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Actuator Saturated Fuzzy Controller Design for Interval Type-2 Takagi-Sugeno Fuzzy Models with Multiplicative Noises

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Abstract: In many practical systems, stochastic behaviors usually occur and need to be considered in the controller design. To ensure the system performance under the effect of stochastic behaviors, the controller may become bigger even beyond the capacity of practical applications. Therefore, the actuator saturation problem also must be considered in the controller design. The type-2 Takagi-Sugeno (T-S) fuzzy model can describe the parameter uncertainties more completely than the type-1 T-S fuzzy model for a class of nonlinear systems. A fuzzy controller design method is proposed in this paper based on the Interval Type-2 (IT2) T-S fuzzy model for stochastic nonlinear systems subject to actuator saturation. The stability analysis and some corresponding sufficient conditions for the IT2 T-S fuzzy model are developed using Lyapunov theory. Via transferring the stability and control problem into Linear Matrix Inequality (LMI) problem, the proposed fuzzy control problem can be solved by the convex optimization algorithm. Finally, a nonlinear ship steering system is considered in the simulations to verify the feasibility and efficiency of the proposed fuzzy controller design method.

Keywords: interval type-2 T-S fuzzy model; fuzzy control; multiplicative noises; actuator saturation constraint

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1. Introduction

In 1965, the concept of fuzzy sets was firstly proposed by Zadeh [1]. The Takagi-Sugeno (T-S) fuzzy model has been proposed as an effective modeling method for complex nonlinear systems based on fuzzy sets. Via expressing nonlinear systems into many linear subsystems with IF-THEN fuzzy rule, a wide class of linear control theory can be applied for the T-S fuzzy models. In [2–6], it is witnessed that the T-S fuzzy model-based control theory has already been applied in many engineering systems successfully. The T-S fuzzy model can also be used to solve the actuator saturation problem for nonlinear systems [7–9]. In order to develop the fuzzy controller design method, a so-called "Parallel Distributed Compensation" (PDC) method was proposed for the T-S fuzzy model [10–13]. However, the state of practical engineer systems or controllers may differ from the original due to service life and frequency. Thus, parameter uncertainties have attracted more and more attention and need to be considered in the controller design method. Zadeh has expanded the type-1 fuzzy sets to the type-2 fuzzy sets [14]. It is noted that the type-2 fuzzy sets can construct the parameter uncertainties more completely than the type-1 fuzzy sets. With type-2 fuzzy sets, the type-2 T-S fuzzy model was proposed to solve nonlinear systems' control problem with parameter uncertainties, which has been detailed in [15]. In this paper, the type-2 T-S fuzzy model is utilized to represent stochastic nonlinear systems with parameter uncertainties.

Based on the type-2 fuzzy sets, the type-2 fuzzy system has been proposed as a more general model description for nonlinear systems since the type-1 fuzzy system is a special case of the type-2 fuzzy system [16]. Unlike the type-1 fuzzy sets in which the membership grade is crisp, the membership grade of type-2 fuzzy sets is also fuzzy. In other words,

the membership grade of type-1 fuzzy sets also has its membership function, which is called secondary membership function, in type-2 fuzzy sets. By using the secondary membership function, the uncertainties of type-1 fuzzy sets can be modeled. The description of type-2 fuzzy sets consists of the upper bound and lower bound of the membership function. The region between them, which describes the information of parameter uncertainties, is called Footprint of Uncertainty (FOU) [17–19]. According to the secondary membership function of the type-2 fuzzy sets, it can furtherly be divided into general type-2 fuzzy sets [20,21] and Interval Type-2 (IT2) fuzzy sets [22–25]. It is known that the secondary membership grades of general type-2 fuzzy sets are the value from 0 to 1 [19], which causes the complexity of the calculation. To reduce the calculation in the fuzzy controller design process, the IT2 fuzzy sets are applied to establish the T-S fuzzy model in this paper. However, the IT2 fuzzy systems are still much more complicated than the type-1 fuzzy systems. Much research has put effort into designing the description method for the IT2 fuzzy systems [26–28]. In [29], the technique has been extended to the T-S fuzzy model and applied to the practical inverted pendulum system successfully. Thus, the IT2 T-S fuzzy system proposed in [29] is considered, in this paper, to develop the fuzzy controller design method.

In order to improve the system performance under the stochastic behavior effects, a more significant control force is usually necessary for the controller design. However, there are some limitations in all control actuation devices, which may be provided as force, torque, stroke, voltage, and so on [30]. Because of the limitations, the control force applied to the system cannot achieve the expected performance or even cause instability, called actuator saturation. In [31], it can be found that the control problem subject to actuator saturation should be discussed with the displacement limit of the device of the active suspension system. In [32], the rate saturation, which is a critical problem in operating the aircraft, will contribute to the onset of pilot-induced oscillations and become the cause of airplane crashes. Thus, the control problem considering the actuator saturation has become an inevitable issue for the practical systems. To avoid the excessive controller force that will damage the system components, the performance requirement of actuator saturation also needs to be considered in the controller design. It is known that the saturation problem of the actuator is generally considered via designing the low gain control law or estimating the domain of attraction in the presence of actuator saturation [33]. In order to develop the actuator saturation in the controller design method, the saturation function of an actuator can be characterized based on the convex hull of linear combinations of linear functions and saturation functions [34]. In this paper, the saturation function is designed inside a specific nonlinear saturation sector. The control method subject to the actuator saturation has also been combined with the type-1 T-S fuzzy systems successfully [33]. In order to solve the parameter uncertainties in practical stochastic nonlinear systems, the type-2 T-S fuzzy system is applied. However, the control input might become bigger since the system's performance requirements need to be achieved. By extending the application of actuator saturation to the type-2 T-S fuzzy system, the saturation problem can also be solved for the nonlinear systems with parameter uncertainties [23]. In [23], the saturation problem was solved effectively by designing the control law to reduce the control gain. Thus, the actuator saturation is considered with the IT2 T-S fuzzy systems in this paper.

The contributions of this paper are described as follows. In the practical system, the stochastic behaviors are inevitable and necessary to be considered in the controller design method. The control problem of stochastic nonlinear systems becomes an important issue. Based on the type-1 T-S fuzzy model, more and more researchers have developed the controller design method of stochastic nonlinear systems [35,36]. Moreover, the fuzzy control methods are also designed based on the IT2 T-S fuzzy model to solve the control problem of stochastic nonlinear systems with parameter uncertainties [37,38]. In [39], it is seen that the IT2 fuzzy controller design method has been applied to the nonlinear truck-trailer system successfully, and efficient results can be obtained. To achieve the performance requirement under the effects of stochastic behaviors, the control force may become bigger even over the limit of practical systems. Because of this reason, the constraint of actuator

saturation is also necessary to be combined in the fuzzy controller design method. Many control theories have already been proposed with the type-1 T-S fuzzy model [9,40]. Based on the IT2 T-S fuzzy model, several efficient fuzzy controller design methods subject to actuator saturation also can be provided [23,41,42]. Considering the performance requirements of stochastic behaviors and actuator saturation, various controller design methods have been proposed based on the type-1 fuzzy model [33,43,44]. In [33,43,44], it is obvious that stochastic nonlinear systems' performance under the stochastic behavior effects is improved, and the control gain can be suppressed under the actuator saturation constraint. Thus, the required system performance can be obtained by applying the cost-effective control force. However, the control problem of parameter uncertainties, which may cause by perturbations or modeling error, is also unavoidable in practical systems. The controller design method subject to actuator saturation for nonlinear systems with stochastic behaviors and parameter uncertainties based on the IT2 T-S fuzzy model is hardly discussed in the existing paper. In this paper, a new IT2 T-S fuzzy controller design method is provided to solve the control problem for nonlinear systems with stochastic behaviors and parameter uncertainties. To present purposes and contributions more clearly, the block diagram is given in Figure 1.

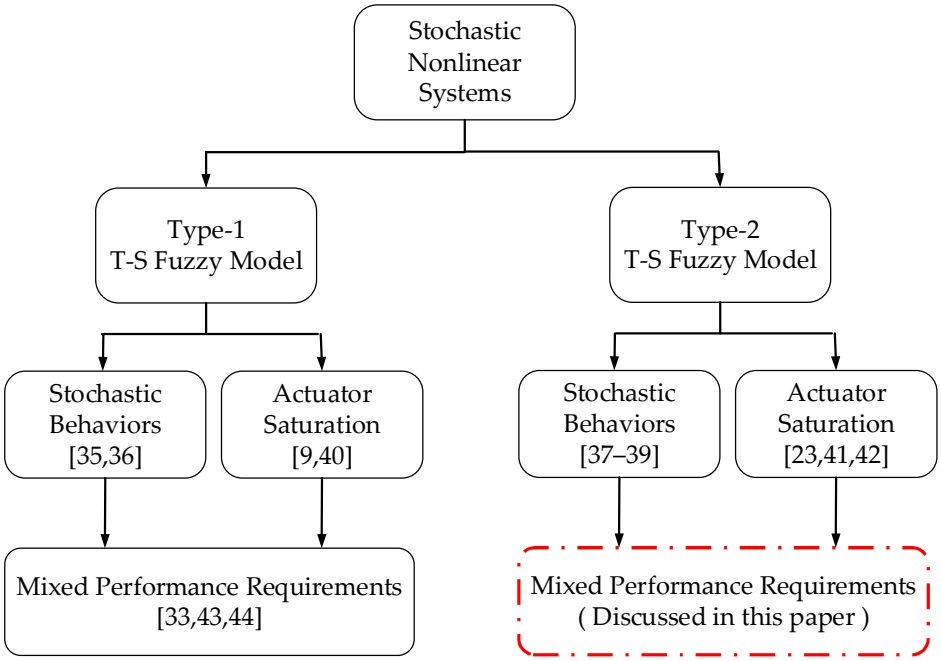


Figure 1. Motivation and purpose of this paper.

This paper proposes a fuzzy controller design method for stochastic nonlinear systems with parameter uncertainties subject to actuator saturation. Firstly, a class of stochastic nonlinear systems is modeled into the IT2 T-S fuzzy system via the expression of multiplicative noises. Then, the fuzzy controller, which satisfies the actuator saturation constraint, is designed to close the loop of the IT2 type-2 T-S fuzzy system by the PDC method. Based on the Lyapunov stability theory, some sufficient conditions are derived to guarantee the stability of the closed-loop IT2 T-S fuzzy model system. In order to solve the control problem via the convex optimization algorithm, the sufficient conditions are derived into the form of Linear Matrix Inequality (LMI). To process the derivation problem in stability analysis, Young's inequality is also utilized [45]. Thus, the stability and the performance requirement of actuator saturation can be achieved by satisfying these sufficient conditions. Finally, a practical nonlinear ship steering system is applied to verify the applicability and feasibility of the fuzzy controller design method developed in this paper.

The structure of this paper is presented as follows. In Section 2, a class of stochastic nonlinear systems is expressed as an IT2 T-S fuzzy model with multiplicative noises, and definitions and lemmas are introduced. In Section 3, sufficient conditions are derived for ensuring the stability and performance requirement of actuator saturation of the IT2 T-S fuzzy model. In Section 4, a nonlinear ship steering system is applied to verify the applicability and efficiency of the proposed fuzzy control method. At last, some conclusions of the proposed fuzzy controller design method and the simulation results are provided in Section 5.

2. System Descriptions and Problem Statements

In this section, the IT2 T-S fuzzy model is considered to discuss the control problem subject to actuator saturation for a class of stochastic nonlinear systems. The stochastic behaviors of the system are expressed as the multiplicative noises in this paper. Then, the IT2 T-S fuzzy model with multiplicative noises is presented as follows:

$$\begin{aligned} &\text{Plant Rule } i: \\ &\text{IF } \rho_1(t) \text{ is } \overset{\sim}{\text{M}}{}_1^i \text{ and } \rho_2(t) \text{ is } \overset{\sim}{\text{M}}{}_2^i \text{ and } \ldots \text{ and } \rho_z(t) \text{ is } \overset{\sim}{\text{M}}{}_z^i, \\ &\text{Then } \dot{x}(t) = \mathbf{A}_i x(t) + \mathbf{B}_i \widetilde{u}(t) + \widehat{\mathbf{A}}_i x(t)\omega(t) \end{aligned} \tag{1}$$

where $\rho_1(t), \rho_2(t), \ldots, \rho_z(t)$ are the premise variables, z is the number of premise variables, $i = 1, 2, \ldots, \Phi$ and Φ is the number of fuzzy rules, $\overset{\sim}{\text{M}}{}_z^i$ is an IT2 fuzzy set of rule i, $x(t) \in \Re^{n_x}$ is the state vector, $\widetilde{u}(t) = \left[\widetilde{u}_1(t), \cdots, \widetilde{u}_{n_u}(t)\right]^{\text{T}} = [sat(u_1(t)), \cdots, sat(u_{n_u}(t))]^{\text{T}}$ and $sat(u_{n_u}(t))$ is $n_u - th$ saturated control input vector, $\omega(t)$ is a scalar zero-mean white noise satisfying the property $E\{\omega(t)x(t)\} = 0$. $\mathbf{A}_i$, $\mathbf{B}_i$ and $\widehat{\mathbf{A}}_i$ are constant matrices. For each fuzzy rule of (1), the firing strength can be presented for interval fuzzy sets as follows.

$$W_i(\rho(t)) = [\underline{w}_i(\rho(t)), \overline{w}_i(\rho(t))], \text{ for } i = 1, 2, \cdots, \Phi \tag{2}$$

where $\underline{w}_i(\rho(t)) = \prod_{j=1}^{z} \underline{\mu}_{\tilde{M}{}_j^i}(\rho_j(t)) \geq 0$ denotes the lower grade of membership, $\overline{w}_i(\rho(t)) = \prod_{j=1}^{z} \overline{\mu}_{\tilde{M}{}_j^i}(\rho_j(t)) \geq 0$ denotes the upper grade of membership, $\underline{\mu}_{\tilde{M}{}_j^i}(\rho_j(t)) \geq 0$ denotes the lower bound of membership function, $\overline{\mu}_{\tilde{M}{}_j^i}(\rho_j(t)) \geq 0$ denotes the upper bound of the membership function. Referring to [29], the following IT2 T-S fuzzy model can be inferred from (1).

$$\dot{x}(t) = m\frac{\sum_{i=1}^{\Phi} \underline{w}_i(\rho(t))\left\{\mathbf{A}_i x(t) + \mathbf{B}_i \widetilde{u}(t) + \widehat{\mathbf{A}}_i x(t)\omega(t)\right\}}{\sum_{i=1}^{\Phi} \underline{w}_i(\rho(t))} + n\frac{\sum_{i=1}^{\Phi} \overline{w}_i(\rho(t))\left\{\mathbf{A}_i x(t) + \mathbf{B}_i \widetilde{u}(t) + \widehat{\mathbf{A}}_i x(t)\omega(t)\right\}}{\sum_{i=1}^{\Phi} \overline{w}_i(\rho(t))} \tag{3}$$

where m and n are the tuning parameters for determining the lower and upper bounded systems.

In this paper, the performance of actuator saturation is considered to limit the control input such that the practical nonlinear systems can avoid the damage caused by the exceed control force. Thus, the definition related to actuator saturation is given as follows:

Definition 1. *Considering the saturated control input $\widetilde{u}_\alpha(t)$ in the IT2 T-S fuzzy model (3), the performance requirement of actuator saturation can be defined as follows:*

$$\widetilde{u}_\alpha(t) = sat(u_\alpha(t)) = \begin{cases} u_{\alpha H} & if \quad u_{\alpha H} < u_\alpha \\ u_\alpha & if \quad u_{\alpha L} < u_\alpha < u_{\alpha H} \\ u_{\alpha L} & if \quad u_\alpha < u_{\alpha L} \end{cases} \tag{4}$$

where $u_{\alpha L}$ and $u_{\alpha H}$ denote the lower and upper constraints of the control input, which satisfying $u_{\alpha L} < 0 < u_{\alpha H}$ and $\alpha = 1, 2, \cdots, n_u$.

To develop the stability analysis with actuator saturation, the following process is needed. Considering the saturated actuator defined in (4), one can obtain

$$\|\widetilde{u}(t)\| \leq \|u(t)\| \tag{5}$$

According to [33], a parameter ε can be given to satisfy the condition $0 < \varepsilon < 1$ such that the saturation map *sat* is inside the sector $(\varepsilon, 1)$. Then, the following relationship can be obtained with the parameter.

$$(1-\varepsilon)\|u(t)\| \geq (1-\varepsilon)\|\widetilde{u}(t)\| \tag{6}$$

$$(1+\varepsilon)\|u(t)\| \geq (1+\varepsilon)\|\widetilde{u}(t)\| \tag{7}$$

$$(1+\varepsilon)\|u(t)\| + (1-\varepsilon)\|u(t)\| \geq 2\|\widetilde{u}(t)\| \tag{8}$$

From inequalities (6) to (8), the following relationship is also obtained.

$$\frac{1-\varepsilon}{2}\|\widetilde{u}(t)\| \geq \|\widetilde{u}(t) - \frac{1+\varepsilon}{2}u(t)\| \tag{9}$$

Under the conditions $u_{\alpha \mathrm{L}} \leq \varepsilon u_\alpha$ and $u_{\alpha \mathrm{H}} \geq \varepsilon u_\alpha$, the relation $\frac{u_{\alpha \mathrm{L}}}{\varepsilon} \leq u_\alpha \leq \frac{u_{\alpha \mathrm{H}}}{\varepsilon}$ can be inferred from (9). Thus, the following condition can also be inferred by setting $u_{\alpha \mathrm{H}} = -u_{\alpha \mathrm{L}}$.

$$|u_\alpha| \leq \frac{u_{\alpha \mathrm{H}}}{\varepsilon} \tag{10}$$

According to (10), the following inequality can be derived.

$$\left(\widetilde{u}(t) - \frac{1+\varepsilon}{2}u(t)\right)^{\mathrm{T}}\left(\widetilde{u}(t) - \frac{1+\varepsilon}{2}u(t)\right) \leq \left(\frac{1-\varepsilon}{2}\right)^2 u^{\mathrm{T}}(t)u(t) \tag{11}$$

It was noted that the inequality (11) is important in the derivation of stability analysis in this paper. In order to consider the actuator saturation for the IT2 T-S fuzzy model (3), the sector parameter ε is combined into (3) as follows:

$$\begin{aligned}\dot{x}(t) = m\frac{\sum_{i=1}^{\Phi}\underline{w}_i(\rho(t))\left\{\mathbf{A}_i x(t) + \mathbf{B}_i\left(\frac{1+\varepsilon}{2}u(t)\right) + \mathbf{B}_i\left(\widetilde{u}(t) - \frac{1+\varepsilon}{2}u(t)\right) + \widehat{\mathbf{A}}_i x(t)\omega(t)\right\}}{\sum_{i=1}^{\Phi}\underline{w}_i(\rho(t))} \\ + n\frac{\sum_{i=1}^{\Phi}\overline{w}_i(\rho(t))\left\{\mathbf{A}_i x(t) + \mathbf{B}_i\left(\frac{1+\varepsilon}{2}u(t)\right) + \mathbf{B}_i\left(\widetilde{u}(t) - \frac{1+\varepsilon}{2}u(t)\right) + \widehat{\mathbf{A}}_i x(t)\omega(t)\right\}}{\sum_{i=1}^{\Phi}\overline{w}_i(\rho(t))}\end{aligned} \tag{12}$$

Then, the PDC method is applied to design the fuzzy controller for the IT2 T-S fuzzy model (12). Via the PDC method, the fuzzy controller corresponding to each subsystem is designed, and the overall fuzzy controller is obtained through the "blending" process. The IT2 T-S fuzzy controller design sequence is presented as follows:

$$\begin{aligned}&\text{Controller Rule } i: \\ &\text{IF } \rho_1(t) \text{ is } \overset{\sim}{\mathrm{M}}_1^i \text{ and } \rho_2(t) \text{ is } \overset{\sim}{\mathrm{M}}_2^i \text{ and} \ldots \text{and } \rho_z(t) \text{ is } \overset{\sim}{\mathrm{M}}_z^i, \\ &\text{Then } u(t) = \mathbf{F}_i x(t)\end{aligned} \tag{13}$$

where $\mathbf{F}_i$ are feedback gains. Via the same method of the IT2 T-S fuzzy model (1–3), the controller (13) is represented as

$$u(t) = m\frac{\sum_{i=1}^{\Phi} \underline{w}_i(\rho(t))\mathbf{F}_i x(t)}{\sum_{i=1}^{\Phi} \underline{w}_i(\rho(t))} + n\frac{\sum_{i=1}^{\Phi} \overline{w}_i(\rho(t))\mathbf{F}_i x(t)}{\sum_{i=1}^{\Phi} \overline{w}_i(\rho(t))} \tag{14}$$

Substituting (14) into (12), the fuzzy controller (14) is applied to control the IT2 T-S fuzzy system (12) with saturated control input $\widetilde{u}(t)$ as follows:

$$\begin{aligned}\dot{x}(t) = \frac{\sum_{i,j,l,q}^{\Phi} g_{ijlq}(\rho(t))}{\sum_{i,j,l,q}^{\Phi} g_{ijlq}(\rho(t))}\Bigg\{ & m\Bigg(\mathbf{G}_{ilq}x(t) + \mathbf{B}_i\Big(\widetilde{u}(t) - \Big(\tfrac{1+\varepsilon}{2}\Big)\big(m\mathbf{F}_l + n\mathbf{F}_q\big)x(t)\Big) + \widehat{\mathbf{A}}_i x(t)\omega(t)\Bigg) \\ & + n\Bigg(\Big(\mathbf{G}_{ilq}x(t) + \mathbf{B}_i\Big(\widetilde{u}(t) - \Big(\tfrac{1+\varepsilon}{2}\Big)\big(m\mathbf{F}_l + n\mathbf{F}_q\big)x(t)\Big)\Big)dt + \widehat{\mathbf{A}}_j x(t)\omega(t)\Bigg)\Bigg\}\end{aligned} \tag{15}$$

where $g_{ijlq}(\rho(t)) = \underline{w}_i(\rho(t))\overline{w}_j(\rho(t))\underline{w}_l(\rho(t))\overline{w}_q(\rho(t))$, $\mathbf{G}_{vps} = \mathbf{A}_v + \mathbf{B}_v\Big(\frac{1+\varepsilon}{2}\Big)\big(m\mathbf{F}_p + n\mathbf{F}_s\big)$. Then, the closed-loop IT2 T-S fuzzy system (15) can be represented as

$$\begin{aligned}\dot{x}(t) = & \frac{\sum_{i=1}^{\Phi}\sum_{j=1}^{\Phi} g_{ijjj}(\rho(t))}{\sum_{i=1}^{\Phi}\sum_{j=1}^{\Phi} g_{ijjj}(\rho(t))}\left\{\begin{array}{l} m\Big(\mathbf{G}_{ijj}x(t) + \mathbf{B}_i\mathbf{R}_{jj}(t) + \widehat{\mathbf{A}}_i x(t)\omega(t)\Big) \\ +n\Big(\mathbf{G}_{jjj}x(t) + \mathbf{B}_j\mathbf{R}_{jj}(t) + \widehat{\mathbf{A}}_j x(t)\omega(t)\Big)\end{array}\right\} \\ & +2\frac{\sum_{i=1}^{\Phi}\sum_{i<l}^{\Phi}\sum_{l=1}^{\Phi} g_{ijll}(\rho(t))}{\sum_{i=1}^{\Phi}\sum_{i<l}^{\Phi}\sum_{l=1}^{\Phi} g_{ijll}(\rho(t))}\left\{\begin{array}{l} m\Big(\big(\mathbf{\Psi}_{ijj} + \mathbf{\Psi}_{ill}\big)x(t) + \frac{\mathbf{B}_i}{2}\mathbf{R}_{jj}(t) + \frac{\mathbf{B}_i}{2}\mathbf{R}_{ll}(t) + \widehat{\mathbf{A}}_i x(t)\omega(t)\Big) \\ +n\Big(\Big(\mathbf{\Psi}_{jll} + \mathbf{\Psi}_{ljj}\Big)x(t) + \frac{\mathbf{B}_j}{2}\mathbf{R}_{ll}(t) + \frac{\mathbf{B}_l}{2}\mathbf{R}_{jj}(t) + \Big(\frac{\widehat{\mathbf{A}}_j + \widehat{\mathbf{A}}_l}{2}\Big)x(t)\omega(t)\Big)\end{array}\right\} \\ & +2\frac{\sum_{i=1}^{\Phi}\sum_{j=1}^{\Phi}\sum_{l<q}^{\Phi}\sum_{q=1}^{\Phi} g_{ijlq}(\rho(t))}{\sum_{i=1}^{\Phi}\sum_{j=1}^{\Phi}\sum_{l<q}^{\Phi}\sum_{q=1}^{\Phi} g_{ijlq}(\rho(t))}\left\{\begin{array}{l} m\Big(\Big(\mathbf{\Psi}_{jll} + \mathbf{\Psi}_{jqq}\Big)x(t) + \frac{\mathbf{B}_j}{2}\mathbf{R}_{ll}(t) + \frac{\mathbf{B}_j}{2}\mathbf{R}_{qq}(t) + \widehat{\mathbf{A}}_i x(t)\omega(t)\Big) \\ +n\Big(\Big(\mathbf{\Psi}_{jll} + \mathbf{\Psi}_{jqq}\Big)x(t) + \frac{\mathbf{B}_j}{2}\mathbf{R}_{ll}(t) + \frac{\mathbf{B}_j}{2}\mathbf{R}_{qq}(t) + \widehat{\mathbf{A}}_j x(t)\omega(t)\Big)\end{array}\right\}\end{aligned} \tag{16}$$

where $\mathbf{\Psi}_{vps} = \frac{\mathbf{A}_v}{2} + \frac{\mathbf{B}_v}{2}\Big(\frac{1+\varepsilon}{2}\Big)\big(m\mathbf{F}_p + n\mathbf{F}_s\big)$ and $\mathbf{R}_{ps}(t) = \widetilde{u}(t) - \Big(\frac{1+\varepsilon}{2}\Big)\big(m\mathbf{F}_p + n\mathbf{F}_s\big)x(t)$.

Based on the closed-loop IT2 T-S fuzzy system (16), the stability analysis and synthesis are carried on by Lyapunov stability theory with actuator saturation presented in Definition 1. To complete the derivations of the stability conditions for the proposed fuzzy control problem, the following lemma is useful and introduced as follows:

Lemma 1. [45] *Given the matrices* $\mathbf{X}$, $\mathbf{Y}$ *and adjusting parameters* δ, *where* $\delta > 0$, *Young's inequality is expressed as*

$$\mathbf{X}^{\mathrm{T}}\mathbf{Y} + \mathbf{Y}^{\mathrm{T}}\mathbf{X} \leq \delta\mathbf{X}^{\mathrm{T}}\mathbf{X} + \delta^{-1}\mathbf{Y}^{\mathrm{T}}\mathbf{Y} \tag{17}$$

Based on the IT2 T-S fuzzy system (16) and introduced definition and lemma, some sufficient conditions are derived to achieve the stability of the considered stochastic nonlinear systems in the next section. The stability conditions are derived based on the Lyapunov stability theory. Besides, the actuator saturation constraint is also considered in the derivations of the stability conditions.

3. Stability Analysis for IT2 T-S Fuzzy System Subject to Actuator Saturation

In this section, the stability analysis and the IT2 T-S fuzzy controller design are proposed for the IT2 T-S fuzzy system (16). The system performance under the effect of multiplicative noises should be guaranteed in the proposed fuzzy controller design

process. Moreover, the control gain can be limited by satisfying the saturated actuator $\widetilde{u}_\alpha(t)$ defined in (4).

Theorem 1. *The IT2 T-S fuzzy system (16) is stable in the sense of mean square and satisfies the actuator saturation if there exist a positive definite matrix* $\mathbf{P}^{\mathrm{T}} = \mathbf{P} > 0$ *and feedback gains* $\mathbf{F}_i$ *such that the following sufficient conditions are satisfied.*

$$He\{\mathbf{P}(m\mathbf{G}_{ijj} + n\mathbf{G}_{jjj})\} + \left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)^{\mathrm{T}}\mathbf{P}\left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right) + \delta\mathbf{P}\Xi_{ij}\Xi_{ij}^{\mathrm{T}}\mathbf{P} + \delta^{-1}\left(\tfrac{1-\varepsilon}{2}\right)^2(m+n)^2\mathbf{F}_j^{\mathrm{T}} < 0 \quad \text{for } i = 1\ldots\Phi,\ j = l = q = 1\ldots\Phi \tag{18}$$

$$\begin{aligned} He\left\{\mathbf{P}\left(m\Psi_{ijj} + m\Psi_{ill} + n\Psi_{jll} + n\Psi_{ljj}\right)\right\} + \left(m\widehat{\mathbf{A}}_i + n\tfrac{\widehat{\mathbf{A}}_j}{2} + n\tfrac{\widehat{\mathbf{A}}_l}{2}\right)^{\mathrm{T}}\mathbf{P}\left(m\widehat{\mathbf{A}}_i + n\tfrac{\widehat{\mathbf{A}}_j}{2} + n\tfrac{\widehat{\mathbf{A}}_l}{2}\right) \\ +\tfrac{1}{4}\delta\mathbf{P}\Xi_{il}\Xi_{il}^{\mathrm{T}}\mathbf{P} + \delta^{-1}\left(\tfrac{1-\varepsilon}{2}\right)^2(m+n)^2\mathbf{F}_j^{\mathrm{T}}\mathbf{F}_j + \tfrac{1}{4}\delta\mathbf{P}\Xi_{ij}\Xi_{ij}^{\mathrm{T}}\mathbf{P} + \delta^{-1}\left(\tfrac{1-\varepsilon}{2}\right)^2(m+n)^2\mathbf{F}_l^{\mathrm{T}}\mathbf{F}_l < 0 \\ \text{for } i,j = 1\ldots\Phi-1,\ l = q = 2\ldots\Phi\ (j<l) \end{aligned} \tag{19}$$

$$\begin{aligned} He\left\{\mathbf{P}\left(m\Psi_{ill} + m\Psi_{iqq} + n\Psi_{jll} + n\Psi_{jqq}\right)\right\} + \left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)^{\mathrm{T}}\mathbf{P}\left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right) \\ +\tfrac{1}{2}\delta\mathbf{P}\Xi_{ij}\Xi_{ij}^{\mathrm{T}}\mathbf{P} + \delta^{-1}\left(\tfrac{1-\varepsilon}{2}\right)^2(m+n)^2\mathbf{F}_l^{\mathrm{T}}\mathbf{F}_l + \delta^{-1}\left(\tfrac{1-\varepsilon}{2}\right)^2(m+n)^2\mathbf{F}_q^{\mathrm{T}}\mathbf{F}_q < 0 \\ \text{for } i,j = 1\ldots\Phi,\ l = 1\ldots\Phi-1,\ q = 2\ldots\Phi\ (l<q) \end{aligned} \tag{20}$$

$$(m+n)\left(\mathbf{F}_i^{(\alpha)}\right)\mathbf{P}^{-1}\left(\mathbf{F}_i^{(\alpha)}\right)^{\mathrm{T}}(m+n) - \left(\frac{u_{\alpha\mathrm{H}}}{\varepsilon}\right)^2 \le 0 \tag{21}$$

where Ψ_{ijj}, Ψ_{jjj}, Ψ_{ill}, Ψ_{jll}, Ψ_{ljj}, Ψ_{iqq}, Ψ_{jqq} are matrices defined in (16), $\Xi_{gf} = \left(m\mathbf{B}_g + n\mathbf{B}_f\right)$ and $He\{[\bullet]\} = [\bullet] + [\bullet]^{\mathrm{T}}$.

Proof. Based on the closed-loop form IT2 T-S fuzzy system (15), the derivative of Lyapunov function, which is selected as $V(x(t)) = x^{\mathrm{T}}(t)\mathbf{P}x(t)$, can be obtained by Itô's formula [25] as follows:

$$\begin{aligned} \dot{V}(x(t)) &= \frac{\sum_{i,j,l,q}^{\Phi} g_{ijlq}(\rho(t))}{\sum_{i,j,l,q}^{\Phi} g_{ijlq}(\rho(t))} \left\{ \begin{array}{l} \left(\begin{array}{l} m\left(\mathbf{G}_{ilq}x(t) + \mathbf{B}_i\mathbf{R}_{lq}(t) + \widehat{\mathbf{A}}_i x(t)\omega(t)\right) \\ +n\left(\mathbf{G}_{jlq}x(t) + \mathbf{B}_j\mathbf{R}_{lq}(t) + \widehat{\mathbf{A}}_j x(t)\omega(t)\right) \end{array} \right)^{\mathrm{T}} \mathbf{P}x(t) \\ +x^{\mathrm{T}}(t)\mathbf{P}\left(\begin{array}{l} m\left(\mathbf{G}_{ilq}x(t) + \mathbf{B}_i\mathbf{R}_{lq}(t) + \widehat{\mathbf{A}}_i x(t)\omega(t)\right) \\ +n\left(\mathbf{G}_{jlq}x(t) + \mathbf{B}_j\mathbf{R}_{lq}(t) + \widehat{\mathbf{A}}_j x(t)\omega(t)\right) \end{array} \right) \\ +\left(\left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)x(t)\right)^{\mathrm{T}}\mathbf{P}\left(\left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)x(t)\right) \end{array} \right\} \\ &= \frac{\sum_{i,j,l,q}^{\Phi} g_{ijlq}(\rho(t))}{\sum_{i,j,l,q}^{\Phi} g_{ijlq}(\rho(t))} \left\{ \begin{array}{l} \left(m\left(\mathbf{G}_{ilq}x(t) + \mathbf{B}_i\mathbf{R}_{lq}(t)\right) + n\left(\mathbf{G}_{jlq}x(t) + \mathbf{B}_j\mathbf{R}_{lq}(t)\right)\right)^{\mathrm{T}}\mathbf{P}x(t) \\ +x^{\mathrm{T}}(t)\mathbf{P}\left(m\left(\mathbf{G}_{ilq}x(t) + \mathbf{B}_i\mathbf{R}_{lq}(t)\right) + n\left(\mathbf{G}_{jlq}x(t) + \mathbf{B}_j\mathbf{R}_{lq}(t)\right)\right) \\ +\left(\left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)x(t)\right)^{\mathrm{T}}\mathbf{P}\left(\left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)x(t)\right) \\ +2x^{\mathrm{T}}(t)\mathbf{P}\left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)x(t)\omega(t) \end{array} \right\} \\ &= V_\Gamma(x(t)) + \frac{\sum_{i,j=1}^{\Phi} g_{ij}(\rho(t))}{\sum_{i,j=1}^{\Phi} g_{ij}(\rho(t))}\left\{2x^T(t)\mathbf{P}\left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)x(t)\omega(t)\right\} \end{aligned} \tag{22}$$

where $g_{ij}(\rho(t)) = \underline{w}_i(\rho(t))\overline{w}_j(\rho(t))$ and

$$V_{\Gamma}(x(t)) = \frac{\sum\limits_{i,j,l,q}^{\Phi} g_{ijlq}(\rho(t))}{\sum\limits_{i,j,l,q}^{\Phi} g_{ijlq}(\rho(t))} \left\{ \begin{array}{l} \Big(m\Big(\mathbf{G}_{ilq}x(t) + \mathbf{B}_i\mathbf{R}_{lq}(t)\Big) + n\Big(\mathbf{G}_{jlq}x(t) + \mathbf{B}_j\mathbf{R}_{lq}(t)\Big)\Big)^{\mathrm{T}}\mathbf{P}x(t) \\ +x^{\mathrm{T}}(t)\mathbf{P}\Big(m\Big(\mathbf{G}_{ilq}x(t) + \mathbf{B}_i\mathbf{R}_{lq}(t)\Big) + n\Big(\mathbf{G}_{jlq}x(t) + \mathbf{B}_j\mathbf{R}_{lq}(t)\Big)\Big) \\ +\Big(\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)x(t)\Big)^{\mathrm{T}}\mathbf{P}\Big(\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)x(t)\Big) \end{array} \right\} \tag{23}$$

Then, the following relationship can be obtained by the representation method of the IT2 T-S fuzzy system (16).

$$V_{\Gamma}(x(t)) = V_{\Gamma 1}(x(t)) + V_{\Gamma 2}(x(t)) + V_{\Gamma 3}(x(t))$$

where

$$\begin{aligned} V_{\Gamma 1}(x(t)) &= \frac{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi} g_{ijjj}(\rho(t))}{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi} g_{ijjj}(\rho(t))} \left\{ \begin{array}{l} He\Big\{\big(m\big(\mathbf{G}_{ijj}x(t) + \mathbf{B}_i\mathbf{R}_{jj}(t)\big) + n\big(\mathbf{G}_{jjj}x(t) + \mathbf{B}_j\mathbf{R}_{jj}(t)\big)\big)^{\mathrm{T}}\mathbf{P}x(t)\Big\} \\ +\Big(\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)x(t)\Big)^{\mathrm{T}}\mathbf{P}\Big(\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)x(t)\Big) \end{array} \right\} \\ &= \frac{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi} g_{ijjj}(\rho(t))}{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi} g_{ijjj}(\rho(t))} \left\{ \begin{array}{l} x^{\mathrm{T}}(t)\Big(He\big\{\mathbf{P}\big(m\mathbf{G}_{ijj} + n\mathbf{G}_{jjj}\big)\big\} + \Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)^{\mathrm{T}}\mathbf{P}\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)\Big)x(t) \\ +He\big\{x^{\mathrm{T}}(t)\mathbf{P}\big(m\mathbf{B}_i + n\mathbf{B}_j\big)\mathbf{R}_{jj}(t)\big\} \end{array} \right\} \end{aligned} \tag{24}$$

$$\begin{aligned} V_{\Gamma 2}(x(t)) &= \frac{\sum\limits_{i=1}^{\Phi}\sum\limits_{j<l}^{\Phi}\sum\limits_{l=1}^{\Phi} g_{ijll}(\rho(t))}{\sum\limits_{i=1}^{\Phi}\sum\limits_{j<l}^{\Phi}\sum\limits_{l=1}^{\Phi} g_{ijll}(\rho(t))} \left\{ \begin{array}{l} He\left\{ \begin{pmatrix} m\Big(\big(\mathbf{\Psi}_{ijj} + \mathbf{\Psi}_{ill}\big)x(t) + \frac{\mathbf{B}_i}{2}\mathbf{R}_{jj}(t) + \frac{\mathbf{B}_i}{2}\mathbf{R}_{ll}(t)\Big) \\ +n\Big(\Big(\mathbf{\Psi}_{jll} + \mathbf{\Psi}_{ljj}\Big)x(t) + \frac{\mathbf{B}_j}{2}\mathbf{R}_{ll}(t) + \frac{\mathbf{B}_j}{2}\mathbf{R}_{jj}(t)\Big) \end{pmatrix}^{\mathrm{T}}\mathbf{P}x(t) \right\} \\ +\Big(\Big(m\widehat{\mathbf{A}}_i + n\frac{\widehat{\mathbf{A}}_j}{2} + n\frac{\widehat{\mathbf{A}}_l}{2}\Big)x(t)\Big)^{\mathrm{T}}\mathbf{P}\Big(\Big(m\widehat{\mathbf{A}}_i + n\frac{\widehat{\mathbf{A}}_j}{2} + n\frac{\widehat{\mathbf{A}}_l}{2}\Big)x(t)\Big) \end{array} \right\} \\ &= \frac{\sum\limits_{i=1}^{\Phi}\sum\limits_{j<l}^{\Phi}\sum\limits_{l=1}^{\Phi} g_{ijll}(\rho(t))}{\sum\limits_{i=1}^{\Phi}\sum\limits_{j<l}^{\Phi}\sum\limits_{l=1}^{\Phi} g_{ijll}(\rho(t))} \left\{ \begin{array}{l} x^{\mathrm{T}}(t)\left(\begin{array}{c} He\Big\{\mathbf{P}\Big(m\big(\mathbf{\Psi}_{ijj} + \mathbf{\Psi}_{ill}\big) + n\Big(\mathbf{\Psi}_{jll} + \mathbf{\Psi}_{ljj}\Big)\Big)\Big\} \\ +\Big(m\widehat{\mathbf{A}}_i + n\frac{\widehat{\mathbf{A}}_j}{2} + n\frac{\widehat{\mathbf{A}}_l}{2}\Big)^{\mathrm{T}}\mathbf{P}\Big(m\widehat{\mathbf{A}}_i + n\frac{\widehat{\mathbf{A}}_j}{2} + n\frac{\widehat{\mathbf{A}}_l}{2}\Big) \end{array} \right)x(t) \\ +He\big\{x^{\mathrm{T}}(t)\mathbf{P}\mathbf{\Xi}_{il}\mathbf{R}_{jj}(t)\big\} + He\big\{x^{\mathrm{T}}(t)\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{R}_{ll}(t)\big\} \end{array} \right\} \end{aligned} \tag{25}$$

$$\begin{aligned} V_{\Gamma 3}(x(t)) &= \frac{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi}\sum\limits_{l<q}^{\Phi}\sum\limits_{q=1}^{\Phi} g_{ijlq}(\rho(t))}{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi}\sum\limits_{l<q}^{\Phi}\sum\limits_{q=1}^{\Phi} g_{ijlq}(\rho(t))} \left\{ \begin{array}{l} He\left\{ \begin{pmatrix} m\Big(\big(\mathbf{\Psi}_{ill} + \mathbf{\Psi}_{iqq}\big)x(t) + \frac{\mathbf{B}_i}{2}\mathbf{R}_{ll}(t) + \frac{\mathbf{B}_i}{2}\mathbf{R}_{qq}(t)\Big) \\ +n\Big(\Big(\mathbf{\Psi}_{jll} + \mathbf{\Psi}_{jqq}\Big)x(t) + \frac{\mathbf{B}_j}{2}\mathbf{R}_{ll}(t) + \frac{\mathbf{B}_j}{2}\mathbf{R}_{qq}(t)\Big) \end{pmatrix}^{\mathrm{T}}\mathbf{P}x(t) \right\} \\ +\Big(\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)x(t)\Big)^{\mathrm{T}}\mathbf{P}\Big(\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)x(t)\Big) \end{array} \right\} \\ &= \frac{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi}\sum\limits_{l<q}^{\Phi}\sum\limits_{q=1}^{\Phi} g_{ijlq}(\rho(t))}{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi}\sum\limits_{l<q}^{\Phi}\sum\limits_{q=1}^{\Phi} g_{ijlq}(\rho(t))} \left\{ \begin{array}{l} x^{\mathrm{T}}(t)\left(\begin{array}{c} He\Big\{\mathbf{P}\Big(m\big(\mathbf{\Psi}_{ill} + \mathbf{\Psi}_{iqq}\big) + n\Big(\mathbf{\Psi}_{jll} + \mathbf{\Psi}_{jqq}\Big)\Big)\Big\} \\ +\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)^{\mathrm{T}}\mathbf{P}\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big) \end{array} \right)x(t) \\ +He\big\{x^{\mathrm{T}}(t)\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{R}_{ll}(t)\big\} + He\big\{x^{\mathrm{T}}(t)\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{R}_{qq}(t)\big\} \end{array} \right\} \end{aligned} \tag{26}$$

Then, the stability criteria for the IT2 T-S fuzzy system (16) can be proposed with (23)–(26) as follows. Firstly, applying the inequalities (17) in Lemma 1 and (11) to (24), one can obtain

$$\begin{aligned} V_{\Gamma 1}(x(t)) &\le \frac{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi} g_{ijjj}(\rho(t))}{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi} g_{ijjj}(\rho(t))} \left\{ \begin{array}{l} x^{\mathrm{T}}(t)\Big(He\big\{\mathbf{P}\big(m\mathbf{G}_{ijj} + n\mathbf{G}_{jjj}\big)\big\} + \Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)^{\mathrm{T}}\mathbf{P}\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)\Big)x(t) \\ +\delta x^{\mathrm{T}}(t)\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{\Xi}_{ij}^{\mathrm{T}}\mathbf{P}x(t) + \delta^{-1}\mathbf{R}_{jj}^{\mathrm{T}}(t)\mathbf{R}_{jj}(t) \end{array} \right\} \\ &\le \frac{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi} g_{ijjj}(\rho(t))}{\sum\limits_{i=1}^{\Phi}\sum\limits_{j=1}^{\Phi} g_{ijjj}(\rho(t))} \left\{ x^{\mathrm{T}}(t)\left(\begin{array}{c} He\big\{\mathbf{P}\big(m\mathbf{G}_{ijj} + n\mathbf{G}_{jjj}\big)\big\} + \Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big)^{\mathrm{T}}\mathbf{P}\Big(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\Big) \\ +\delta\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{\Xi}_{ij}^{\mathrm{T}}\mathbf{P} + \delta^{-1}\Big(\frac{1-\varepsilon}{2}\Big)^2(m+n)^2\mathbf{F}_j^{\mathrm{T}}\mathbf{F}_j \end{array} \right)x(t) \right\} \end{aligned} \tag{27}$$

It is obvious that if the condition (18) is satisfied by Theorem 1, then the situation $V_{\Gamma 1}(x(t)) < 0$ can also be achieved by the relationship (27). Via a similar process, the following relationship is also obtained from (25):

$$\begin{aligned} V_{\Gamma 2}(x(t)) &\le \frac{\sum_{i=1}^{\Phi}\sum_{j<l}^{\Phi}\sum_{l=1}^{\Phi} g_{ijjl}(\rho(t))}{\sum_{i=1}^{\Phi}\sum_{j<l}^{\Phi}\sum_{l=1}^{\Phi} g_{ijjl}(\rho(t))} \left\{ \begin{array}{l} x^{\mathrm{T}}(t)\left(\begin{array}{c} He\Big\{\mathbf{P}\Big(m\big(\mathbf{\Psi}_{ijj}+\mathbf{\Psi}_{ijl}\big)+n\Big(\mathbf{\Psi}_{jjl}+\mathbf{\Psi}_{ljj}\Big)\Big)\Big\} \\ +\left(m\widehat{\mathbf{A}}_i+n\frac{\widehat{\mathbf{A}}_j}{2}+n\frac{\widehat{\mathbf{A}}_l}{2}\right)^{\mathrm{T}}\mathbf{P}\left(m\widehat{\mathbf{A}}_i+n\frac{\widehat{\mathbf{A}}_j}{2}+n\frac{\widehat{\mathbf{A}}_l}{2}\right) \end{array} \right)x(t) \\ +\frac{1}{4}\delta x^{\mathrm{T}}(t)\mathbf{P}\mathbf{\Xi}_{il}\mathbf{\Xi}_{il}^{\mathrm{T}}\mathbf{P}x(t)+\frac{1}{4}\delta x^{\mathrm{T}}(t)\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{\Xi}_{ij}^{\mathrm{T}}\mathbf{P}x(t) \\ +\delta^{-1}\mathbf{R}_{jj}^{\mathrm{T}}(t)\mathbf{R}_{jj}(t)+\delta^{-1}\mathbf{R}_{ll}^{\mathrm{T}}(t)\mathbf{R}_{ll}(t) \end{array} \right\} \\ &\le \frac{\sum_{i=1}^{\Phi}\sum_{j<l}^{\Phi}\sum_{l=1}^{\Phi} g_{ijjl}(\rho(t))}{\sum_{i=1}^{\Phi}\sum_{j<l}^{\Phi}\sum_{l=1}^{\Phi} g_{ijjl}(\rho(t))} \left\{ x^{\mathrm{T}}(t)\left(\begin{array}{l} He\Big\{\mathbf{P}\Big(m\big(\mathbf{\Psi}_{ijj}+\mathbf{\Psi}_{ijl}\big)+n\Big(\mathbf{\Psi}_{jjl}+\mathbf{\Psi}_{ljj}\Big)\Big)\Big\} \\ +\left(m\widehat{\mathbf{A}}_i+n\frac{\widehat{\mathbf{A}}_j}{2}+n\frac{\widehat{\mathbf{A}}_l}{2}\right)^{\mathrm{T}}\mathbf{P}\left(m\widehat{\mathbf{A}}_i+n\frac{\widehat{\mathbf{A}}_j}{2}+n\frac{\widehat{\mathbf{A}}_l}{2}\right) \\ +\frac{1}{4}\delta\mathbf{P}\mathbf{\Xi}_{il}\mathbf{\Xi}_{il}^{\mathrm{T}}\mathbf{P}+\frac{1}{4}\delta\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{\Xi}_{ij}^{\mathrm{T}}\mathbf{P}+\delta^{-1}\left(\frac{1-\varepsilon}{2}\right)^2(m+n)^2\mathbf{F}_j^{\mathrm{T}}\mathbf{F}_j \\ +\delta^{-1}\left(\frac{1-\varepsilon}{2}\right)^2(m+n)^2\mathbf{F}_l^{\mathrm{T}}\mathbf{F}_l \end{array} \right)x(t) \right\} \end{aligned} \tag{28}$$

From (26), one can obtain

$$\begin{aligned} V_{\Gamma 3}(x(t)) &\le \frac{\sum_{i=1}^{\Phi}\sum_{j=1}^{\Phi}\sum_{l<q}^{\Phi}\sum_{q=1}^{\Phi} g_{ijlq}(\rho(t))}{\sum_{i=1}^{\Phi}\sum_{j=1}^{\Phi}\sum_{l<q}^{\Phi}\sum_{q=1}^{\Phi} g_{ijlq}(\rho(t))} \left\{ \begin{array}{l} x^{\mathrm{T}}(t)\left(\begin{array}{c} He\Big\{\mathbf{P}\Big(m\big(\mathbf{\Psi}_{ijl}+\mathbf{\Psi}_{iqq}\big)+n\Big(\mathbf{\Psi}_{jjl}+\mathbf{\Psi}_{jqq}\Big)\Big)\Big\} \\ +\left(m\widehat{\mathbf{A}}_i+n\widehat{\mathbf{A}}_j\right)^{\mathrm{T}}\mathbf{P}\left(m\widehat{\mathbf{A}}_i+n\widehat{\mathbf{A}}_j\right) \end{array} \right)x(t) \\ +\frac{1}{2}\delta x^{\mathrm{T}}(t)\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{\Xi}_{ij}^{\mathrm{T}}\mathbf{P}x(t)+\delta^{-1}\mathbf{R}_{ll}^{\mathrm{T}}(t)\mathbf{R}_{ll}(t)+\delta^{-1}\mathbf{R}_{qq}^{\mathrm{T}}(t)\mathbf{R}_{qq}(t) \end{array} \right\} \\ &\le \frac{\sum_{i=1}^{\Phi}\sum_{j=1}^{\Phi}\sum_{l<q}^{\Phi}\sum_{q=1}^{\Phi} g_{ijlq}(\rho(t))}{\sum_{i=1}^{\Phi}\sum_{j=1}^{\Phi}\sum_{l<q}^{\Phi}\sum_{q=1}^{\Phi} g_{ijlq}(\rho(t))} \left\{ x^{\mathrm{T}}(t)\left(\begin{array}{l} He\Big\{\mathbf{P}\Big(m\big(\mathbf{\Psi}_{ijl}+\mathbf{\Psi}_{iqq}\big)+n\Big(\mathbf{\Psi}_{jjl}+\mathbf{\Psi}_{jqq}\Big)\Big)\Big\} \\ +\left(m\widehat{\mathbf{A}}_i+n\widehat{\mathbf{A}}_j\right)^{\mathrm{T}}\mathbf{P}\left(m\widehat{\mathbf{A}}_i+n\widehat{\mathbf{A}}_j\right)+\frac{1}{2}\delta\mathbf{P}\mathbf{\Xi}_{ij}\mathbf{\Xi}_{ij}^{\mathrm{T}}\mathbf{P} \\ +\delta^{-1}\left(\frac{1-\varepsilon}{2}\right)^2(m+n)^2\mathbf{F}_l^{\mathrm{T}}\mathbf{F}_l+\delta^{-1}\left(\frac{1-\varepsilon}{2}\right)^2(m+n)^2\mathbf{F}_q^{\mathrm{T}}\mathbf{F}_q \end{array} \right)x(t) \right\} \end{aligned} \tag{29}$$

Thus, if the conditions (19)–(20) are satisfied by Theorem 1, $V_{\Gamma 2}(x(t)) < 0$ and $V_{\Gamma 3}(x(t)) < 0$ can also be achieved. From the above results, it is known that the condition $V_{\Gamma}(x(t)) < 0$ is guaranteed by the conditions (18)–(20).

Then, taking the expectation to (22), one can find the following relation due to the properties $E\{x(t)\omega(t)\} = 0$.

$$E\Big\{\dot{V}(x(t))\Big\} = E\{V_{\Gamma}(x(t))\} \tag{30}$$

Based on the above statements, it has been proven that the condition $V_{\Gamma}(x(t)) < 0$ is satisfied by sufficient conditions (18)–(20) proposed in Theorem 1. Via the relationship (30), $E\Big\{\dot{V}(x(t))\Big\} < 0$ is also achieved by $E\{V_{\Gamma}(x(t))\} < 0$. Referring to [46], the IT2 T-S fuzzy system (16) is said to be stable in the sense of mean square via satisfying the conditions (18)–(20).

In addition to the stability, the actuator saturation requirement is also considered to limit the control input of the IT2 T-S fuzzy system (16). Thus, the proof of the actuator saturation constraint (21) is given as follows. Firstly, an ellipsoid D_1 is defined as follows:

$$D_1 = \Big\{x(t)\Big|x^{\mathrm{T}}(t)\mathbf{P}x(t) \le 1\Big\} \tag{31}$$

From [33], it is known that if the sufficient conditions (18)–(20) are satisfied by Theorem 1, the ellipsoid (31), which is inside the domain of attraction, is contractively invariant. Then, the following condition of actuator saturation is obtained by (10) and (14).

$$\left| \left(m \frac{\sum_{i=1}^{\Phi} \underline{w}_i(\rho(t))}{\sum_{i=1}^{\Phi} \underline{w}_i(\rho(t))} + n \frac{\sum_{i=1}^{\Phi} \overline{w}_i(\rho(t))}{\sum_{i=1}^{\Phi} \overline{w}_i(\rho(t))} \right) \left(\mathbf{F}_i^{(\alpha)} x(t) \right) \right| \leq \frac{u_{\alpha \mathrm{H}}}{\varepsilon} \tag{32}$$

where $\mathbf{F}_i^{(\alpha)}$ denotes the α-th row of $\mathbf{F}_i$. Referring to [33], the condition can be derived as follows if (32) is satisfied with $\frac{\sum_{i=1}^{\Phi} \underline{w}_i(\rho(t))}{\sum_{i=1}^{\Phi} \underline{w}_i(\rho(t))} = \frac{\sum_{i=1}^{\Phi} \overline{w}_i(\rho(t))}{\sum_{i=1}^{\Phi} \overline{w}_i(\rho(t))} = 1$.

$$D_2 = \left\{ x(t) \middle| x^{\mathrm{T}}(t)(m+n) \left(\mathbf{F}_i^{(\alpha)} \right)^{\mathrm{T}} \left(\mathbf{F}_i^{(\alpha)} \right) (m+n) x(t) \leq \left(\frac{u_{\alpha \mathrm{H}}}{\varepsilon} \right)^2 \right\} \tag{33}$$

Based on the above results, it is known that $x(t) \in D_1 \subset D_2$ is required for satisfying the actuator saturation. Then, the equivalent condition for $x(t) \in D_1 \subset D_2$ can be obtained by (31) and (33) as follows:

$$(m+n) \left(\mathbf{F}_i^{(\alpha)} \right) \mathbf{P}^{-1} \left(\mathbf{F}_i^{(\alpha)} \right)^{\mathrm{T}} (m+n) \leq \left(\frac{u_{\alpha \mathrm{H}}}{\varepsilon} \right)^2 \tag{34}$$

(34) of actuator saturation can also be satisfied. Then, the performance requirement of the actuator saturation (4) is achieved for the IT2 T-S fuzzy system (16) by the condition (32). Based on the above results, the IT2 T-S fuzzy system (16) can achieve asymptotically stability in the sense of mean square and the actuator saturation constraint via satisfying the sufficient conditions (18)–(20) proposed in Theorem 1. □

Remark 1. *The purpose of Theorem 1 is to develop the stability analysis and fuzzy controller design for nonlinear stochastic systems with parameter uncertainties. First, the system parameter uncertainties were considered in representing the IT2 T-S fuzzy model (16). The stochastic behaviors presented as multiplicative noises were also considered in the stability analysis by Itô's formula (22). It was noted that the control input might become bigger when the designers considered the above performance requirements simultaneously. Via the application of inequality (11), the performance constraint of actuator saturation (4) has been combined into the stability analysis to avoid the situation that the control inputs exceed the limit of the practical systems. Applying Young's inequality in Lemma 1, the derivations of sufficient conditions (18)–(20) have also been solved successfully. Thus, the fuzzy controller design problem subject to system parameter uncertainties, multiplicative noises, and actuator saturation requirements has been studied in Theorem 1 based on the IT2 T-S fuzzy model.*

The conditions derived in Theorem 1 are not linear matrix inequalities. It is difficult to solve the conditions of Theorem 1 using numerical analysis methods. In order to solve the above control problem by the convex optimization algorithm, the stability conditions (18)–(21) are transferred into LMI form in the following theorem.

Theorem 2. *The closed-loop IT2 T-S fuzzy system (16) is stable in the sense of mean square and satisfying the actuator saturation constraint if there exist a positive definite matrix* $\mathbf{Q}^{\mathrm{T}} = \mathbf{Q} > 0$ *and feedback gains* $\mathbf{K}_i$ *such that the following sufficient conditions are satisfied.*

$$\begin{bmatrix} \boldsymbol{\Theta}_{a11} & * & * \\ \left(\frac{1-\varepsilon}{2}\right)(m+n)\mathbf{K}_j & -\delta\mathbf{I}_{n_u\times n_u} & * \\ \left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)\mathbf{Q} & \mathbf{0}_{n_x\times n_x} & -\mathbf{Q} \end{bmatrix} < 0 \quad \text{for } i = 1\ldots\Phi,\ j = l = q = 1\ldots\Phi \tag{35}$$

$$\begin{bmatrix} \boldsymbol{\Theta}_{b11} & * & * & * \\ \left(\frac{1-\varepsilon}{2}\right)(m+n)\mathbf{K}_j & -\delta\mathbf{I}_{n_u\times n_u} & * & * \\ \left(\frac{1-\varepsilon}{2}\right)(m+n)\mathbf{K}_l & \mathbf{0}_{n_u\times n_u} & -\delta\mathbf{I}_{n_u\times n_u} & * \\ \left(m\widehat{\mathbf{A}}_i + n\left(\frac{\widehat{\mathbf{A}}_j+\widehat{\mathbf{A}}_l}{2}\right)\right)\mathbf{Q} & \mathbf{0}_{n_x\times n_u} & \mathbf{0}_{n_x\times n_u} & -\mathbf{Q} \end{bmatrix} < 0 \quad \text{for } i,j = 1\ldots\Phi-1,\ l = q = 2\ldots\Phi\ (j<l) \tag{36}$$

$$\begin{bmatrix} \boldsymbol{\Theta}_{c11} & * & * & * \\ \left(\frac{1-\varepsilon}{2}\right)(m+n)\mathbf{K}_l & -\delta\mathbf{I}_{n_u\times n_u} & * & * \\ \left(\frac{1-\varepsilon}{2}\right)(m+n)\mathbf{K}_q & \mathbf{0}_{n_u\times n_u} & -\delta\mathbf{I}_{n_u\times n_u} & * \\ \left(m\widehat{\mathbf{A}}_i + n\widehat{\mathbf{A}}_j\right)\mathbf{Q} & \mathbf{0}_{n_x\times n_u} & \mathbf{0}_{n_x\times n_u} & -\mathbf{Q} \end{bmatrix} < 0 \quad \text{for } i,j = 1\ldots\Phi,\ l = 1\ldots\Phi-1,\ q = 2\ldots\Phi\ (l<q) \tag{37}$$

$$\begin{bmatrix} -\left(\frac{1+\varepsilon}{2}\right)^2\left(\frac{\mu_{\alpha H}}{\varepsilon}\right)^2 & (m+n)\left(\frac{1+\varepsilon}{2}\right)\mathbf{K}_i^{(\alpha)} \\ * & -\mathbf{Q} \end{bmatrix} \le 0 \tag{38}$$

where

$$\boldsymbol{\Theta}_{a11} = \delta\boldsymbol{\Xi}_{ij}\boldsymbol{\Xi}_{ij}^{\mathrm{T}} + He\left\{(m\mathbf{A}_i + n\mathbf{A}_j)\mathbf{Q} + \left(\frac{1+\varepsilon}{2}\right)\left(\left(m^2+mn\right)\mathbf{B}_i + \left(nm+n^2\right)\mathbf{B}_j\right)\mathbf{K}_j\right\},$$

$$\begin{aligned}\boldsymbol{\Theta}_{b11} &= \tfrac{1}{4}\delta\boldsymbol{\Xi}_{il}\boldsymbol{\Xi}_{il}^{\mathrm{T}} + \tfrac{1}{4}\delta\boldsymbol{\Xi}_{ij}\boldsymbol{\Xi}_{ij}^{\mathrm{T}} + He\left\{\left(m\mathbf{A}_i + n\tfrac{\mathbf{A}_j}{2} + n\tfrac{\mathbf{A}_l}{2}\right)\mathbf{Q} + \left(\tfrac{1+\varepsilon}{2}\right)\left(\left(m^2+mn\right)\tfrac{\mathbf{B}_i}{2} + \left(nm+n^2\right)\tfrac{\mathbf{B}_l}{2}\right)\mathbf{K}_j\right.\\ &\left. + \left(\tfrac{1+\varepsilon}{2}\right)\left(\left(m^2+mn\right)\tfrac{\mathbf{B}_i}{2} + \left(nm+n^2\right)\tfrac{\mathbf{B}_j}{2}\right)\mathbf{K}_l\right\},\end{aligned}$$

$$\begin{aligned}\boldsymbol{\Theta}_{c11} &= \tfrac{1}{2}\delta\boldsymbol{\Xi}_{ij}\boldsymbol{\Xi}_{ij}^{\mathrm{T}} + He\left\{(m\mathbf{A}_i + n\mathbf{A}_j)\mathbf{Q} + \left(\tfrac{1+\varepsilon}{2}\right)\left(\left(m^2+mn\right)\tfrac{\mathbf{B}_i}{2} + \left(nm+n^2\right)\tfrac{\mathbf{B}_j}{2}\right)\mathbf{K}_l\right.\\ &\left. + \left(\tfrac{1+\varepsilon}{2}\right)\left(\left(m^2+mn\right)\tfrac{\mathbf{B}_i}{2} + \left(nm+n^2\right)\tfrac{\mathbf{B}_j}{2}\right)\mathbf{K}_q\right\},\end{aligned}$$

$\mathbf{K}_j = \mathbf{F}_j\mathbf{Q}$, $\mathbf{Q} = \mathbf{P}^{-1}$, and $*$ denotes the transposed element in the symmetric position.

Proof. It is obvious that the stability conditions (18)–(20) can be obtained from (35)–(37) by applying Schur complement and pre-and-post-multiplying the matrix $\mathbf{P}^{-1}$. Moreover, the condition (40) can be obtained from the actuator saturation constraint (21) as follows. Firstly, applying Schur complement to (21), one can obtain

$$\begin{bmatrix} -\left(\frac{u_{\alpha H}}{\varepsilon}\right)^2 & (m+n)\mathbf{F}_i^{(\alpha)} \\ * & -\mathbf{P} \end{bmatrix} \le 0 \tag{39}$$

Then, the condition (38) can be obtained from (39) by pre-and-post multiplying $diag\left\{\frac{1+\varepsilon}{2}, \mathbf{P}^{-1}\right\}$. Via the setting $\mathbf{Q} = \mathbf{P}^{-1}$ and $\mathbf{K}_i = \mathbf{F}_i\mathbf{Q}$, sufficient conditions (18)–(20) are achieved if the conditions (35)–(37) are satisfied. The actuator saturation constraint (21) can also be satisfied due to the condition (38). Therefore, from Theorem 1, it can be said that the IT2 T-S fuzzy system (16) is asymptotically stable in the sense of mean square subject to the actuator saturation by satisfying the stability conditions (35)–(38) proposed in Theorem 2. □

Remark 2. *The purpose of Theorem 2 is to convert the sufficient conditions (18)–(21) proposed in Theorem 1 into the LMI problems. Then, the sufficient conditions (35)–(38) presented in Theorem 2 can be easily solved by the convex optimization algorithm.*

In order to show the feasibility and efficiency of the proposed fuzzy controller design method, a practical nonlinear ship steering systems' control problem is considered in the next section. By solving the stability conditions derived in Theorem 2, an actuator saturated fuzzy controller can be designed to stabilize the nonlinear ship steering system that was represented by the IT2 T-S fuzzy system.

4. Simulation of Nonlinear Ship Steering System

On the unpredicted ocean, how to control the ship with accurate positioning and heading is always an important issue. Thus, the stochastic behaviors and uncertainties caused by environments are usually considered in the controller design and stability analysis. Based on the IT2 T-S fuzzy model, the saturated fuzzy controller design method proposed in this paper is applied to ensure the performance and stability of the ship steering system in this section. Referring to [47,48], a ship that has a length of 200.6 m and a mass of 73,097.15 kg is considered in this section. Besides, the dynamic equations of the nonlinear ship steering system can be presented as follows:

$$\dot{x}_1(t) = cos(x_3(t))x_4(t) + 0.5x_4(t)\omega(t) - sin(x_3(t))x_5(t) \tag{40}$$

$$\dot{x}_2(t) = sin(x_3(t))x_4(t) + cos(x_3(t))x_5(t) + 1.3x_5(t)\omega(t) \tag{41}$$

$$\dot{x}_3(t) = x_6(t) + 1.5x_6(t)\omega(t) \tag{42}$$

$$\dot{x}_4(t) = -0.0358x_1(t) - 0.0797x_4(t) + 0.9215u_1(t) \tag{43}$$

$$\dot{x}_5(t) = -0.0208x_2(t) - 0.0818x_5(t) - 0.1224x_6(t) + 0.7805u_2(t) + 7.4562u_3(t) \tag{44}$$

$$\dot{x}_6(t) = -0.0394x_2(t) - 0.2254x_5(t) - 0.2468x_6(t) + 1.4811u_2(t) + 7.4562u_3(t) \tag{45}$$

where $x_1(t)$ and $x_2(t)$ denote the ship position on the earth-fixed frame, $x_3(t)$ denotes the ships' yaw angle, $x_4(t)$ and $x_5(t)$ denote the surge and sway motion of ship, $x_6(t)$ denote the yaw angular velocity, $u_1(t)$, $u_2(t)$ and $u_3(t)$ denote the control forces of the thrusters. The details of ship steering systems (40)–(45) can be referred to [49]. To construct the IT2 T-S fuzzy model for nonlinear ship steering system (40)–(45), the membership function is selected as shown in Figure 2. Then, the IT2 T-S fuzzy model with multiplicative noises is presented as follows:

$$\begin{aligned} &\text{Plant Rule } i: \\ &\text{IF } x_3(t) \text{ is } \overset{\sim}{\mathrm{M}}_3^i \\ &\text{Then } \dot{x}(t) = \mathbf{A}_i x(t) + \mathbf{B}_i \widetilde{u}(t) + \widehat{\mathbf{A}}_i x(t)\omega(t) \end{aligned} \tag{46}$$

where subsystem matrices of three fuzzy rules are

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 0 & 0.0349 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0.0349 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -0.0358 & 0 & 0 & -0.0797 & 0 & 0 \\ 0 & -0.0208 & 0 & 0 & -0.0818 & -0.1224 \\ 0 & -0.0394 & 0 & 0 & -0.2254 & -0.2468 \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 0 & 0 & 1 & -0.0349 & 0 \\ 0 & 0 & 0 & 0.0349 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -0.0358 & 0 & 0 & -0.0797 & 0 & 0 \\ 0 & -0.0208 & 0 & 0 & -0.0818 & -0.1224 \\ 0 & -0.0394 & 0 & 0 & -0.2254 & -0.2468 \end{bmatrix},$$

$$\mathbf{A}_3 = \begin{bmatrix} 0 & 0 & 0 & 0.0349 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0.0349 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -0.0358 & 0 & 0 & -0.0797 & 0 & 0 \\ 0 & -0.0208 & 0 & 0 & -0.0818 & -0.1224 \\ 0 & -0.0394 & 0 & 0 & -0.2254 & -0.2468 \end{bmatrix},$$

$$\mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.9215 & 0 & 0 \\ 0 & 0.7802 & 1.4811 \\ 0 & 1.4811 & 7.4562 \end{bmatrix}, \text{ and } \widehat{\mathbf{A}}_1 = \widehat{\mathbf{A}}_2 = \widehat{\mathbf{A}}_3 = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

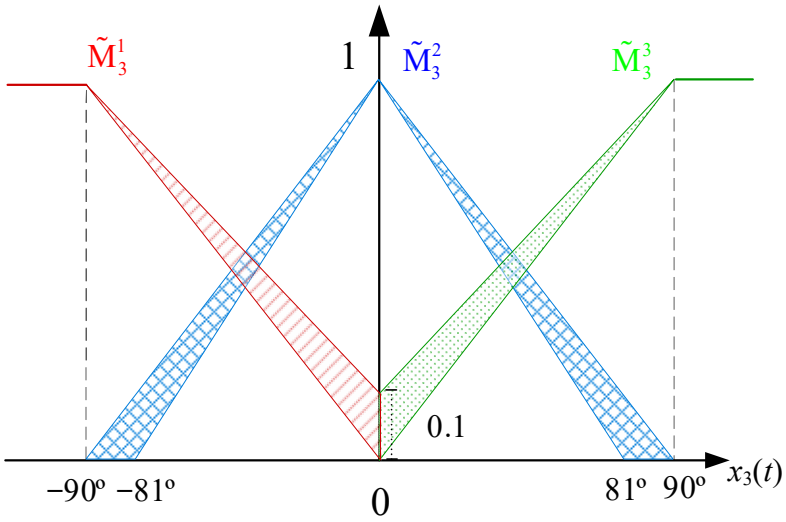


Figure 2. Membership functions of $x_3(t)$.

For a ship on the unpredictable ocean, the positioning system will be affected seriously by the stochastic environment, such as waves, winds, and currents. It is expected that a more significant control force is necessary to be applied when these effects occur. However, the limitations of all actuation devices should be considered for the ship steering system (40)–(46). To deal with this critical issue, the performance constraint of actuator saturation is combined into the development of the proposed fuzzy controller design.

The steering gears are usually employed to be the modern actuator equipment for the ship steering systems. Because of the advantages of eco-friendly, low space occupation, maintenance frequency, and so on, the permanent-magnet linear synchronous actuator was developed to replace the oil-powered actuators [50–52]. Moreover, the entire electrical steering gear system, which is in the digital domain, can solve environmental protection nowadays. For the steering gear system, some problems need to be improved, which can be presented as follows:

1. The difference in the actuator angle and ordered angle: This problem will occur when the wrong or insufficient control adjustment is applied.

2. Unsatisfactory Steering: The breakdown of safety valves or by-pass valves in the steering gear system will cause the problem.
3. Ship movement is beyond the limit: The main reason is the error signal return by the sensor such that the control command is given over the limit of autopilot.

It is known that the saturation of the actuator arises due to the difference between the designed control input and the practical actuator output. In order to solve the actuator saturation problem, it is usually necessary to add a filter between the designed controller and the practical actuator. However, adding filters will increase costs. If we can put the actuator saturation problem directly into the controller design process, the designers will be able to get a practical controller that is direct and cost-saving. Thus, the controller design considering the actuator saturation constraint is an essential issue for the modern ship steering system. Based on the IT2 T-S fuzzy system of ship steering systems (46), the proposed design method is compared with the design method in [29] in this simulation.

Remark 3. *In [29], the fuzzy controller design method is also developed based on the IT2 T-S fuzzy model and PDC method. The difference between the proposed design method and the design method in [29] is described below. In this paper, the stochastic behaviors are considered more in the IT2 T-S fuzzy model. Moreover, the constraint of the actuator saturation is also applied in the proposed fuzzy controller design method to limit the control gain, such that it will not work over the tolerance of the practical systems.*

To carry on the IT2 fuzzy controller design method of both methods, the tuning parameters are selected as $m = 0.7$ and $n = 0.3$. Moreover, actuator saturation constraints are designed for each control input as $u_{1H} = 10$, $u_{2H} = 10$ and $u_{3H} = 10$ of the proposed design method. Solving the sufficient conditions (35–38), the positive definite matrix, and feedback gains can be obtained as follows:

$$\mathbf{F}_1 = \begin{bmatrix} -0.6333 & 0.0020 & -0.0012 & -13.3718 & 0.0480 & -0.0053 \\ -0.0052 & -0.2113 & 0.2931 & -0.0348 & -9.8108 & 1.7339 \\ 0.0002 & 0.0054 & -0.1314 & -0.0054 & 0.2764 & -0.6170 \end{bmatrix} \quad (47)$$

$$\mathbf{F}_2 = \begin{bmatrix} -0.6296 & -0.0000001 & -0.000002 & -13.2836 & 0.00006 & -0.00002 \\ 0.000003 & -0.2110 & 0.2918 & -0.00003 & -9.7859 & 1.7180 \\ 0.000004 & 0.0086 & -0.1313 & 0.00009 & 0.4075 & -0.6152 \end{bmatrix} \quad (48)$$

$$\mathbf{F}_3 = \begin{bmatrix} -0.6333 & -0.0020 & 0.0012 & -13.3714 & -0.0482 & 0.0053 \\ 0.0052 & -0.2113 & 0.2931 & 0.0348 & -9.8108 & 1.7339 \\ -0.0002 & 0.0054 & -0.1314 & 0.0054 & 0.2761 & -0.6170 \end{bmatrix} \quad (49)$$

$$\mathbf{P} = \begin{bmatrix} 0.0362 & -0.0000001 & -0.00000001 & 0.0858 & -0.000001 & -0.00000002 \\ -0.0000001 & 0.0163 & 0.0001 & -0.0000005 & 0.0304 & -0.0005 \\ -0.00000001 & 0.0001 & 0.0298 & -0.0000001 & -0.0034 & 0.0139 \\ 0.0858 & -0.0000005 & -0.0000001 & 1.7177 & -0.00002 & -0.0000004 \\ -0.000001 & 0.0304 & -0.0034 & -0.00002 & 1.2520 & -0.0240 \\ -0.00000002 & -0.0005 & 0.0139 & -0.0000004 & -0.0240 & 0.0950 \end{bmatrix} \quad (50)$$

To demonstrate the advantages of the proposed design method, the design method in [29] is applied to present the comparison results. Thus, the positive definite matrix and control gains of each fuzzy rule can also be obtained by solving the stability conditions proposed in [29] as follows:

$$\mathbf{F}_1 = \begin{bmatrix} -3.0944 & 0.0215 & 0.0000006 & -26.8185 & 0.1558 & -0.000003 \\ -0.0408 & -6.0637 & 0.5137 & -0.2952 & -52.2263 & 0.4122 \\ 0.0081 & 1.2587 & -0.2896 & 0.0586 & 10.8251 & -0.1807 \end{bmatrix} \quad (51)$$

$$\mathbf{F}_2 = \begin{bmatrix} -2.9223 & -0.0002 & 0.0000003 & -25.3278 & -0.0027 & 0.0000002 \\ 0.0005 & -5.7513 & 0.5084 & 0.0051 & -49.5187 & 0.3991 \\ -0.00009 & 1.2012 & -0.2886 & -0.0010 & 10.3261 & -0.1780 \end{bmatrix} \quad (52)$$

$$\mathbf{F}_3 = \begin{bmatrix} -3.0947 & -0.0215 & -0.0000004 & -26.8216 & -0.1574 & 0.000004 \\ 0.0408 & -6.0644 & 0.5137 & 0.2983 & -52.2319 & 0.4122 \\ -0.0081 & 1.2589 & -0.2896 & -0.0593 & 10.8261 & -0.1807 \end{bmatrix} \quad (53)$$

$$\mathbf{P} = \begin{bmatrix} 0.1013 & -0.000000008 & 0.000000002 & 0.7368 & 0.00003 & 0.0000000006 \\ -0.000000008 & 0.1013 & 0.000003 & -0.00003 & 0.7370 & -0.00002 \\ 0.000000002 & 0.000003 & 0.1388 & 0.00000001 & -0.000002 & 0.0567 \\ 0.7368 & -0.00003 & 0.00000001 & 6.3291 & -0.0000007 & 0.00000001 \\ 0.00003 & 0.7370 & -0.000002 & -0.0000007 & -6.3309 & -0.0001 \\ 0.0000000006 & -0.00002 & 0.0567 & 0.00000001 & -0.0001 & 0.1390 \end{bmatrix} \quad (54)$$

In Figure 3, the comparison between the proposed controller design method and the design method of [29] is presented precisely. For the design method of [29], the requirement of stability is considered in the design process, and the control input is designed as $u(t)$ based on the feedback gains of (51)–(53), which is presented on the left-hand side of Figure 3. When actuation devices have actual saturation limits, the control input of the design method of [29] may be too high for the actuator saturation limits. If the control input designed by this control method exceeds the actuator saturation constraint, its control input $u(t)$ may not be used for the practical nonlinear ship steering systems. In many practical systems like the ship steering system (40)–(45), the actuator saturation will deteriorate system performance seriously or cause instability. The actuator saturation requirement is directly combined into the proposed fuzzy controller design method to deal with the problem. Based on the designed fuzzy controller with feedback gains (47)–(49), the saturated control input can be obtained as $\widetilde{u}(t)$, which is shown on the right-hand side of Figure 3. The proposed fuzzy controller design method can meet the actual actuator saturation limits without adding additional filter equipment, and it can be directly used in practical actuation devices for the ship steering systems. The simulation results from the comparisons between the two design methods are presented as follows:

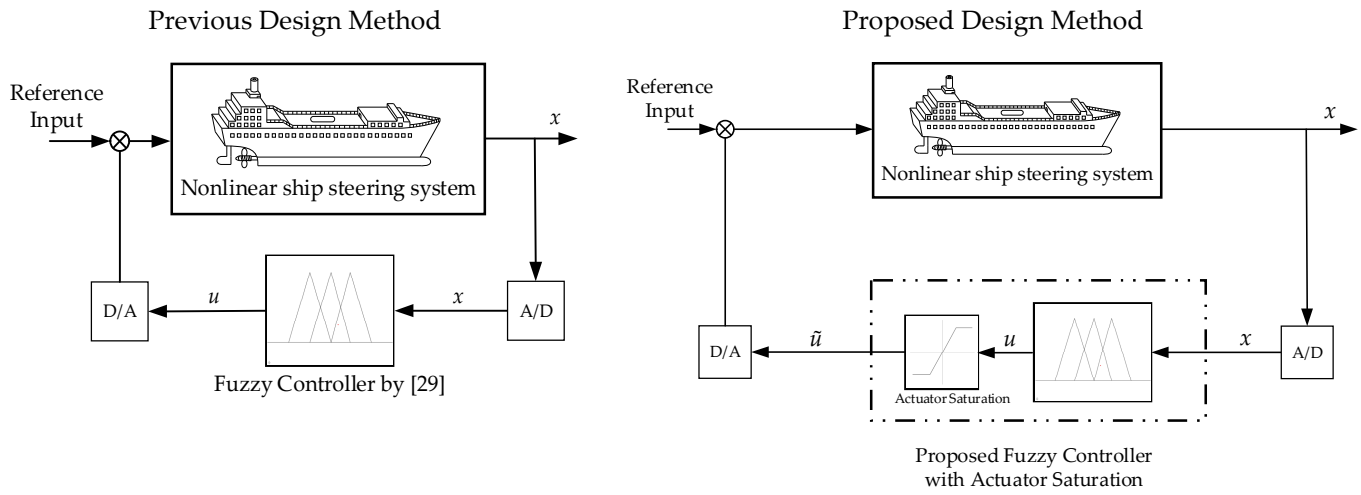


Figure 3. Block-diagrams of the control ship steering system.

Applying the IT2 fuzzy controllers (47)–(49) and (51)–(53) in the form of (13), the responses of ship steering systems (46) are presented in Figures 4–13 by setting the initial conditions $x(0) = \begin{bmatrix} 10 & 10 & 30^o & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$. It is noted that the yaw angle $x_3(t)$ should be constrained in 180º for the ship steering system, which is presented in Figures 6–9. To show the efficiency of the proposed fuzzy controller design method, the variance value

of the zero-mean white noise $\omega(t)$ is given as $\mathbf{W} = 25$. However, the ship steering system will become unstable, controlled by the design method of [29] if the variance value of the zero-mean white noise $\omega(t)$ is given as $\mathbf{W} = 25$. The controlled ship steering system is stable, just in the variance value of the zero-mean white noise $\omega(t)$ below $\mathbf{W} = 8$. Moreover, for the actuator saturation constraint $u_{2H} = 10$, the design method of [29] cannot also find a stable controller for the ship steering systems. It must relax the constraint of actuator saturation value as $u_{2H} = 20$, then the design method of [29] can find a feasible controller. The responses of the control input $u_2(t)$ are presented in Figure 11. Based on the above statement, it is known that the proposed design method can achieve the stability of the ship steering system by tolerating a larger variance value of noise and a smaller bound of actuator saturation constraint.

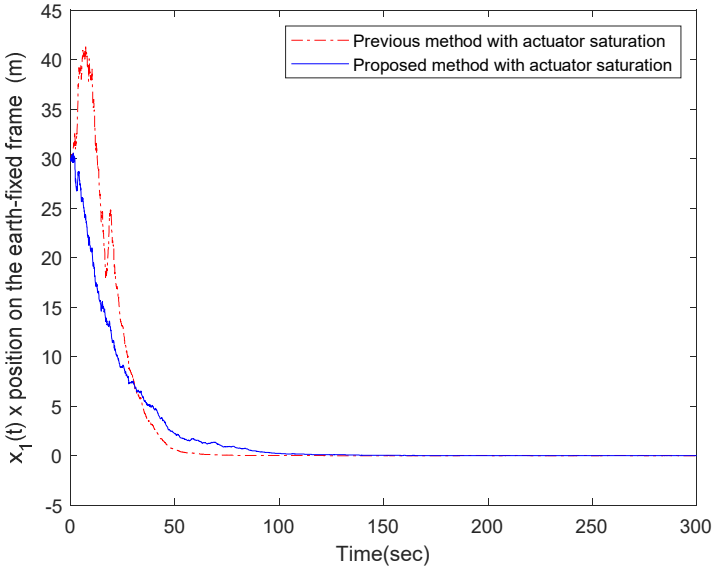


Figure 4. The responses of $x_1(t)$ compared with [29].

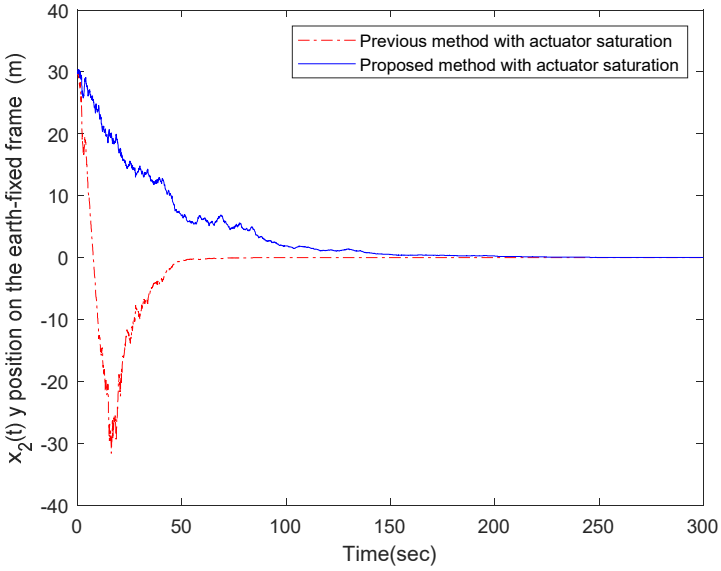


Figure 5. The responses of $x_2(t)$ compared with [29].

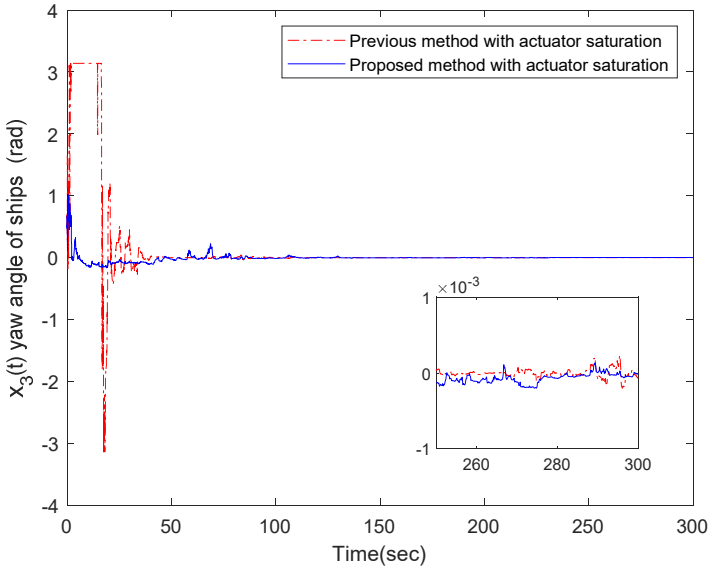


Figure 6. The responses of $x_3(t)$ compared with [29].

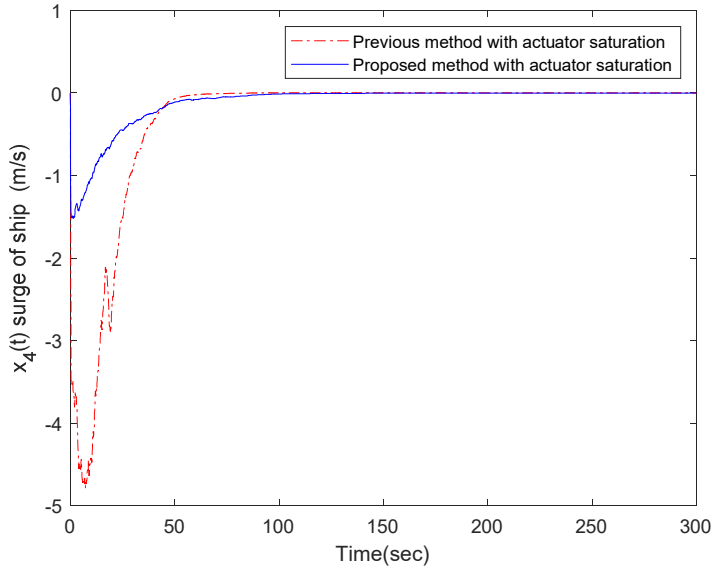


Figure 7. The responses of $x_4(t)$ compared with [29].

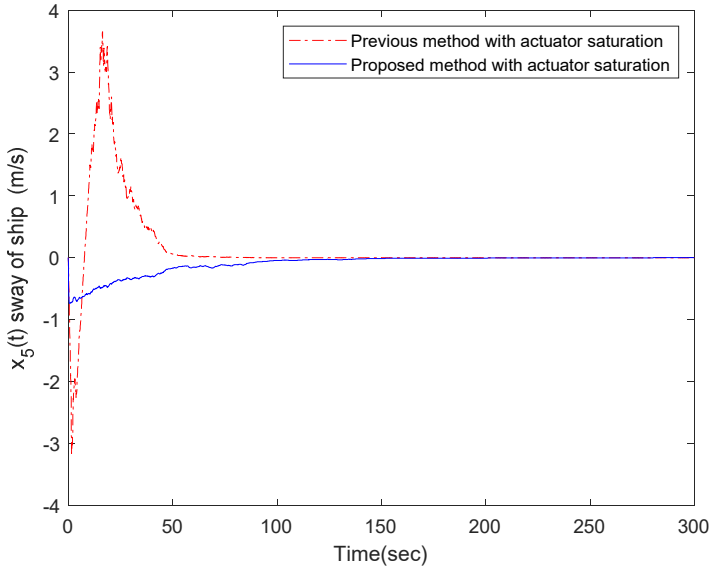


Figure 8. The responses of $x_5(t)$ compared with [29].

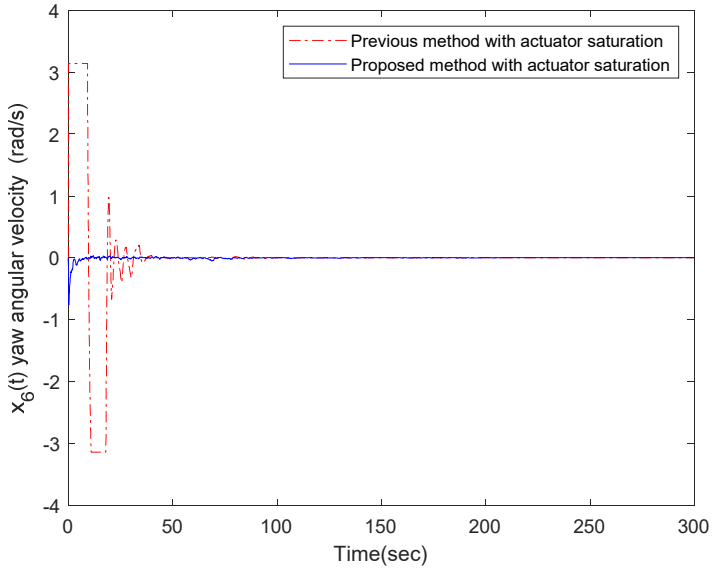


Figure 9. The responses of $x_6(t)$ compared with [29].

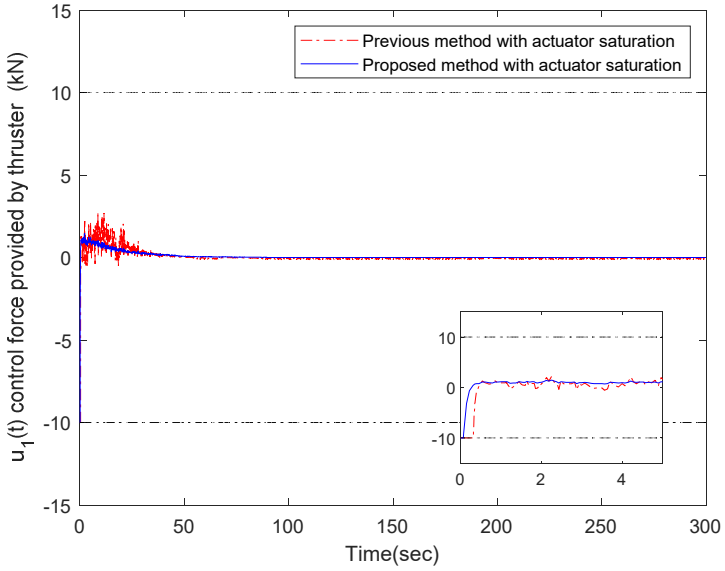


Figure 10. The responses of $u_1(t)$ compared with [29].

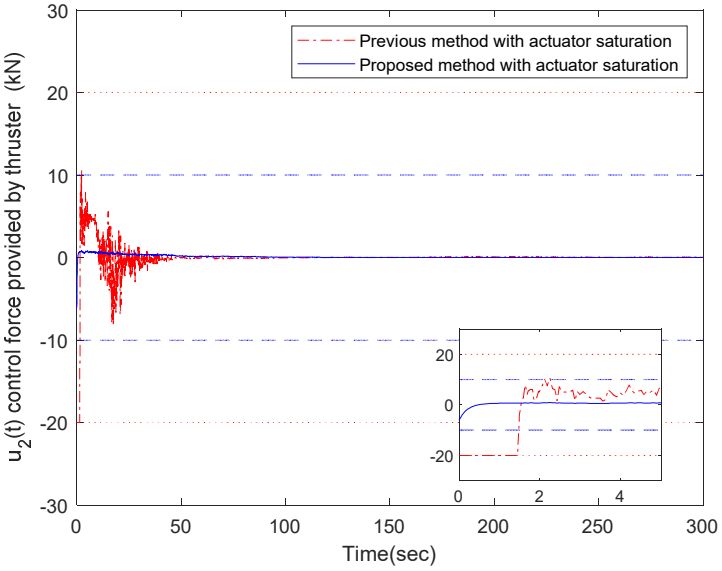


Figure 11. The responses of $u_2(t)$ compared with [29].

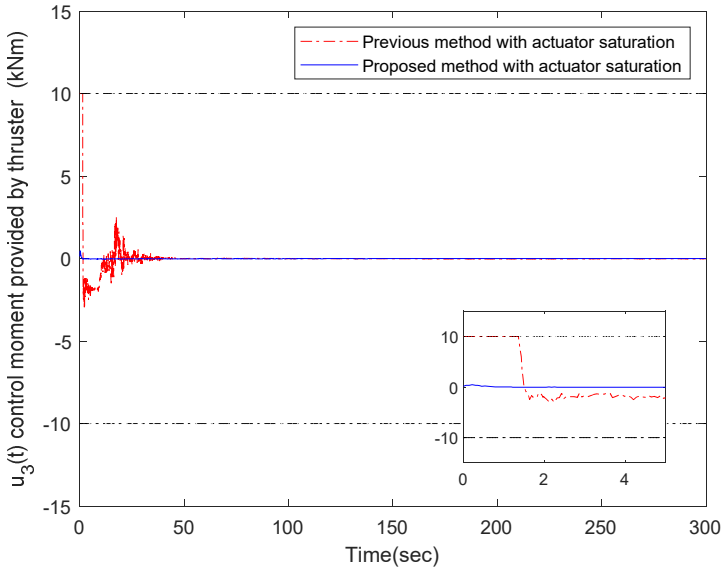


Figure 12. The responses of $u_3(t)$ compared with [29].

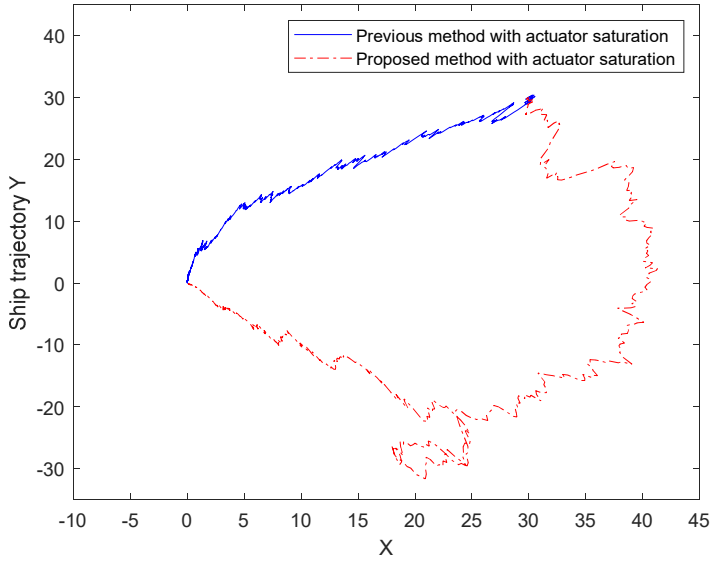


Figure 13. The trajectory of the ship with difference control method.

In Figures 4–9, it is seen that the proposed design method can control the ship steering system to be stable. From the transient responses presented in Figures 4–9, it is clear that the state responses have stochastic behaviors that were affected by the zero-mean white noise $\omega(t)$. Especially, the steady-state responses of the system states also presented the stochastic behaviors that can be referred to in the small diagram embedded in Figure 6. Because the overshoots of transient responses of system states are relatively large, the steady-state responses seem to converge to zero. Moreover, this paper considered the multiplicative noises. The states approach zero in the steady-state response; hence, the value of the noise multiplied by the state will also approach zero. Therefore, the stochastic behavior of the noises is less obvious in the steady-state response. However, the phenomenon of stochastic behaviors still occurs in the steady-state response stage because the system is affected by the white noise $\omega(t)$. Besides, the stochastic behaviors of the controlled ship can also be shown in the trajectory diagram of Figure 13. In Figure 13, the oscillated curves also presented the stochastic behaviors affected by the white noise $\omega(t)$. From the state responses shown in Figures 4–9, one can find that the proposed fuzzy control method provided more smooth and rapid state responses compared with the design method of [29].

Moreover, the performance constraints of the actuator saturation are also satisfied for the proposed design method, which are presented in Figures 10–12. It is noted that the red dash line in Figures 4–12 shows the simulation results obtained by the design method of [29] with the saturation constraint. From these figures, it is known that the responses of each state obtained by the proposed design method are much smaller than those obtained by the method of [29]. Thus, the proposed design method can achieve stability and ensure system performance under the effects of multiplicative noises by the smaller control force. Additionally, the responses of both states and control inputs obtained by the design method of [29] deteriorate seriously if the actuator saturation is constrained in a smaller value. Thus, the IT2 T-S fuzzy controller obtained by [29] cannot maintain the system performance if it needs to satisfy the smaller actuator saturation. From the simulation results presented in Figures 10–12, responses of all control inputs obtained by the proposed design method have smoother convergence. However, the control inputs provided by the design method of [29] with the actuator saturation are oscillating rapidly. The control inputs provided by the design method of [29] have violent oscillations due to the white noise $\omega(t)$. Under the more significant effects of the white noise $\omega(t)$, the proposed fuzzy control method also can provide more rational control inputs than the design method of [29]. To present the above results, the ship trajectory is shown in Figure 13. In Figure 13, the ship is controlled to sail from the position (30 m, 30 m) to the origin. From Figure 13, it can be found that the proposed fuzzy controller design method provided a more rational and smoother trajectory than the design method of [29] for the ship steering systems. Thus, the proposed design method provided a more proper fuzzy controller design approach for the ship steering systems, and the control inputs can also be suppressed in the smaller values.

The control purpose for the ship steering system is to let the ship heading to the desired position and correct the yaw angle faster, to avoid the collision of the ships maneuvering on the unpredictable ocean. Thus, the responses obtained by the proposed design method are even better in Figures 4–6. Moreover, considering the system limit of ships, the performance requirement of the actuator saturation is also ensured. Based on the previous statements, the proposed design method can provide a better fuzzy controller design method for the nonlinear ship steering system to ensure the system performance under the effects of the stochastic behavior and satisfy the actuator saturation based on the IT2 T-S fuzzy model.

Indeed, the control problem of the proposed design method to a ship steering system is just a simulation case study. It is not a practical experiment. Hence, implementing a continuous actuation that generates a large control signal in an infinitesimally short time was not considered in this paper. The continuous actuation investigated by the proposed design method that produces a large control signal in an infinitesimally short time may need some electromechanical conversion devices in practical applications. However, the practical implementation is not the primary purpose of this paper. The main target of this paper is to investigate a useful and valuable theoretical development for the design of actuator saturated fuzzy controller for interval type-2 Takagi-Sugeno fuzzy models with multiplicative noises. Referring to [48–50], it can also be found that the problem of implementation of a continuous actuation that generates a large control signal in an infinitesimally short time for the ship steering systems was also not considered. Hence, the contribution of this paper is a theoretical development, not an experimental development. The experimental development considering the practical implementation of the proposed fuzzy control method for the ship steering systems can be investigated in the future.

5. Conclusions

In this paper, a fuzzy controller design method subject to actuator saturation was proposed for the nonlinear systems with parameter uncertainties based on the IT2 T-S fuzzy model. Firstly, the IT2 T-S fuzzy model with multiplicative noises was used to represent the nonlinear systems. To achieve the performance requirement of actuator saturation, the related constraint was also combined into the stability analysis and synthesis. Based on the Lyapunov stability theory, some sufficient conditions were derived to achieve stability

in the mean square and actuator saturation. By applying the proposed fuzzy controller design method, the simulation results of a practical nonlinear ship steering system were presented. For different ship systems, actuator saturation constraints can be adjusted for the fuzzy controller design method proposed in this paper. Therefore, the most cost-effective controller can be obtained to achieve the performance requirement. The proposed design method can provide better responses for the considered practical nonlinear ship steering system with lower feedback gains and satisfying the actuator saturation constraint. In the future, more performance requirements can be considered into the fuzzy controller design method based on the IT2 T-S fuzzy model.

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