

Article

Numerical Simulation of In Situ Stress Fields in Deep Geotechnical Engineering Using Nonlinear Iterative Inversion

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Abstract

The mechanical behavior of deep rock masses under high-stress conditions exhibits significant nonlinear characteristics. However, current in situ stress field inversion methods typically rely on linear elastic constitutive models and multiple linear regression analysis. By analyzing the results of triaxial stress–strain tests and confining pressure calibration experiments on rocks, and drawing on the nonlinear concepts from the Duncan-Zhang model, a nonlinear characterization function was developed, represented by mean stress p , bulk modulus K , and shear modulus G . The nonlinear elastic constitutive model was integrated into a numerical simulation framework, and a new in situ stress field inversion fitting method based on nonlinear elastic constitutive modeling was proposed. This method uses initial linear iterations followed by multiple nonlinear iterations until convergence is achieved. Applied to the inversion of the deep in situ stress field at the Xishan Iron Mine, the results demonstrate that compared to traditional linear regression-based methods, the errors in mean stress, deviatoric stress, and the Lode parameter were reduced by 58%, 50%, and 22%, respectively, confirming the effectiveness of this method in in situ stress field inversion in rock mechanics.

Keywords: in situ stress field; rock mechanics; nonlinear inversion; numerical simulation; nonlinear iterative fitting

1. Introduction

As resource extraction moves deeper into the Earth's crust, both the in situ stress levels and the properties of the surrounding rock mass change. The fundamental mechanical properties exhibited by rock masses at different depths differ significantly from those observed during shallow mining, and some key mechanical parameters also undergo changes [1]. Deep geotechnical engineering activities face extreme challenges due to high in situ stresses [2], with the nonlinear behavior of rock materials becoming more pronounced. The influence of in situ stress states and their interaction with geological stress environments also becomes more prominent [3]. The concentration, complexity, and significant anisotropy of in situ stress make it an urgent and challenging issue in geotechnical engineering, particularly in regions with active tectonic processes [4,5].

In situ stress is the stress generated within a rock mass due to long-term geological processes, constrained by factors such as boundaries and temperature, and exhibits complex distribution patterns [6,7]. In situ stress is a critical factor affecting underground excavation, design, and stability analysis [8,9]. Seismic activity, rock bursts, and stress-induced



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failures are the primary geotechnical hazards in deep hard rock mining, emphasizing the need for accurate in situ stress measurement and inversion [10,11]. The acquisition of in situ stress information can be achieved through stress measurements and stress field inversion. Current in situ stress measurement methods include hydraulic fracturing and the hollow inclusion method. Lakirouhani and Jolfaei [12] made a significant contribution by integrating fracture mechanics with artificial neural networks to predict maximum horizontal stress from hydraulic fracturing data. Their approach accounts for initial crack length effects and provides a robust framework for enhancing the accuracy of in situ stress estimation. Linear inversion methods for stress fields rely on assumptions about the formation mechanisms and distribution characteristics of the stress field, with actual in situ stress values calculated using linear elastic theory [13]. Zhang et al. [14] used seismic physical parameters to reconstruct the original stress characteristics, inferring the principal stress direction from the mechanism of microseismic sources, and employing a spatial stress vector and average value solving method. They established a regional in situ stress inversion method for deep geotechnical engineering scale rock masses based on microseismic force axis characteristics [15]. Meng et al. [16] improved the representation of both the primary and secondary stress fields by introducing a regression-based inversion model that incorporates a constant term into the stress field formulation. Zhang et al. [17] investigated the stress characteristics of surrounding rock and supporting structures in a tunnel composed of combined hard and soft rock strata under high in situ stress conditions. By measuring stress responses in the surrounding rock, primary support, and secondary lining, their study provides valuable guidance for future tunnel construction and deep geotechnical engineering.

Upon entering the Earth's deeper layers, the elastic modulus of rocks changes non-linearly with stress [18], which contradicts the assumptions of linear elasticity in stress field inversion. To account for rock nonlinearity, Duncan et al. [19] proposed a nonlinear material model in 1970, later modified by Kulhawy et al. [20] in 1972. This model employs a hyperbolic stress–strain relationship and is used for nonlinear elastic analysis in tunnel deformation. A piecewise linear strength model has been proposed for analyzing multiple failure mechanisms in rocks, revealing the intrinsic mechanisms of failure in various rock materials [21]. Kong et al. [22] introduced a three-parameter nonlinear strength criterion for rock materials, with coefficients A, B, and C determined through simple testing, which also have clear physical significance. Gong et al. [23] conducted uniaxial unloading tests on 14 types of rock under different stress levels to study the relationship between rock internal elastic energy, dissipated energy, and total input energy, summarizing the linear storage and energy dissipation characteristics of rock materials. Ding et al. [24] performed finite difference simulations to compute the nonlinear elastic responses of slope models. Xie et al. [25] proposed a constitutive model based on direct shear tests and statistical damage theory to accurately reflect the entire shear stress–displacement relationship in jointed rock. Cabezas et al. [26] extended the standard nonlinear model to account for strength factors in brittle fracture, determining model parameters from complete rock stress–strain curves. Feng et al. [27] developed an incremental elastoplastic constitutive model reflecting shear stress softening in joints, using non-associated flow rules and the change in dilation angle as the potential function to describe the softening characteristics of shear stress post-peak. Wang et al. [28] proposed an enhanced constitutive model for quasi-brittle rocks with localized damage. Xiao et al. [29] presented an analytical solution for in situ stress algorithms considering nonlinear characteristics of rock stress–strain curves, eliminating stress oscillation effects in the strain-hardening zone. Liu et al. [30] developed a three-dimensional nonlinear strength criterion for rocks considering both brittleness and ductility by combining piecewise fitting functions with a multi-parameter generalized

deviation function. Yang et al. [31] explored the use of stress boundary combinations in FLAC^{3D} to reconstruct the initial stress field, validating the feasibility of this method in combination with in situ stress field measurements. Xu et al. [32] proposed a comprehensive analytical method combining rock mechanics testing, numerical simulations, and field engineering validation, verifying its reliability and practicality in deep resource extraction. Yang et al. [33] developed a unified damage–plasticity coupling model to describe the nonlinear mechanical behavior of rocks, including peak plastic deformation and post-peak behavior. Pei et al. [34] proposed a method for estimating in situ stress in tunnels based on multivariate information integration by combining limited in situ stress measurements, stereographic projection techniques, and numerical simulations. The proposed approach provides a useful reference for support design and hazard management of surrounding rock in deep-buried long tunnels.

Linear elastic theory-based in situ stress inversion methods mainly rely on actual in situ stress values and employ mathematical theories or computational models to construct the initial stress field. Alternatively, displacement-based inverse analysis methods based on elastic–plastic finite element computations can be used. Further development and refinement of rock in situ stress field fitting methods are necessary. This study presents a novel approach to the inversion of in situ stress fields in deep geotechnical engineering. The primary innovation lies in integrating a nonlinear elastic constitutive model to address the limitations of conventional linear methods, offering a more accurate representation of rock deformation under varying stress conditions. The methodology combines experimental data, nonlinear fitting, and numerical simulations to derive the stress field more reliably. While traditional methods primarily rely on linear regression for stress field inversion, they often fail to capture the nonlinear behavior of rocks at high stress levels. In contrast, the proposed nonlinear model significantly reduces inversion errors in mean stress, deviatoric stress, and Lode parameters by up to 58%, 50%, and 22%, respectively, compared to traditional methods. This advancement enhances the accuracy and reliability of stress field characterization in deep underground conditions.

2. Proposal for a Nonlinear Elastic Model Considering the Entire Loading Process of Rocks

Internal defects in rocks, such as fractures and interlayers, arise from geological processes or the inherent formation mechanisms of the rock itself. As a result, significant variations exist in the internal structures of both the same rock type and different rock types. Under the influence of various geological forces, the internal defects within the rock exhibit nonlinear variations [35]. While Hooke’s law has been widely applied in the analysis of linear elastic deformation in rock masses, studies on the stress–strain relationships of rocks, even during the elastic deformation phase, consistently reveal pronounced nonlinear behavior.

2.1. Linear and Nonlinear Fitting Methods for Rock Mechanics Parameters

Based on confining pressure calibration tests, the elastic modulus and Poisson’s ratio are typically determined using linear fitting methods applied to stress–strain data. However, at high stress levels, rocks exhibit strong nonlinear characteristics. Therefore, during in situ stress field fitting, it is essential to consider the nonlinear mechanical properties of rocks, where the modulus changes with varying stress levels.

In conventional methods, it is assumed that the rock’s elastic modulus and Poisson’s ratio are constant. Confining pressure is applied to the core samples extracted from boreholes using a confining pressure calibration device, and the strain variations under different confining pressures are recorded. From these results, the elastic modulus and Poisson’s

ratio of the rock at the measurement points can be calculated [36]. The corresponding calculation formula is as follows:

$$E = k_1 \frac{P_0}{\varepsilon_\theta} \frac{2R^2}{R^2 - r^2} \quad (1)$$

$$\nu = \frac{\varepsilon_a}{\varepsilon_\theta} \quad (2)$$

where P_0 represents the confining pressure, while E and ν are the elastic modulus and Poisson's ratio of the core sample, respectively. k_1 is a variable that depends on the core's deformation modulus and the parameters of the measurement equipment, such as the hollow inclusion strain gauge. R and r denote the outer radius of the relieved rock core and the radius of the central borehole, respectively. ε_a and ε_θ represent the average axial strain and average circumferential strain of the core sample, respectively.

In the field of rock nonlinearity, it is unstable to solely consider the nonlinear relationship between the rock's elastic modulus and confining pressure. The modulus of the rock tends to increase with pressure. The quantity that represents the material's stress level in space is the mean stress. In the field of mechanics, mean stress corresponds to volumetric strain, and the modulus associated with volumetric strain is the bulk modulus and shear modulus. Therefore, bulk modulus and shear modulus are used for fitting analysis. Theoretically, the rock's bulk modulus and shear modulus increase with rising stress levels, but they will not continue to increase indefinitely; there must be a critical value. Based on the nonlinear concepts in the Duncan-Zhang model, and considering the phenomenon of varying stress states during rock compression, a hyperbolic model is used to fit the relationship between the bulk modulus and mean stress, as well as between the shear modulus and mean stress [37]. The fitting equation is as follows:

$$K = K_0 + \frac{p}{a + bp} \quad (3)$$

$$G = G_0 + \frac{p}{c + dp} \quad (4)$$

where p represents the mean stress, while a , b , c , and d are parameters related to the initial modulus, ultimate modulus, and the increment of the initial modulus, respectively. K and G denote the bulk modulus and shear modulus of the core sample, while K_0 and G_0 are the bulk modulus and shear modulus when the mean stress is zero.

The bulk modulus K and shear modulus G are calculated based on the elastic modulus E and Poisson's ratio ν obtained from the confining pressure calibration tests, using Equations (5) and (6). During the testing process, the mean stress p in the rock core continuously changes. The corresponding mean stress value for each segment can be determined from the maximum value of the stress-strain curve, as shown in Equation (7).

$$K = \frac{E}{3(1 - 2\nu)} \quad (5)$$

$$G = \frac{E}{2(1 + \nu)} \quad (6)$$

$$p = \frac{2}{3}\sigma_r \quad (7)$$

where σ_r represents the radial stress exerted on the rock core. By rewriting Equations (1) and (2) in incremental form and substituting them into Equations (3) and (4), the following relationship is obtained:

$$K = k_1 \frac{\frac{\Delta\sigma_r}{\Delta\epsilon_\theta} \frac{2R^2}{R^2+r^2}}{3\left(1 + 2\frac{\Delta\epsilon_a}{\Delta\epsilon_\theta}\right)} \quad (8)$$

$$G = k_1 \frac{\frac{\Delta\sigma_r}{\Delta\epsilon_\theta} \frac{2R^2}{R^2-r^2}}{2\left(1 - \frac{\Delta\epsilon_a}{\Delta\epsilon_\theta}\right)} \quad (9)$$

By solving Equations (3), (4), (7)–(9) simultaneously and simplifying, the following relationship is obtained:

$$\Delta(\epsilon_\theta + 2\epsilon_a) = k_1 \frac{(a + bp)\Delta p}{aK_0 + (bK_0 + 1)p} \frac{R^2}{R^2 - r^2} \quad (10)$$

$$\Delta(\epsilon_\theta - \epsilon_a) = k_1 \frac{(c + dp)\Delta p}{cK_0 + (dK_0 + 1)p} \frac{3R^2}{2(R^2 - r^2)} \quad (11)$$

Integrating both sides of Equations (10) and (11) yields the following result:

$$\epsilon_\theta + 2\epsilon_a = k_1 \frac{R^2}{R^2 - r^2} \left[\frac{p}{K_0} - \frac{p}{K_0(bK_0 + 1)} + \frac{a}{(bK_0 + 1)^2} \ln \frac{aK_0 + (bK_0 + 1)p}{aK_0} \right] \quad (12)$$

$$\epsilon_\theta - \epsilon_a = k_1 \frac{3R^2}{2(R^2 - r^2)} \left[\frac{p}{G_0} - \frac{p}{G_0(dG_0 + 1)} + \frac{c}{(dG_0 + 1)^2} \ln \frac{cG_0 + (dG_0 + 1)p}{cG_0} \right] \quad (13)$$

2.2. Mechanical Parameters and Error Analysis Under Linear and Nonlinear Fitting Methods

The test specimens consisted of five types: quartzite, phyllite, magnetite, and granite samples (50×100 mm) from the Xishan Iron Mine, and marble in situ stress-relieved core samples from the Wujic Iron Mine. Triaxial compression tests were conducted on the four rock types from Xishan Iron Mine, while confining pressure calibration tests were performed on the marble from Wujic Iron Mine. This study focuses on presenting the results of the mechanical parameters and the error analysis methods under different fitting approaches. The triaxial tests for the quartzite, phyllite, magnetite, and granite samples from the Xishan Iron Mine were designed with a confining pressure gradient, ranging from 5 MPa to a maximum of 40 MPa. The loading system for the rock mechanics tests is shown in Figure 1.

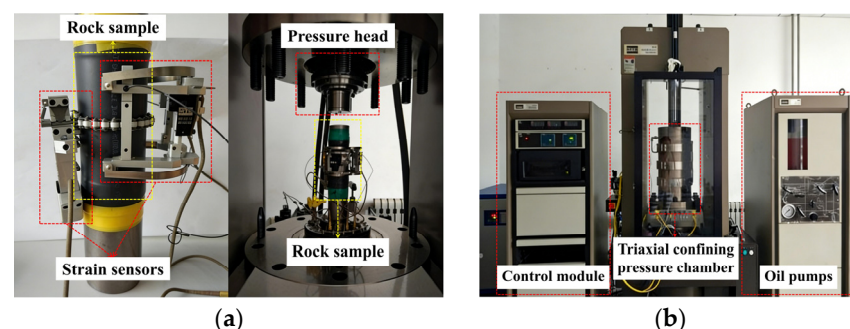


Figure 1. MTS815 rock mechanics testing loading system: (a) Installation of rock samples; (b) triaxial testing machine.

To ensure the reliability and reproducibility of the experimental results, a comprehensive quality assurance (QA) procedure was implemented throughout the rock mechanics

testing program. The QA process included: 1. calibration of all testing apparatus prior to each test series; 2. strict control of specimen moisture content and laboratory environmental conditions, whereby each group of rock specimens was oven-dried at 105 °C for 24 h prior to testing, and all experiments were conducted in a temperature- and humidity-controlled chamber maintained at 20 ± 3 °C and $50\% \pm 10\%$ relative humidity; and 3. testing of multiple specimens for each lithology under identical confining pressure conditions, with at least three replicate specimens tested for each rock type at each confining pressure level. Due to the large number of rock samples tested, only representative stress–strain curves under different lithologies and confining pressures are presented, as shown in Figure 2.

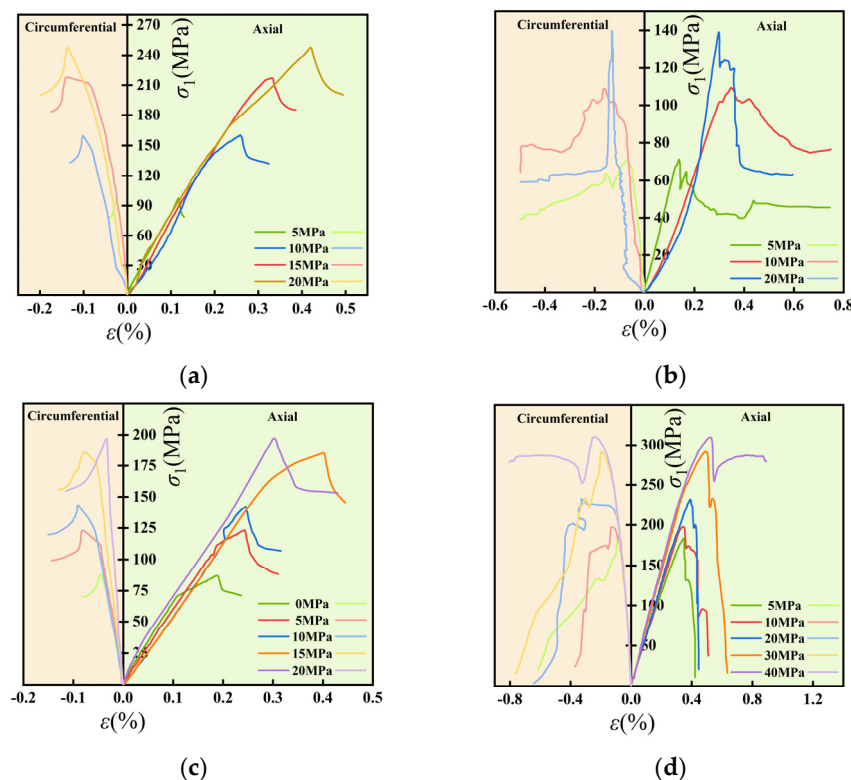


Figure 2. Stress–strain curves for different confining pressures and rock types: (a) Quartzite; (b) phyllite; (c) magnetite; (d) granite.

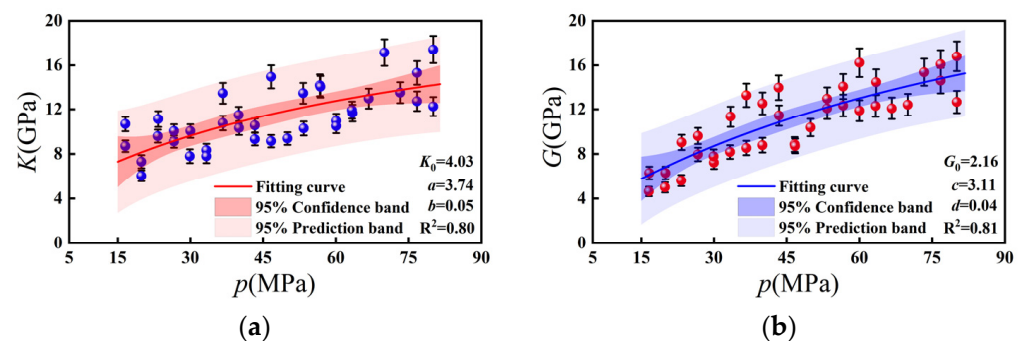
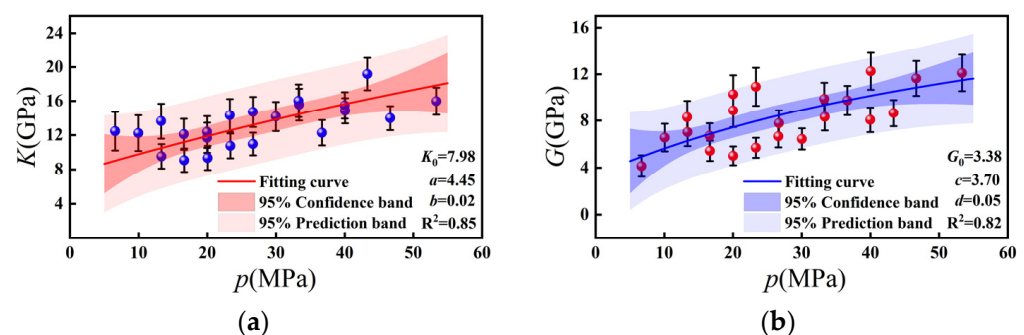
Figure 2 indicates that, during the compaction phase, the nonlinearity of the stress–strain relationship is more pronounced under low axial pressure. As the axial pressure increases and the rock enters the elastic phase, the stress–strain curve becomes approximately linear. The Poisson’s ratio and elastic modulus of the rock are typically calculated by linearly fitting the relevant strain data in this phase. However, some rocks exhibit clear non-linear characteristics, such as the phyllite at a confining pressure of 20 MPa and quartzite at 15 MPa, both of which lack distinct linear segments. Upon entering the yield phase, the stress–strain relationship further transitions towards curvature, with the elastic–plastic boundary marking the starting point of this change. In the post-peak phase, when the rock reaches its triaxial compressive strength, the specimen fails, and the bearing capacity rapidly declines, exhibiting significant nonlinearity.

To provide a more intuitive understanding of the lithology of the four rock types from the Xishan Iron Mine, a summary of their main mineral compositions and experimentally measured mechanical strengths is presented in Table 1.

Table 1. Mineral composition and strength characteristics of the four rock types.

Rock Type	Main Mineral Composition	Peak Compressive Strength						
		0 MPa	5 MPa	10 MPa	15 MPa	20 MPa	30 MPa	40 MPa
quartzite	quartz (>90%), minor mica	-	98	160	218	248	-	-
phyllite	quartz, sericite, chlorite	-	72	110	141	-	-	-
magnetite	magnetite, quartz, amphibole	88	124	142	186	197	-	-
granite	quartz, feldspar, biotite	-	184	198	-	233	292	310

To further verify that the mechanical parameters of different rock samples exhibit significant nonlinear characteristics with varying stress levels, it is important to consider that simply studying the nonlinear relationship between the rock's elastic modulus and confining pressure is not stable enough. The modulus of the rock tends to increase with rising stress levels. Therefore, it is necessary to examine the relationship between rock mechanical parameters and the mean stress level, incorporating the spatial stress perspective. The axial stress–strain curves from the aforementioned tests were used to determine the elastic modulus and Poisson's ratio through piecewise linear fitting. This approach allowed for the determination of the relationships between the rock's bulk modulus K and mean stress p , as well as the shear modulus G and mean stress p . The fitting was conducted using Equations (3) and (4), and the results are presented in Figures 3–6. The error bars represent the mean values of the bulk modulus K and shear modulus G at each mean stress level p , expressed as $\pm$ one standard deviation. The relatively small error bars for quartzite and granite indicate good experimental repeatability. Magnetite also exhibits comparatively small variability, whereas the slightly larger error bars observed for phyllite reflect its greater structural heterogeneity.

**Figure 3.** The nonlinear relationship between the bulk modulus K , shear modulus G , and mean stress p for quartzite: (a) K - p ; (b) G - p . Error bars represent $\pm$ one standard deviation of replicate measurements.**Figure 4.** The nonlinear relationship between the bulk modulus K , shear modulus G , and mean stress p for phyllite: (a) K - p ; (b) G - p . Error bars represent $\pm$ one standard deviation of replicate measurements.

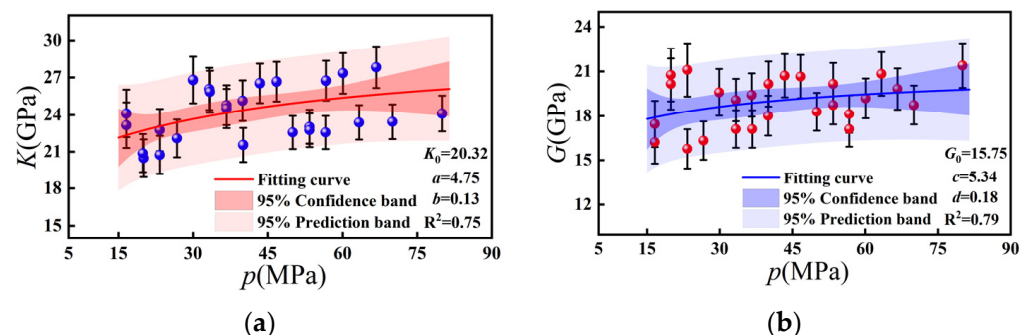


Figure 5. The nonlinear relationship between the bulk modulus K , shear modulus G , and mean stress p for magnetite: (a) K - p ; (b) G - p . Error bars represent $\pm$ one standard deviation of replicate measurements.

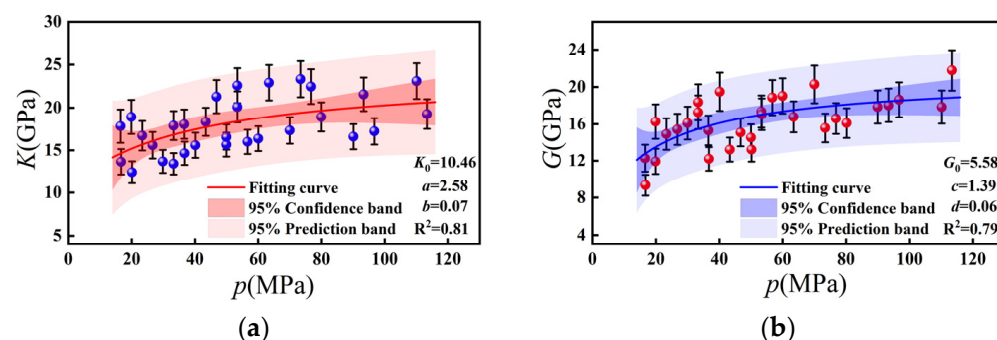


Figure 6. The nonlinear relationship between the bulk modulus K , shear modulus G , and mean stress p for granite: (a) K - p ; (b) G - p . Error bars represent $\pm$ one standard deviation of replicate measurements.

For the bulk modulus K , the coefficient of variation is approximately 8% in the low-mean-stress region and 7% in the high-mean-stress region. For the shear modulus G , the coefficient of variation is approximately 9% in the low-mean-stress region and 8% in the high-mean-stress region. The slightly larger coefficients of variation for G reflect its greater sensitivity to microstructural features. The consistently small error bars throughout confirm the homogeneous crystalline nature of quartzite and the high quality of the experimental data.

Due to the well-developed phyllite and inherent anisotropy of phyllite, the coefficients of variation are substantially larger than those for quartzite. For the bulk modulus K , the coefficient of variation ranges from 18% in the low-mean-stress region to 10% in the high-mean-stress region. For the shear modulus G , which is more sensitive to fabric, the coefficient of variation ranges from 22% in the very low-mean-stress region to 13% in the high-mean-stress region. The large scatter in both K and G values at similar mean stress levels quantitatively demonstrates the significant influence of rock fabric on mechanical parameter variability. The decreasing coefficient of variation with increasing stress confirms the progressive closure of phyllite planes under high confining pressure.

Magnetite exhibits low variability characteristic of dense crystalline rocks. For the bulk modulus K , the coefficient of variation is approximately 8% in the low-mean-stress region and 6% in the high-mean-stress region. For the shear modulus G , which is slightly more sensitive to microstructural features, the coefficient of variation is approximately 9% in the low-mean-stress region and 7% in the high-mean-stress region. The small scatter in both parameters confirms the homogeneous nature of magnetite, with variability comparable to that of quartzite and significantly lower than that of anisotropic rocks like phyllite.

Granite exhibits intermediate variability characteristic of polymineralic crystalline rocks. For the bulk modulus K (a), the coefficient of variation is approximately 11% in the

low-mean-stress region and 9% in the high-mean-stress region. For the shear modulus G (b), which is more sensitive to mineral boundaries, the coefficient of variation is approximately 12% in the low-mean-stress region and 10% in the high-mean-stress region. The variability is higher than that of homogeneous rocks like quartzite and magnetite, but significantly lower than that of anisotropic rocks like phyllite. The scatter at similar mean stress levels reflects the heterogeneous mineral composition of granite, where local variations in quartz, feldspar, and mica content influence the mechanical response.

The triaxial compression tests conducted on the 50×100 mm rock samples demonstrate that their mechanical parameters exhibit significant nonlinear behavior as the stress level changes. To further investigate the effect of changes in rock stress state on its nonlinear characteristics, biaxial loading tests were conducted on large-scale in situ stress-relieved core samples. The test object was the in situ stress-relieved marble core from the Wuji Iron Mine in China, located at a depth of 300 m. A self-developed high-confining-pressure calibration system, shown in Figure 7, was used. The high-pressure calibration system consists of long-term loading electric control system, fully automated pressure verification device, high-pressure oil pump, three-way control valve, confining pressure calibration chamber, rubber sleeve, and several connecting pipes. It can provide up to 200 MPa of confining pressure with an accuracy of 0.001 MPa and can maintain high-pressure load for up to 180 days.

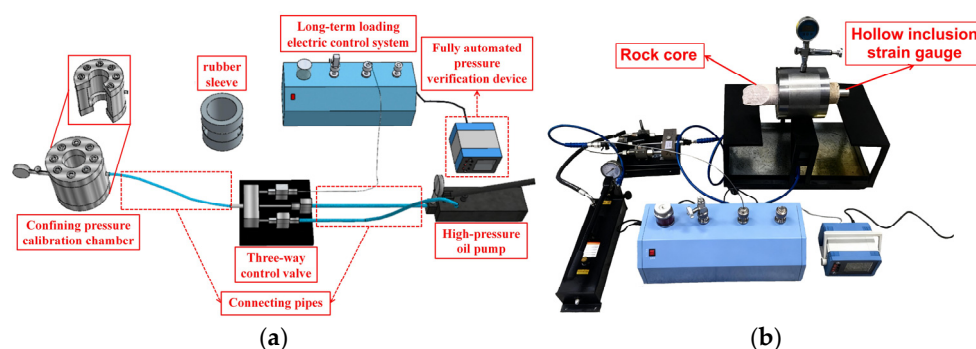


Figure 7. The high-confining-pressure calibration test system: (a) The components of the system; (b) the high-confining-pressure calibration test.

The confining pressure calibration test was conducted by applying confining pressure to the rock core after it was relieved on-site. Real-time data acquisition from the hollow inclusion strain gauge sensor inside the core's small borehole was used to establish the relationship between confining pressure and strain. The test was designed with a confining pressure gradient of 2 MPa, and the maximum confining pressure was set at 16 MPa. The final experimental data were obtained from three repeated tests, all conducted under controlled environmental conditions with a constant temperature of 20 ± 3 °C and relative humidity maintained at $50\% \pm 10\%$. In the conventional linear fitting method, the experimental data obtained from the confining pressure calibration test were plotted as shown in Figure 8. A linear fit was applied to the experimental data points. Using Equations (1) and (2), the elastic modulus of the marble core was determined to be 32.25 GPa, from which the corresponding strain values, $\varepsilon_\theta + 2\varepsilon_a$ and $\varepsilon_\theta - \varepsilon_a$, at different mean stress levels, were calculated.

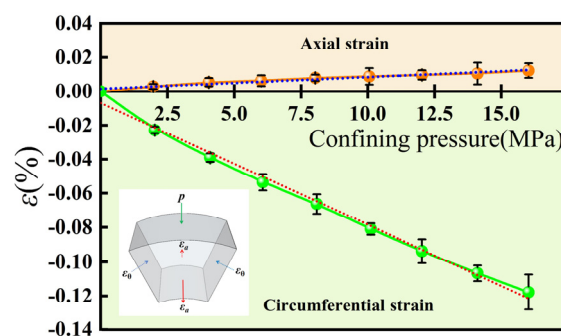


Figure 8. The linear fitting results for marble.

For the nonlinear fitting method, each segment of the polyline in Figure 8 was selected, and using Equations (1) and (2), the elastic modulus and Poisson's ratio corresponding to the mean stress at the midpoint of each segment were calculated. The elastic modulus E increases progressively with the rise in stress level, ranging from 26.24 GPa to 37.76 GPa. Subsequently, the corresponding bulk modulus and shear modulus for each segment were determined. The fitting was then carried out using Equations (3) and (4) to obtain the nonlinear relationship between the bulk modulus and shear modulus with respect to mean stress. Based on this nonlinear relationship, and using Equations (12) and (13), the strain values $\varepsilon_\theta + 2\varepsilon_a$ and $\varepsilon_\theta - \varepsilon_a$ at different mean stress levels were calculated. The fitting results are shown in Figure 9.

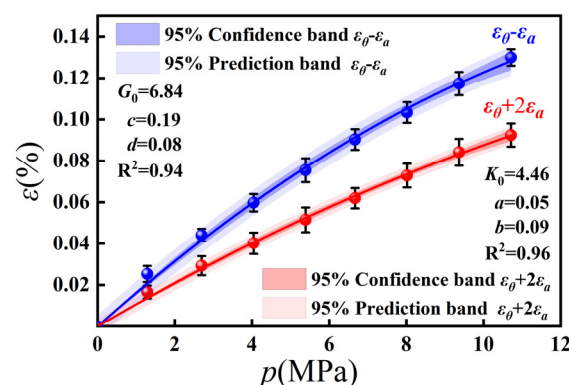


Figure 9. The nonlinear fitting results for marble.

Figure 9 demonstrates that the two nonlinear fitting curves achieve coefficients of determination (R^2) of 0.94 and 0.96, respectively, indicating high fitting accuracy. Both the 95% confidence bands and prediction bands are relatively narrow, with all measured data points falling within the prediction bands, suggesting low uncertainty in the computational estimates and reliable results. The error bars are uniformly distributed without systematic deviation, reflecting stable and controllable fitting and measurement errors. This confirms that incorporating the nonlinear characteristics of rocks enables a more accurate description of the stress–strain relationship.

The differences between the measured values and those obtained from both linear and nonlinear fitting methods for $\varepsilon_\theta + 2\varepsilon_a$ and $\varepsilon_\theta - \varepsilon_a$, together with the corresponding mechanical parameters, are summarized in Table 2. Specifically, Δ_1 denotes the difference between the measured $\varepsilon_\theta + 2\varepsilon_a$ and the value predicted by linear fitting; Δ_2 represents the difference between the measured $\varepsilon_\theta - \varepsilon_a$ and the linear fitting result; Δ_3 indicates the difference between the measured $\varepsilon_\theta + 2\varepsilon_a$ and the nonlinear fitting result; and Δ_4 corresponds to the difference between the measured $\varepsilon_\theta - \varepsilon_a$ and the nonlinear fitting

result. For clarity and ease of comparison, all strain values in Table 2 are expressed in microstrain ($\mu\epsilon$).

Table 2. Comparison of linear and nonlinear fitting results for marble.

p (MPa)	Test		E (GPa)	ν	Linear Fitting				E (GPa)	ν	Nonlinear Fitting			
	$\epsilon_\theta + 2\epsilon_a$ ($\mu\epsilon$)	$\epsilon_\theta - \epsilon_a$ ($\mu\epsilon$)			$\epsilon_\theta + 2\epsilon_a$ ($\mu\epsilon$)	Δ_1	$\epsilon_\theta - \epsilon_a$ ($\mu\epsilon$)	Δ_2			$\epsilon_\theta + 2\epsilon_a$ ($\mu\epsilon$)	Δ_3	$\epsilon_\theta - \epsilon_a$ ($\mu\epsilon$)	Δ_4
10.67	925	1296	32.35	0.1	912	13	1271	25	37.76	0.07	932	−8	1298	−2
9.39	843	1169			802	41	1118	51	37.29	0.08	831	12	1170	0
8.02	728	1034			686	42	955	79	36.68	0.08	723	5	1031	4
6.72	619	899			574	44	801	98	35.91	0.09	619	0	895	4
5.39	514	751			461	53	643	109	34.85	0.09	512	3	752	−1
4.05	404	599			346	58	482	117	33.28	0.1	401	2	601	−2
2.73	289	439			234	55	326	113	30.88	0.12	291	−2	442	−3
1.33	166	254			114	52	159	95	26.24	0.14	166	0	250	3

As shown in Table 2, the average error between the strain values obtained from nonlinear fitting and the experimentally measured strains is reduced by up to 97.23% compared to the linear fitting method. Therefore, with the change in stress levels, the modulus of the rock is not a fixed value but rather a variable that exhibits nonlinear variation with the stress level. The nonlinear fitting method is more suitable for describing the true mechanical parameters as they change with stress. During the in situ stress field fitting process, the accuracy of the fitted stress field is directly influenced by the input rock parameters at different stress levels. It should be noted that the strain values presented in Table 2 are direct measurements from confining pressure calibration tests using high-precision strain gauges. The apparently low Poisson's ratio values reflect the marble cores' exceptionally intact, dense structure with minimal microcracks and very low porosity, which inherently influences the stress–strain response. These values have been verified through multiple test repetitions.

To further assess the statistical significance of the two fitting methods, an analysis of variance (ANOVA) is conducted on the residuals Δ_1 , Δ_2 , Δ_3 , Δ_4 presented in Table 2. The residuals for $\epsilon_\theta + 2\epsilon_a$ and $\epsilon_\theta - \epsilon_a$ are treated as a combined dataset for the calculation. The results show that the sum of squares due to error (SSE) for the linear fit (SSE = 79,887) is substantially larger than that for the nonlinear fit (SSE = 309). The distribution characteristics of the residuals further indicate that the linear fit residuals exhibit a systematic deviation, increasing significantly as the confining pressure decreases. In contrast, the nonlinear fit residuals are randomly scattered around zero. Therefore, the fitting algorithm incorporating nonlinear characteristics not only yields values numerically closer to the measurements but is also statistically superior to the traditional linear model.

3. The Principle of In Situ Stress Field Iterative Fitting Analysis Considering the Nonlinear Characteristics of Rock Masses

3.1. In Situ Stress Measurement

The deformation and failure of rocks are directly related to the stress field, and the inversion of rock mass properties and in situ stress field is a key focus and challenge in the field of geotechnical engineering [38]. The selected engineering background for this in situ stress field fitting is the Xishan Iron Mine in Anshan City, Liaoning Province, China. The method used for in situ stress measurement at this mine is the hydraulic fracturing technique, which has become a well-established method commonly employed to determine in situ stresses at depth [39]. Measurement data from 23 points in the ZKB19 and ZKB39 boreholes were used as the basis for the stress field fitting. (For convenience in data processing, points 1–12 correspond to the ZKB19 measurement data, while points 13–23

correspond to the ZKB39 measurement data.) The coordinates for the ZKB19 borehole are (1190.2355, −609.85, 29.707), with a drilling inclination of 75°, and the coordinates for the ZKB39 borehole are (311.2172, −48.743, 24.213), also with a drilling inclination of 75°. The arrangement of the in situ stress measurement boreholes is shown in Figure 10. The specific measurement data are presented in Table 3.

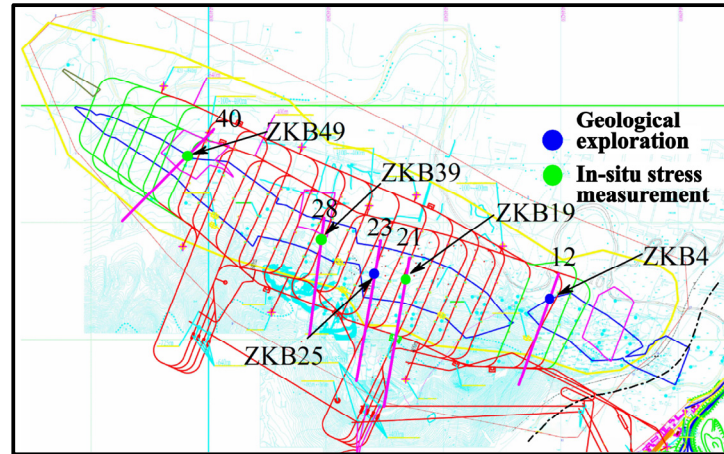


Figure 10. Arrangement of the in situ stress measurement boreholes.

Table 3. In situ stress measurement results from the ZKB19 and ZKB39 boreholes.

Borehole	Measurement Point	Measured Section Depth (m)	Principal Stress (MPa)			Direction of Fracture
			σ_1	σ_2	σ_3	
ZKB19	1	334.80	10.63	8.70	7.16	N65° E
	2	353.40	13.61	9.19	8.66	
	3	381.30	13.32	9.91	8.66	
	4	399.90	12.99	10.40	8.91	
	5	418.50	11.97	10.88	8.93	
	6	437.10	13.23	11.36	9.91	
	7	465.00	12.78	12.09	10.48	N69° E
	8	492.90	14.62	12.82	11.36	
	9	511.50	17.11	13.30	12.64	
	10	539.40	17.47	14.02	12.96	
	11	567.30	18.72	14.75	13.60	
	12	595.20	19.54	15.48	14.40	
ZKB39	13	348.94	15.86	13.33	10.12	N73.5° E
	14	423.66	20.07	17.20	12.29	
	15	507.72	23.24	18.70	14.72	
	16	554.42	29.33	21.15	16.08	
	17	629.14	33.38	23.55	18.25	
	18	657.16	35.74	24.38	19.06	N68.8° E
	19	759.90	38.21	26.73	22.04	
	20	881.32	45.44	29.90	25.56	
	21	900.00	47.68	30.33	26.10	
	22	918.68	47.96	31.20	26.64	
	23	937.36	48.98	32.24	27.18	

To provide a more intuitive understanding of the in situ stress distribution in the Xishan Iron Mine, the stress measurement data from boreholes ZKB19 and ZKB39 are compiled and plotted in Figure 11 to illustrate the variation of in situ stress with depth,

where σ_H denotes the maximum horizontal principal stress, σ_h denotes the minimum horizontal principal stress, and σ_v denotes the vertical principal stress.

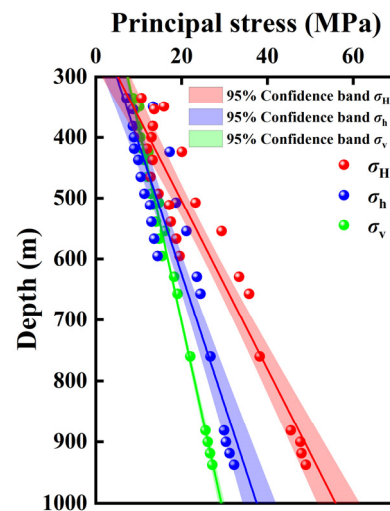


Figure 11. Variation of the principal stresses with depth in the Xishan Iron Mine.

Figure 11 indicates that the horizontal stresses in each subregion exhibit a gradually increasing trend with depth, although some scatter in the measurement results is observed. Within the measured depth range, the magnitude of σ_H remains below 50 MPa. Linear regression analysis further illustrates the variation of the principal stresses with depth. The shaded areas represent the 95% confidence band for the linear regression of the three principal stress components, suggesting that the in situ stress measurement data from the Xishan Iron Mine are relatively reliable, while also indicating that the in situ stress level at the mine is relatively high.

3.2. Nonlinear Mechanical Parameters of Rock Masses

The composition of rock masses includes discontinuities (structural surfaces) and intact rock material. Due to their different formation processes, the physical and mechanical properties of these discontinuities and intact rock material are quite complex. The mechanical properties of rock masses are closely related to the discontinuities, intact rock material, and the environment in which the rock mass exists. Therefore, the physical and mechanical properties of rock material differ from those of the entire rock mass, and the mechanical parameters of rock material cannot be directly applied to in situ stress field fitting. The *E-Hoek* method is used to determine the mechanical parameters of rock masses [40], while the physical and mechanical parameters of rock material are obtained through linear fitting of laboratory test data. The specific equations for calculating the rock mass mechanical parameters are as follows:

$$E_{rm} = E_i \left[0.02 + \frac{1 - D/2}{1 + e^{((60+15D-GSI)/11)}} \right] \quad (14)$$

where E_{rm} represents the elastic modulus of the rock mass, E_i represents the elastic modulus of the rock material, GSI is the geological strength index, and D is the parameter for rock mass disturbance. Since the rock mass is assumed to be undisturbed by excavation, D is set to 0.

Assuming that the density and Poisson's ratio remain constant, the mechanical parameters of the rock mass can be calculated using Equation (14). The rock mass mechanical parameters are presented in Table 4. The GSI values for each lithology presented in Table 4 were determined based on comprehensive field geological investigations and systematic

discontinuity statistics at the Xishan Iron Mine site, including joint spacing, surface conditions, and weathering degree observed in exploration adits and drill cores. The qualitative assessment followed the guidelines established by *E-Hoek* method for estimating *GSI* from rock mass observations.

Table 4. Rock mass physical and mechanical parameters.

Lithology	Density (g/cm ³)	<i>GSI</i>	<i>D</i>	Poisson's Ratio	Elastic Modulus (GPa)	Bulk Modulus (GPa)	Shear Modulus (GPa)
quartzite	2.67	47	0	0.19	16.87	9.07	7.09
phyllite	2.72	56	0	0.24	24.12	15.46	9.73
magnetite	3.39	60	0	0.21	41.22	23.69	17.03
granite	2.64	57	0	0.2	26.31	14.62	10.96
shale	2.58	44	0	0.2	3.41	1.89	1.42

3.3. Iterative Fitting Algorithm for the In Situ Stress Field Considering Nonlinear Characteristics

The formation of the initial in situ stress field in rock masses is influenced by various factors, including the rock mass's self-weight, lithology, topography, geological structure, rock mass temperature, and groundwater. Among these, self-weight and geological structure have the greatest impact on the formation of the initial stress field, while the influence of other factors is relatively minor.

As the stress level changes, the modulus of the rock mass is not a fixed value but a variable that exhibits nonlinear variation with the stress level [18]. When fitting in situ stress using the multiple linear regression method, there is a certain error between the fitted and measured in situ stress. Some of this error arises from the unreasonable setting of rock mass mechanical parameters. Therefore, a nonlinear elastic model-based in situ stress field inversion approach is proposed, incorporating linear initial values, nonlinear piecewise iteration, and a three-parameter control index based on mean stress, deviatoric stress, and Lode parameters to fit the nonlinear rock mass in situ stress field.

Based on the experimental data obtained from rock mechanics tests conducted on core samples of four lithologies, namely quartzite, phyllite, magnetite, and granite, collected from the Xishan Iron Mine, the nonlinear relationship between K - p and G - p is derived based on Equations (3) and (4). The nonlinear relationship between the bulk modulus K and shear modulus G with respect to mean stress p for these four rock types is obtained. Due to the low strength and fragility of shale, no shale cores were used for the rock mechanics experiments, and therefore, the nonlinear relationship between the bulk modulus, shear modulus, and mean stress for shale could not be determined. In the simulation, it is assumed that the bulk modulus and shear modulus of shale remain unchanged. This assumption is geologically justified based on the site-specific conditions of the Xishan Iron Mine. According to field investigations, the shale layers in the study area are exclusively distributed within the shallow overburden, with a maximum burial depth of less than 100 m. At such shallow depths, the in situ stress levels are relatively low, and the stress-induced nonlinear variation of mechanical parameters is expected to be negligible. The simplification maintains computational efficiency without compromising the accuracy of the nonlinear analysis for the deeply buried competent rocks that govern the stress field distribution.

The bulk modulus K and shear modulus G of the rock mass change nonlinearly with mean stress p . Therefore, in the process of fitting the in situ stress field, for the same rock type, the bulk modulus and shear modulus vary with different stress levels. In the numerical simulation, it is necessary to assign parameters to each model element based on

different stress levels and lithology to make the fitted in situ stress field closer to the actual in situ stress field. The main steps are as follows:

1. Based on the Newton–Raphson method for root-finding, the initial stress state of the rock mass is obtained using the multiple linear regression method;
2. Based on the K - p and G - p relationships for each rock type in the model, the new bulk modulus K and shear modulus G are assigned to the elements, and the in situ stress field is fitted again;
3. The calculation stops when the correlation coefficient between the fitted in situ stress values ($\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}$) at two consecutive stress measurement points and the measured values does not improve by more than 0.00001; otherwise, the process in step 2 is repeated using the current fitted stress field.

To provide a more intuitive representation of the in situ stress field fitting process, it is presented in the form of a structured algorithm flowchart, as shown in Figure 12.

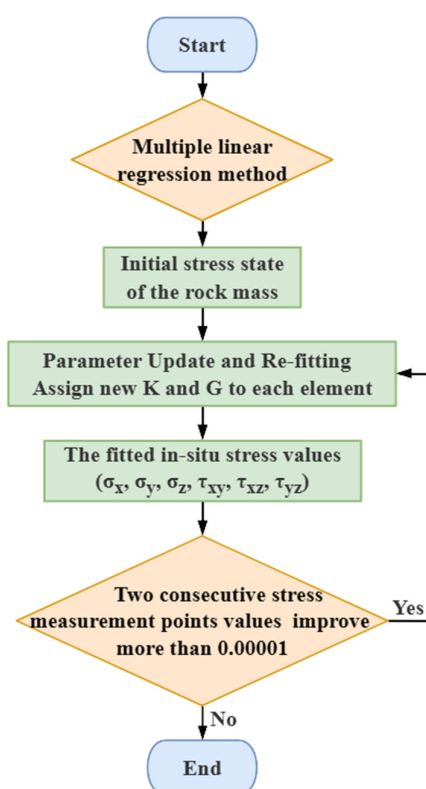


Figure 12. Logical structure flowchart of the in situ stress field fitting process.

In FLAC^{3D} (v.5.01), new constitutive models can be implemented either through C++ programming or through parameter adjustment during simulation. This study adopts the latter approach. The proposed nonlinear elastic model is achieved by real-time monitoring of element mean stress using FLAC^{3D} built-in HISTORY and TABLE commands, with dynamic adjustment of bulk modulus and shear modulus according to the stress-dependent relationships established from laboratory testing. Throughout this procedure, FLAC^{3D} standard linear elastic constitutive model serves as the base framework, while nonlinear behavior is achieved through real-time monitoring and iterative parameter updates. This implementation strategy avoids complex C++ programming while effectively capturing the stress-dependent nonlinear characteristics of the rock mass.

4. Inversion Analysis of the Initial Three-Dimensional In Situ Stress Field of Rock Masses Using Different Fitting Methods

4.1. Model Boundary and Initial Conditions

The in situ stress field fitting for the Xishan Iron Mine was conducted using the FLAC^{3D} software. The model consists of six layers of rock, as shown in Figure 13. From top to bottom, these layers are quartzite, shale, quartzite, phyllite, magnetite, and granite. In the model, the positive x-direction corresponds to the east of the engineering site, the positive y-direction corresponds to the north, and the positive z-direction is vertically upward. The model has a length of 1297 m in the x-direction, 708 m in the y-direction, and a vertical range from 1100 m to 1186 m in the z-direction.

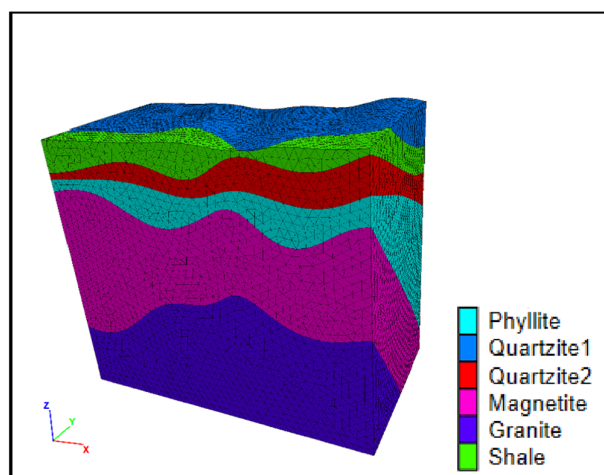


Figure 13. Three-dimensional numerical model.

The numerical model is driven by both gravitational loading and tectonic stress boundary conditions. Following the multiple linear regression approach widely adopted in geotechnical engineering, the complex in situ stress field is decomposed into six fundamental loading modes: self-weight and five tectonic displacement components. The model's upper surface representing the ground surface is set as a free surface, while normal displacements are constrained on the other five boundaries, allowing tangential movement during stress equilibration [41].

Ground topography was incorporated based on survey data from the Xishan Iron Mine exploration campaign, discretized into the FLAC^{3D} grid with 34,143 nodes and 184,908 elements, ensuring realistic representation of the terrain effects on stress distribution.

The complete initial stress field is obtained through multiple linear regression using the 23 measured in situ stress data points and laboratory-derived rock parameters. This approach solves for influence coefficients analogous to the Newton–Raphson iterative method, ensuring the fitted stress field optimally matches the measured values.

4.2. Inversion of the Three-Dimensional In Situ Stress Field Using the Conventional Linear Fitting Method

The simulation assumes that the factors influencing the formation of the initial in situ stress field in the rock mass are self-weight and geological structural movements. The physical and mechanical parameters of the rock are obtained through linear fitting methods based on laboratory tests, and the rock mass mechanical parameters are calculated using the *E-Hoek* method. The in situ stress at each point in the stress field is taken as a linear function

of the self-weight stress and the stress generated by each unit of structural movement, as follows:

$$\sigma = S_{zz} + a_1 S_{xx} + a_2 S_{yy} + a_3 S_{xy} + a_4 S_{yx} + a_5 S_{xz} + a_6 S_{yz} \quad (15)$$

where S_{zz} represents the self-weight stress, while S_{xx} , S_{yy} , S_{xy} , S_{yx} , S_{xz} , S_{yz} denote the stresses generated by the unit displacement of structural movements in the corresponding directions. a_1 , a_2 , a_3 , a_4 , a_5 , a_6 are the influence coefficients of the various structural movement factors.

A least squares fitting analysis was performed to determine the influence coefficients of the structural movements, which were then used to calculate the corresponding fitted in situ stress at each measurement point. The results of the influence coefficients and the correlation coefficients between the fitted in situ stress values and the measured values are shown in Table 5. For clarity, based on the six stress components σ_x , σ_y , σ_z , τ_{xy} , τ_{xz} , τ_{yz} obtained at each measurement point from the in situ stress field fitting, the three principal stresses σ_1 , σ_2 , σ_3 are calculated. The corresponding equations for the principal stresses are given as follows:

$$\begin{aligned} &\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + \\ &\left(\sigma_y\sigma_z + \sigma_z\sigma_x + \sigma_x\sigma_y - \tau_{yz}^2 - \tau_{xz}^2 - \tau_{xy}^2\right)\sigma - \\ &\left(\sigma_x\sigma_y\sigma_z - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2 + 2\tau_{yz}\tau_{xz}\tau_{xy}\right) = 0 \end{aligned} \quad (16)$$

Table 5. Table of influence coefficients and correlation coefficients.

Coefficient	a_1	a_2	a_3	a_4	a_5	a_6
value	79.3716	−27.4128	−161.5766	−94.9463	−67.2327	−19.1606
correlation coefficient	0.8942					

The comparison between the measured in situ stress and the fitted in situ stress is shown in Table 6.

Table 6. Comparison between the measured in situ stress and the linear fitted values.

Measurement Point	Actual Value			Fitted Value		
	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)
1	10.63	8.70	7.16	16.35	13.01	3.55
2	13.61	9.19	8.66	17.68	15.53	4.85
3	13.32	9.91	8.66	18.28	15.69	4.86
4	12.99	10.40	8.91	20.25	15.46	5.68
5	11.97	10.88	8.93	21.50	17.01	6.09
6	13.23	11.36	9.91	21.51	17.04	6.90
7	12.78	12.09	10.48	23.41	17.84	7.61
8	14.62	12.82	11.36	24.36	18.92	7.84
9	17.11	13.30	12.64	26.12	19.48	8.61
10	17.47	14.02	12.96	28.29	20.65	9.68
11	18.72	14.75	13.60	28.79	21.71	10.56
12	19.54	15.48	14.40	30.78	22.46	11.66
13	15.86	13.33	10.12	14.65	10.30	2.51

Table 6. Cont.

Measurement Point	Actual Value			Fitted Value		
	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)
14	20.07	17.20	12.29	21.72	13.83	9.34
15	23.24	18.70	14.72	24.99	18.03	12.55
16	29.33	21.15	16.08	26.92	20.27	14.03
17	33.38	23.55	18.25	28.04	22.97	14.86
18	35.74	24.38	19.06	28.38	23.59	15.04
19	38.21	26.73	22.04	30.27	24.63	16.12
20	45.44	29.90	25.56	32.35	25.52	17.20
21	47.68	30.33	26.10	32.78	25.74	17.34
22	47.96	31.20	26.64	32.43	25.64	17.29
23	48.98	32.24	27.18	33.08	25.87	17.65

4.3. Iterative Fitting and Computational Analysis of the Rock Mass In Situ Stress Field Considering Nonlinear Characteristics

During the fitting process of the rock mass in situ stress field considering nonlinear characteristics, the difference between the measured in situ stress at each measurement point and the fitted in situ stress obtained from the previous iteration is taken as the dependent variable. In the first iteration, this dependent variable is defined as the difference between the measured in situ stress at each measurement point and the fitted in situ stress obtained using the multiple linear regression method. The stress values generated by unit structural displacement under the updated mechanical parameters are taken as the independent variables. The iterative difference calculation formula is given as follows:

$$\Delta\sigma = \sigma_{\text{Actual measured value}} - \sigma_{\text{Fitted value}} \quad (17)$$

The three-dimensional in situ stress field, incorporating the nonlinear elastic model constitutive approach, was embedded into the numerical simulation. An iterative fitting algorithm for the in situ stress field, considering nonlinear characteristics, was applied. After four iterations, the influence coefficients a_i and the correlation coefficients are presented in Table 7. The measured principal stress values and the principal stress values obtained from nonlinear fitting are shown in Table 8.

Table 7. Influence coefficients and correlation coefficients from multiple iterations.

Number of Iterations	a_1	a_2	a_3	a_4	a_5	a_6	Correlation Coefficient
first iteration	3.4975	−2.5489	−61.8723	−36.5573	−2.4524	−6.0011	0.9287
second iteration	−0.4029	0.0671	5.6001	3.5994	−0.4078	0.0541	0.9314
third iteration	−0.0077	0.0012	0.0567	0.0798	−0.0075	0.0010	0.9314
fourth iteration	$−6.8 \times 10^{-5}$	6.3×10^{-5}	0.0004	0.0005	$−5.9 \times 10^{-5}$	8.9×10^{-5}	0.9314

Table 8. Comparison between the measured in situ stress values and the nonlinear fitted values.

Measurement Point	Measured In Situ Stress (MPa)			First Iteration (MPa)			Second Iteration (MPa)			Third Iteration (MPa)			Fourth Iteration (MPa)		
	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3
1	10.6	8.7	7.2	14.9	10.5	4.4	13.6	10.6	5.7	13.6	10.6	5.7	13.6	10.6	5.7
2	13.6	9.2	8.7	16.0	12.9	4.7	15.0	11.6	6.0	15.0	11.6	6.0	15.0	11.7	6.0

Table 8. Cont.

Measurement Point	Measured In Situ Stress (MPa)			First Iteration (MPa)			Second Iteration (MPa)			Third Iteration (MPa)			Fourth Iteration (MPa)		
	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3	σ_1	σ_2	σ_3
3	13.3	9.9	8.7	16.3	13.2	4.7	15.6	12.6	5.6	15.6	12.6	5.6	15.6	12.6	5.6
4	13.0	10.4	8.9	16.7	13.9	5.5	15.6	13.9	6.2	15.6	13.9	6.2	15.6	13.9	6.2
5	12.0	10.9	8.9	17.3	14.6	5.1	16.0	13.7	6.1	16.0	13.7	6.1	16.0	13.7	6.1
6	13.2	11.4	9.9	19.3	15.3	5.8	17.5	14.4	7.4	17.5	14.4	7.4	17.5	14.4	7.4
7	12.8	12.1	10.5	20.5	15.1	6.5	18.8	14.7	7.9	18.8	14.6	7.9	18.8	14.7	7.9
8	14.6	12.8	11.4	20.2	16.5	6.6	18.5	15.6	8.0	18.5	15.6	8.0	18.5	15.6	8.0
9	17.1	13.3	12.6	22.3	17.3	7.5	20.1	16.7	8.9	20.1	16.7	8.9	20.1	16.7	8.9
10	17.5	14.0	13.0	23.4	18.0	8.6	21.8	16.0	9.2	21.8	16.0	9.2	21.8	16.0	9.2
11	18.7	14.8	13.6	25.0	18.5	9.7	22.2	17.4	11.0	22.1	17.4	11.0	22.2	17.4	11.0
12	19.5	15.5	14.4	25.1	19.4	10.6	22.8	18.4	11.6	22.8	18.4	11.6	22.8	18.4	11.6
13	15.9	13.3	10.1	15.1	10.5	3.1	15.3	11.4	4.3	15.3	11.4	4.3	15.3	11.4	4.3
14	20.1	17.2	12.3	23.1	13.6	8.2	22.8	14.7	9.3	22.8	14.7	9.3	22.8	14.7	9.3
15	23.2	18.7	14.7	27.1	18.2	12.1	26.9	18.3	12.1	26.9	18.3	12.1	26.9	18.3	12.1
16	29.3	21.2	16.1	29.1	20.9	14.6	28.9	20.9	13.7	28.9	21.0	13.7	28.9	21.0	13.6
17	33.4	23.6	18.3	29.7	24.3	15.4	29.6	24.4	15.4	29.6	24.4	15.4	29.6	24.4	15.4
18	35.7	24.4	19.1	31.8	25.2	16.5	32.7	25.2	15.5	32.7	25.2	15.6	32.7	25.2	15.5
19	38.2	26.7	22.0	33.3	26.6	18.5	34.5	26.5	18.0	34.5	26.5	18.0	34.5	26.6	18.0
20	45.4	29.9	25.6	37.3	27.4	21.4	38.3	27.3	22.6	38.3	27.4	22.6	38.4	27.4	22.6
21	47.7	30.3	26.1	39.7	27.7	21.4	40.2	27.6	22.5	40.2	27.6	22.5	40.2	27.6	22.4
22	48.0	31.2	26.6	39.2	27.5	21.3	40.5	27.4	23.6	40.5	27.4	23.6	40.5	27.5	23.6
23	49.0	32.2	27.2	40.9	28.7	21.6	41.9	27.6	23.7	41.9	27.6	23.7	41.9	27.6	23.7

4.4. Numerical Simulation Results

Considering the influence of rock mass self-weight, the upper surface of the model is set as a free surface, as it represents the ground surface, and no constraints are applied to it. Constraints are applied to the other five boundary surfaces of the model, restricting the normal displacement while allowing the nodes in the tangential direction to move freely. Gravity is applied to the model for calculation, and the computation stops when the ratio of unbalanced forces within the model reaches 10^{-5} . This threshold is the default convergence criterion in FLAC^{3D} for static analysis and is commonly accepted in geotechnical engineering practice. It ensures that subsequent calculation steps result in negligible changes to the stress and displacement fields, thereby assuring the reliability of the numerical results [32]. The results of the calculation are shown in Figure 14.

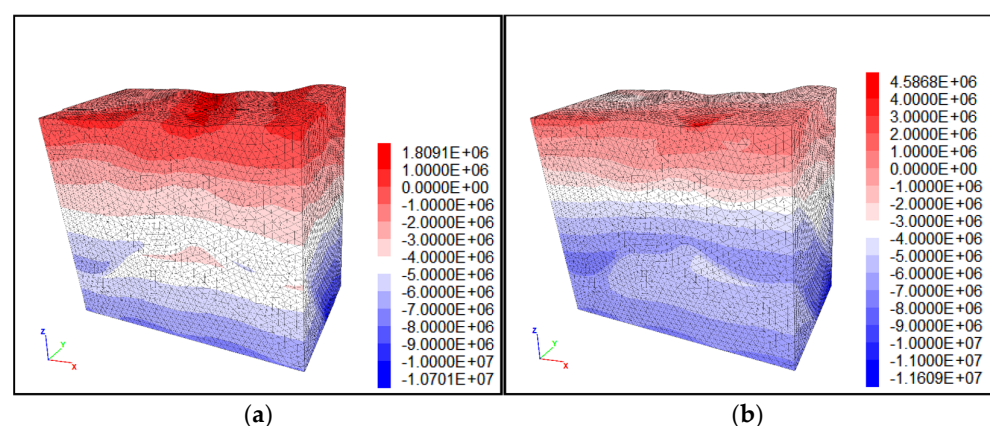


Figure 14. Cont.

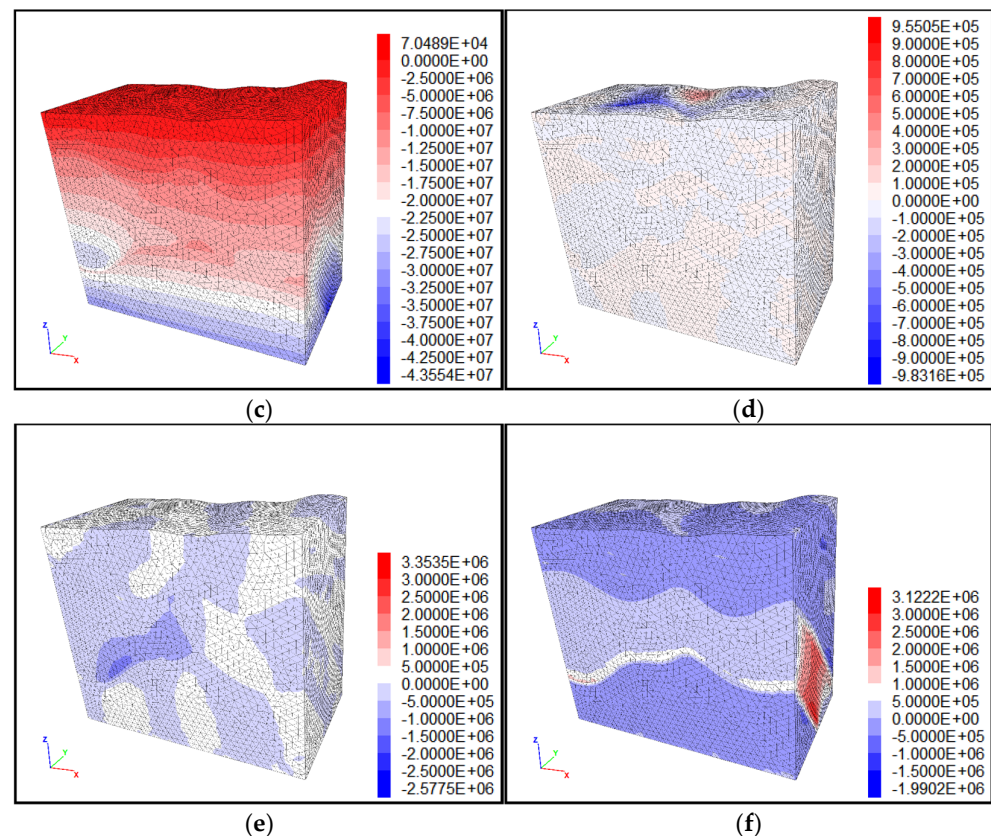


Figure 14. Stress distribution under the effect of self-weight stress and structural movement: (a) Stress distribution in the XX direction; (b) stress distribution in the YY direction; (c) stress distribution in the ZZ direction; (d) stress distribution in the XY direction; (e) stress distribution in the XZ direction; (f) stress distribution in the YZ direction.

From the stress contour map obtained through the simulation analysis, it can be observed that when considering the effects of rock mass self-weight and structural movement, both the vertical principal stress and horizontal principal stress increase with depth. At the bottom of the model, the stress values reach their maximum.

5. Discussion

To clarify the methodology, the 23 in situ stress data points used in this study are independent field measurements obtained from hydraulic fracturing tests in boreholes ZKB19 and ZKB39 at the Xishan Iron Mine. The rock mechanical parameters were determined from independent laboratory tests on core samples, combined with engineering geological data from the study area. These measured stresses serve as target values for the multiple linear regression and subsequent nonlinear iterations. The comparison between fitted and measured stresses presented throughout this paper is designed to evaluate the accuracy improvement achieved by the proposed nonlinear iterative method relative to conventional linear regression.

To more comprehensively evaluate the proximity between the fitted in situ stress and the measured in situ stress under the two fitting methods, a comparison is made between the fitted and measured in situ stresses to analyze their correlation. In the rock mass, the in situ stress at any given point can be represented by the three principal stresses σ_1 , σ_2 , and σ_3 . The mean stress p , the second deviatoric stress invariant J_2 , and the Lode parameter μ are directly related to the three principal stresses. Therefore, the evaluation of the fitting focuses on the three principal stresses, mean stress p , the second deviatoric stress invariant

J_2 , and the Lode parameter μ . The calculation methods for p , J_2 , and μ are provided in Equations (18), (19) and (20), respectively.

$$p = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3} \quad (18)$$

$$J_2 = \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \quad (19)$$

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \quad (20)$$

In the field of rock mechanics, error analysis typically considers both the mean absolute error and the mean squared error [42]. The calculation formulas for these are provided in Equations (21) and (22). Based on the principal stresses σ_1 , σ_2 , and σ_3 , using Equations (18)–(22), the mean absolute error m and mean squared error n between the fitted and measured values are calculated. The results of the error comparison are presented in Table 9.

$$m = \frac{1}{N} \sum_{i=1}^N |Actual\ measured\ value - Fitted\ value| \quad (21)$$

$$n = \frac{1}{N} \sum_{i=1}^N (Actual\ measured\ value - Fitted\ value)^2 \quad (22)$$

Table 9. Comparison of the error between the fitted in situ stress values and the measured values under different methods.

Fitting Method	m			n		
	p (MPa)	J_2 (MPa)	μ	p (MPa)	J_2 (MPa)	μ
linear fitting	4.50	49.42	0.59	27.44	2981.92	1.14
nonlinear fitting	first iteration	2.29	35.12	0.51	7.85	1422.37
	second iteration	1.90	24.88	0.46	5.60	783.81
	third iteration	1.89	24.85	0.46	5.58	783.51
	fourth iteration	1.89	24.85	0.46	5.58	783.51

Table 9 shows that, compared to the results from the linear regression method, the in situ stress field fitting values exhibit a 58% reduction in mean stress error, a 50% reduction in deviatoric stress error, and a 22% reduction in the Lode parameter error. Based on the variations in the three consideration indices, mean stress p , second deviatoric stress invariant J_2 , and Lode parameter μ , the first iteration of the iterative fitting algorithm for the in situ stress field, which takes nonlinear characteristics into account, results in the greatest reduction in the error between the fitted and measured in situ stress values. The nonlinear method demonstrates significant advantages across all three evaluation indices. The fitted in situ stress values become closer to the measured values. As the number of iterations increases, the error between the fitted and measured in situ stress values gradually approaches a stable value. This indicates that the in situ stress field obtained using this method converges to a stable solution, confirming that this fitting method improves the inversion accuracy of the in situ stress field.

Considering the constraints of data availability from a single engineering site, future work should extend the validation of the proposed nonlinear iterative inversion method to other deep engineering projects, including deep tunnels, non-metallic mines, and complex lithological combinations such as soft–hard rock interbedded formations, to further demonstrate its universality and robustness. It is important to note that the current model does not account for unsaturated conditions, high ground temperatures, or groundwater

effects. In the Xishan Iron Mine study area, groundwater content is relatively low, and moderate runoff conditions minimize these influences. Future research will expand the nonlinear inversion method to saturated rock masses and incorporate temperature effects to evaluate their impact on the accuracy of the in situ stress field under more complex hydro-thermal-mechanical coupling conditions.

Additionally, the confining pressure range of the triaxial compression tests (5–40 MPa) is lower than the in situ stress levels at the deepest measurement points (up to 937 m depth). This limitation stems from the equipment constraints of the MTS815 testing system, which cannot safely apply confining pressures exceeding 40 MPa. Therefore, the nonlinear K - p and G - p relationships were calibrated within this tested range, and extrapolating to higher stresses introduces some uncertainty. Future studies should utilize high-pressure triaxial apparatus to validate the model's applicability under deeper, higher-stress conditions.

It is also noted that the current evaluation primarily focuses on mean absolute error and mean squared error to demonstrate the superiority of the nonlinear iterative method over the linear approach. While residual distribution analysis could provide further insights, it falls outside the scope of this study. Future work may incorporate such diagnostics to refine the inversion model and assess its predictive uncertainty.

6. Conclusions

In situ stress inversion is fundamental to the analysis and design of underground engineering and is a prerequisite for the excavation and design of various underground projects. Currently, in situ stress measurements are typically point-based; however, to obtain the overall in situ stress distribution in an engineering area, fitting of the in situ stress field is required. This paper addresses the nonlinear characteristics of deep rock masses and, based on in situ stress measurement data, develops a numerical model. It proposes an iterative fitting algorithm for the rock mass in situ stress field, considering nonlinear characteristics, with linear initial values, nonlinear piecewise iteration, and three control parameters: mean stress, deviatoric stress, and Lode parameter. The main conclusions are as follows:

1. Rock mass mechanical parameters in deep underground environments exhibit pronounced stress-dependent nonlinear behavior that cannot be adequately represented by constant elastic properties. Laboratory tests on multiple representative lithologies confirm that both elastic modulus and Poisson's ratio vary significantly with stress level. Nonlinear fitting based on stress-dependent bulk and shear moduli provides a more accurate description of rock deformation than conventional linear fitting, demonstrating the necessity of incorporating nonlinear mechanical behavior in in situ stress analysis for deep underground engineering.
2. A nonlinear iterative inversion framework for in situ stress fields was developed by embedding stress-dependent constitutive relationships into numerical modeling and iterative fitting procedures. The proposed method integrates linear regression based initial stress estimation with nonlinear parameter updating governed by mean stress, the second deviatoric stress invariant, and the Lode parameter. The iterative process exhibits stable convergence characteristics, with most stress corrections occurring in the initial iteration, indicating stable numerical performance and effective convergence behavior.
3. Compared with the traditional linear regression-based inversion methods, the proposed nonlinear approach significantly enhances the accuracy and reliability of in situ stress field characterization for deep underground engineering. Quantitative error analysis reveals substantial reductions in errors associated with mean stress, the second deviatoric stress invariant, and the Lode parameter, highlighting the practical

advantages of the method for engineering design and safety assessment. The proposed nonlinear numerical simulation method provides an accurate and scalable approach for in situ stress field inversion under complex geological and mechanical conditions.

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