma_s + 2\sigma_f) - (\sigma_s - \sigma_f)\varphi}$$
(10)

The volume fraction of the nanoparticles is denoted by φ in Equations (6)–(10), and subscripts *s* and *f* refer to solid and fluid, respectively. Table 1 shows the thermophysical characteristics of the base fluid and solid nanoparticles.

Properties	ho (kg/m ³)	Cp (J/kgK)	<i>K</i> (W/mK)	σ (Ω .m) $^{-1}$
Water (H_2O)	997.1	4179	0.613	$5 imes 10^{-2}$
Gold (Au)	19,300	129.1	318	$4.52 imes 10^7$
Silver (Ag)	10,500	235	429	$6.30 imes 10^7$

Table 1. Thermophysical characteristics of the base fluid and solid nanoparticles.

In the following equations, we seek a similarity solution [6] for Equations (1)–(4),

$$u = axf'(\eta), \ v = axg(\eta), \ w = -\sqrt{av_f} f(\eta), \\ \eta = z\sqrt{\frac{a}{v_f}}, \ \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}$$
(11)

The steadiness Equation (1) is identically satisfied in the preceding equations, whereas Equations (1)–(5) become:

$$\frac{1}{\left(1-\varphi\right)^{5/2}}f^{\prime\prime\prime} = \left(1-\varphi+\varphi\,\frac{\rho_s}{\rho_{bf}}\right)\left(f^{\prime 2}-ff^{\prime\prime}-2g\lambda\right) + \frac{\sigma_{nf}}{\sigma_f}M\lambda f^{\prime} \tag{12}$$

$$\frac{1}{(1-\varphi)^{\frac{5}{2}}}g'' = \left(1-\varphi+\varphi\,\frac{\rho_s}{\rho_{bf}}\right)\left(gf'-fg'+2\lambda f'\right) + \frac{\sigma_{nf}}{\sigma_f}M\lambda g\tag{13}$$

$$\theta'' \left[\frac{k_{nf}}{k_f} + Rd \left(1 + (\theta_w - 1) \right)^3 \right] + \Pr\left[1 - \varphi + \varphi \frac{(\rho C_p)_s}{(\rho C_p)_f} \right] f \theta' = 0$$
(14)

with conditions as follows:

$$f(0) = 0, g(0) = 0, f'(0) = 1, \theta(0) = 1, f'(\infty) = 0, g(\infty) = 0, \theta(\infty) = 0$$
(15)

 $Pr = \frac{(\mu c_p)_f}{k_f}$ represents the Prandtl number of base fluid, $Rd = \frac{16\sigma^* T_{\infty}^3}{3k^* k_f}$ is the radiation constraint, $M = \frac{\sigma B_o^2}{\rho_f \Omega}$ is the magnetic field constraint, $\lambda = \frac{\Omega}{a}$ is the proportion of rotation to the enlarging surface rate, and θ_w is the temperature ratio parameter [52].

The skin friction coefficients C_{fx} and C_{fy} , and Nusselt number Nu_x are of physical relevance, and are defined as follows:

$$c_{fx} = \frac{\tau_{wx}}{\rho_f u_w^2}, \ c_{fy} = \frac{\tau_{wy}}{\rho_f u_w^2}, \ Nu_x = \frac{xq_w}{k_f (T_w - T_\infty)},$$
(16)

The wall shear stress is supplied by $\tau_{wx} = \tau_{zx}|_{z=0}$ and $\tau_{wy} = \tau_{zy}|_{z=0}$, and q_w is the wall temperature flux given by:

$$\tau_{wx} = \mu_{nf} \frac{\partial u}{\partial z} \Big|_{z=0}, \ \tau_{wy} = \mu_{nf} \frac{\partial v}{\partial z} \Big|_{z=0}, \ q_w = -k_{nf} \frac{\partial T}{\partial z} \Big|_{z=0} + q_r \Big|_{z=0}$$
(17)

$$(Re_{x})^{\frac{1}{2}}C_{fx} = \frac{1}{(1-\varphi)^{5/2}} f''(0), \quad (Re_{x})^{\frac{1}{2}}C_{fy} = \frac{1}{(1-\varphi)^{5/2}} g'(0)$$

$$Nu_{x}(Re_{x})^{-1/2} = -\left(\frac{k_{nf}}{k_{f}} + Rd\theta_{w}^{3}\right) \theta'(0)$$
(18)

where $Re_x = \frac{u_w x}{v_f}$ is the Reynold's number.

The rotational flow and temperature transmission of *Ag*-water and *Au*-water ferrofluid, generated by a linear enlarging surface, were investigated in this research using the Tiwari and Das model [49]. The numerical results of Equations (12)–(14), under the conditions of Equation (15), were calculated in MATLAB using the bvp4c method. The volume portion was assumed to be in the range of $0 \le \varphi \le 0.2$ in all findings (as above from 0.2 is not feasible due to aggregation), and the Prandtl number used for water was Pr = 6.2. We associated pure base fluid using Wang's [1] and Nazar et al.'s [3] studies and found an extremely high agreement with them (Table 2).

λ –	Wan	Wang [1]		et al. [3]	Present Results	
	f"(0)	g ['] (0)	f ["] (0)	g ['] (0)	f ["] (0)	g'(0)
0	-1	0	-1	0	-1.00426	0
0.5	-1.1384	-0.5128	-1.1384	-0.5128	-1.17189	-0.5488
1.0	-1.3250	-0.8371	-1.3250	-0.8371	-1.3582	-0.8589
2.0	-1.6523	-1.2873	-1.6523	-1.2873	-1.68033	-1.3027

Table 2. For specific situations ($\varphi = 0$, M = 0), the present results are compared to prior research.

3. Results and Discussion

A numerical calculation for steady three-dimensional flow was executed to analyze the outcomes of the current problem. The numerical values of the local Nusselt number $Nu_x(Re_x)^{-1/2}$ for some values of the parameters are computed in Tables 3 and 4. The magnitude of $Nu_x(Re_x)^{-1/2}$ decays severely with the volume fraction φ when a large wall-to-ambient temperature ratio is considered. When the magnetic parameter M value rises, the Lorentz force acts perpendicularly on the surface, resulting in the decay in the Nusselt number in the absence of radiation, and it grows when radiation exists. Similar behavior can be noted from the tables for rotation parameters. As rotation rises, the Nusselt number decreases for the absence and increases for the existence of radiation. Moreover, $Nu_x(Re_x)^{-1/2}$ has an inverse and nonlinear relation with the radiation parameter Rd. This outcome is true for both linear and nonlinear cases and holds in linear and nonlinear situations.

Table 3. Values of the local Nusselt number (for *Ag*-water nanofluid) $Nu_x(Re_x)^{-1/2}$ for different values of embedded constraints.

	М	λ Rd=		<i>Rd</i> =1		
φ			Rd= 0	Linear Radiation	Nonlinear Radiation	
					θ_w =1.1	θ_w =1.8
0.1	1	0	2.27905	2.27843	2.3246	2.67737
		0.5	1.92681	1.92617	1.94786	2.10176
		1.0	1.64197	1.64138	1.65357	1.76274
		2.0	1.30861	1.30813	1.31688	1.42612
0.1	0	1	1.6446	1.63871	1.65072	1.75894
	0.5		1.64404	1.64006	1.65216	1.76085
	1.0		1.64346	1.64138	1.65357	1.76274
	2.0		1.64226	1.64395	1.65631	1.76642
0	1	0.5	3.97155	3.70572	3.69339	3.65956
0.05			3.25756	3.06632	3.05024	2.95741
0.1			2.65786	2.5617	2.55231	2.50946
0.2			1.71642	1.80762	1.81973	1.93974

The impact of the magnetic field constraint *M* on the dimensionless *x* component and *y* component of velocity is shown in Figures 2 and 3. The function $g(\eta)$ has a parabolic profile with a negative value, indicating flow in the negative *y* direction. The flow loses speed in both the *x* and *y* directions as *M* increases. In both types of nanofluid, the existence of magnetic field *M* limits fluid mobility, resulting in a smaller boundary layer and increased magnetic field intensity.

				<i>Rd</i> =1		
φ	Μ	λ	Rd= 0	Linear Radiation	Nonlinear Radiation	
					θ_w =1.1	θ_w =1.8
0.1	1	0	2.41145	2.41145	2.4653	2.88787
		0.5	2.14057	2.14057	2.17291	2.39552
		1.0	1.88024	1.88024	1.89916	2.03691
		2.0	1.53027	1.53027	1.54042	1.64452
0.1	0	1	1.6446	1.8777	1.89631	2.03194
	0.5		1.64404	1.87899	1.89776	2.03446
	1.0		1.64346	1.88024	1.89916	2.03691
	2.0		1.64226	1.88258	1.9018	2.04162
0	1	0.5	3.97717	3.70938	3.69688	3.66193
0.05			3.38361	3.2549	3.24742	3.2149
0.1			2.86298	2.8489	2.84918	2.86158
0.2			1.98224	2.13649	2.15484	2.30469

Table 4. Values of the Nusselt number (for *Au*-water nanofluid) $Nu_x(Re_x)^{-1/2}$ for changed values of embedded constraints.



Figure 2. The effect of parameter *M* on $f'(\eta)$.



Figure 3. The effect of parameter *M* on $g(\eta)$.

The influence of ratio λ on the *x* and *y* components of velocity is shown in Figures 4 and 5. When compared to the rotation rate, a larger value of λ implies lesser

stretching rates (along the x axis). As a result, when λ increases, the *x* component of velocity $f'(\eta)$ is inversely proportional to λ and the strength of the *y* component of velocity $g(\eta)$ rises. The behavior is similar for both nanofluids.



Figure 4. The effect of parameter λ on $f'(\eta)$.



Figure 5. The effect of parameter λ on $g(\eta)$.

The skin friction coefficients $Re_x^{1/2} C_{fx}$ and $Re_x^{1/2} C_{fy}$ are shown in Figures 6 and 7 for varying values of M and volume fraction φ , respectively. Because fluid exerts stress on the stretched wall, the values of skin friction $Re_x^{1/2} C_{fx}$ and $Re_x^{1/2} C_{fy}$ are negative. The magnitudes of $Re_x^{1/2} C_{fx}$ and $Re_x^{1/2} C_{fy}$ diminish as M increases because a transverse magnetic field has a propensity to produce a Lorentz force, which opposes transference. For both types of nanofluid, this results in a decrease in flow speed and an increase in surface shear stress. Figure 8 depicts the effect of volume fraction φ on the temperature. Due to the consideration of a large φ , the high viscous force creates a thicker thermal barrier, as predicted. This can also be noted as the wall temperature gradient increases.



Figure 6. The effect of parameter *M* on $Re_x^{1/2} C_{fx}$.



Figure 7. The effect of parameter *M* on $Re_x^{1/2} C_{fy}$.



Figure 8. The effect of parameter φ on $\theta(\eta)$.

The effect of the magnetic field M on temperature θ is shown in Figure 9. The temperature rises due to the resistance linked with the Lorentz force caused by the applied magnetic field. As a result, as M increases, the temperature increases and the thermal boundary layer thickens. On a thermal boundary layer, the behavior of ratio λ is qualitatively comparable to that of M (Figure 10). When it changes, however, the consequences are more noticeable.



Figure 9. The effect of parameter *M* on $\theta(\eta)$.



Figure 10. The effect of parameter λ on $\theta(\eta)$.

Figure 11 depicts the temperature profile for different values of the temperature ratio parameter θ_w . When θ_w increases, unlike the linear radiation scenario, the profile changes from a regular form to an S-shaped thicker profile. The thermal diffusivity (α_{eff}) is a function of temperature and controls the enlarging of the thermal boundary layer. Because the sheet is warmer than the fluid, α_{eff} is higher adjacent to the sheet than at the fluid layer. As a result, increasing the wall-to-ambient temperature ratio parameter θ_w tends to lower the temperature gradient at the surface, resulting in the inflection point. When θ_w is sufficient enough, the concavity of temperature functionality changes in [0, 1). In the case of linear radiation heat transmission, however, the concavity has been retained. As θ_w becomes greater, the temperature gradient between the wall and the environment increases, resulting in a thicker thermal frontier layer. These findings are consistent with those in the literature.



Figure 11. The effect of parameter θ_w on $\theta(\eta)$.

The effects of the radiation parameter Rd are shown in Figures 12 and 13 for both linear and nonlinear radiative heat flows. When the radiation parameter Rd increases in the linear radiation scenario, the temperature wall slope approaches a constant finite value. In the event of nonlinear radiation, this effect is not retained. When a larger radiation parameter Rd is considered, the temperature θ seems to be higher. Furthermore, compared to linear radiation, temperature θ increases gradually with nonlinear radiation. We may deduce from Figure 12 that the outcomes of linear and nonlinear radiation are equal only when Rd small and θ_w is near the unit. As the radiation parameter steadily raises, the variation between linear and nonlinear radiation grows.



Figure 12. The effect of parameter *Rd* linear on $\theta(\eta)$.

Figure 14 demonstrates that when the volume fraction φ is raised, the local Nusselt number $Nu_x(Re_x)^{-1/2}$ exhibits an inverse relationship. Notably, the temperature transmission rate from the sheet changes substantially with *M*.



Figure 13. The effect of parameter *Rd* nonlinear on $\theta(\eta)$.



Figure 14. The effect of *M* on $Nu_x(Re_x)^{-1/2}$.

4. Conclusions

In this study, we investigated the magnetic and radiative effects on three-dimensional rotating nanofluid flow. The Tiwari and Das [49] model was used to investigate the rotational flow and temperature transmission of *Ag*-water and *Au*-water ferrofluids across a deformable surface. A numerical technique was used in MATLAB to solve the proposed nonlinear boundary value problem. The following are some of the most important findings of this study:

- 1. The local Nusselt number increases as magnetic field *M* increases;
- 2. The *y* component of velocity is negative for both types of nanofluids, and the velocity distribution is parabolic;
- 3. The parameter λ has fundamentally different impacts on the *x* and *y* components of velocity;
- 4. The velocity distributions f' and g both decay when a large magnetic field M is applied;
- 5. The magnetic field and the volume fraction have inverse relationships with the skin friction coefficient and local Nusselt numbers, respectively;
- 6. The parameter λ enhances the local Nusselt number as it increases;
- 7. *Ag*-water nanofluid has a high-temperature transfer constant compared to *Au*-water nanofluid.

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