

Article

Mechanisms of Liquid Imbibition in Coal Controlled by Physical Structural Characteristics: A Theoretical and Experimental Study

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Abstract

Coal seam water injection is a widely used technique for dust suppression in coal mining. However, its effectiveness is often limited in low-permeability coal seams with poor wettability, primarily due to an unclear understanding of the wetting mechanism. To address this issue, a spontaneous imbibition model of a curved capillary bundle incorporating surface roughness elements was established based on the Hagen–Poiseuille equation. The accuracy of the proposed model was validated through experimental measurements, and the effects of different structural parameters on liquid imbibition behavior were systematically investigated. The results indicate that an increase in pore area fractal dimension significantly enhances the spontaneous imbibition capacity of water within coal pores. Moreover, the relative roughness inside pores increases with the minimum pore diameter, leading to a higher imbibition height. This suggests that increased relative roughness strengthens capillary attraction and promotes water migration in the pore structure. The proposed model provides a theoretical description applicable to the wetting stage of coal seam water injection and offers valuable insights for improving dust suppression efficiency in low-permeability coal seams.

Keywords: mine dust control; coal seam water injection; imbibition model; surface morphology; fractal dimension



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1. Introduction

Coal seam water injection, as a primary dust suppression measure in coal mining, involves injecting pressurized water into the seam in advance through boreholes, allowing the water to fully infiltrate the coal seam, thereby effectively reducing dust generation during mining [1–5]. The imbibition process in coal seam water injection is essentially the migration of liquid driven by capillary forces and wettability through pores and fractures into the coal matrix. This process directly determines the distribution range and the duration of water retention in the seam [6–8]. Good imbibition capacity ensures that water can be distributed more uniformly and penetrate deeper into the coal, thereby disrupting the surface structure of dust particles, enhancing wettability, and suppressing dust generation and dispersion.

The imbibition process is regulated by multiple factors, including temperature, pressure, fluid properties, mineral composition, pore characteristics, and initial moisture state [9–11]. Various experimental methods have been employed to investigate the mechanism of spontaneous imbibition in coal. Yue et al. [12], based on a combination of dimensionless analysis and electrode monitoring techniques, clarified the coupling mechanism of water–gas migration during imbibition. Zhou et al. [13] optimized the determination method of surface relaxation rate in low-permeability coal using nuclear magnetic resonance (NMR) technology, revealing the dynamic distribution of water in multiscale pores. Lu et al. [14], by combining X-ray μ -CT imaging and scanning electron microscopy, elucidated the mechanism by which pore structure and mineral composition influence imbibition behavior and demonstrated that an increase in inert components significantly enhances imbibition height and wettability. Xu et al. [15], through NMR dynamic monitoring, found that dense sandstone systems exhibit remarkable hydrophilicity, with the imbibition process primarily governed by micropores. In terms of theoretical analysis, extensive studies have been conducted on capillary flow in porous media, leading to the development of a variety of theoretical models. The Lucas–Washburn equation, established in the early 19th century, laid the theoretical foundation for liquid flow in capillaries. Subsequent researchers expanded upon this model in multiple aspects. Ramon et al. [16] introduced interfacial phase transition effects into capillary flow dynamics, improving the model by accounting for evaporation and condensation mass transfer. Fries et al. [17], from a momentum balance perspective, derived an analytical solution that not only extended the applicability of the Lucas–Washburn equation but also quantitatively evaluated the error range when gravity was neglected. Later, Wang et al. [18] established analytical solutions for spontaneous imbibition in single fractures under both gravity and non-gravity conditions, and further developed a semi-analytical model for wetting-phase imbibition in gas-saturated fractured porous media, ultimately deriving precise expressions for wetting-phase mass variation under gravity. Qin et al. [19] proposed a novel pore-network model for porous media, which can accurately predict imbibition rates and saturation–time curves under varying viscosity ratios and wettability conditions and distinguished between spontaneous and quasi-static gas imbibition mechanisms in terms of pore filling. Xue et al. [20] integrated fractal theory, the law of flow proposed by Poiseuille, and Darcy’s law to establish a fractal permeability model. From a theoretical perspective, they analyzed the mechanical response mechanism of tortuous capillaries in porous media and elucidated the dynamic evolution relationship between their geometric morphology and permeability.

The above studies have investigated the imbibition phenomenon within porous media and its influencing factors from both experimental and theoretical perspectives. Although these efforts have greatly enhanced the understanding of capillary flow and spontaneous imbibition in porous materials, the actual process of water injection in coal seams is considerably more complex. During coal seam water injection, the migration behavior of water within the fracture–pore network of coal porous media is affected by multiple factors, such as coal structural characteristics, in situ stress, and gas pressure. In addition, coal particles generated during drilling operations, as well as those inherently present within the coal structure, may be carried by the flowing water into seepage channels. This results in significant accumulation of water-borne coal particles within fractures and pores, forming a packed state. The coupled migration and interaction between water and coal particles consequently influence the transport behavior and wetting performance during coal seam water injection. Under such complex particle packing conditions, the imbibition process within coal cannot be adequately described by the classical capillary tube model. To address this limitation, this study establishes a fractal imbibition model applicable to different packed coal particle states. Based on the traditional capillary bundle imbibition model,

a roughness correction factor is introduced to account for the influence of physical structural characteristics on imbibition behavior. The proposed model incorporates structural parameters, including the pore area fractal dimension, tortuosity fractal dimension, and relative roughness. The accuracy of the theoretical model is validated through spontaneous imbibition experiments, particle size analysis, and atomic force microscopy measurements. The findings provide new physical insights into the effects of particle packing states and coal physical structural characteristics on water injection wetting performance in coal seams and offer theoretical support for improving dust suppression efficiency.

2. Construction of Wetting Model Based on Fractal Characteristics of Coal Structure

2.1. Construction of Surface Roughness Elements

Since the surface of coal particles is rough, the surface of the capillary bundle model formed by the accumulation of coal particles should also be rough. Therefore, a certain number of frustum-shaped elements of different sizes were introduced on the inner wall of the capillary to represent surface roughness. In this study, the rough elements are assumed to be frustums, with the upper radius denoted as r_1 and the lower radius as r_2 , and it is assumed that $\theta = 60^\circ$. The corresponding height $h = \sqrt{3}(r_2 - r_1)$ can then be calculated, as shown in Figure 1. It should be emphasized that the truncated cone geometry is an equivalent representation rather than the actual shape of the coal. The purpose is to introduce a mathematically tractable roughness element with a gradually varying cross-section to approximate the volumetric occupation effect of irregular asperities.

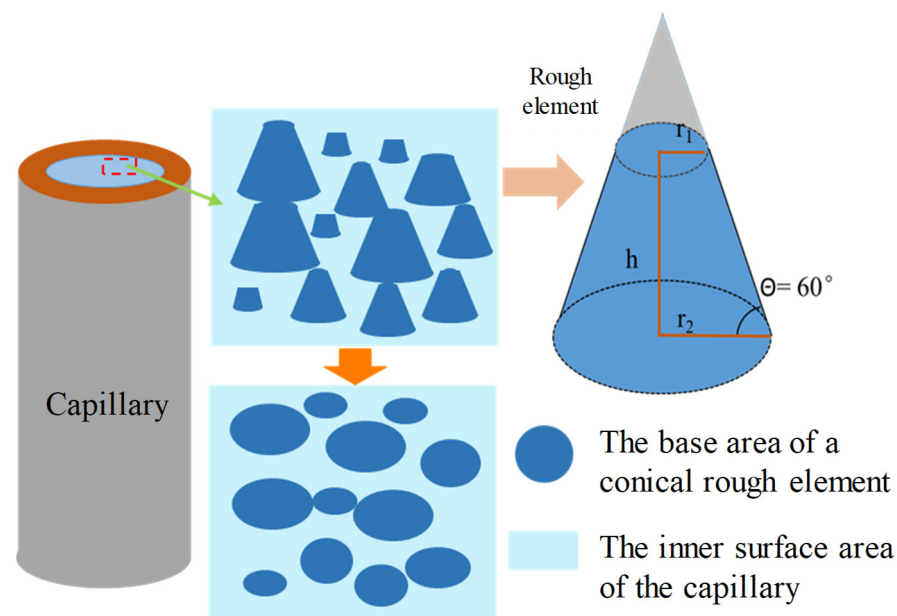


Figure 1. Schematic diagram of the distribution of rough elements.

The base area of the frustum-shaped element can be expressed as follows:

$$S = \pi r_2^2 = \pi \xi^2 r_1^2 \quad (1)$$

The volume of the frustum-shaped element can be expressed as follows:

$$V = \frac{1}{3} \pi h (r_1^2 + r_1 r_2 + r_2^2) = \frac{\sqrt{3}}{3} \pi r_1^3 (\xi - 1) (1 + \xi + \xi^2) \quad (2)$$

By integrating Equations (1) and (2) within the range from the minimum base diameter d_{min} to the maximum base diameter d_{max} , the total base area and total volume of the rough elements can be obtained. The probability density distribution function and the calculation method of the fractal dimension for the rough elements are consistent with those of the capillary bundle.

$$S_1 = - \int_{r_{1,min}}^{r_{1,max}} \pi \xi^2 r_1^2 dN = \pi \xi^2 \frac{Dr_{1,max}^2}{2-D} (1 - \alpha^{2-D}) \quad (3)$$

$$V_1 = - \int_{r_{1,min}}^{r_{1,max}} V dN = \frac{\sqrt{3}}{3} \pi D (\xi - 1) (1 + \xi + \xi^2) r_{1,max}^3 \frac{1 - \alpha^{3-D}}{3-D} \quad (4)$$

Here, α denotes the ratio of the minimum to the maximum base radius of rough elements within the same capillary, expressed as ($\alpha = \frac{r_{1,min}}{r_{1,max}}$), $D = 2 - \frac{\ln \varphi_s}{\ln \alpha}$, and φ_s represents the ratio of the sum of the base areas of the rough elements to the total inner surface area of the capillary.

$$S_1 = \pi \xi^2 \frac{Dr_{1,max}^2}{2-D} (1 - \varphi_s) \quad (5)$$

The surface area of the capillary wall can be expressed as follows:

$$S_t = \frac{S}{\varphi_s} = \frac{\pi}{\varphi_s} \xi^2 \frac{Dr_{1,max}^2}{2-D} (1 - \varphi_s) \quad (6)$$

In this study, it is assumed that, due to self-similarity, the relative roughness of capillaries with different diameters remains the same, i.e., $\frac{(h_{max})_{\lambda_{min}}}{(h_{max})_{\lambda}} = \frac{\lambda_{min}}{\lambda}$. Moreover, since $h = \sqrt{3}(r_2 - r_1) = \sqrt{3}(\xi - 1)r_1$, it follows that $\frac{(r_{1,max})_{\lambda_{min}}}{(r_{1,max})_{\lambda}} = \frac{\lambda_{min}}{\lambda}$. Therefore, it can be concluded that:

$$h_{eff} = \frac{V_t}{S_t} = \frac{\sqrt{3}}{3} \frac{(\xi - 1)(1 + \xi + \xi^2)}{\xi^2} \frac{2-D}{3-D} \frac{r_{1,max}(1 - \alpha^{3-D})\varphi_s}{(1 - \varphi_s)} = \frac{(1 + \xi + \xi^2)\lambda(h_{max})_{\lambda_{min}}}{3\xi^2(\lambda_{min})} \frac{2-D}{3-D} \frac{(1 - \alpha^{3-D})\varphi_s}{(1 - \varphi_s)} \quad (7)$$

The relative roughness can be expressed as follows:

$$\varepsilon = \frac{2h_{eff}}{\lambda} = \frac{2(1 + \xi + \xi^2)(h_{max})_{\lambda_{min}}}{3\xi^2(\lambda_{min})} \frac{2-D}{3-D} \frac{(1 - \alpha^{3-D})\varphi_s}{(1 - \varphi_s)} \quad (8)$$

2.2. Spontaneous Imbibition Model Considering Rough Capillary Bundles

2.2.1. Model Construction

Imbibition is a ubiquitous phenomenon in nature. During coal seam water injection, coal acts as a porous medium with a complex pore structure. The seepage process occurs through numerous pore channels within the coal, where water migrates driven by capillary forces, gradually replacing the original solid–gas interface with a solid–liquid interface and thereby wetting the coal matrix. The main principle of the spontaneous imbibition experiment is that water flows through curved capillary channels under the combined effects of gravity and capillary forces. Based on fractal theory and the capillary bundle model, a capillary bundle model considering roughness and tortuosity is derived. Fractal theory and capillary bundle models have been proven to be effective approaches for characterizing the pore structure of porous media. In this study, the porous medium is assumed to consist of a bundle of capillaries with different radii. The number of capillaries follows a fractal distribution and can be expressed using fractal theory as follows:

The imbibition of fluid in a single capillary, under the combined action of capillary pressure P_c (as shown in Equation (9)) and gravity, is described by the modified Hagen–Poiseuille equation (Equation (10)) [21].

$$P_c = 4\sigma \cos \theta / \lambda \quad (9)$$

$$q(\lambda) = \frac{\pi}{128} \frac{\lambda_c^4}{\mu L_t(\lambda)} \left(\frac{4\sigma \cos \theta}{\lambda} - \rho g L_0 \right) \quad (10)$$

Here, q denotes the flow rate in a single capillary channel, $L_t(\lambda)$ is the actual length of the capillary, σ is the surface tension of deionized water, with $\sigma = 0.072$ N/m, μ is the dynamic viscosity coefficient of water; ρ is the density of water, with $\rho = 1$ g/cm³, g is the gravitational acceleration, taken as $g = 9.8$ m/s², L_0 is the imbibition height at time T , and λ_c^4 represents the diameter of the capillary bundle imbibition channel, considering roughness.

$$\lambda_c = (1 - \varepsilon)\lambda \quad (11)$$

It should be noted that surface roughness may influence capillary transport through two primary mechanisms: (1) increasing hydraulic resistance by enlarging the flow path length and inducing additional viscous dissipation, and (2) modifying the effective contact angle at the solid–liquid interface. In the present model, roughness is incorporated into Equation (11) through an effective capillary diameter correction, expressed as $\lambda_c = (1 - \varepsilon)\lambda$.

For imbibition through a bundle of curved capillaries, by combining Equations $-dN = D_f \lambda_{max}^{D_f - (D_f + 1)}$ and $L_t(\lambda) = (\lambda)^{1-D_T} L_0^{D_T}$ [22,23], and integrating Equation (11) from the minimum capillary diameter λ_{min} to the maximum capillary diameter λ_{max} , the total flow rate Q can be obtained, as shown in Equation (12).

$$Q = \int_{\lambda_{min,c}}^{\lambda_{max,c}} q(\lambda) dN = \frac{\pi \sigma \cos \theta D_f \lambda_{max,c}^{2+D_T}}{32 \mu L_0^{D_T}} \frac{1 - \beta^{2+D_T-D_f}}{2 + D_T - D_f} - \frac{\rho g \pi L_0^{1-D_T} D_f \lambda_{max,c}^{3+D_T}}{128 \mu} \frac{1 - \beta^{3+D_T-D_f}}{3 + D_T - D_f} \quad (12)$$

In the equation, D_f is the fractal dimension of the pore area, and D_T is the fractal dimension of tortuosity. $\beta = \frac{\lambda_{min,c}}{\lambda_{max,c}}$, $D_T = 1 + \frac{\ln \tau_{av}}{\ln \left(\frac{L_0}{\lambda} \right)}$, $\tau_{av} = [1 + \sqrt{1 - \varphi} / 2 + \sqrt{1 - \varphi} \sqrt{(1 / \sqrt{1 - \varphi} - 1)^2 + 1/4} / (1 - \sqrt{1 - \varphi})] / 2$ [24].

The analytical relationship among the fractal dimension of pore area, the porosity of fractal porous media, and the self-similar scale can be expressed as Equation (13) [25]

$$D_f = D - \frac{\ln \varphi}{\ln \beta} \quad (13)$$

In this study, a two-dimensional plane is mainly used as the cross-sectional view of pore structures in particle packing. Therefore, the planar dimension is taken as $D = 2$, and the following equation can be obtained:

$$D_f = 2 - \frac{\ln \varphi}{\ln \beta} \quad (14)$$

where φ is the porosity.

$$\varphi = 1 - [(m / \rho_c) / (S l_0)] \quad (15)$$

In the equation, m is the mass of coal particles required for the experiment, ρ_c is the true density of coal powder, s is the base area of the coal particle column, with $s = 0.3848$ cm², and l_0 is the height of the coal particle column, with $l_0 = 3.5$ cm.

By integrating the circular area formula, the total cross-sectional area A_p of a bundle of curved capillaries can be obtained [26]:

$$A_p = \int_{\lambda_{\min,c}}^{\lambda_{\max,c}} \frac{\pi \lambda^2}{4} dN = \frac{\pi D_f \lambda_{\max,c}^2 (1 - \varphi)}{4(2 - D_f)} \quad (16)$$

By substituting Equations (12) and (16) into the expression of the actual average imbibition velocity given in Equation (17), the relationship between the average imbibition rising velocity v_s and the capillary diameter can be obtained, as shown in Equation (17).

$$V_s = \frac{Q}{A_p} = \frac{dL}{dT} \quad (17)$$

$$V_s = \frac{Q}{A_p} = \frac{\sigma \cos \theta \lambda_{\max,c}^{D_T} (2 - D_f)}{8\mu L_0^{D_T} (1 - \varphi)} \frac{1 - \beta^{2+D_T-D_f}}{2 + D_T - D_f} - \frac{\rho g L_0^{1-D_T} \lambda_{\max,c}^{1+D_T} (2 - D_f)}{32\mu (1 - \varphi)} \frac{1 - \beta^{3+D_T-D_f}}{3 + D_T - D_f} = \frac{dL}{dT} \quad (18)$$

By setting $dL_0/dT = 0$, the theoretical steady-state imbibition height L_e and its relationship with the capillary diameter can be obtained, as shown in Equation (19) [27,28].

$$L_e = \frac{4\sigma \cos \theta}{\rho g \lambda_{\max,c}} \frac{1 - \beta^{2+D_T-D_f}}{1 - \beta^{3+D_T-D_f}} \frac{3 + D_T - D_f}{2 + D_T - D_f} \quad (19)$$

By substituting Equation (11) into Equation (19), Equation (20) can be obtained, which represents the equilibrium imbibition height considering rough and curved capillaries.

$$L_e = \frac{4\sigma \cos \theta}{\rho g (1 - \varepsilon) \lambda_{\max}} \frac{1 - \beta^{2+D_T-D_f}}{1 - \beta^{3+D_T-D_f}} \frac{3 + D_T - D_f}{2 + D_T - D_f} \quad (20)$$

2.2.2. Capillary Channel Theoretical Model Based on Coal Particle Characteristics

The key parameters for obtaining the value of L_e in Equation (20) are the maximum and minimum capillary diameters, $\lambda_{\max}$ and $\lambda_{\min}$. However, it is difficult to directly measure the diameters of capillary bundles experimentally. The flow channels are often approximated as curved capillary channels, whose diameters are related to the packing shape and particle size of coal. Accordingly, some researchers have proposed theoretical calculation methods to estimate pore diameters based on solid particle parameters [22]. For particles of different sizes, the imbibition channels formed under different packing arrangements vary, resulting in different effective diameters. In this study, three packing arrangements—triangular packing, square packing, and random packing—are selected as the research objects, as illustrated in Figure 2.

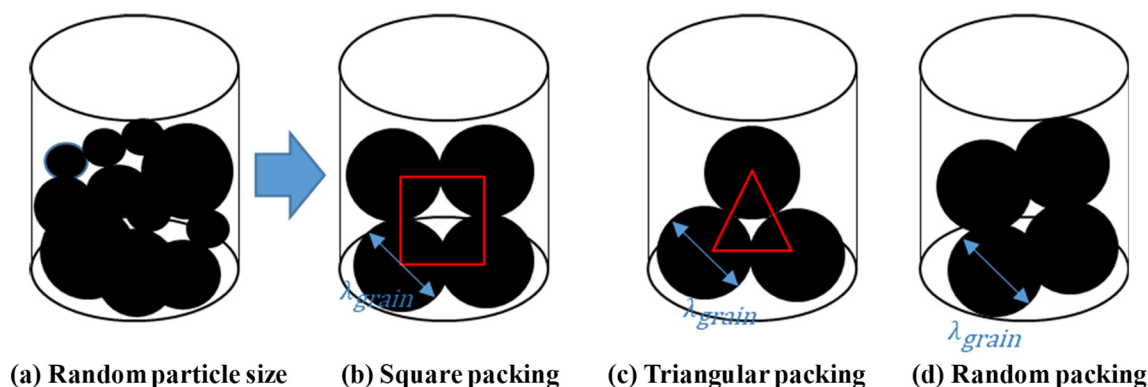


Figure 2. Particle packing state.

Square packing, as shown in Figure 2, forms flow channels through the pores created by four particles of the same diameter. These channels are irregular, so the central pore is considered as the flow channel and assumed to have a circular interface. The channel diameter is represented by an equivalent diameter.

(1) Maximum pore model for square packing: As illustrated in Figure 3, the relationship between the maximum pore area A_{max1} and the porosity φ is given by the following equation:

$$A_{max1} = \frac{\pi}{4} \lambda_{max1}^2 = \frac{\pi}{4} \lambda_{grain}^2 \frac{\varphi}{1 - \varphi} \quad (21)$$

$$\lambda_{max1} = \lambda_{grain} \sqrt{\frac{\varphi}{1 - \varphi}} \quad (22)$$

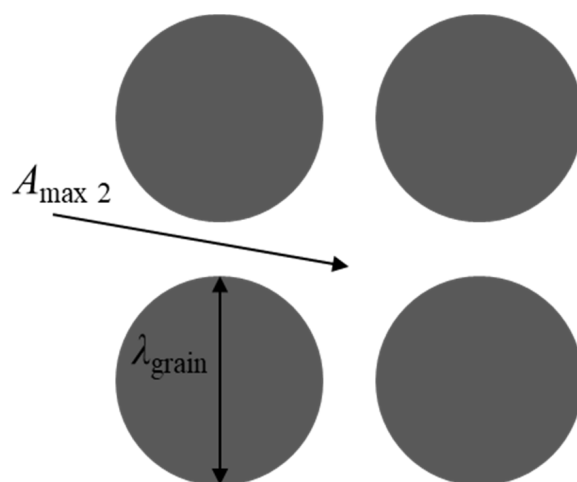


Figure 3. Maximum pore model for square packing of solid particles.

(2) Minimum pore and capillary diameter: As shown in Figure 4, the pore area A_{min} formed by particle packing has a certain relationship with the cross-sectional area of the solid particles, as expressed in Equation (23). Based on this relationship, the minimum capillary diameter λ_{min} can be calculated, as given in Equation (24).

$$A_{min} = \frac{\pi}{4} \lambda_{min}^2 = \frac{\sqrt{3}}{4} \lambda_{grain}^2 - \frac{\pi}{8} \lambda_{grain}^2 \quad (23)$$

$$\lambda_{min} = \frac{\lambda_{grain}}{\sqrt{\pi}} \sqrt{\sqrt{3} - \frac{\pi}{2}} \quad (24)$$

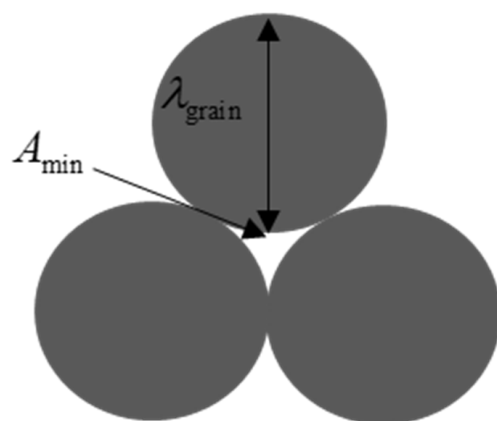


Figure 4. Minimum pore model formed in particle packing.

The maximum pore model for triangular packing is shown in Figure 5. The relationship between the maximum pore area A_{max2} and the porosity φ is given by the following equation:

$$A_{max2} = \frac{\pi}{4} \lambda_{max2}^2 = \frac{\pi}{8} \lambda_{grain}^2 \frac{\varphi}{1-\varphi} \quad (25)$$

$$\lambda_{max2} = \lambda_{grain} \sqrt{\frac{\varphi}{2(1-\varphi)}} \quad (26)$$

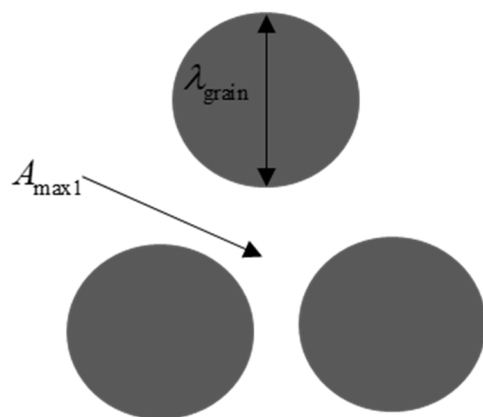


Figure 5. Maximum pore model formed in the triangular packing arrangement of solid particles.

(3) Random packing of coal particles: In the naturally packed state, all coal particles are randomly arranged. It is assumed that all particles have the same diameter, represented by an equivalent particle diameter λ_{grain} . In this natural state, the imbibition channels are more complex, and a simple derivation is not feasible. Therefore, the pore-specific surface area is introduced, as shown in Equation (27), where λ_3 denotes the capillary diameter and L_3 denotes the capillary length.

$$S_{por} = \frac{S_1}{V_1} = \frac{\pi \lambda_3 L_3}{\frac{1}{4} \pi \lambda_3^2 L_3} = \frac{4}{\lambda_3} \quad (27)$$

It can be assumed that, within a certain scale, the total pore surface area S_1 is equal to the total particle surface area S_2 [29–31], i.e.,

$$S_1 = S_2 \quad (28)$$

Meanwhile, according to the definition of porosity, within the same scale, the pore volume V_1 and the particle volume V_2 satisfy the following relationship:

$$\frac{V_1}{V_2} = \frac{\varphi}{1-\varphi} \quad (29)$$

Based on Equations (28) and (29), the specific surface area of pores S_{por} in the porous medium can be obtained as follows:

$$S_{por} = \frac{S_1}{V_1} = \frac{S_2}{V_2 \cdot \frac{\varphi}{1-\varphi}} = \frac{\pi \lambda_{grain}^2}{\frac{1}{6} \pi \lambda_{grain}^3 \cdot \frac{\varphi}{1-\varphi}} = \frac{6}{\lambda_{grain}} \frac{1-\varphi}{\varphi} \quad (30)$$

By substituting into Equation (27), it can be obtained as follows:

$$\lambda_{max3} = \lambda_3 = \frac{2}{3} \lambda_{grain} \frac{\varphi}{1-\varphi} \quad (31)$$

3. Experimental Validation of the Spontaneous Imbibition Model

3.1. Determination of Key Parameters in the Theoretical Model of Spontaneous Imbibition

3.1.1. Coal Sample Preparation and Parameter Measurement

The coal samples used in this experiment were obtained from Xinglongzhuang Mine. Prior to testing, the lump coal samples were crushed into powder. After crushing, the coal samples were sieved using mesh screens of different sizes and divided into four groups: 40–60 mesh, 60–80 mesh, 80–100 mesh, and 100–120 mesh. Following sieving, the samples were dried in an oven at 105 °C for 2 h, then cooled and sealed in airtight bags for preservation. The particle size distribution of the coal samples was determined using a Malvern 2000 laser particle size analyzer. The test results are shown in Table 1.

Table 1. Particle size characteristic parameters of coal samples.

Coal Sample	Particle Size Range	Characteristic Particle Size/ μm					$S_{\text{solid}}/\mu\text{m}^{-1}$
		Minimum Particle Size	Maximum Particle Size	Median Particle Size	Average Particle Size of Surface Area	Volumetric Average Particle Size	
XLZ	40~60	168.05	808.30	617.85	344.60	384.37	0.0192
	60~80	143.05	326.02	322.69	218.00	236.39	0.0217
	80~100	106.05	383.00	212.40	169.77	213.26	0.0221
	100~120	93.10	227.21	113.05	94.19	114.90	0.0237

By measuring the particle size distribution, the results show that the particle size of coal powder in the 40–60 mesh group ranges from 168.05 to 808.30 μm , that in the 60–80 mesh group ranges from 143.05 to 326.02 μm , that in the 80–100 mesh group ranges from 106.05 to 383.00 μm , and that in the 100–120 mesh group ranges from 93.10 to 227.21 μm . The effective porosity of the coal samples was calculated using Equation (15), and the results are presented in Table 2.

Table 2. Parameters related to porosity calculation.

Coal Sample	Particle Size Range	m/g	$\rho_c/\text{g}/\text{cm}^3$	φ
XLZ	40~60	1.453	1.36	0.2068
	60~80	1.420	1.36	0.2248
	80~100	1.505	1.36	0.1754
	100~120	1.436	1.36	0.2161

3.1.2. Determination of Capillary Radius

Equations (21)–(31) establish a functional relationship between coal particle characteristics and capillary diameter. According to these equations, the capillary diameter can be further determined based on particle characteristic parameters. However, under actual water injection mining conditions, the particle size of the filling state in fractures and pores at certain specific locations within the coal body generally falls within a fixed range, with a large difference between the maximum and minimum particle sizes, which makes it difficult to accurately reflect the real situation. Therefore, in this study, the characteristic particle size of coal, λ_{grain} , was obtained using the following method, and combined with Equations (21)–(31) to derive the capillary diameter for further analysis.

The minimum particle size of coal used in the experiment was selected as the equivalent particle size, denoted as $\lambda_{\text{grain}, 1}$, the maximum particle size of coal used in the

experiment was selected as the equivalent particle size, denoted as $\lambda_{\text{grain}, 2}$. Since the surface area of coal particles affects the contact between water and coal powder particles, smooth spherical particles must be consistent with the coal particles used in the experiment. Based on the calculation formula for surface area, an equivalent particle size can be obtained as $\lambda_{\text{grain}, 3} = 6/S_{\text{solid}}$, where S_{solid} represents the specific surface area of coal powder measured in the particle size analysis, which is defined as $\lambda_{\text{grain}, 3}$. The equivalent particle size $\lambda_{\text{grain}, 4}$ refers to the median size of the coal particles, corresponding to 50% of the particle size distribution. The equivalent particle size $\lambda_{\text{grain}, 5}$ is defined as the surface-area-weighted mean particle size from the particle size distribution test. The equivalent particle size $\lambda_{\text{grain}, 6}$ is defined as the volume-weighted mean particle size obtained from the particle size distribution test.

Based on the statistical data of the six equivalent particle sizes, the corresponding imbibition parameters, including D_f , $\lambda_{\text{max}1}$, $\lambda_{\text{max}2}$, $\lambda_{\text{max}3}$, λ_{min} , were obtained; the results are shown in Table 3. Regarding the relationship between the three types of pore fractal dimensions and the imbibition height, this study carried out a theoretical analysis of the three maximum pore sizes in combination with numerical simulation results. The results indicate that the maximum pore size occurs when particles are arranged in a square packing configuration.

Table 3. Calculated imbibition parameters of XLZ coal sample.

Particle Size Range	φ	$\lambda_{\text{grain}}/\mu\text{m}$	$\lambda_{\text{max}1}/\mu\text{m}$	$\lambda_{\text{max}2}/\mu\text{m}$	$\lambda_{\text{max}3}/\mu\text{m}$	$\lambda_{\text{min}}/\mu\text{m}$	D_f
40~60	0.2068	168.05	85.806	48.127	29.208	6.139	1.234
	0.2068	808.30	412.720	231.485	140.491	29.531	1.234
	0.2068	312.50	159.563	89.495	54.315	11.417	1.234
60~80	0.2248	143.05	77.033	42.225	27.655	5.226	1.285
	0.2248	426.02	229.414	125.753	82.360	15.564	1.285
	0.2248	276.49	148.891	81.614	53.452	10.101	1.285
80~100	0.1754	106.05	48.910	28.518	15.038	3.874	1.127
	0.1754	383.00	176.641	102.995	54.311	13.992	1.127
	0.1754	261.49	120.600	70.319	37.080	9.553	1.127
100~120	0.2161	80.1	42.056	23.311	14.720	2.926	1.261
	0.2161	227.21	119.295	66.125	41.757	8.301	1.261
	0.2161	210.16	110.343	61.163	38.623	7.678	1.261

3.1.3. Determination of Relative Roughness

The surface morphology of coal samples was characterized using atomic force microscopy (AFM). Representative AFM images of the tested coal samples are shown in Figure 6. For each group of coal samples, three images were selected from different observation angles in order to comprehensively capture the surface morphological features. The scanning area is $3 \mu\text{m} \times 3 \mu\text{m}$.

From the AFM images, it can be observed that the internal surfaces of the channels formed by particle packing exhibit pronounced protrusions, which can be approximated as truncated-cone-shaped roughness elements. Moreover, the degree of surface roughness of the coal compacts varies significantly with particle size. As the particle diameter decreases, the surface of the compact tends to become smoother, and the surface morphology appears more uniform.

To better capture the actual distribution of surface roughness elements and quantify their areal proportion, the AFM images were further processed using ImageJ, version 1.53u (National Institutes of Health, Bethesda, MD, USA). The average proportions of protrusions for the four groups of coal samples were calculated to be 14.742%, 9.931%, 9.801%, and 8.261%, respectively. These results demonstrate that the proportion of roughness

elements differs under different particle size packing conditions. Based on the average protrusion proportions obtained from AFM images, the values were substituted into Equations (1)–(8) to calculate the relative roughness for subsequent analysis. As shown in Table 4, the proportion of surface protrusions decreases progressively with decreasing coal particle size, and the relative roughness also changes accordingly with the variation in the minimum pore size formed by particle packing. These findings indicate that particle size is closely associated with the development of surface roughness elements, which in turn affects the imbibition height of coal samples.

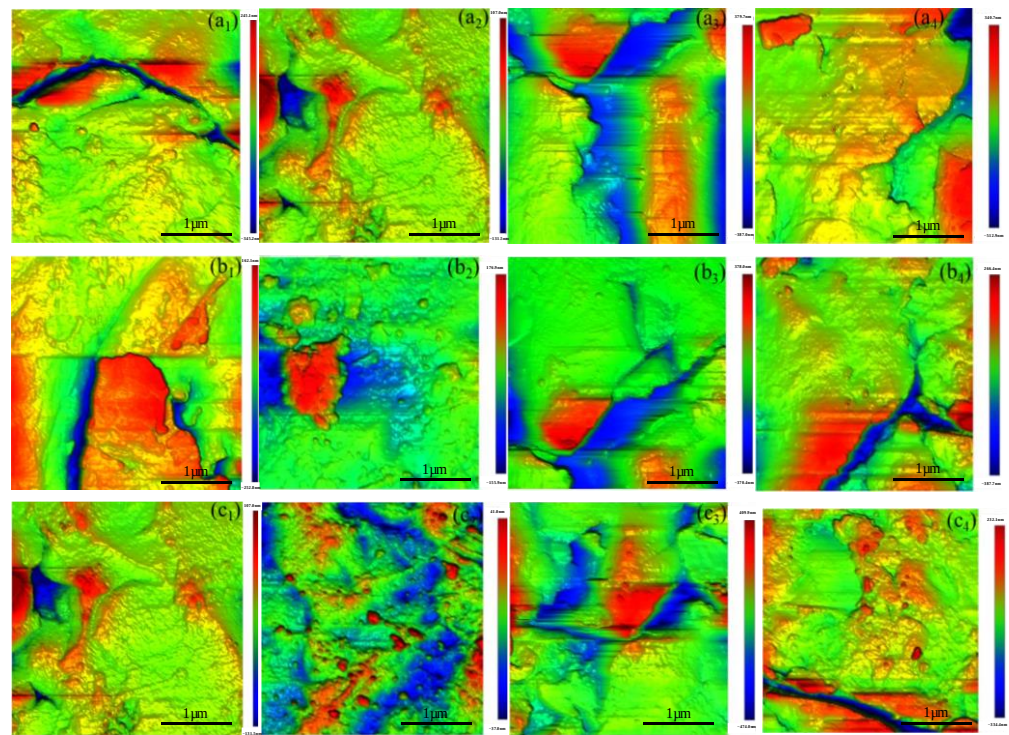


Figure 6. AFM 2D scanning images of coal samples: (a1,b1,c1) Sample 1#; (a2,b2,c2) Sample 2#; (a3,b3,c3) Sample 3#; (a4,b4,c4) Sample 4#.

Table 4. Calculated relative roughness of coal samples.

Particle Size Range	φ_s	D	$\lambda_{min}/\mu\text{m}$	ε
40~60	0.14742	1.584	22.573	0.012383
			12.589	0.006906
			14.042	0.007703
			5.226	0.00325
60~80	0.09931	1.499	15.564	0.009678
			10.101	0.006281
			3.874	0.001237
			13.992	0.004469
80~100	0.09801	1.496	9.553	0.003051
			2.926	0.000613
			8.301	0.00174
			7.67819	0.001609

The main apparatus for the spontaneous imbibition experiment is shown in Figure 7. A glass tube with an inner diameter of 7 mm was used, with the bottom sealed using filter paper. Coal powder was filled to a height of 80 mm and then compacted by vibration to 60 mm, forming a structurally uniform coal powder column. When the bottom of the column came into contact with deionized water, the experiment commenced, and the liquid

rise height (imbibition height H) and corresponding time (T) were recorded continuously until the imbibition height reached equilibrium.

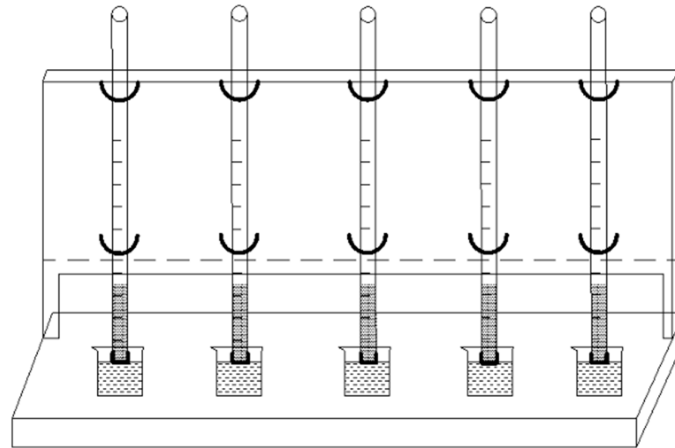


Figure 7. Spontaneous imbibition experimental setup.

The H^2 – T relationship curve was plotted based on the results of the spontaneous imbibition experiment, as shown in Figure 8. From Figure 8, it can be observed that the square of the imbibition height exhibits a good linear relationship with time, consistent with the Lucas–Washburn equation. The slope of the fitted curve for each coal sample is influenced by factors such as the capillary shape coefficient of the packed particle system and the average capillary radius, ranging from 0.6199 to 2.296; that is, the difference between the maximum and minimum slopes is relatively small. In contrast, the intercepts of the fitted curves differ significantly, indicating that the surface properties at the bottom of the coal particle column have a substantial effect on the initial stage of the imbibition process. Within the specified mesh sizes, the slopes of the fitted curves relating imbibition height to time are all positive, demonstrating that the imbibition rate of coal particles with larger diameters follows a regular and predictable trend.

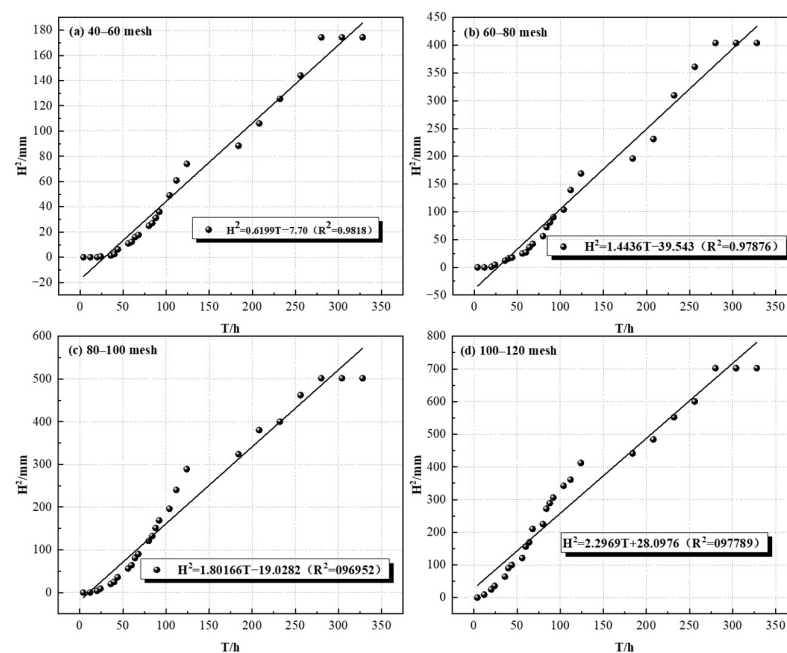


Figure 8. Relationship between imbibition height and time in the spontaneous imbibition experiment of different mesh size samples. (a) 40–60 mesh; (b) 60–80 mesh; (c) 80–100 mesh; (d) 100–120 mesh.

3.2. Comparison Between the Theoretical Imbibition Model and Experimental Data

Based on the derived spontaneous imbibition theoretical model, the theoretical imbibition heights were calculated and plotted, as shown in Figure 9. From Figure 9, it is clearly observed that the imbibition height of coal samples increases progressively as particle size decreases. This theoretical result indicates that, regardless of the type of equivalent particle size used for the coal sample, the trend remains consistent: smaller coal particles correspond to higher imbibition heights and improved wettability. This can be explained by the fact that packing channels formed by smaller particles are narrower and have lower internal roughness, resulting in capillary forces that dominate over gravitational resistance, thereby increasing imbibition height and enhancing coal wettability.

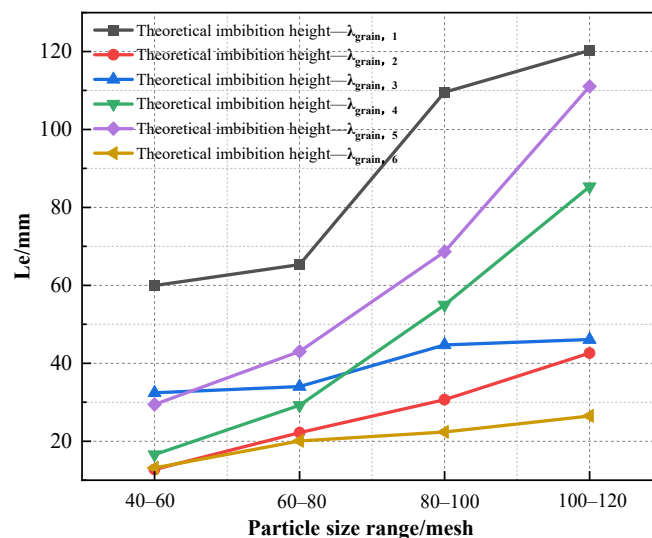


Figure 9. Theoretical values of imbibition height.

To further evaluate the reliability of the theoretical values, the experimental imbibition heights obtained from the spontaneous imbibition tests were compared with the theoretical predictions to assess the accuracy of the model. The comparison between the theoretical and measured imbibition heights is presented in Figure 10. Among the equivalent particle sizes, the stable height calculated using $\lambda_{\text{grain},2}$ —corresponding to the largest particle size in each coal sample group—shows the closest agreement with the experimental results. This indicates that, in spontaneous imbibition experiments, the largest particle size of each coal sample group should be selected as the equivalent particle size. In packed coal particle columns, finer particles fill the voids between larger particles and tend to aggregate within the capillary channels, which are the primary sites for imbibition. Smooth surfaces in close contact hinder imbibition. However, in actual imbibition processes, the complex particle morphology and surface roughness prevent full adhesion between particles, making it difficult to form continuous imbibition spaces. Additionally, smaller particles often occupy these spaces, increasing the overall volume of the imbibition channels. Therefore, the larger particles play a decisive role in defining the imbibition channels. In summary, the equivalent particle size in the imbibition model should be chosen as the largest particle size, which is consistent with both theoretical predictions and experimental results. However, since the equivalent particle size $\lambda_{\text{grain},2}$ obtained from particle size analysis is slightly larger than the actual largest particles, the experimental imbibition heights are slightly lower than the theoretical values. This comparison indicates that, when calculating theoretical imbibition heights, using the maximum particle size as the equivalent particle size provides a reasonable reference.

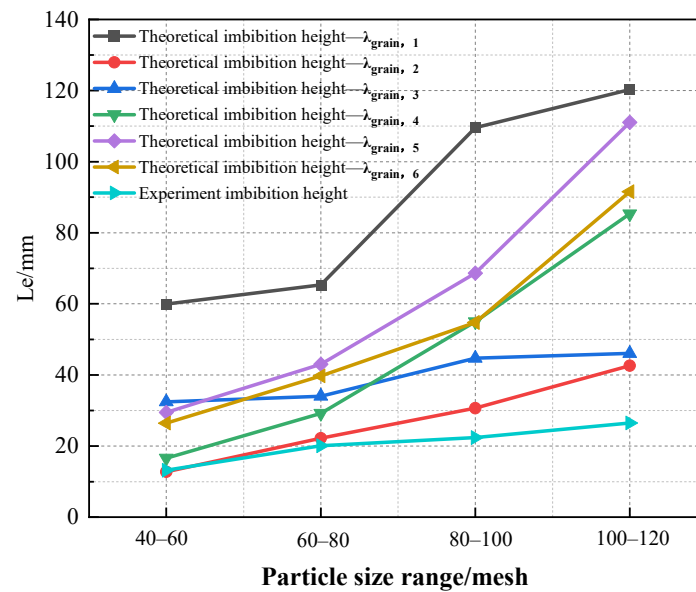


Figure 10. Comparison of imbibition heights of coal samples.

It should be noted that the experimental validation in this study was conducted using a packed coal powder column in a glass tube. Although this experimental system differs from intact in situ coal seams, lacking a consolidated coal matrix and complex fracture–pore networks, during practical coal seam water injection, particularly in the late stage, water can carry coal powder particles that accumulate locally, resulting in incomplete contact between water and the coal seam and thereby affecting wetting efficiency. Under such conditions, capillary-driven imbibition within the granular medium becomes the dominant transport mechanism. Therefore, the fractal-based imbibition model proposed in this study is primarily applicable to the particle-accumulated regions formed during the late stage of water injection. It should also be noted that the current model does not explicitly account for natural fracture networks or poro-mechanical coupling in intact coal seams. Extending the proposed fractal imbibition framework to large-scale, fracture-dominated coal systems remains a subject for further investigation.

4. Analysis of the Main Factors Affecting Coal Particle Wetting and Imbibition

During the model construction, the coal structure was characterized by introducing the pore area fractal dimension (D_f) and relative roughness. The pore area fractal dimension D_f is used to describe the two-dimensional integrity and space-filling capacity of pore cross-sections formed by particle packing, representing the geometric complexity of the inter-particle pore structure. As the proportion of pores in the cross-section increases, the influence of irregular internal boundaries on the liquid imbibition channels becomes more significant, making the pore morphology increasingly complex. From the perspective of fluid flow, D_f governs the statistical distribution of pore sizes through fractal scaling relations. An increase in D_f indicates a higher probability of larger effective pore radii and an enlarged pore cross-sectional area. Although D_f cannot directly quantify topological connectivity, it reflects the degree of structural continuity and space-filling characteristics at the representative scale, thereby statistically improving the availability of effective flow channels. As shown in Figure 11, both the imbibition height and D_f exhibit a gradual increasing trend. This phenomenon mainly arises because, with increasing porosity, the space-filling degree of the pore cross-section increases, leading to a higher fractal dimension D_f . Meanwhile, the enlargement of pore scale increases the equivalent capillary radius and

reduces hydraulic resistance, thereby enhancing the liquid penetration capacity and increasing the imbibition height. Therefore, the synchronous growth of D_f and imbibition height is primarily attributed to the regulation of hydraulic behavior by changes in pore structural scale, rather than to a direct representation of connectivity by the fractal dimension.

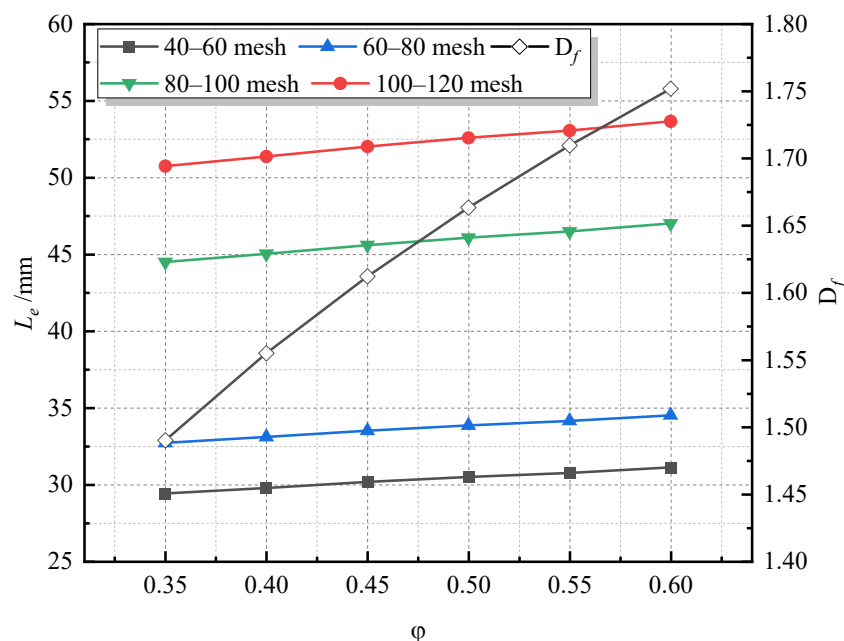


Figure 11. Effect of porosity and fractal dimension on imbibition height.

Relative roughness is defined as the ratio of the height of the roughness elements inside the channel to the pore diameter, and it is commonly used to characterize the degree of irregularity of the inner wall surface of a capillary channel. As the roughness of the channel wall increases, the internal geometry of the capillary becomes more complex, and the surface asperities become more pronounced, thereby altering the effective geometric structure of the imbibition pathway. With increasing relative roughness, the effective radius of the capillary decreases. Since capillary pressure is inversely proportional to the pore radius, a reduction in pore size leads to an enhancement of the capillary driving force. Meanwhile, according to the Hagen–Poiseuille relationship, the flow resistance is inversely proportional to the fourth power of the pore radius; thus, a decrease in pore size also significantly increases the viscous resistance during flow. However, it should be noted that viscous resistance primarily affects the transient dynamic behavior of the imbibition process, whereas the steady-state imbibition height is determined by the balance between capillary pressure and gravity under the condition of $dL/dt = 0$. Therefore, within the parameter range considered in this study, the liquid imbibition process is governed by capillary-dominated control. The enhancement of capillary pressure induced by an increase in relative roughness plays a dominant role in the steady-state equilibrium, resulting in a gradual increase in the imbibition height with increasing relative roughness. As shown in Figure 12, when the minimum pore diameter gradually increases, the scale of the capillary channel enlarges, the relative roughness decreases, and the internal channel structure tends to become more regular and simplified. Conversely, a reduction in pore diameter causes the capillary channels formed by packed particles to exhibit more complex and irregular geometries, thereby increasing the overall structural complexity. The increase in relative roughness enhances the capillary driving force, thus promoting spontaneous water imbibition in the pores and increasing the final steady-state imbibition height.

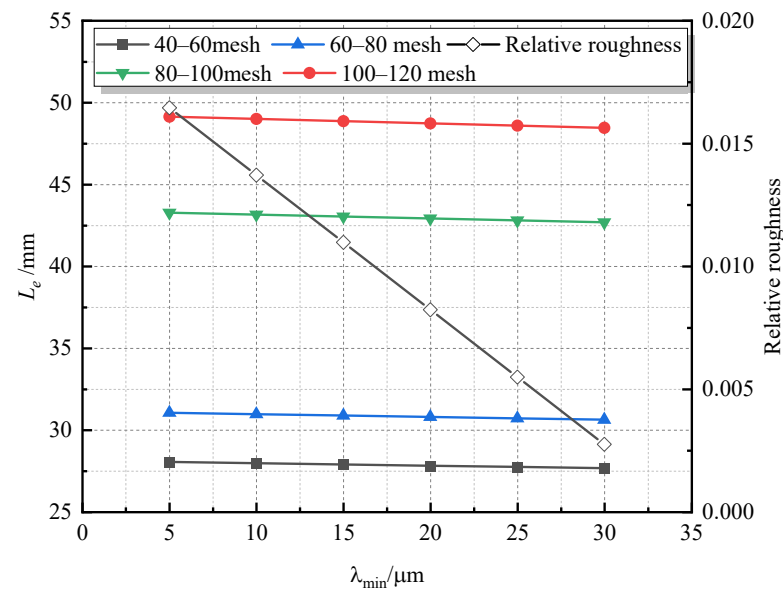


Figure 12. Effect of minimum pore diameter and relative roughness on imbibition height.

5. Conclusions

This study investigated the influence mechanisms of surface morphological features on coal wetting characteristics. By combining fractal geometry theory with particle packing models, the traditional Hagen-Poiseuille (H-P) equation was modified to develop a new theoretical model for spontaneous imbibition. The model was further validated through experiments, and the effects of various structural parameters on imbibition height were analyzed. The main conclusions are summarized as follows:

- (1) Based on fractal geometry theory, and by integrating the H-P equation with the capillary bundle model, a curved capillary imbibition model incorporating relative roughness was established. All model parameters possess clear physical meanings. Compared with traditional capillary models, the proposed model better reflects practical conditions, providing theoretical predictions with higher accuracy and applicability.
- (2) The spontaneous imbibition experiment was used to validate the theoretical model. The results indicate that the stable imbibition height calculated using the equivalent particle diameter $\lambda_{\text{grain},2}$ is closest to the experimental measurements, reflecting real conditions more accurately. Among six equivalent particle diameter selection methods, using the smallest particle size $\lambda_{\text{grain},1}$ yields relatively higher stable imbibition heights, suggesting that coal particles with diameters of approximately 90–168 μm are more effective in promoting water wetting within fissures compared to larger particles.
- (3) Analysis of the factors influencing coal particle wetting shows that: with increasing porosity, the fractal dimension of the pore area increases, enhancing the imbibition effect of the pores, and when the minimum pore diameter increases, the relative roughness inside the pores also rises, leading to a simultaneous increase in imbibition height. The capillary forces within the pores are strengthened, resulting in a more pronounced attraction of water.

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