

Article

Vertical Discretization Analysis of Tunnel Face Stability in Deep Tunnels

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Abstract

Existing methods for assessing the stability of deep tunnel face rarely account for the weakening effect of rock mass parameters caused by excavation disturbance. This paper employs a vertical discretization method to divide the rigid failure body into vertical strip elements with fixed horizontal widths. By considering the weakening effect of rock mass parameters, a stability analysis model for the tunnel face is established. The equivalent cohesion and internal friction angle of the rock mass are obtained using the Hoek–Brown criterion and the equivalent Mohr–Coulomb transformation. Combined with the disturbance weakening factor, these yield the equivalent rock mass parameters after disturbance. Stability is solved using limit analysis and the principle of virtual power. The accuracy of the established model is verified through numerical simulation, demonstrating that the proposed analytical approach requires only about 90 s per run compared to approximately 7 h for 3D numerical models. The results indicate that the importance of parameters, in descending order under the specified reference conditions for deep-buried tunnels, is $GSI > D_r > h_1 > m_i$, where GSI play a dominant role. Excavation disturbance significantly reduces rock mass strength numerically. Assessing GSI and controlling the blasting disturbance are key to ensuring the stability of deep tunnels.

Keywords: limit analysis method; deep tunnel; face stability; discretization techniques

1. Introduction

As China's transportation infrastructure expands into high-altitude, high-stress regions in the west, deep tunnel has become a core component of tunnel construction projects. The drill-and-blast method, as the primary construction technique for rock excavation in underground structures, offers significant advantages such as strong adaptability and low excavation costs, and is therefore widely used in the construction of deep tunnel. Under the disturbance caused by drilling and blasting, deep tunnel subjected to high rock stress undergo significant stress adjustments. Their stability must be ensured by applying support force to the tunnel face. If the support force is insufficient or excessive, it can easily trigger engineering disasters such as rock bursts, splitting, and collapses, as shown in Figure 1 [1,2]. Therefore, quantitatively assessing the stability of the tunnel face under deep-buried conditions is crucial for ensuring construction safety. Furthermore, in deep-buried, high-stress environments, the geotechnical properties and failure mechanisms of rock masses exhibit significant complexity. The intense impact loads generated by drilling and blasting not only



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directly damage the excavation face, but also create damage zones within the surrounding rock, leading to a weakening of rock mass parameters [3]. This makes traditional evaluation models limited in reflecting the evolution of tunnel face instability caused by drilling and blasting [4].

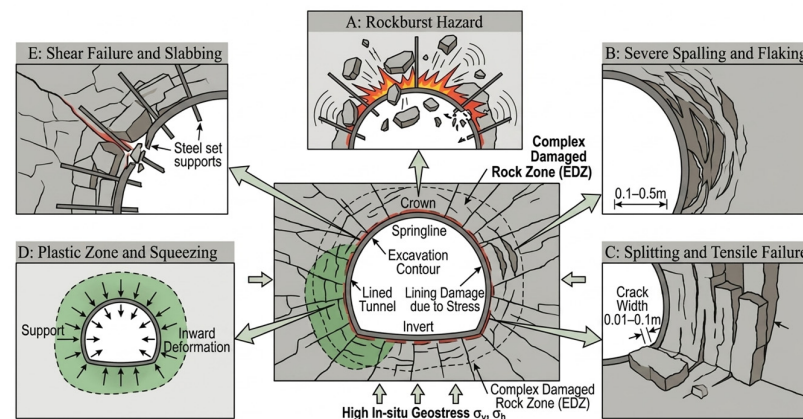


Figure 1. Integrated model of deep tunnel failure modes: (A) Explosive Rock burst. (B) Tangential Spalling. (C) Extensional Splitting. (D) Plastic Zone Squeezing. (E) Cross-cutting Shear Failure.

The limit analysis method is an effective approach used by many scholars to analyze tunnel face stability. Yang [5] applied the upper and lower bound theorems to solve the bearing capacity problem of circular foundations; by analyzing energy dissipation, they derived the upper bound solution for bearing capacity, and by constructing a static allowable stress field based on the principle of stress superposition, they obtained the lower bound solution, thereby significantly advancing research on bearing capacity analysis using the upper and lower bound theorems in China. Leca and Dormieux [6] developed a failure mechanism based on regular geometric shapes to explain the failure modes of tunnel working faces and obtained active and passive support forces from the perspective of kinematic limit analysis. Mollon et al. [7] proposed a three-dimensional rotational failure mechanism within the kinematic framework of limit analysis theory; they improved the original translational failure mechanism using spatial discretization techniques, and the method of generating slip surfaces point by point can be applied to spatially variable soil masses, opening up broad prospects for the application of limit analysis methods in analyzing tunnel stability. Pan and Dias [8] conducted seepage analysis of the entire tunnel face based on the discretization mechanism proposed by Mollon et al., established a tunnel face collapse model, and predicted the support force on the tunnel face using discretized planar triangular patches. Such studies not only demonstrate the effectiveness of limit analysis in assessing face stability, but also indicate that the discretized failure mechanism can more efficiently evaluate the ultimate support force of the tunnel face. However, while existing discretization mechanisms have significantly improved the precision of slip surfaces, they are computationally expensive and primarily limited to homogeneous soils. Furthermore, traditional limit analysis models typically treat the surrounding rock as an isotropic, homogeneous medium. Consequently, they neglect the spatial weakening of rock mechanical parameters within the excavation damage zone (EDZ) induced by drill-and-blast disturbances. Therefore, it is of great significance to develop an analytical model that preserves the computational advantages of discretization mechanisms while incorporating the effects of the excavation damage zone on face stability. Scholars have noted and studied the damage zones induced during the excavation of deep tunnel using the drill-and-blast method, as well as the resulting weakening of rock mass parameters. Read and Siren et al. [9,10] proposed a theoretical framework for the evolution of damage within the surrounding rock, while Malmgren et al. [11] quantitatively demonstrated through in situ

acoustic testing that the drill-and-blast method leads to a significant weakening of the elastic modulus of the surrounding rock. Perras and Diederichs [12] pointed out that under deep-buried conditions, traditional empirical formulas struggle to accurately characterize the secondary damage to rock mass caused by excavation. Existing research indicates that the strength of rock mass disturbed by drilling and blasting does not drop abruptly, but rather exhibits a gradual transition from severely damaged to weakly damaged and finally to undisturbed rock mass, depending on the distance from the excavation face to the tunnel face. Most existing limit analysis models treat the rock mass as completely undisturbed [13], or consider blasting but assign a constant, uniform disturbance parameter to the entire failure zone [14], which fails to account for the weakening effect of rock mass parameters caused by drilling and blasting. We note the method for quantifying rock mass parameter weakening proposed by Dai et al. [15], which introduces a weakening coefficient D_r to quantitatively characterize the degree of rock mass weakening. Combined with limit analysis methods, they proposed a vertical strip discretization method to flexibly handle the spatially non-uniform distribution of rock mass parameters. By calculating the parameters at different disturbance distances and assigning them as material properties in Abaqus, the weakening effect is reproduced. This approach overcomes, to some extent, the limitation of traditional upper-bound theories in handling parameter gradients, providing practical reference value and preliminary assessment for actual engineering construction. The relevant process is shown in Figure 2.

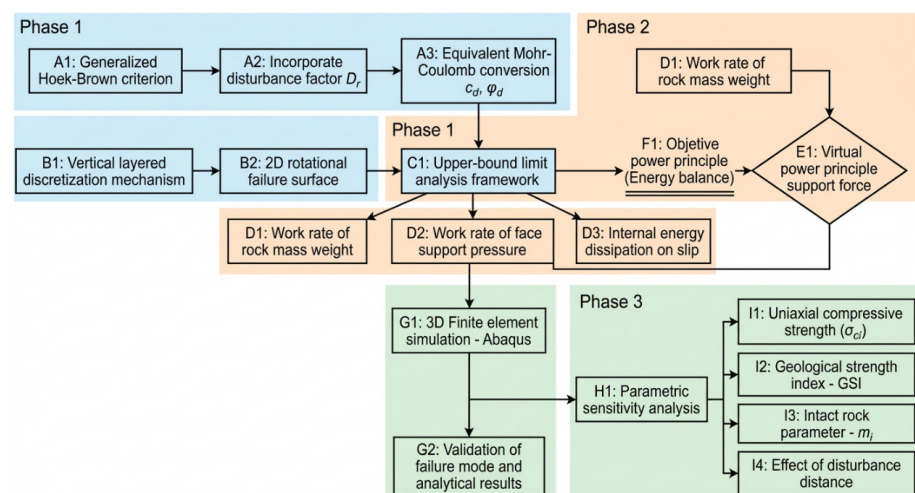


Figure 2. Vertical discrete analysis flowchart.

2. Theoretical Model and Basic Assumptions

2.1. Vertical-Strip Rotational Mechanism and Basic Assumptions

This section presents the proposed vertical-strip discretization framework for tunnel-face stability analysis. The analytical model is established under the following assumptions. First, the failure of the tunnel face is idealized as a two-dimensional rotational mechanism [16], which is mainly applicable to deep tunnels with approximately uniform longitudinal conditions and limited end effects. Second, the failure body is treated as a rigid block rotating with an angular velocity of ω about a rotation center $O(x_0, y_0)$ located behind the tunnel face [17]. Third, the failure body is discretized into equal-width vertical strips, and no relative motion is allowed between adjacent strips; meanwhile, static compatibility between strips is assumed. Fourth, the present formulation is established for dry or well-drained rock masses, and the effects of pore-water pressure and seepage force are not considered.

In the adopted coordinate system, the tunnel face is located at $x = 0$, with the x -axis pointing along the excavation direction and the y -axis pointing vertically upward. The bottom and top points of the tunnel face are denoted by $B(0,0)$ and $A(0,D)$, respectively, where D is the tunnel diameter. The potential failure surface is defined by a discrete series of points extending from the crown point A and the invert point B , and terminates at point E , as shown in Figure 3. For convenience in computation, each strip element is idealized as a right-angled trapezoid with width Δx .

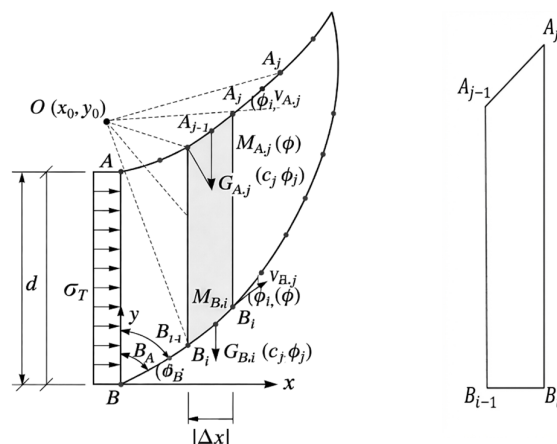


Figure 3. Vertical layered discretization analysis model for the stability of deep tunnel face.

2.2. Equivalent Conversion of Rock Mass Strength Parameters

The Hoek–Brown criterion accurately characterizes the nonlinear strength curve. Parameters such as σ_{ci} and m_i , GSI , D are utilized to calculate rock parameters including m_b , s and a , which effectively describe the nonlinear strength envelope of jointed rock masses. In contrast, the Mohr–Coulomb criterion yields a straight line in the shear stress–normal stress plane, making it more suitable for practical engineering applications. Since most limit analysis frameworks are formulated using Mohr–Coulomb parameters, namely cohesion c and internal friction angle φ . In this paper, based on the generalized H-B criterion, the concept of an equivalent M-C criterion is introduced. The H-B envelope is approximated as a straight line, whose intercept and slope correspond to the equivalent cohesion c and internal friction angle φ .

The expression for the generalized H-B rule in principal stress space is

$$\sigma'_1 = \sigma'_3 + \sigma_{ci} \left(m_b \frac{\sigma'_3}{\sigma_{ci}} + s \right)^a \quad (1)$$

Let $\sigma'_3 = 0$; then, the uniaxial compressive strength of the rock mass, σ_{cm} can be expressed as

$$\sigma_{cm} = \sigma_{ci} s^a \quad (2)$$

As noted by Hoek (1983) [18], the uniaxial tensile strength is equivalent to the biaxial tensile strength. By setting $\sigma'_1 = \sigma'_3 = \sigma_{tm}$ in the preceding equation, the uniaxial tensile strength σ_{tm} of the rock mass can be derived as follows:

$$\sigma_{tm} = \frac{s\sigma_{ci}}{m_b} \quad (3)$$

The Mohr–Coulomb criterion is expressed in the shear stress–normal stress plane as

$$\tau_n = c + \sigma_n \tan \varphi \quad (4)$$

Based on Equations (3) and (4), the above expression can be written as

$$\sigma'_1 = \sigma'_3 \frac{1 + \sin \varphi}{1 - \sin \varphi} + \frac{2c \cdot \cos \varphi}{1 - \sin \varphi} \quad (5)$$

The fundamental idea of the equivalent conversion is to use an M-C line to fit as closely as possible to the H-B curve within an appropriate confining force range $[\sigma_3^{\min}, \sigma_3^{\max}]$. Hoek et al. [19] pointed out that, compared to the instantaneous equivalent method that draws a tangent at a single point, it is more reasonable to use an average linear fit. Through extensive numerical studies, Hoek et al. [19] proposed an empirical formula for the upper confining force $\sigma_3^{\max}$ applicable to tunnel problems:

$$\sigma_3^{\max} = 0.47 \sigma_{cm} \left(\frac{\sigma_{cm}}{\gamma H} \right) \quad (6)$$

According to the Hoek–Brown criterion, the rock strength parameters (m_b , s , a) after perturbation attenuation can be calculated using the Geological Strength Index GSI [20] and intact rock parameters m_i [21]:

$$\begin{cases} m_b = m_i \exp\left(\frac{GSI-100}{28-14D}\right) \\ s = \exp\left(\frac{GSI-100}{9-3D}\right) \\ a = \frac{1}{2} + \frac{1}{6} \left(e^{-GSI/15} - e^{-20/3} \right) \end{cases} \quad (7)$$

Hoek et al. [19] provided closed-form expressions for the equivalent c and φ obtained from a linear fit of the H-B curve:

$$c = \frac{\sigma_{ci}[(1+2a)s + (1-a)m_b\sigma_{3n}](s + m_b\sigma_{3n})^{a-1}}{(1+a)(2+a)\sqrt{1 + \left(6am_b(s + m_b\sigma_{3n})^{a-1}/(1+a)(2+a)\right)}} \quad (8)$$

$$\varphi = \sin^{-1} \left(\frac{3am_b(s + m_b\sigma_{3n})^{a-1}}{(1+a)(2+a) + 3am_b(s + m_b\sigma_{3n})^{a-1}} \right) \quad (9)$$

where

$$\sigma_{3n} = \frac{\sigma_3^{\max}}{\sigma_{ci}} \quad (10)$$

$$\sigma = \sigma_{ci} \frac{[m_b + 4s - a(m_b - 8s)](m_b/4 + s)^{a-1}}{2(a+1)(a+2)} \quad (11)$$

In the limit analysis of this paper, the cohesion c and the angle of internal friction φ are primarily used [22].

2.3. Weakening of Rock Mass Strength Parameters

In drill-and-blast construction of deep tunnels, the strength degradation of the tunnel face and surrounding rock mass is not uniformly distributed but exhibits distinct spatial characteristics. As demonstrated by Dai et al. [15], the shock wave energy generated by blasting attenuates rapidly with increasing propagation distance, resulting in varying degrees of crack propagation and structural damage in the surrounding rock. To accurately characterize this deterioration effect, this paper incorporates an Excavation Damage Zone (EDZ) sub-model. Under the combined influence of drilling and blasting loads and instantaneous stress redistribution, the surrounding rock can typically be divided into three main zones from the excavation boundary outwards, as shown in Figure 4:

1. **Highly Damaged Zone (HDZ):** The area immediately adjacent to the cutting face and the excavation contour line. Within this zone, the rock mass is subjected to high-force shock waves from the near-field of the blast, causing primary joints to open violently and accompanied by a large number of newly formed tangential cracks [23]. The rock mass structure is nearly fragmented, and strength degradation is most severe.
2. **Weakly Damaged Zone (WDZ):** Located outside the HDZ, where shock wave energy has diminished. Damage in this zone is primarily characterized by the expansion of existing fractures induced by stress redistribution. Rock strength is reduced but the rock mass retains a certain degree of self-stabilizing capacity.
3. **Undisturbed Rock Mass:** Beyond the excavation influence zone, the rock mass retains its natural mechanical properties.

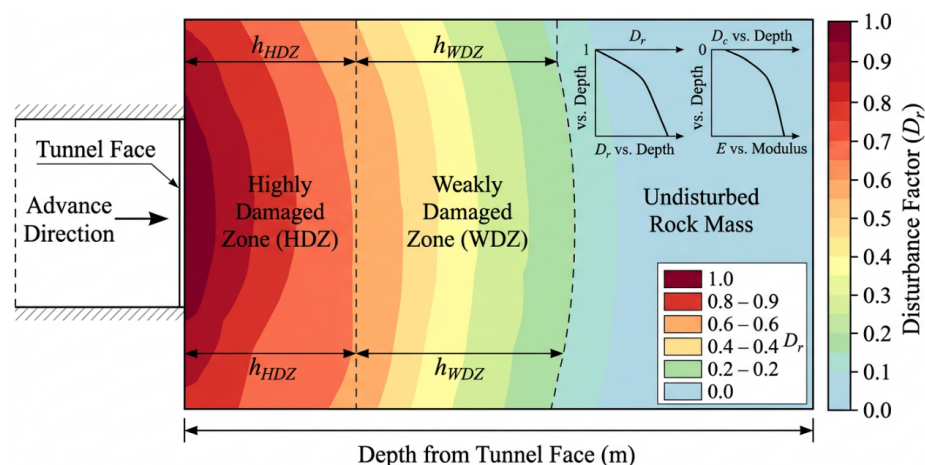


Figure 4. Distribution of excavation damage areas.

The aforementioned HDZ and WDZ are collectively referred to as the EDZ. In the subsequent stability analysis of this paper, the depth of the EDZ and the degree of strength decay within it are key factors affecting the stability of the tunnel face.

To quantitatively characterize the aforementioned weakening effect, this study employs a perturbation factor D_r to correct the rock mass strength parameters. Based on the improved scheme proposed by Dai et al., the perturbation factor is no longer a single empirical constant, but rather a piecewise function related to the excavation distance d , with the following mathematical expression:

$$D_r = \begin{cases} 1.0 & d \leq h_{HDZ} \\ 1.0 - \frac{d - h_{HDZ}}{h_{WDZ}} & h_{HDZ} < d < h_{EDZ} \\ 0 & d \geq h_{EDZ} \end{cases} \quad (12)$$

where d represents the distance from the excavation contour, h_{HDZ} is the distribution depth of the highly damaged zone (within which $D_r = 1$), h_{EDZ} is the distribution depth of the total damaged zone, and h_{WDZ} is the distribution depth of the weak damaged zone; the distribution depths of each zone satisfy $h_1 = h_{EDZ} = h_{WDZ} + h_{HDZ}$.

This functional model clearly reflects the process by which the weakening effect of rock mass parameters gradually diminishes with increasing distance [24]. When the distance from the excavation contour exceeds the distribution depth of the total damage zone, D_r decreases to 0 and remains constant, indicating that the rock mass has returned to its original, undisturbed strength state, as shown in Figure 5. After introducing the disturbance factor D_r , the equivalent strength parameters c and ϕ for different disturbance zones can be obtained by combining Equation (8) through (12).

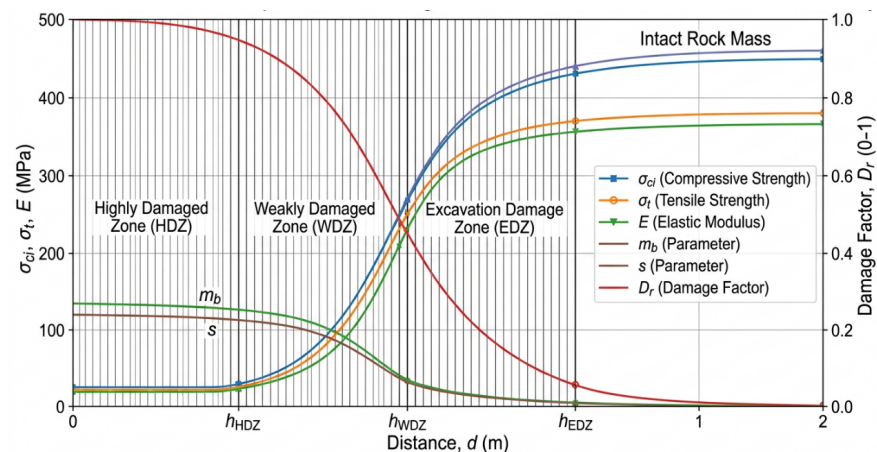


Figure 5. Variation in rock mass properties and damage factor with distance from excavation boundary.

Through the above Equations, the physical damage caused by drilling and blasting disturbances is converted into parameter weakening within the mechanical model.

The advantage of this approach lies in the fact that, in subsequent parameter sensitivity analyses, by altering the value of h_1 , one can quantitatively investigate the influence of changes in the disturbance range on the stability of the tunnel face [25].

To integrate this damage model into the proposed Mohr–Coulomb-based upper-bound limit analysis framework, the degraded parameters must be substituted into the equivalent conversion formulas in Section 2.2 (Equations (8)–(11)) to obtain the damage-equivalent cohesion c_{eq}^D and equivalent internal friction angle φ_{eq}^D . In subsequent upper-bound stability analysis calculations for the tunnel face, the shear strength parameters on the slip surface are updated using these degraded equivalent parameters (c_{eq}^D , φ_{eq}^D). This approach, which combines parameter equivalence with the weakening effect, retains the advantages of the Hoek–Brown criterion in describing the nonlinear characteristics of rock masses and excavation damage, while also meeting the computational requirements of limit analysis theory for linear yield criteria. Ultimately, it provides a more accurate representation of face instability mechanisms during drill-and-blast construction.

3. Working Equations

3.1. Self-Weight Power

Under the rigid-body rotation assumption, the velocity at any point can be expressed [26]. Since the rock mass is assumed to be anhydrous, the self-weight power can be obtained by summing the product of the strip area and the horizontal distance from the center of mass to the rotation axis for each strip. The calculation process is outlined below.

Consider the vertical strip at the position i , $x \in [x_i, x_{i+1}]$, and define the width:

$$\Delta x_i = x_{i+1} - x_i \quad (13)$$

Introduce the dimensionless parameter:

$$t = \frac{x - x_i}{\Delta x_i} \quad (14)$$

The upper boundary is derived from the sliding surface formula:

$$\begin{aligned}
 y(x) &= y_i + \frac{y_{i+1}-y_i}{x_{i+1}-x_i}(x-x_i) \\
 &= y_i + \frac{y_{i+1}-y_i}{\Delta x_i}(x-x_i) \\
 &= y_i + (y_{i+1}-y_i)t
 \end{aligned} \tag{15}$$

The area of the vertical trapezoidal element ΔG_i is defined as [27]

$$\begin{aligned}
 \Delta G_i &= \int_{x_i}^{x_{i+1}} y(x) dx \\
 &= \int_0^1 y(x(t)) \frac{dx}{dt} dt \\
 &= \Delta x_i \int_0^1 [y_i + (y_{i+1}-y_i)t] dt \\
 &= \frac{y_{i+1}+y_i}{2} \Delta x_i
 \end{aligned} \tag{16}$$

The coordinates of the centroid x_{Mi} are defined as

$$x_{Mi} = \frac{1}{\Delta G_i} \int_{x_i}^{x_{i+1}} xy(x) dx \tag{17}$$

Let

$$I_i = \int_{x_i}^{x_{i+1}} xy(x) dx \tag{18}$$

Similarly, using the substitution $x = x_i + t\Delta x_i$, $dx = \Delta x_i dt$ and substituting into Equation (18), we obtain

$$\begin{aligned}
 I_i &= \int_0^1 x(t)y(x(t)) \frac{dx}{dt} dt \\
 &= \int_0^1 [x_i + t\Delta x_i][y_i + (y_{i+1}-y_i)t] \Delta x_i dt
 \end{aligned} \tag{19}$$

Expand the integrand completely:

$$[x_i + t\Delta x_i][y_i + (y_{i+1}-y_i)t] = x_i y_i + x_i(y_{i+1}-y_i)t + y_i \Delta x_i t + (y_{i+1}-y_i)\Delta x_i t^2 \tag{20}$$

Thus,

$$\begin{aligned}
 I_i &= \Delta x_i \left[x_i y_i \cdot 1 + x_i(y_{i+1}-y_i) \cdot \frac{1}{2} + y_i \Delta x_i \cdot \frac{1}{2} + (y_{i+1}-y_i)\Delta x_i \cdot \frac{1}{3} \right] \\
 &= \Delta x_i x_i y_i + \frac{1}{2} \Delta x_i x_i (y_{i+1}-y_i) + \frac{1}{2} y_i \Delta x_i^2 + \frac{1}{3} (y_{i+1}-y_i) \Delta x_i^2
 \end{aligned} \tag{21}$$

Substitute Equations (18) and (21) into Equation (17):

$$\begin{aligned}
 x_{Mi} &= \frac{I_i}{\Delta G_i} \\
 &= \frac{\Delta x_i x_i y_i + \frac{1}{2} \Delta x_i x_i (y_{i+1}-y_i) + \frac{1}{2} y_i \Delta x_i^2 + \frac{1}{3} (y_{i+1}-y_i) \Delta x_i^2}{\frac{y_i+y_{i+1}}{2} \Delta x_i}
 \end{aligned} \tag{22}$$

Simplify by canceling out the common factors in both the numerator and denominator Δx_i :

$$x_{Mi} = \frac{x_i y_i + \frac{1}{2} x_i (y_{i+1}-y_i) + \frac{1}{2} y_i \Delta x_i + \frac{1}{3} (y_{i+1}-y_i) \Delta x_i}{\frac{y_i+y_{i+1}}{2}} \tag{23}$$

Then move the denominator to the numerator:

$$x_{Mi} = \frac{2}{y_i + y_{i+1}} \left[x_i y_i + \frac{1}{2} x_i (y_{i+1}-y_i) + \frac{1}{2} y_i \Delta x_i + \frac{1}{3} (y_{i+1}-y_i) \Delta x_i \right] \tag{24}$$

Expand and simplify the terms in the numerator:

$$x_i y_i + \frac{1}{2} x_i (y_{i+1}-y_i) = \frac{1}{2} x_i y_i + \frac{1}{2} x_i y_{i+1} \tag{25}$$

$$\frac{1}{2}y_i\Delta x_i + \frac{1}{3}(y_{i+1} - y_i)\Delta x_i = \Delta x_i \left(\frac{1}{6}y_i + \frac{1}{3}y_{i+1} \right) \quad (26)$$

Substitute Equations (25) and (26) into Equation (24):

$$\begin{aligned} x_{Mi} &= \frac{2}{y_i + y_{i+1}} \left[\frac{1}{2}x_i y_i + \frac{1}{2}x_i y_{i+1} + \Delta x_i \left(\frac{1}{6}y_i + \frac{1}{3}y_{i+1} \right) \right] \\ &= \frac{1}{y_i + y_{i+1}} \left[x_i y_i + x_i y_{i+1} + 2\Delta x_i \left(\frac{1}{6}y_i + \frac{1}{3}y_{i+1} \right) \right] \\ &= \frac{1}{y_i + y_{i+1}} \left[x_i y_i + x_i y_{i+1} + \Delta x_i \left(\frac{1}{3}y_i + \frac{2}{3}y_{i+1} \right) \right] \end{aligned} \quad (27)$$

Since

$$x_{i+1} = x_i + \Delta x_i, \quad (28)$$

Δx_i can be written as $(x_{i+1} - x_i)$, which is substituted into Equation (27):

$$\begin{aligned} x_{Mi} &= \frac{1}{y_i + y_{i+1}} \left[x_i y_i + x_i y_{i+1} + \frac{1}{3}x_{i+1} y_i + \frac{2}{3}x_{i+1} y_{i+1} - \frac{1}{3}x_i y_i - \frac{2}{3}x_i y_{i+1} \right] \\ &= \frac{1}{y_i + y_{i+1}} \left[\frac{2}{3}x_i y_i + \frac{1}{3}x_i y_{i+1} + \frac{1}{3}x_{i+1} y_i + \frac{2}{3}x_{i+1} y_{i+1} \right] \\ &= \frac{1}{3(y_i + y_{i+1})} [2x_i y_i + x_i y_{i+1} + x_{i+1} y_i + 2x_{i+1} y_{i+1}] \end{aligned} \quad (29)$$

We finally obtain the closed-form expression for the coordinates of the centroid of the vertical strip element x :

$$x_{Mi} = \frac{2x_i y_i + x_i y_{i+1} + x_{i+1} y_i + 2x_{i+1} y_{i+1}}{3(y_i + y_{i+1})} \quad (30)$$

Combining the above expressions to solve for the self-weight power of the rock mass, the self-weight of the vertical strip at i is

$$\Delta G_i \gamma = \gamma \Delta G_i \quad (31)$$

The vertical velocity at the center of mass is

$$v_{y,Mi} = \omega(x_{Mi} - x_0) \quad (32)$$

The external power due to the self-weight of this strip is defined as

$$\Delta P_{G,i} = \gamma \Delta G_i v_{y,Mi} = \gamma \omega \Delta G_i (x_{Mi} - x_0) \quad (33)$$

Substituting Equations (16) and (30) into Equation (33):

$$\begin{aligned} \Delta P_{G,i} &= \gamma \omega \left[\frac{y_i + y_{i+1}}{2} \Delta x_i \right] \left[\frac{2x_i y_i + x_i y_{i+1} + x_{i+1} y_i + 2x_{i+1} y_{i+1}}{3(y_i + y_{i+1})} - x_0 \right] \\ &= \gamma \omega \frac{x_{i+1} - x_i}{6} [2x_i y_i + x_i y_{i+1} + x_{i+1} y_i + 2x_{i+1} y_{i+1} - 3x_0(y_i + y_{i+1})] \end{aligned} \quad (34)$$

The total self-weight power is the sum of the self-weight powers of all vertical strip elements:

$$P_G = \sum_{i=0}^{N-1} \Delta P_{G,i} \quad (35)$$

It can also be expressed as

$$P_G = \gamma \omega \sum_{i=0}^{N-1} \Delta G_i (x_{Mi} - x_0) \quad (36)$$

3.2. Power of Support Force at the Face

Calculate the power of the support force applied to the working face. Let the applied support force σ_T act on the working face $x = 0$. Since the position of the support force is a defined straight line, the power can be obtained by integration. For any point $P(x_P, y_P)$ on the working face, the vector $\vec{OP} = (x_P - x_0, y_P - y_0)$, and the polar angle θ defined as the angle between the $\vec{OP}$ and x axes, then B, A correspond respectively to

$$\begin{aligned}\theta_B &= (0 - x_0, 0 - y_0) = (-x_0, -y_0) \\ \theta_A &= (0 - x_0, d - y_0) = (-x_0, d - y_0)\end{aligned}\quad (37)$$

Therefore, a point on the palm surface can be represented by the angle parameter $\theta \in [\theta_B, \theta_A]$ as

$$P(\theta) = \begin{cases} x(\theta) = 0 \\ y(\theta) = y_0 - x_0 \tan \theta \end{cases} \quad (38)$$

When a rigid body rotates about point O with angular velocity ω , the velocity of any point $P(x, y)$ is

$$\vec{v}(x, y) = \omega[-(y - y_0), (x - x_0)] \quad (39)$$

On the working face $x = 0$; substituting this into the above equation yields

$$\begin{aligned}v_x(\theta) &= \omega x_0 \tan \theta \\ v_y(\theta) &= \omega(0 - x_0) = -\omega x_0\end{aligned}\quad (40)$$

Integrating along the line containing the working face from B to A yields the total external power exerted by the support on the rigid body:

$$\begin{aligned}W_T &= \int_{\theta_B}^{\theta_A} dW_T \\ &= -\sigma_T \omega x_0^2 \int_{\theta_B}^{\theta_A} \tan \theta \sec^2 \theta d\theta \\ &= -\sigma_T \omega x_0^2 \frac{\tan^2 \theta_A - \tan^2 \theta_B}{2} \\ &= \frac{\sigma_T \omega x_0^2}{2} (\tan^2 \theta_B - \tan^2 \theta_A)\end{aligned}\quad (41)$$

Since $\tan^2 \theta_A > \tan^2 \theta_B$, it follows that $W_T < 0$, which aligns with the physical significance of applying support force to prevent face collapse and resist relative movement of the rock mass, thereby performing negative work. According to the principle of upper bounds in the analysis of work done by external forces,

$$P_G = P_D + P_T \quad (42)$$

The power of the support force is typically considered positive, representing one of the external energy losses. For convenience, we define it as

$$P_T = -W_T \quad (43)$$

From Equation (41), we obtain

$$\begin{aligned}P_T &= \frac{\sigma_T \omega x_0^2}{2} (\tan^2 \theta_A - \tan^2 \theta_B) \\ &= \frac{\sigma_T \omega x_0^2}{2} \left[\frac{(d - y_0)^2}{x_0^2} - \frac{y_0^2}{x_0^2} \right] \\ &= \frac{\sigma_T \omega}{2} (d^2 - 2dy_0)\end{aligned}\quad (44)$$

Since numerical simulations typically assume $d = 10$ m, substituting into (44) yields

$$P_T = \sigma_T \omega (50 - 10y_0) \quad (45)$$

3.3. Energy Dissipation at the Slip Surface

In the upper-bound analysis, plastic dissipation occurs only at the slip surface. Under operating conditions, the soil slides along the slip surface. The magnitude of the shear force per unit length on the slip surface (shear strength) is τ , and the relative tangential sliding velocity of the soil on both sides of the slip surface is v_s . Therefore, the power dissipated per unit time along the slip surface length ds at that point is

$$dP_D = \tau(s)v_s(s)ds \quad (46)$$

$$P_D = \int \tau(s)v_s(s)ds \quad (47)$$

As previously discussed, assuming the slip surface consists of two broken lines: one passing through the set of points $\{A_0, A_1, \dots, A_m\}$, forming the line segment $\overline{A_{j-1}A_j} = 1, \dots, m$; and the other passing through the set of points $\{B_0, B_1, \dots, B_n\}$, forming the line segment $\overline{B_{i-1}B_i} = 1, \dots, n$. Each unit segment undergoes shear slip, contributing a portion of the dissipated power. The rigid body rotates about $O(x_0, y_0)$ with an angular velocity of ω . The vector from any point $P(x, y)$ relative to O is

$$\vec{OP} = (x - x_0, y - y_0) \quad (48)$$

Then the rotational velocity of the rigid body is

$$\vec{v}(P) = \omega[-(y - y_0), (x - x_0)] \quad (49)$$

The direction is tangential and perpendicular to $\vec{OP}$. Taking a line segment $A_{j-1}A_j$ as an example, the coordinates of its endpoints are

$$A_{j-1}(x_{A,j-1}, y_{A,j-1}), A_j(x_{A,j}, y_{A,j}) \quad (50)$$

Segment length:

$$|A_{j-1}A_j| = \sqrt{(x_{A,j} - x_{A,j-1})^2 + (y_{A,j} - y_{A,j-1})^2} \quad (51)$$

The coordinates of the midpoint $M_{A,j}(x_{MA,j}, y_{MA,j})$ are

$$x_{MA,j} = \frac{x_{A,j-1} + x_{A,j}}{2}, y_{MA,j} = \frac{y_{A,j-1} + y_{A,j}}{2} \quad (52)$$

Assuming the length of the discretized segment is sufficiently small, the variation in shear slip velocity along it is negligible. Consequently, the velocity at the midpoint can represent the slip velocity of the entire segment. The radial distance from the center of rotation to this midpoint is given by

$$r_j = \sqrt{(x_{MA,j} - x_0)^2 + (y_{MA,j} - y_0)^2} \quad (53)$$

The tangential sliding velocity at the midpoint is

$$v_{s,A,j} = \omega r_j \cos \varphi_j \quad (54)$$

Similarly, for the other slip line $\overline{B_{i-1}B_i}$:

$$v_{s,B,i} = \omega r_i \cos \varphi_i \quad (55)$$

In upper-bound limit analysis considering a purely cohesive rock mass, the shear stress along the slip surface is typically equated to its ultimate cohesion c . Accordingly, the shear stresses acting on segments $\overline{A_{j-1}A_j}$ and $\overline{B_{i-1}B_i}$ are defined as

$$\tau_{A,j} = c_j, \tau_{B,i} = c_i \quad (56)$$

For the first segment j , $\overline{A_{j-1}A_j}$:

$$\begin{aligned} \Delta P_{D,A,j} &= \tau_{A,j} \cdot v_{s,A,j} \cdot |A_{j-1}A_j| \\ &= c_j \cdot (\omega r_j \cos \varphi_j) \cdot |A_{j-1}A_j| \\ &= \omega c_j |A_{j-1}A_j| r_j \cos \varphi_j \end{aligned} \quad (57)$$

Similarly, for the second slip line $\overline{B_{i-1}B_i}$, its dissipated power is

$$\Delta P_{D,B,i} = \omega c_i |B_{i-1}B_i| r_i \cos \varphi_i \quad (58)$$

The total dissipated power at the slip surface is the sum of the power for each segment:

$$\begin{aligned} P_D &= \sum_{j=1}^m \Delta P_{D,A,j} + \sum_{i=1}^n \Delta P_{D,B,i} \\ &= \sum_{j=1}^m \omega c_j |A_{j-1}A_j| r_j \cos \varphi_j + \sum_{i=1}^n \omega c_i |B_{i-1}B_i| r_i \cos \varphi_i \end{aligned} \quad (59)$$

Rearranging yields

$$P_D = \omega \sum_{j=1}^m c_j |A_{j-1}A_j| r_j \cos \varphi_j + \omega \sum_{i=1}^n c_i |B_{i-1}B_i| r_i \cos \varphi_i \quad (60)$$

In particular, r_i, r_j represents the distance from the midpoint of each segment to the center of rotation $O(x_0, y_0)$:

$$\begin{aligned} r_i &= \sqrt{\left(\frac{x_{B,i-1} + x_{B,i}}{2} - x_0\right)^2 + \left(\frac{y_{B,i-1} + y_{B,i}}{2} - y_0\right)^2} \\ r_j &= \sqrt{\left(\frac{x_{A,j-1} + x_{A,j}}{2} - x_0\right)^2 + \left(\frac{y_{A,j-1} + y_{A,j}}{2} - y_0\right)^2} \end{aligned} \quad (61)$$

In summary,

$$\begin{cases} P_G = \gamma \omega \sum_{i=0}^{N-1} \Delta G_i (x_{Mi} - x_0) \\ P_T = \frac{\sigma_T \omega x_0^2}{2} (\tan^2 \theta_A - \tan^2 \theta_B) \\ P_D = \omega \sum_{j=1}^m c_j |A_{j-1}A_j| r_j \cos \varphi_j + \omega \sum_{i=1}^n c_i |B_{i-1}B_i| r_i \cos \varphi_i \end{cases} \quad (62)$$

Substituting into the energy balance Equation (42) yields

$$\begin{aligned} &\gamma \omega \sum_{i=0}^{N-1} \Delta G_i (x_{Mi} - x_0) \\ &= \omega \sum_{j=1}^m c_j |A_{j-1}A_j| r_j \cos \varphi_j + \omega \sum_{i=1}^n c_i |B_{i-1}B_i| r_i \cos \varphi_i + \frac{\sigma_T \omega x_0^2}{2} (\tan^2 \theta_A - \tan^2 \theta_B) \end{aligned} \quad (63)$$

Further solving Equation (63) yields

$$\sigma_T = \frac{\gamma \sum_{i=0}^{N-1} \Delta G_i (x_{Mi} - x_0) - \sum_{j=1}^m c_j |A_{j-1} A_j| r_j \cos \varphi_j - \sum_{i=1}^n c_i |B_{i-1} B_i| r_i \cos \varphi_i}{\frac{x_0^2}{2} (\tan^2 \theta_A - \tan^2 \theta_B)} \quad (64)$$

For a homogeneous rock-soil mass with uniform strength parameters, Equation (63) can be simplified to

$$\sigma_T = \frac{\gamma \sum_{i=0}^{N-1} \Delta G_i (x_{Mi} - x_0) - c \cos \varphi \left(\sum_{j=1}^m |A_{j-1} A_j| r_j \sum_{i=1}^n |B_{i-1} B_i| r_i \right)}{\frac{x_0^2}{2} (\tan^2 \theta_A - \tan^2 \theta_B)} \quad (65)$$

In this study, a computational program was developed using MATLAB (R2018a) to optimize the objective function described above. The aim is to identify the location of the most critical slip surface and the corresponding upper bound of the ultimate support force. Ultimately, under the geometric constraints of Equation (67), the objective function can be expressed as

$$\sigma_T = \max[\sigma_T | P_G = P_D + P_T] \quad (66)$$

$$s.t. \begin{cases} 0 < \theta_B < \frac{\pi}{2} \\ \theta_B < \theta_A < \frac{\pi}{2} \end{cases} \quad (67)$$

3.4. Convergence Assessment of the Vertical Strip Discretization

Because the proposed analytical model is established based on the vertical strip discretization method, a convergence assessment is strictly required to investigate the effect of strip density on computational accuracy. A baseline analytical case (tunnel burial depth $C = 30$ m, tunnel diameter $D = 10$ m, rock mass density $\gamma = 20$ kN/m³, uniaxial compressive strength $\sigma_{ci} = 2$ MPa, disturbance factor $D_r = 0.5$, Geological Strength Index $GSI = 20$, Hoek–Brown material constant $m_i = 10$, yielding an ultimate support force of 60.80 kPa) is selected to demonstrate how the predicted result varies with different vertical strip widths (Δx). The calculation results for convergence assessment are summarized in Table 1. As presented in Table 1, a larger vertical strip width (Δx) leads to a greater relative error compared to the continuous solution. As Δx decreases, the calculated ultimate support force converges rapidly. Because the MATLAB algorithm employed in this study directly solves the continuous analytical equations (equivalent to $\Delta x \rightarrow 0$), the reported results are theoretically converged, thereby avoiding mesh-dependency errors.

Table 1. Convergence assessment of the Ultimate Support Force with varying vertical strip widths.

Strip Width Δx (m)	σ_T (kPa)	Relative Difference
1.0	65.36	7.50%
0.5	63.17	3.90%
0.1	61.35	0.90%
Continuous Limit ($\Delta x \rightarrow 0$)	60.80	-(Reference)

4. Numerical Simulation Verification

To further verify the accuracy of the proposed upper-bound analysis model, which relies on vertical strip discretization and accounts for the weakening effect of excavation disturbance, we conducted three-dimensional [28,29] numerical simulations using the finite element software Abaqus 6.14-4. The simulations aimed to evaluate the stability of the tunnel face in a deep tunnel and compare the simulation results and failure modes with the theoretical analytical results presented in this paper. The model configuration is shown in Figure 6. Based on existing methods, we can validate the results by comparing them with

those from two-dimensional rotational failure mechanism analyses using the finite element software Abaqus. Pan et al. [8] and Man et al. [30,31] have demonstrated that finite element software is suitable for calculating the stability of tunnel face conditions.

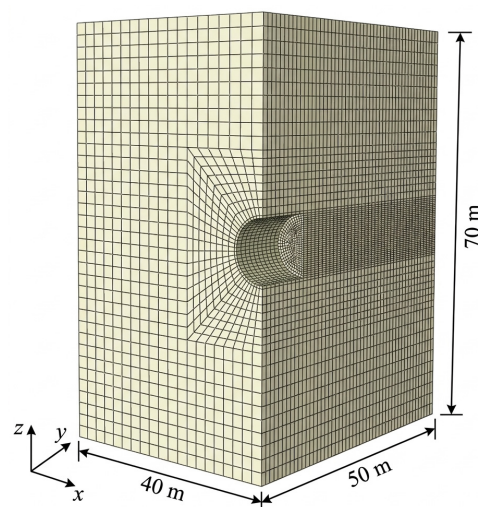


Figure 6. Numerical model.

The model employs C3D8R hexahedral reduced-integration elements for meshing, with local mesh refinement applied to the tunnel face and the excavation disturbance zone [32]. To establish boundary-independence, the numerical model dimensions are sized as $40\text{ m} \times 50\text{ m} \times 70\text{ m}$ ($x \times y \times z$), ensuring that the lateral and bottom boundaries are located at least $3D$ to $4D$ away from the tunnel center. This effectively eliminates the influence of boundary constraints on mechanical behavior. Furthermore, the number of grid cells at this stage is sufficient in terms of accuracy to analyze the specified operating conditions [31]. The model boundaries are constrained by normal displacement, with the bottom fully fixed. The tunnel has a diameter of $D = 10\text{ m}$ and a burial depth of $C = 30\text{ m}$. The rock mass is modeled using an ideal elastic-plastic constitutive model following the Mohr–Coulomb yield criterion. A shotcrete lining, simulated as elastic shell elements with a thickness of 0.2 m , is applied to the excavated sections. During the simulation, the lining advances with the excavation, leaving only the cutting face unsupported or under a preset support force, which realistically mimics the dynamic excavation-support cycle.

To account for the disturbance effects caused by drilling and blasting, the Abaqus model employs a method of assigning different weakening parameters to each layer after excavation. Based on the calculated equivalent parameters (c , φ , E , etc.) corresponding to varying disturbance factors D_r , the rock mass ahead of the tunnel face was geometrically divided into several 0.5-m -thick vertical slices. For each layer, uniform initial parameters are used prior to excavation, which are gradually replaced with calculated weakened parameters after excavation to reflect the parameter weakening effect caused by the drill-and-blast method.

Finally, the excavation and support loading sequence is systematically executed in four steps: (1) Initial stress balance: Gravity is applied and run until convergence. (2) Excavation: The rock elements inside the tunnel are deactivated. (3) Support application: A uniform virtual support force is applied precisely onto the tunnel face. (4) Stepwise reduction: The applied support force is gradually reduced while monitoring the horizontal displacement at the center of the tunnel face. The ultimate limit state is definitively identified when the horizontal displacement experiences a sudden macroscopic mutation without numerical convergence, indicating complete tunnel face failure. The support force at this exact mutation point is recorded as the ultimate support pressure.

Furthermore, the morphology of the plastic slip zone in front of the tunnel face revealed by the numerical simulation closely matches the failure surface contour constructed by the vertical discretization mechanism in this study, further validating the rationality of the theoretical model, as shown in Figure 7. As Çetindemir [33] emphasizes, although no single constitutive model can capture all the complexities of geotechnical material behavior, the Mohr–Coulomb model remains the most universal and fundamental model for understanding soil–structure interaction. Based on this concept, the analytical framework and numerical model in this study represent different conceptual stages of the same physical process. The analytical framework reveals the physical mechanism of damage induced by excavation through a transition from the Hoek–Brown criterion to the equivalent Mohr–Coulomb criterion with a weakening effect, and assigns the weakened material properties to the corresponding disturbed zones in the idealized elastoplastic model. Therefore, the agreement between the analytical and numerical results is primarily attributed to the mechanical mechanisms.

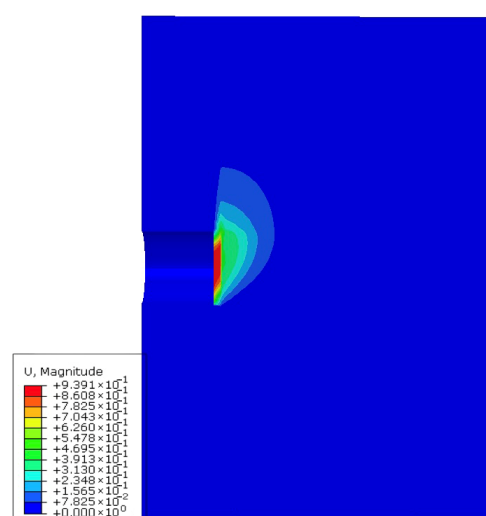


Figure 7. Numerical simulation displacement contour plots.

Figure 8 shows the curve of horizontal displacement at the center of the working face as a function of applied support force, derived from numerical simulations. As depicted in Figure 8, during the initial stage when the support force is gradually reduced, the deformation of the working face accumulates slowly [34]. At this point, the surrounding rock is primarily in the elastic deformation stage, and the working face remains stable. However, when the support force decreases to a certain critical value, a distinct inflection point appears in the curve. The minimum virtual support force can be defined as the value at which the horizontal displacement suddenly increases significantly [8,30]. Based on this, the critical value is identified as 61.2 kPa, which closely matches the theoretical solution of 60.8 kPa obtained using the vertical strip discretization theoretical model adopted in this study. This simulation result not only intuitively reflects the mechanical process of face instability but also demonstrates the accuracy of the method proposed in this study in predicting the ultimate support force.

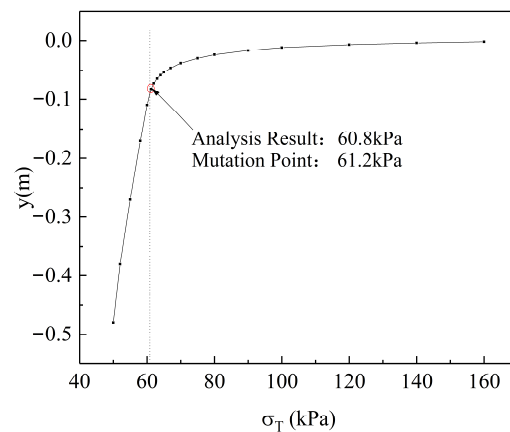


Figure 8. Comparison of numerical simulation and analytical results.

To fundamentally verify the mathematical accuracy of the proposed vertical discretization framework, the present model was degenerated and quantitatively compared with the classical published benchmark documented by Senent et al. (2013) [35]. By excluding the excavation disturbance function and transforming the Hoek–Brown criterion into an equivalent homogeneous Mohr–Coulomb medium, the geometric and geotechnical parameters were set identically to the benchmark: tunnel cover depth $C = 20$ m, diameter $D = 10$ m, Uniaxial compressive strength of rock $\sigma_{ci} = 1$ MPa, Geological Strength Index $GSI = 10$, Hoek–Brown material constant $m_i = 5$, and unit weight $\gamma = 25$ kN/m³. Under these specific conditions, the ultimate support force calculated by our MATLAB program is 48.8 kPa. This value is in close agreement with the theoretical upper limit of 49.5 kPa calculated using the model proposed [35]. This detailed quantitative comparison rigorously validates the intrinsic computational reliability of the proposed theoretical search algorithm before introducing complex disturbance effects.

In terms of computational efficiency, the vertical discretization analytical model developed in this study demonstrates significant advantages. For the deep tunnel case analyzed in Figure 8, the numerical simulation in the traditional 3D finite element software Abaqus took approximately 7 h, whereas the analysis model written in MATLAB took only about 90 s. Both simulations were conducted on a PC equipped with an Intel Core i7–12700 CPU and 32 GB of RAM, demonstrating that the proposed analytical solution method is significantly more efficient.

5. Parameter Sensitivity Analysis

As mentioned above, based on the theoretical model and optimization algorithm established earlier for tunnel excavation, this section performs a parameter sensitivity analysis on the ultimate support force at the tunnel face of a deep tunnel. The baseline settings are as follows: $C = 30$ m, $d = 10$ m, $\gamma = 20$ kN/m³, $GSI = 20$, $D_r = 0.5$, $m_i = 10$.

Figure 9a illustrates the effect of the disturbance factor D_r for drill-and-blast excavation on the ultimate support force under different uniaxial compressive strengths σ_{ci} . As can be clearly seen from the above figure, σ_T exhibits a significant nonlinear positive correlation with D_r ; that is, the more severe the disturbance, the poorer the stability of the tunnel working face. Taking $\sigma_{ci} = 3000$ kPa as an example, when the operating conditions deteriorate from undisturbed ($D_r = 0$) to severely disturbed ($D_r = 1.0$), the required support force increases from 34.5 kPa to 66.7 kPa. The mechanical mechanism behind this phenomenon lies in the fact that stress unloading and dynamic impact caused by drilling and blasting excavation disrupt the original structural integrity of the surrounding rock, leading to a significant reduction in the equivalent cohesion on the slip surface. To compensate for the loss of internal dissipated power and maintain the balance of the

virtual work equations, higher active support forces must be applied to the tunnel face. This principle quantitatively highlights the importance of controlling blasting quality and minimizing rock mass damage during the construction of a deep tunnel. Figure 9b illustrates the influence of the geological strength index GSI . This provides a macroscopic characterization of the degree of joint development and surface weathering in the rock mass. Calculations indicate that, for the same strength, the smaller the value of GSI , the greater the support force required for the tunnel, and the slope of the curve increases sharply as GSI decreases. When GSI decreases from 35 to 15, the ultimate support force exhibits a trend of exponential increase. This indicates that when traversing fault zones or extremely weak rock formations, the surrounding rock is highly prone to losing its self-stabilizing capacity and is at high risk of collapse, necessitating the advance installation of high-strength support measures. Conversely, when the rock mass quality is good, the ultimate support force approaches 0. This indicates that under such geological conditions, the internal forces within the rock mass are sufficient to resist the weight of the rock, and the face possesses good self-stabilization capacity; in actual construction, support can be reduced as appropriate. Figure 9c illustrates the effect of the intact rock material constant m_i in the Hoek–Brown criterion on the support force. The m_i depends on the mineral composition of the rock and macroscopically reflects the rock's hardness and its internal friction characteristics. The calculation results indicate that the ultimate support force increases as the value of m_i decreases. This is because smaller values of m_i typically correspond to weak, cohesive rocks such as mudstone, which experience a more severe reduction in equivalent shear strength after the same degree of excavation disturbance. Compared to D_r and GSI , the influence of m_i on support force is relatively milder. Figure 9d illustrates the nonlinear effect of the disturbance distance h_1 on support force. Considering the spatial extension characteristics of the excavation damage zone (EDZ) in actual drill-and-blast construction, the disturbance distance is defined as the thickness of the damage zone extending from the cutting face into the rock mass. The calculation results indicate that as h_1 increases from 0 m to 4 m, the ultimate support force exhibits a consistent upward trend. The mechanical mechanism underlying this is that an increase in damage depth expands the volume of the potential collapse mass in a weakened state. In the vertical discretization model, this implies that more discrete strip elements traverse the low-strength zone, causing the resistance power on the slip surface to decrease, which in turn increases the critical support requirement. This conclusion not only validates the necessity of adopting the equivalent nonlinear H-B criterion but also provides theoretical support for construction in high-stress zones of deep tunnels.

Figure 10 illustrates the effect of the disturbance factor D_r on the ultimate support force σ_T at the working face under different parameter combinations. As shown in the Figure, the ultimate support force σ_T exhibits a nonlinear increasing trend with an increase in the disturbance factor. A comparison of the sub-Figures reveals that when the rock mass strength parameters (σ_{ci} , GSI , m_i) are low, the overall position of the curve is higher, and the slope of the curve increases more steeply with an increase in the disturbance factor (e.g., $GSI = 15$ in Figure 10b). This indicates that poorer rock masses are more sensitive to blasting disturbances, and their actual support requirements increase significantly as the degree of disturbance increases. Therefore, when constructing in weak and fractured surrounding rock, strict control of blasting disturbances is crucial for maintaining the stability of the working face.

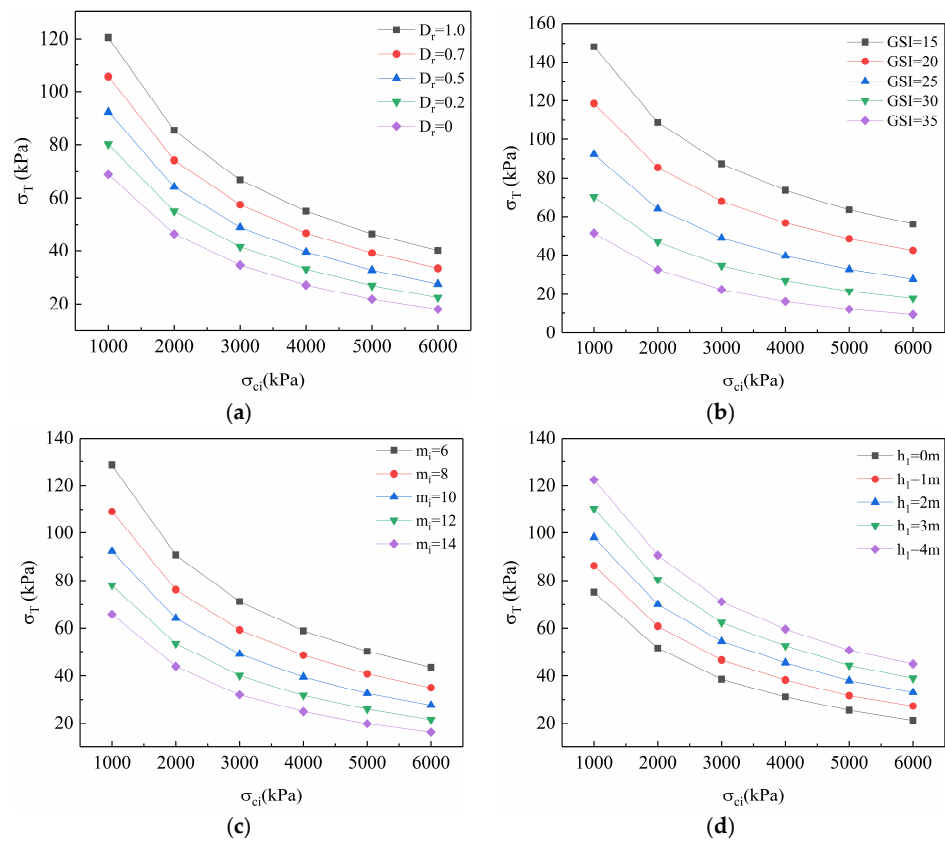


Figure 9. Effect of Rock Uniaxial Compressive Strength σ_{ci} on the Ultimate Support Force σ_T under different conditions in Drill-and-Blast Excavation. The variables are (a) D_r ; (b) GSI; (c) m_i ; (d) h_1 .

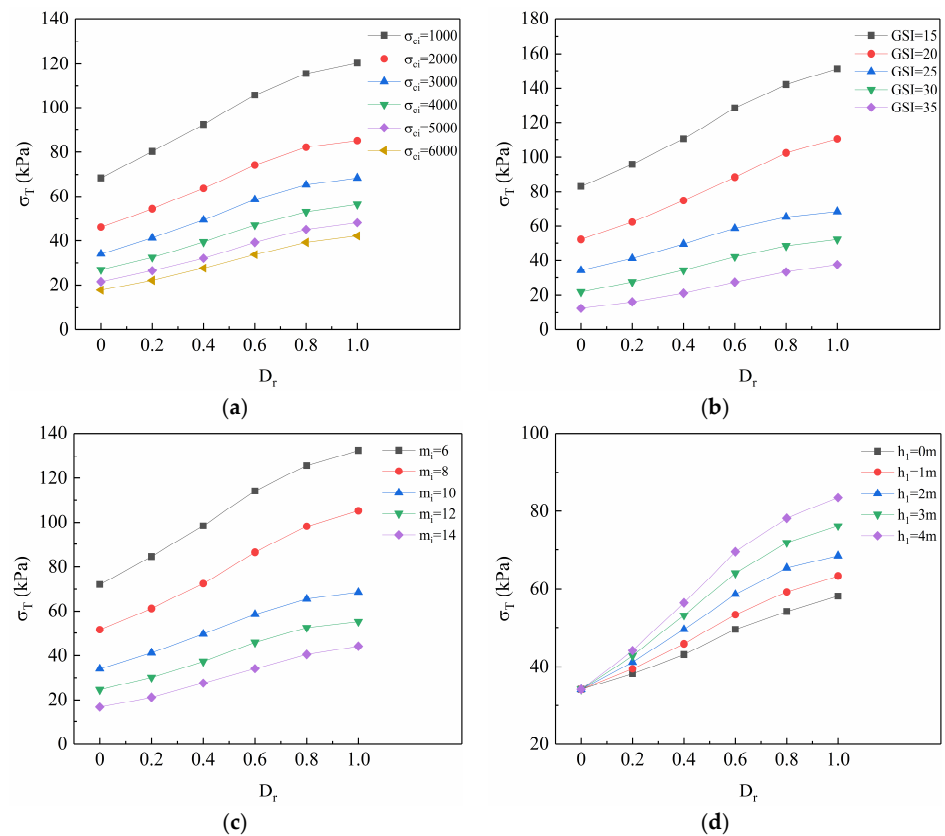


Figure 10. Effect of the Disturbance Factor D_r on the Ultimate Support Force σ_T under different conditions in Drill-and-Blast Excavation. The variables are (a) σ_{ci} ; (b) GSI; (c) m_i ; (d) h_1 .

Figure 11 illustrates the relationship between the geological strength index GSI and the ultimate support force. As GSI increases, σ_T exhibits a distinct nonlinear downward trend. As GSI increases, the curves gradually converge and flatten out, with σ_T approaching 0. This indicates that rock masses with good integrity essentially possess self-stabilizing capacity and are less affected by other parameters, whereas fractured rock masses with well-developed joints pose an extremely high risk of instability.

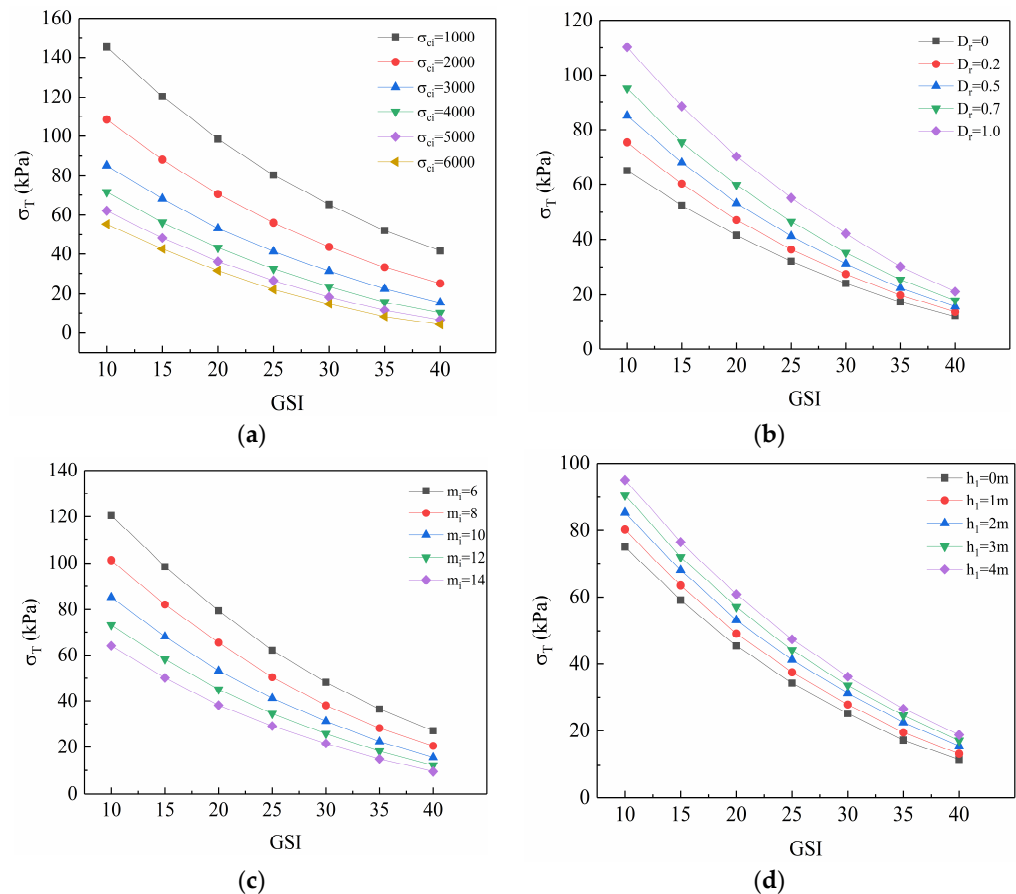


Figure 11. Effect of the geological strength index GSI on Ultimate Support Force σ_T under different conditions in Drill-and-Blast Excavation. The variables are (a) σ_{ci} ; (b) GSI ; (c) m_i ; (d) h_1 .

Figure 12 shows the relationship between the material constant m_i for intact rock and the ultimate support force. Similar to the influence pattern of GSI , σ_T decreases nonlinearly with increasing m_i , but its curve slope is relatively gentle, indicating lower sensitivity. This result suggests that soft rock has relatively weak shear resistance and requires greater support force at the working face after excavation unloading.

Figure 13 illustrates the spatial influence of the excavation damage zone depth (h_1) on the ultimate support force. As shown, excluding the undisturbed condition ($D_r = 0$), σ_T exhibits a nonlinear increasing trend with increasing h_1 . In the initial stage of depth increase, the support force rises rapidly; as the depth continues to increase, the growth rate of the curve gradually slows down. This indicates that the extension of the damaged zone toward greater depths increases the volume of potentially unstable rock mass, thereby raising the support requirements at the tunnel face. Therefore, during tunnel excavation, blasting parameters must be optimized to limit the depth of the damaged zone h_1 , thus ensuring tunnel construction safety.

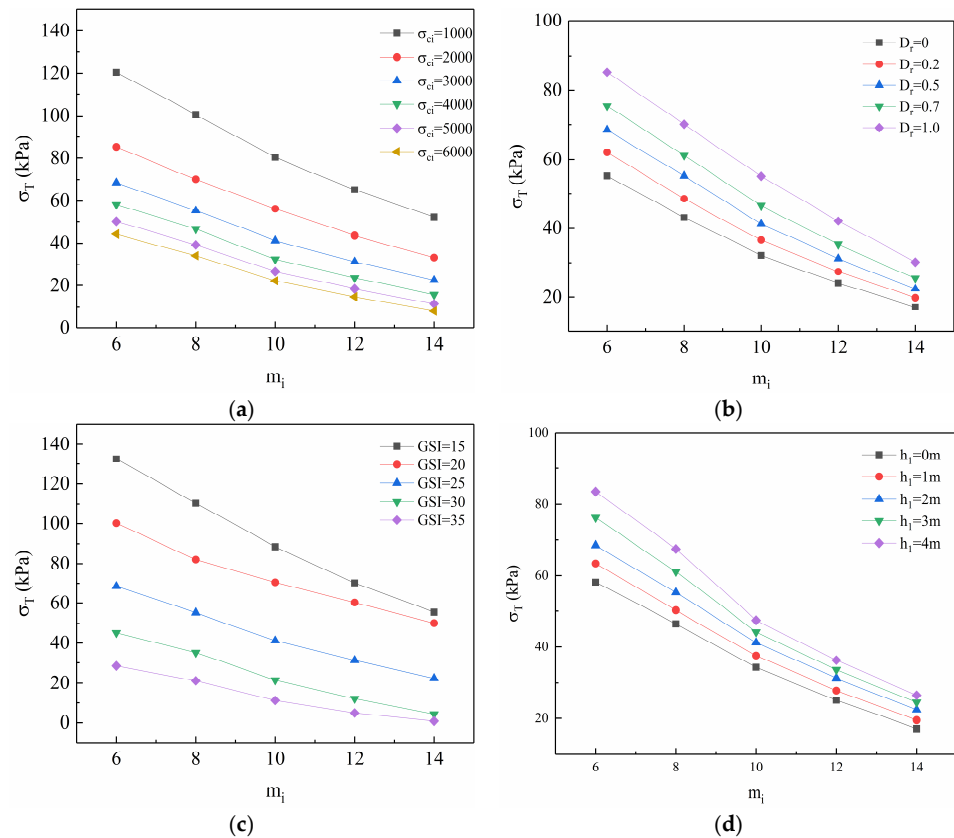


Figure 12. Effect of the material constant m_i on the Ultimate Support Force σ_T under different parameter combinations in Drill-and-Blast Excavation. The variables are (a) σ_{ci} ; (b) D_r ; (c) GSI; (d) h_1 .

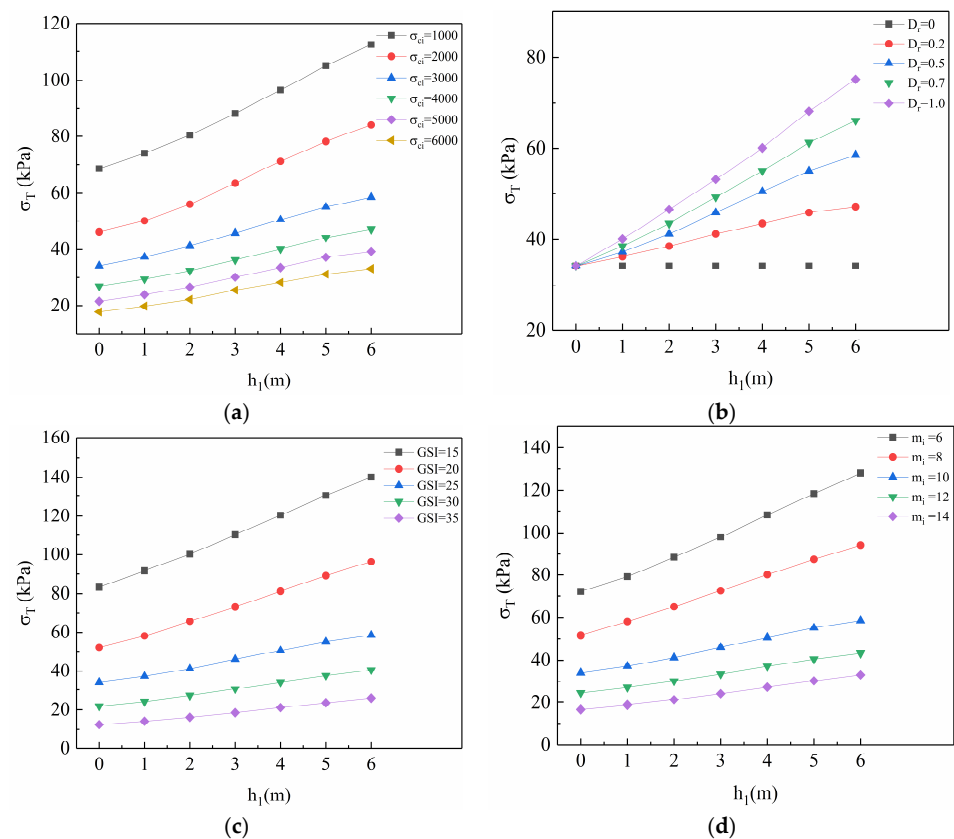


Figure 13. Effect of the excavation damage zone depth h_1 on the Ultimate Support Force σ_T under different parameter combinations. The variables are (a) σ_{ci} ; (b) D_r ; (c) GSI; (d) m_i .

To further quantitatively evaluate the sensitivity weights of the aforementioned four parameters on the stability of the tunnel face, specific operating conditions ($\sigma_{ci} = 3000$ kPa, $D_r = 0.5$, $GSI = 10$, $m_i = 10$, $h_1 = 4$ m) were selected for sensitivity analysis. Based on this benchmark, each parameter was shifted 20% in an unfavorable direction, and the percentage increase in the ultimate support force was recorded. The results are shown in Table 2.

Table 2. Quantitative comparison of parameter sensitivity for Limit Support Force.

Parameter Variable	Original Parameter Value	Change in Parameter	σ_T Initial Value (kPa)	σ_T Change (kPa)	σ_T Percentage Change
GSI	10	8	73.358	151.561	106.6
D_r	0.5	0.6		138.283	88.5
h_1 (m)	4	4.8		128.568	75.3
m_i	10	8		100.510	37.0

Combining the results from the table above, the sensitivity of the four parameters to the stability of the working face, from strongest to weakest given the specified parameters for the deep-buried tunnel, is as follows: $GSI > D_r > h_1 > m_i$. Among these, GSI exhibits the strongest sensitivity. This conclusion is highly consistent with the mathematical expression of the Hoek–Brown criterion [19]. In the generalized H-B criterion, GSI is the exponential term controlling the rock mass structural parameters; even a slight change in this parameter can cause an order-of-magnitude change in the macroscopic strength of the rock mass. The excavation disturbance factor D_r exhibits the second-highest sensitivity; it directly reflects the degree of physical damage to the surrounding rock integrity caused by drilling and blasting, leading to a significant reduction in equivalent cohesion. The disturbance distance h_1 determines the spatial extent of the weakened zone and also exhibits high sensitivity to support strength. On contrast, the intact rock parameter m_i , as a material constant reflecting rock friction characteristic, has a relatively weaker impact on the ultimate support force when changed individually. Therefore, accurate investigation of GSI and strict control of D_r are critical in deep tunnel construction. Given the highly nonlinear nature of the Hoek–Brown criterion, the calculated support force is extremely sensitive to input uncertainties. To mitigate engineering risks associated with parameter fluctuations, it is recommended to adopt a critical parameter combination for support design: utilizing the lower-bound field values for resistance parameters (e.g., GSI , m_i) and the upper-bound values for disturbance parameters (e.g., D_r , h_1). This conservative approach ensures a sufficient safety margin for the evaluated ultimate support force.

6. Conclusions

This paper proposes a vertical strip discretization analysis method that accounts for the weakening effect of excavation disturbance on the stability of the tunnel face in deep tunnels. By combining the Hoek–Brown criterion with the equivalent Mohr–Coulomb transformation and incorporating disturbance factors, equivalent strength parameters c and φ are derived for different disturbance zones. A target function for the ultimate support force at the tunnel face is derived, and a two-dimensional rotational failure mechanism accounting for the weakening effect of disturbance is established. The results are validated through comparison with existing literature and finite element numerical simulations. The conclusions are as follows:

- (1) The theoretical model established in this study, based on vertical strip discretization, effectively characterizes the non-logarithmic spiral failure pattern ahead the tunnel face under drilling and blasting disturbances. The theoretically derived ultimate

support force obtained from theoretical calculations and the predicted shape of the plastic slip zone show excellent agreement with the simulation results from the Abaqus 3D finite element software. Furthermore, whereas traditional finite element iterative computations typically demand several hours, this analytical model, based on a MATLAB optimization algorithm, requires only about 90 s for a single calculation. This significantly enhances the computational efficiency of stability assessments and renders it highly viable for real-time field applications.

- (2) Geological conditions and excavation disturbances exert significant nonlinear effects on the ultimate support force at the tunnel face. Quantitative sensitivity comparison results indicate that under the specified tunnel condition, the influence weights of various parameters on the ultimate support force, from strongest to weakest, are: $GSI > D_r > h_1 > m_i$. Among these, the geological strength index GSI , as the index term controlling rock mass strength, exhibits the strongest sensitivity. Specifically, quantitative analysis reveals that a 20% deterioration in GSI leads to a 106.6% increase in the ultimate support force, strongly validating its dominant role.
- (3) The excavation disturbance factor D_r and disturbance distance h_1 have a significant impact on stability. Drilling and blasting disturbances not only numerically weaken rock mass strength but also spatially expand the volume of potential collapse zones in a weakened state. In a vertical discretization model, this directly results in a longer path of work for the slip surface as it traverses low-strength zones, causing a significant decrease in internal shear work efficiency and thereby increasing the critical support requirements.

Based on the aforementioned mechanical principles, during the actual construction of deep-buried rock tunnels, efforts should be intensified to explore and predict geological strength GSI ; simultaneously, blasting designs should be rigorously optimized to minimize disturbance factors D_r and reduce the depth of the excavation damage zone h_1 , thereby effectively preventing and controlling face instability hazards. The core parameters required for this method can be obtained as follows: the value of m_i can be determined by consulting relevant literature [36] or based on empirical values, the GSI value can be automatically calculated using machine vision methods [37] or determined by consulting relevant quantification charts, σ_{ci} can be obtained through on-site measurements or laboratory tests, while D_r and h_1 can be measured via ultrasonic testing [15]; by substituting these parameters, the method can be applied at the construction site. However, quantifying rock mass parameters in actual construction is challenging, and results derived from these parameters inherently carry uncertainty. Consequently, in our future research, we will focus on investigating the collapse risks associated with parameter uncertainty.

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