

Article

Design Method and Performance Analysis of the Variable-Radius and Variable-Pitch Rotor for Screw Vacuum Pumps

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Abstract

Variable cross-section rotors demonstrate significant potential for enhancing screw vacuum pump performance. This study proposes a variable-radius and variable-pitch screw rotor with a seven-segment fully smooth profile, accompanied by its parametric design methodology. Corresponding clearance design methods are provided, resulting in optimized clearance distribution. A thermodynamic model incorporating four leakage channels was developed. This model effectively simulates screw vacuum pump performance and has been experimentally validated. Systematic analysis was conducted on the effects of key parameters on pump geometric characteristics and performance, with comparative studies against two traditional constant cross-section rotors. Results demonstrate that the proposed design method enables rapid and precise generation of new 3D rotor models. The clearance optimization results validate the design expectations. The variable-radius design achieves cross-sectional variation, and its combination with variable pitch produces a dual internal compression effect, to which the variable radius contributes more significantly. With an increasing cone angle, the internal volume ratio rises significantly. Compared with conventional constant cross-section rotors, the rotor demonstrates superior performance in internal volume ratio, sealing characteristics, and structural integrity, notably cutting shaft power by 52.9% versus equal-pitch rotors. These findings provide an effective solution for developing high-performance screw vacuum pumps.

Keywords: variable cross-section rotor; profile; clearance distribution; thermodynamic model; screw vacuum pump



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1. Introduction

As a representative type of positive displacement machinery, the dry twin-screw vacuum pump has been widely adopted in fields such as the chemical [1] and aerospace [2] industries, owing to its outstanding pumping capacity, high reliability, and oil-free operation. The core of its performance lies in the precise design of a pair of intermeshing screw rotors, whose geometry directly determines the pump's efficiency, energy consumption, and operational stability [3]. Variable-pitch screw rotors are considered an effective approach to enhance pump performance through internal compression and have been extensively used; however, their compression capability is constrained by structural strength and manufacturability [4]. In recent years, variable cross-section screw rotors have attracted increasing

attention for their ability to substantially increase the internal volume ratio, emerging as a key direction for next-generation screw rotor development with the potential to significantly improve overall pump performance, efficiency, and cleanliness standards [5,6].

Increasing the internal volume ratio is an effective way to enhance the performance of screw vacuum pumps [7]. Wang et al. [8,9] proposed a screw rotor with a larger internal volume ratio, which has lower power consumption and is beneficial for achieving a higher ultimate vacuum. Wang et al. [10,11] demonstrated that CSP-generated screw rotors achieved full smoothness and correct meshing. Zhao et al. [12] used MATLAB to plot the performance curves of the MGLS vacuum pump and compared them with those of the CLS vacuum pump. Hsieh et al. [13] designed a screw claw-type rotor profile using the theory of gearing and the equation of undercutting to improve gas sealing and reduce carryover. D. Pfaller et al. [14] presented a method for the energetic optimization of screw spindle vacuum pumps and investigated optimized rotor pitch distributions. Rane et al. [15] developed a deforming grid method for variable geometry screw rotors. CFD results demonstrated steeper pressure rise attributed to shortened sealing lines and reduced blow-hole areas. Zhang et al. [16] established conical rotor profiles with adjustable flow field clearances. Li et al. [17] developed a mathematical model for conical rotors and analyzed the influence of key parameters. Hou et al. [18] established a mathematical model for variable cross-section screw rotors and developed a new variably-sectioned rotor using the proposed methodology. Hu et al. [19] established a mathematical model for conical rotor surface generation by decomposing it into spherical profile and conical helix components. Wang et al. [20] proposed a variable cross-section screw rotor, which exhibits better overall performance in terms of the built-in volume ratio, gas sealing, and mechanical properties.

Analysis of gas flow and thermodynamic processes in screw vacuum pumps is an effective means to study their performance [21,22]. Li et al. [23] proposed a simulation method integrating the chamber model and thermal resistance network approach, enabling effective prediction of component temperatures under varying inlet pressures and rotational speeds. Li et al. [24] modeled the dynamic characteristics of dry screw vacuum pumps by incorporating forces induced by gas, gears, bearings, and rotor gravity. Wen et al. [25] developed a mathematical model to investigate the effect of liquid refrigerant injection (LRI) on screw compressor performance, demonstrating that LRI significantly reduces discharge temperature. Ali Moghaddam et al. [26] employed an overset grid method for flow simulation in twin-screw pumps and systematically investigated clearance flow characteristics. T. Ohbayashi et al. [27] developed and experimentally validated an analytical model for screw vacuum pump performance. K. Hanjalić et al. [28,29] developed a numerical model applicable to screw compressors and expanders, incorporating fluid leakage, injection, heat transfer, and real fluid behavior. Flow separation and gas backflow are key mechanisms leading to performance degradation and aerodynamic instabilities in internal flow systems, which significantly affect the efficiency and pumping speed of screw vacuum pumps [30,31]. Advanced numerical techniques for accurately resolving leakage flows in screw machines with complex geometries have been demonstrated using customized mesh generation methods [32].

For the manufacturing of screw rotors, A. Arifin et al. [33,34] proposed a cutting tool profile with a chip breaker for machining variable-pitch screw rotors. V.T. Tran et al. [35] constructed a 3D model of a CNC-machined variable-pitch rotor to study its operational load and deformation. It is worth noting that the thermodynamic model developed in this study has potential for further refinement to enhance its predictive accuracy. Data-driven and AI techniques have demonstrated powerful capabilities in constructing multi-objective optimization models, enabling efficient exploration of complex fluid dynamics design

spaces and providing strong support for the development of high-performance screw vacuum pumps [36,37].

Currently, screw vacuum pumps mainly employ constant cross-sectional rotors [38]. Variable-pitch rotors exhibit higher energy efficiency than equal-pitch rotors due to their higher internal volume ratio [14]. Conventional variable-pitch rotors are generally confined to a range of 1–7 [39,40]. Exceeding this ratio leads to insufficient tooth width at the discharge side, causing increased leakage, compromised structural strength, and manufacturing challenges. Variable cross-section screw rotors are considered a promising solution to overcome these limitations owing to their structural merits [16]. However, existing studies on conical rotors have primarily focused on geometric description [16,17], lacking comprehensive performance analysis, experimental validation, and systematic design methodologies.

To overcome these limitations, this study proposes a variable-radius and variable-pitch screw rotor with several innovations. First, the rotor employs a seven-segment fully smooth profile, replacing point contact with surface-to-surface sealing for enhanced leakage control. Second, a complete parametric design methodology is developed, with the profile family generated using independently developed software—enabling rapid and accurate 3D rotor generation that is not reported in previous conical rotor studies. Third, a comprehensive thermodynamic model incorporating four leakage channels is established and experimentally validated, providing a reliable foundation for performance prediction. Fourth, systematic performance analysis examines the individual and combined effects of cone angle, pitch variation, and clearance, offering quantitative design insights and optimization guidance.

This work successfully demonstrates a working conical rotor design with experimental validation and comprehensive performance characterization. By combining variable-radius and variable-pitch to create a dual-compression effect, the proposed rotor achieves significantly higher internal volume ratios while maintaining structural integrity, addressing the fundamental limitation of conventional variable-pitch designs. These findings provide important theoretical support for the engineering application and further development of variable cross-section screw rotors.

2. Design Method of the Variable-Radius and Variable-Pitch Screw Rotor

2.1. Cross-Sectional Profile

As shown in Figure 1, this paper proposes a design method for a seven-segment fully smooth screw rotor profile. The profile contour consists of seven sequentially connected smooth curve segments: an Involute Segment *AB*, a Backing Circular Arc Segment *BC*, an Addendum Circular Arc Segment *CD*, a Tip Circular Arc Segment *DE*, an Epicycloid Offset Curve Segment *EF*, a Dedendum Circular Arc Segment *FG*, and a Circular Conjugate Curve Segment *GA*. The proposed fully smooth profile design effectively eliminates the sharp cusps inherent in the conventional Quimby-tooth rotor profile, thereby preventing localized excessive wear and deformation induced by cusp-associated stress concentration [35]. Furthermore, the conventional point-to-surface clearance is replaced with an enhanced surface-to-surface clearance configuration, offering superior sealing performance.

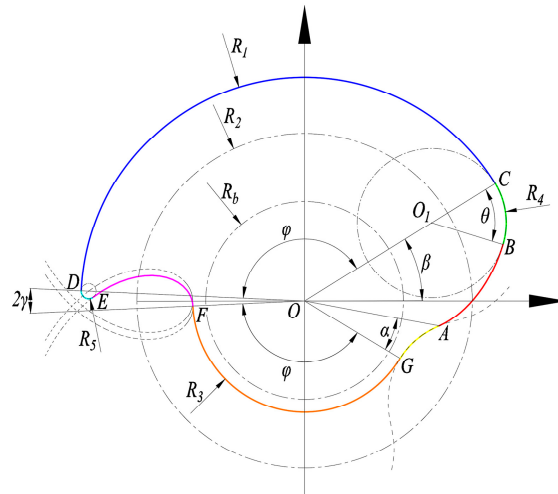


Figure 1. Seven-segment fully smooth screw rotor profile.

The parametric equations for each segment are defined as follows:

- (1) The Involute Segment AB ensures proper gear-like meshing and transmission of motion between rotors:

$$\begin{bmatrix} x_{AB}(t) \\ y_{AB}(t) \end{bmatrix} = \begin{bmatrix} R_b(\cos(t - \alpha) + t\sin(t - \alpha)) \\ R_b(\sin(t - \alpha) - t\cos(t - \alpha)) \end{bmatrix} \quad (1)$$

where the Involute initial angle α is given by

$$\alpha = \tan\left(\arccos\frac{2R_b}{R_1 + R_3}\right) - \arccos\frac{2R_b}{R_1 + R_3} \quad (2)$$

- (2) Backing Circular Arc Segment BC:

$$\begin{bmatrix} x_{BC}(t) \\ y_{BC}(t) \end{bmatrix} = \begin{bmatrix} m + R_4\cos(t) \\ n - R_4\sin(t) \end{bmatrix} \quad (3)$$

where (m, n) denote the coordinates of the center O_1 :

$$m = (R_1 - R_4)\cos(\beta) \quad (4)$$

$$n = (R_1 - R_4)\sin(\beta) \quad (5)$$

and the divergence angle β is

$$\beta = \tan\left(\frac{\arccos(R_b)}{\sqrt{R_b^2 + \left(\sqrt{(R_1 - R_4)^2 - R_b^2} + R_4\right)^2}}\right) - \arccos\frac{R_b}{R_1 - R_4} - \alpha \quad (6)$$

- (3) Addendum Circular Arc Segment CD:

$$\begin{bmatrix} x_{CD}(t) \\ y_{CD}(t) \end{bmatrix} = \begin{bmatrix} R_1\cos(t) \\ R_1\sin(t) \end{bmatrix} \quad (7)$$

- (4) Tip Circular Arc Segment
- DE*
- :

$$\begin{bmatrix} x_{DE}(t) \\ y_{DE}(t) \end{bmatrix} = \begin{bmatrix} R_5 \cos(t) \\ R_5 \sin(t) \end{bmatrix} \quad (8)$$

- (5) The Epicycloid Offset Curve Segment
- EF*
- replaces traditional point contact with surface-to-surface sealing, which is the geometric origin of the enhanced sealing performance:

$$\begin{bmatrix} x_{EF}(t) \\ y_{EF}(t) \end{bmatrix} = - \begin{bmatrix} 2R_2 \cos(t) - (R_1 - R_5) \cos(2t) + \frac{R_5(R_2 \cos(t) - (R_1 - R_5) \cos(2t))}{\sqrt{(R_1 - R_5)^2 + R_2^2 - 2R_2(R_1 - R_5) \cos(t)}} \\ 2R_2 \sin(t) - (R_1 - R_5) \sin(2t) - \frac{R_5(-R_2 \sin(t) + (R_1 - R_5) \sin(2t))}{\sqrt{(R_1 - R_5)^2 + R_2^2 - 2R_2(R_1 - R_5) \cos(t)}} \end{bmatrix} \quad (9)$$

- (6) Dedendum Circular Arc Segment
- FG*
- :

$$\begin{bmatrix} x_{FG}(t) \\ y_{FG}(t) \end{bmatrix} = \begin{bmatrix} R_3 \cos(t) \\ R_3 \sin(t) \end{bmatrix} \quad (10)$$

- (7) Circular Conjugate Curve Segment
- GA*
- :

$$\begin{bmatrix} x_{GA}(t) \\ y_{GA}(t) \end{bmatrix} = - \begin{bmatrix} m \cos 2\varphi - n \sin 2\varphi + R_4 \cos(2\varphi + t) + 2R_2 \cos \varphi \\ m \sin 2\varphi - n \cos 2\varphi + R_4 \sin(2\varphi + t) + 2R_2 \sin \varphi \end{bmatrix} \quad (11)$$

where φ is defined as

$$\varphi = \pi - \frac{\arcsin(2n \cos(t) - 2m \sin(t))}{2R_2} - \theta \quad (12)$$

2.2. Software Parametric Modeling and Meshing Relationship

The manual process involved in traditional 3D modeling is inefficient and prone to errors. For the variable-radius and variable-pitch rotor, geometric accuracy is critically dependent on the density of cross-sections along the axial direction. To overcome this limitation, this paper introduces an innovative approach: the automation of rotor cross-section generation via Python programming, using PyCharm Community Edition 2023.3.4. This framework codifies the complex and repetitive task of profile generation, enabling the rapid creation of large datasets of high-precision 2D cross-sections. These sections facilitate the efficient construction of accurate 3D mathematical models of the variable-radius and variable-pitch rotor.

The parametric equations involve ten geometric parameters: $R_1, R_2, R_3, R_4, R_5, R_b, \alpha, \beta, \gamma, \theta$. The pitch radius R_2 is determined by the addendum (R_1) and dedendum (R_3) radius:

$$R_2 = (R_1 + R_3)/2 \quad (13)$$

Angular parameters α and β are explicitly defined by Equations (2) and (6). Angle γ is given by

$$2\gamma = -\arctan\left(\frac{(R_1 - R_5) \cos(t)}{(R_1 - R_5) \sin(t)}\right) \quad (14)$$

The central angle θ is determined by the position of center $O_1 = (m, n)$.

The five key radius parameters serve distinct functional roles in rotor profile design. The R_1 defines the outer diameter and directly determines the suction volume and compression ratio. The R_2 remains constant to ensure proper meshing. The R_3 defines the root diameter and affects tooth root strength. The R_4 controls flank curvature and influences

sealing line length, where smaller values improve sealing but increase stress concentration. Finally, the R_5 determines the rotor tip shape and affects leakage characteristics; together with R_4 , it ensures a smooth profile transition and proper clearance distribution.

Comprehensive analysis reveals that the overall shape and evolution of the rotor profile are predominantly governed by four key geometric parameters: R_1 , R_3 , R_4 , R_5 . Leveraging this insight, parametric modeling software was developed. This tool generates the corresponding rotor cross-section contour in real time upon user input of parameter values, significantly enhancing design efficiency.

Table 1 explicitly defines the theoretical profile composition and its corresponding meshing relation.

Table 1. Meshing relation of the seven-segment fully smooth screw rotor.

Male Rotor	Female Rotor	Mesh Clearance
Involute Segment AB	Involute Segment BA	
Backing Circular Arc Segment BC	Circular Conjugate Curve Segment AG	Δ_3
Addendum Circular Arc Segment CD	Dedendum Circular Arc Segment GF	Δ_2
Tip Circular Arc Segment DE	Epicycloid Offset Curve Segment FE	
Epicycloid Offset Curve Segment EF	Tip Circular Arc Segment ED	Δ_4
Dedendum Circular Arc Segment FG	Addendum Circular Arc Segment DC	
Circular Conjugate Curve Segment GA	Backing Circular Arc Segment CB	Δ_3

2.3. Conical Helix and Axially Variable Cross-Section

As shown in Figure 2, the helical curve of the screw rotor proposed in this study is a conical helix characterized by both variable radius and variable pitch. The design induces a continuous axial decrease in the cross-sectional radius and the pitch from the inlet to the outlet, resulting in an overall conical shape. The equation of the three-dimensional conical helix is

$$\begin{cases} X(\phi) = (R_{1\text{ in}} - Z(\phi) * \tan(\alpha_c)) \cos \phi \\ Y(\phi) = (R_{1\text{ in}} - Z(\phi) * \tan(\alpha_c)) \sin \phi \\ Z(\phi) = \frac{1}{2\pi} \int_0^\phi p(\phi) d\phi \end{cases} \quad (15)$$

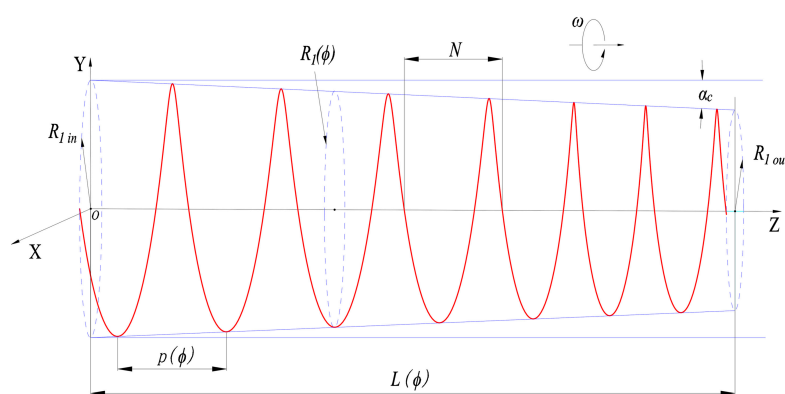


Figure 2. The conical helix of the variable-radius and variable-pitch screw rotors.

The equation of the cone angle α_c is

$$\alpha_c = \arctan\left(\frac{R_{1\text{ in}} - R_{1\text{ out}}}{L}\right) \quad (16)$$

where $R_{1\text{ in}}$ represents the addendum circle radius on the inlet side; $R_{1\text{ out}}$ represents the addendum circle radius on the outlet side; $p(\phi)$ represents the pitch function varying with the rotation angle ϕ .

As shown in Figure 3, the variable cross-section design is achieved by modifying key radius. After determining the profile and establishing its parametric modeling method in the software, a series of axially variable cross-sectional profiles can be generated for the desired variable-radius and variable-pitch screw rotor by integrating the preset conical helical design. The rotor proposed in this paper is intended for use in twin-screw pumps; therefore, the center distance between the two screw rotors remains constant throughout, i.e., the pitch circle radius R_2 is invariant. The addendum circle radius R_1 determines the size of the end-face cross-section, and its axial variation, $R_1(\phi)$, can be defined according to design requirements. Based on the relationship $2 \times R_2 = R_1 + R_3$, the axial variation of R_3 , $R_3(\phi)$, can be derived. Once $R_1(\phi)$ and $R_3(\phi)$ are established, appropriate values for $R_4(\phi)$ and $R_5(\phi)$ are selected to ensure that a closed, continuous cross-sectional profile can be correctly generated at any axial position. As shown in Figure 3, from the inlet side to the outlet side (i.e., axially from 0 to the total rotor length L), the typical design involves $R_1(\phi)$ gradually decreasing, $R_3(\phi)$ gradually increasing, with equal variation amplitudes for $R_1(\phi)$ and $R_3(\phi)$, ensuring that $R_2(\phi)$ remains constant. Meanwhile, $R_4(\phi)$ decreases and $R_5(\phi)$ increases along the axial direction.

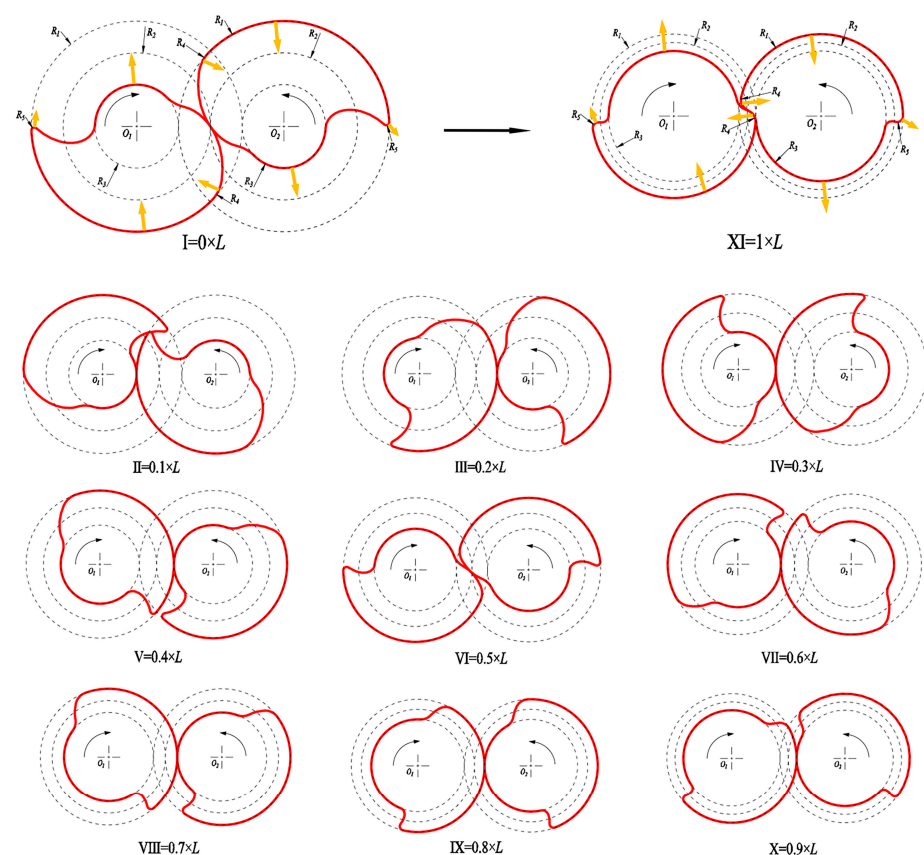


Figure 3. Axial variable cross-sections of variable-radius and variable-pitch screw rotors.

2.4. Rotor Modeling Method and Clearance Design

As shown in Figure 4, the three-dimensional model of the variable-radius and variable-pitch screw rotor is constructed by performing a blending sweep operation of the previously generated axial variable cross-section (red) along the guiding conical helix path (blue). By modulating the variation laws of the pitch and cross-sectional geometry, rotor configura-

tions that meet diverse design requirements can be efficiently generated. The rotor pair (Z_1 and Z_2) share identical geometric definitions of the profile family and conical helical path, differing only in their handedness.

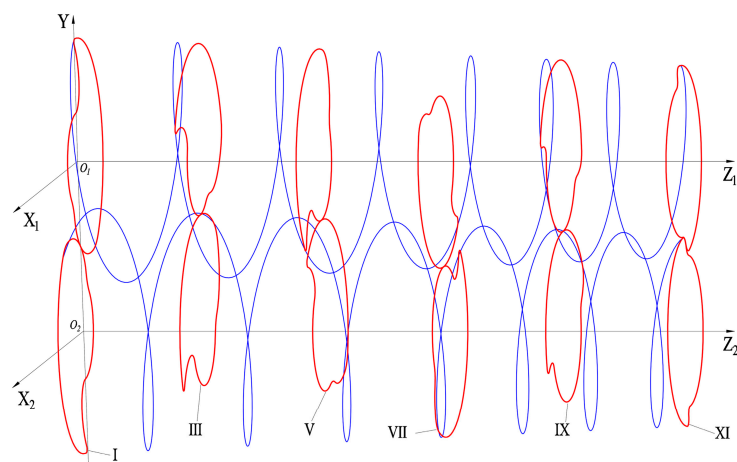


Figure 4. The variable-radius and variable-pitch screw rotors generated by hybrid sweeping.

Clearance distribution critically affects the performance and reliability of screw vacuum pumps, necessitating consideration of its three-dimensional distribution during the modeling phase. For the rotor fabricated via the surface–volume hybrid sweeping method, the actual clearance is achieved by reserving a nominal clearance on the 2D theoretical profile. Among these, the tooth clearance (Δ_3) and tooth side clearance (Δ_4) are influenced by the helix inclination, decreasing from the suction side with a larger pitch toward the discharge side with a smaller pitch [9]. Moreover, due to higher operating temperatures and more pronounced thermal deformation on the discharge side, additional clearance allowance is required to prevent mechanical friction between rotors. To overcome the aforementioned limitations, this study employs the screw rotor modeling method illustrated in Figure 5. The feature of this approach is that four types of clearances are first used to generate corresponding 3D surfaces. These 3D surfaces are then combined and filled to form the solid rotor model.

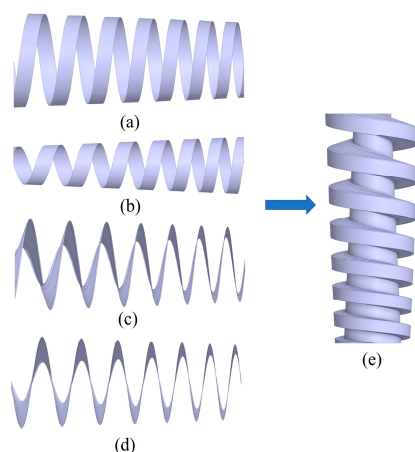


Figure 5. Rotors generated by hybrid sweeping and filling: (a) tooth top 3D surface; (b) radial 3D surface; (c) tooth 3D surface; (d) tooth side 3D surface; (e) solid model of the rotor generated by filling.

The adopted curve–surface–solid hybrid sweeping strategy is centered on precisely configuring clearances through the geometric adjustment of 3D surfaces. To address the control challenge posed by the Δ_3 and the Δ_4 , Figure 6 illustrates the solution: assigning a

nominal clearance Δ_N by axially translating specific surfaces. Owing to the very high helix angle α_s (approaching 90°) in this region, the axial translation amount, as per Equation (17), directly approximates the actual normal dimension of the clearance Δ_A .

$$\Delta_A = \Delta_N \times \cos(\alpha_s) \quad (17)$$

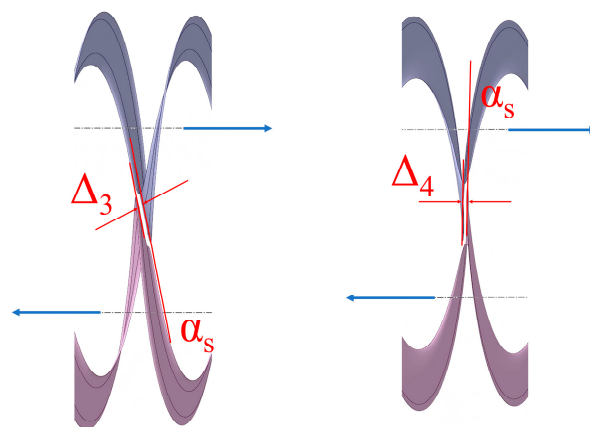


Figure 6. The actual clearance design of the Δ_3 and Δ_4 .

Figure 7 presents the clearance distribution achieved using the described method, with a designed nominal value of 0.25 mm for all clearances. The results show that the measured values of all four clearances remain around 0.25 mm, with the maximum deviation not exceeding ± 0.02 mm, indicating that the actual clearance distribution closely aligns with the design intent. Specifically, the Δ_1 and the Δ_2 exhibit highly uniform distributions. In contrast, the Δ_3 and the Δ_4 show a slight increasing trend along the axis from the suction side toward the discharge side. This trend is primarily attributed to the decreased pitch and increased helix angle near the discharge side, which leads to a marginally larger actual geometric clearance in that region. This characteristic can be utilized for clearance compensation against thermal deformation.

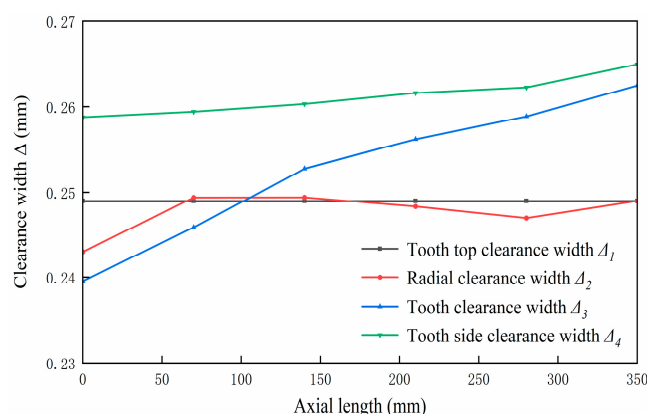


Figure 7. Actual clearance distribution results.

3. Thermodynamic Mathematical Model and Experimental Verification

3.1. Working Principle of the Screw Vacuum Pump

The key performance parameters of screw vacuum pumps primarily include pumping speed, power consumption, and ultimate vacuum pressure. Clearances significantly influence pump performance, as they induce leakage that directly reduces effective pumping speed. As shown in Figure 8, four clearance types exist: tooth top clearance (Δ_1), radial

clearance (Δ_2), tooth clearance (Δ_3), and tooth side clearance (Δ_4). The contact line length of these leakage channels is a crucial geometric parameter governing leakage rates; shorter contact lines effectively suppress leakage and enhance pumping efficiency. To model leakage flow rates, it is essential to simplify the complex geometry of these channels into standard configurations, such as parallel plates, cylindrical surfaces, and narrow slits for analytical computation.

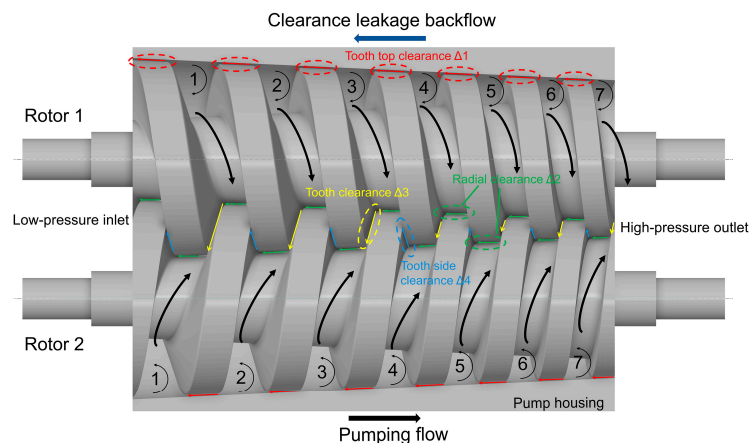


Figure 8. Clearance leakage model of the variable-radius and variable-pitch screw rotors in vacuum pumps.

Among these, the crucial parameter is the internal volume ratio ν , defined as follows:

$$\nu = \frac{V_{out}}{V_{in}} \quad (18)$$

where V_{in} denotes the suction volume over one complete cycle; V_{out} denotes the discharge volume over one complete cycle.

3.2. Clearance Leakage Model

3.2.1. Tooth Top Clearance Leakage Model

As shown in Figure 9a, the tooth top clearance length L_1 takes the form of an annular clearance, which can be determined by the following equation:

$$L_1 = 2 \times \left(1 - \frac{1}{\pi} \arccos \frac{A}{2 \times R_1} \right) \sqrt{\pi^2 (2 \times R_1)^2 - \lambda^2} \quad (19)$$

where A is the center distance between the screw rotors; λ is the lead of the screw rotor.

The cross-sectional area of this leakage channel can be expressed as the product of its length and width:

$$A_1 = L_1 \times \Delta_1 \quad (20)$$

Since gas leakage between the successive stages of the tooth top region tends to follow the shortest flow path, the depth of leakage channel L_1 is defined as the normal thickness of the tooth top surface, which can be given by

$$h_1 = \frac{2\pi B R_1}{\sqrt{(2 \times R_1)^2 - (\lambda/\pi)^2}} \quad (21)$$

where B denotes the width of the tooth top of the rotor.

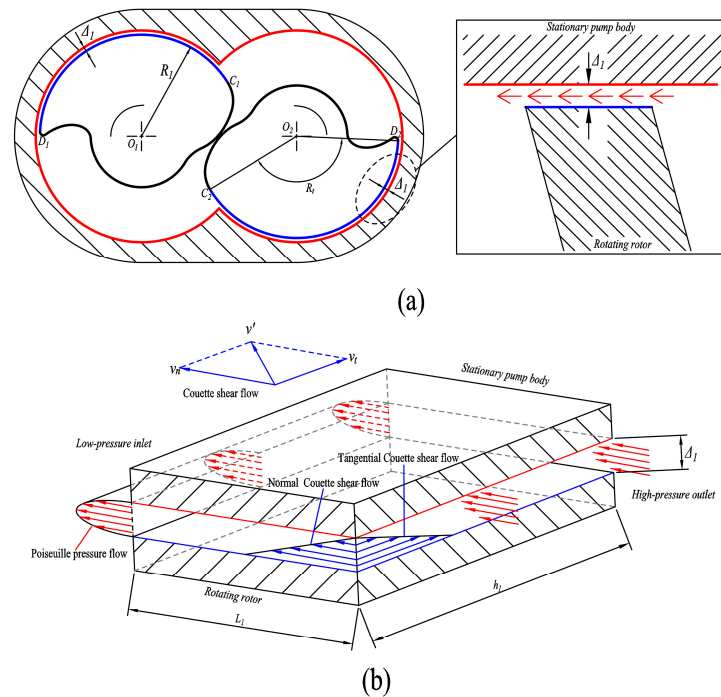


Figure 9. Tooth top clearance leakage model: (a) tooth top clearance leakage simplified as a parallel-plate slit model; (b) schematic diagram of gas backflow in the tooth top clearance leakage channel.

As shown in Figure 9a, since both the length L_1 and the depth h_1 of the leakage channel are significantly greater than its width (i.e., the Δ_1), the curvature effects of the annular geometry are negligible. Consequently, this annular leakage channel may be simplified for calculation purposes as a parallel-plate slit model.

As shown in Figure 9b, the gas flow within the tooth top clearance leakage channel consists of two components: Poiseuille pressure-driven flow and Couette shear-driven flow. The Poiseuille pressure-driven flow is caused by the pressure difference between the gas in the tooth grooves on both sides of the screw rotor and represents the dominant contribution to the backflow leakage. The Couette shear-driven flow arises from the velocity difference between the high-speed rotating tooth top surface and the stationary inner wall of the pump chamber. Due to the inclined angle of the helical tooth top surface, the Couette shear-driven flow can be decomposed into tangential and normal components relative to the tooth top surface. Based on the superposition principle of gas flow, the leakage rates of the two flow types are calculated separately and then summed.

The mass flow rate q_{MC} of backflow gas caused by the Couette shear-driven flow is given by

$$q_{MC} = \frac{\pi n}{60} \times \frac{\mu}{RT_{i+1}} \times \frac{2PR_1\Delta_1L_1}{\sqrt{\lambda^2 + (2R_1\pi)^2}} \times (P_i + P_{i+1}) \quad (22)$$

In the equation, P_i denotes the gas pressure on the suction side of the tooth top clearance; P_{i+1} denotes the gas pressure on the discharge side of the tooth top clearance; μ is the molar mass of the gas; R is the universal gas constant; and T_{i+1} is the gas temperature on the discharge side of the tooth top clearance.

The mass flow rate q_{MP} of backflow gas caused by the Poiseuille pressure-driven flow is given by

$$q_{MP} = \frac{\Delta_1^3 L_1}{24} \times \frac{\mu}{\eta RT_{i+1}} \times (P_{i+1}^2 - P_i^2) \quad (23)$$

where η is the dynamic viscosity of the gas ($\text{N}\cdot\text{s}/\text{m}^2$).

3.2.2. Clearance Leakage Model Between Rotors

As shown in Figure 10a, the length L_2 of the radial clearance leakage channel is equal to the axial width B of the tooth top. Consequently, its leakage area A_2 is given by the following equation:

$$A_2 = L_2 \cdot \Delta_2 = B \cdot \Delta_2 \quad (24)$$

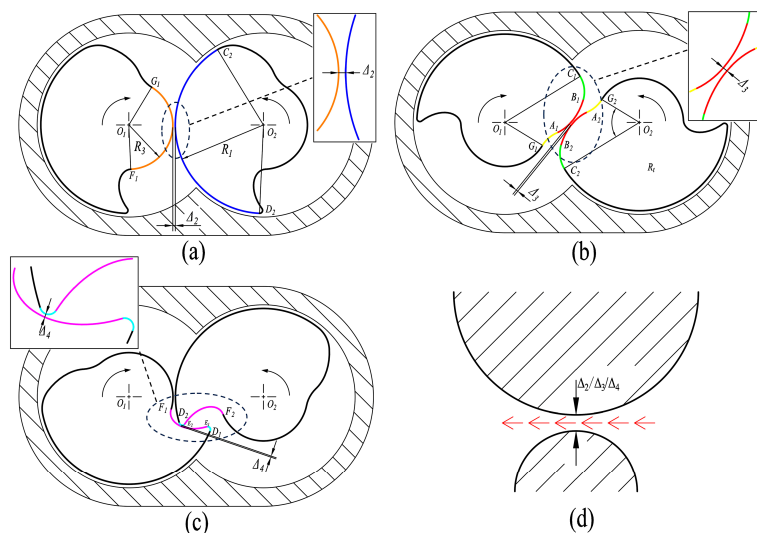


Figure 10. (a) Radial clearance leakage; (b) tooth clearance leakage; (c) tooth side clearance leakage; (d) simplified model based on the orifice–nozzle theory.

As shown in Figure 10b, the leakage area A_3 of the tooth clearance leakage channel is calculated as follows:

$$A_3 = L_3 \cdot \Delta_3 = \sqrt{\left(\frac{\eta_p}{2\pi}\lambda\right)^2 + (R_1 - R_3)^2} \cdot \Delta_3 \quad (25)$$

where η_p is the involute phase angle.

As shown in Figure 10c, the area A_4 of the tooth side clearance channel is calculated as follows:

$$A_4 = L_4 \cdot \Delta_4 = R_1 \cdot \frac{\arccos(R_2/R_1)}{\sin(\beta_i)} \cdot \Delta_4 \quad (26)$$

where β_i is the helix angle.

As shown in Figure 10, these clearance channels share the following common characteristics: the cross-sectional shape of each leakage channel is that of a Laval nozzle, which is wide at both sides and narrow in the middle; the narrowest part corresponds to the contact line of the two rotors; the depth of the leakage channel can be regarded as zero; the gas flow in these channels is all jet flow, so all can be calculated based on the orifice–nozzle (Saint-Venant/Wantzel) model [41].

Under this condition, the mass flow rate q_{MH} of backflow gas generated through the three leakage channels can be expressed as follows:

$$q_{MH} = \alpha_{fc} \times (A_2 + A_3 + A_4) \times \left(\frac{P_{i+1}}{P_i}\right)^{\frac{1}{k}} \times \frac{P_{i+1}}{R \cdot T_{i+1}} \times \sqrt{\frac{2k}{k-1} \cdot R \cdot T_{i+1} \cdot \left(1 - \left(\frac{P_{i+1}}{P_i}\right)^{\frac{k-1}{k}}\right)} \quad (27)$$

$$x = \frac{P_{i+1}}{P_i} > x_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

$$q_{MH} = \alpha_{fc} \times (A_2 + A_3 + A_4) \times \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \times \frac{P_{i+1}}{R \cdot T_{i+1}} \times \sqrt{\frac{2k}{k+1} \cdot R \cdot T_{i+1}} \quad (28)$$

$$x = \frac{P_{i+1}}{P_i} \leq x_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

In the equation, k is the adiabatic index of the gas; α_{fc} is the flow coefficient; x is the ratio of gas pressure between adjacent stages; x_c is the critical pressure ratio.

The leakage models presented assume laminar, continuum flow. This assumption is justified for the majority of the operating range, where clearance heights are small ($\Delta = 0.25$ mm) and Knudsen numbers remain below 0.01 for pressures above 10 Pa [6]. At very low pressures (<10 Pa), rarefaction effects may become non-negligible. However, the good agreement between model predictions and experimental measurements at 10 Pa suggests that the continuum-based models remain reasonably accurate, consistent with findings in similar vacuum pump studies [27]. The orifice–nozzle model explicitly accounts for compressibility through the adiabatic index k and critical pressure ratio x_c following established methods [41]. Regarding turbulence, the parallel-plate slit model for tooth top clearance assumes laminar flow based on low Reynolds numbers due to the small clearance height; at higher inlet pressures ($>10^4$ Pa), localized turbulence may occur, but laminar models remain valid for the majority of the operating range [26].

3.3. Thermodynamic Mathematical Model of Rotor Operation Process

The thermodynamic mathematical model is founded on the following fundamental assumptions: (1) the pumped gas satisfies the ideal gas equation of state; (2) disregarded rotor-induced stirring effects and related power dissipation; (3) discretized pumping volumes into independent chamber units; (4) uniform chamber gas states without port pressure fluctuations under steady-state conditions.

To assess the validity of the ideal gas assumption, the compressibility factor Z for air was evaluated across the full range of operating pressures (1 Pa to 101,325 Pa) and temperatures (300 K to 450 K) encountered in this study. Using the Peng–Robinson equation of state, Z was found to remain within 0.998 to 1.002 for all conditions. Consequently, the maximum error introduced by the ideal gas assumption is estimated to be less than 0.2%, which is negligible compared to other sources of uncertainty.

The governing equations of this thermodynamic mathematical model comprise the following core formulations. The model incorporates gas state parameters, including pressure P_i , temperature T_i , chamber volume V_i , mass flow rate m_i , and backflow mass flow rate m_{L_i} . These parameters are all constantly changing variables, but they always satisfy the ideal gas state equation:

$$P_i V_i = \frac{R}{\mu} m_i T_i \quad (29)$$

The equation for the rate of change of gas mass in any intermediate gas storage chamber is

$$\frac{dm_i}{d\tau} = \frac{\mu}{R} \cdot \left(\frac{V_i}{T_i} \cdot \frac{dP_i}{d\tau} + \frac{P_i}{T_i} \cdot \frac{dV_i}{d\tau} - \frac{P_i V_i}{T_i^2} \cdot \frac{dT_i}{d\tau} \right) = q_{L_{i+1}} - q_{L_i} \quad (30)$$

The total backflow mass flow rate, q_{L_i} , generated through the four leakage channels, is given by

$$q_{L_i} = q_{\Delta 1} + q_{\Delta 2} + q_{\Delta 3} + q_{\Delta 4} = q_{MC} + q_{MP} + q_{MH} \quad (31)$$

Without considering leakage, the theoretical geometric pumping rate Q_t is

$$Q_t = \frac{nV_1}{60} \quad (32)$$

Based on the thermodynamics of adiabatic compression processes, the theoretical temperature variation between adjacent chambers follows

$$\frac{T_{i+1}}{T_i} = \left(\frac{V_i}{V_{i+1}} \right)^{k-1} \quad (33)$$

Ultimately, the gas in each stage of the pump chamber volume of the screw vacuum pump satisfies the mass continuity equation:

$$m_i(\tau = \tau_0) - m_{L_i} = m_{i+1}(\tau = \tau_0) - m_{L_{i+1}} \quad (34)$$

In the power consumption calculation, only the compression power and exhaust power are considered, while factors such as gas friction, mechanical friction, and gas–solid heat exchange losses are neglected, as expressed by

$$W_i = \left[\sum_{i=1}^{N-1} \left(\frac{k}{k-1} \cdot V_i W_i \cdot \left[\left(\frac{W_{i+1}}{W_i} \right)^{1-\frac{1}{k}} - 1 \right] \cdot 10^{-9} \right) + V_N \cdot (P_{out} - P_N) \cdot 10^{-9} \right] \cdot \frac{n_s}{60} \cdot 10^{-3} \quad (35)$$

This study established a thermodynamic mathematical model of the rotor operation process based on a numerical computing platform. The solution process of the model is as follows: First, the geometric parameters of the screw rotors are input. Subsequently, the backflow mass flow rates of the four types of leakage channels are solved using the Clearance Leakage Model. On this basis, the gas state parameters (e.g., pressure, temperature) under the non-leakage condition are calculated and used as the initial values of the program. Then, considering the influence of leakage, iterative calculations are conducted via the thermodynamic mathematical model of the rotor operation process. Finally, when the relative errors of the obtained pressure and temperature are both less than 0.1%, the calculation results are determined to meet the convergence criterion. Based on this, performance results such as pumping speed curves and power consumption curves under different operating conditions can be obtained. For detailed information, please refer to the Supporting Document.

3.4. Experimental Verification

The overall structure of the screw vacuum pump is shown in Figure 11a. It mainly consists of the following components: a pair of screw rotors, pump chamber, bearings, synchronous gears, water-cooling system for heat dissipation, suction port, discharge port, gas seal sleeves, pressure plates, coupling, sealing rings, connection cover, end covers, and other components. As shown in Figure 11b, the rotor is manufactured from stainless steel. Based on the preset geometric model, it is shaped on a 5-axis turn-mill center into a conical structure and finally finished by grinding. Key dimensions are inspected using a coordinate measuring machine (CMM). Balance weights are integrated on its suction side to ensure dynamic balance and stable operation under high-speed rotation conditions.

To validate the accuracy of the developed thermodynamic model, experimental tests on the pumping speed and shaft power were conducted using Rotor #00. The design parameters of the screw vacuum pump adopted in the model are exactly consistent with those of the experimental prototype, with all key design parameters clearly presented in Section 4 of this paper. Rotor #00 was selected as the validation benchmark because its geometric parameters ($\alpha_c = 2.5^\circ$, $\nu = 4.6$) represent a typical medium-range design within the parameter space explored in Section 4. Pumping speed and shaft power directly reflect the model's fundamental mechanisms, namely clearance leakage and internal compression, and are key parameters characterizing pump performance. Furthermore, the parametric

analysis focuses on comparative analysis, so the conclusions remain robust even in the presence of systematic absolute deviations.

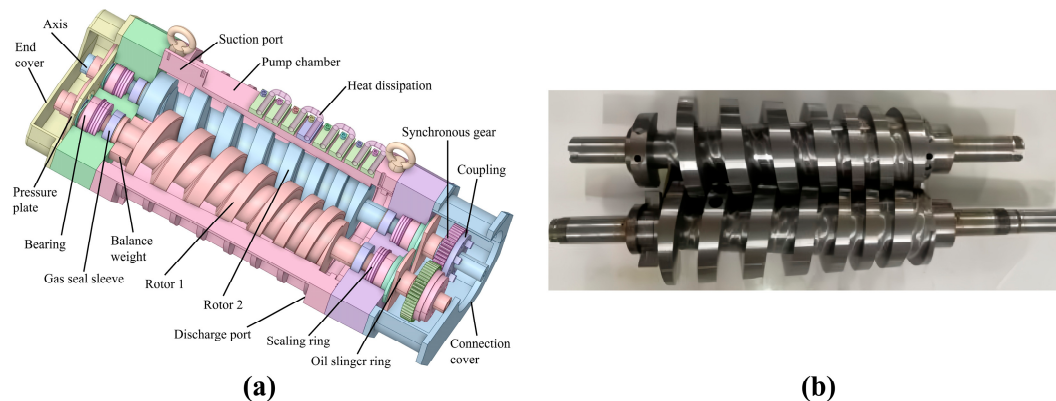


Figure 11. (a) The structure of the screw vacuum pump; (b) the variable-radius and variable-pitch screw rotor.

The experimental setup shown in Figure 12a was employed to measure the pumping speed and power of the screw vacuum pump under varying inlet pressures. It consisted of a screw vacuum pump, a 0.2 m³ test container (ISO 21360 compliant), a mass flowmeter (0–300 m³/h, accuracy $\pm 0.5\%$ FS), a vacuum gauge (0.1 Pa to 100 kPa, accuracy $\pm 0.5\%$), a controller, a motor (0–6 kW), and a motor drive, among other components.

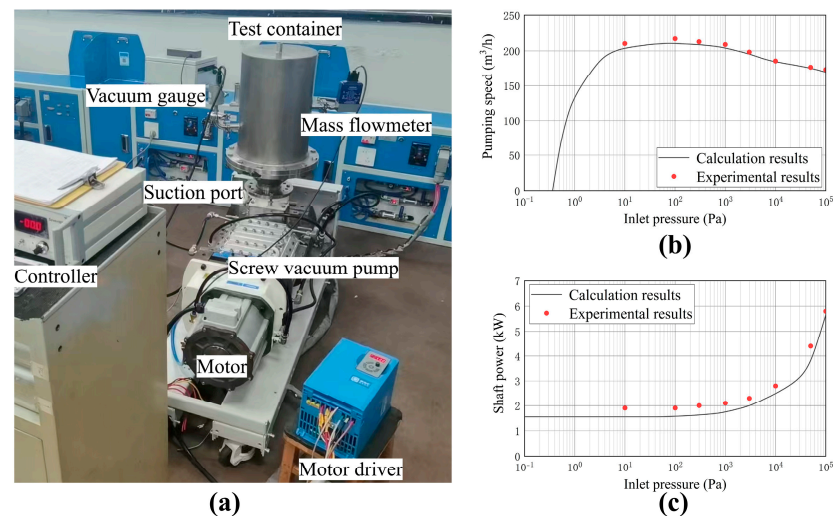


Figure 12. (a) The experiment of the screw vacuum pump; (b) experimental verification of pumping speed; (c) experimental verification of shaft power.

The comparison of the calculation results with the experimental data indicates that the calculated pumping speed is highly consistent with the measured trend, as shown in Figure 12b. As shown in Figure 12c, the calculated shaft power is consistently lower than the experimentally measured values across the entire inlet pressure range. The average relative deviation between the model and experiment is 12.3%, with a maximum deviation of 19.4% observed at the inlet pressure 10 Pa. This discrepancy is primarily attributed to the model's omission of rotor-induced gas stirring losses and mechanical friction in bearings and shaft seals. Importantly, the consistent trend of the deviation indicates that the model accurately captures the underlying thermodynamic behavior. Overall, the calculated and experimental results show consistent trends with acceptable deviations, demonstrating that

the proposed thermodynamic model can be effectively applied to predict the performance of screw vacuum pumps.

4. Performance Analysis and Discussion

In this study, the fixed geometric parameters of the vacuum pump were specified as follows: rotor length $L = 350$ mm, number of stages $N = 7$, center distance $A = 100.5$ mm, and identical suction volume V_{in} . The working parameters were set as follows: a clearance of 0.25 mm, a rotational speed of 5000 rpm, air as the pumped medium, and an inlet gas temperature of 300 K.

4.1. Effect of Variable Radius on Rotor Performance

As shown in Table 2, under identical conditions, only the cross-sectional radius was adjusted. R_1 gradually decreases from the inlet to the outlet.

Since the cone angle α_c is principally determined by R_1 and L , modifying the variable radius also results in variations in the cone angle design. The lower limit of α_c is 0° . The upper limit of α_c is governed by Equation (36). Typically, α_c ranges from 0° to 4° , and in this study, the maximum value is $\alpha_{c\ max} = 3.63^\circ$.

$$\alpha_{c\ max} = \arctan\left(\frac{R_{1\ in} - R_{1\ out}}{L}\right) = \arctan\left(\frac{R_{1\ in} - A/2}{L}\right) \quad (36)$$

As shown in Figure 13, increasing the cone angle α_c from 0.5° to 3.5° (Rotors #11 to #16) results in a significant rise in the internal volume ratio ν from 1.9 to 15.7. This increase drives a reduction in all four leakage areas, with the tooth top clearance A_1 accounting for approximately 85% of the total leakage area, identifying it as the primary leakage path. Notably, the anomalous rise in A_1 at the final stage of Rotor #16 indicates a design boundary where R_1 approaches R_3 —a condition that should be avoided.

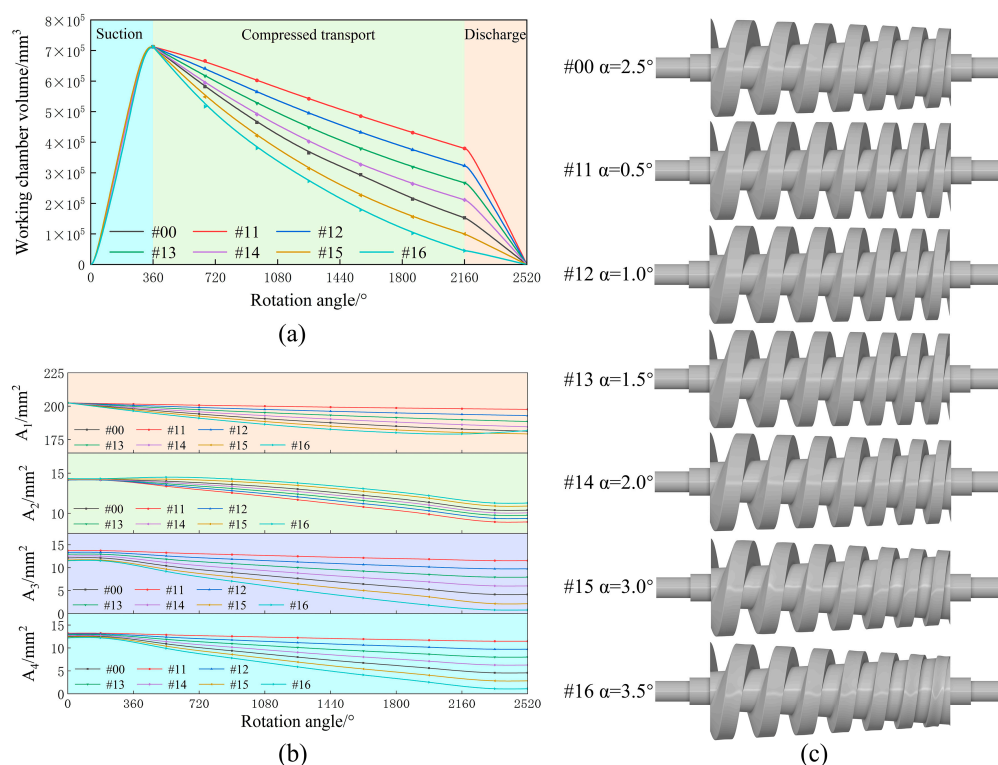
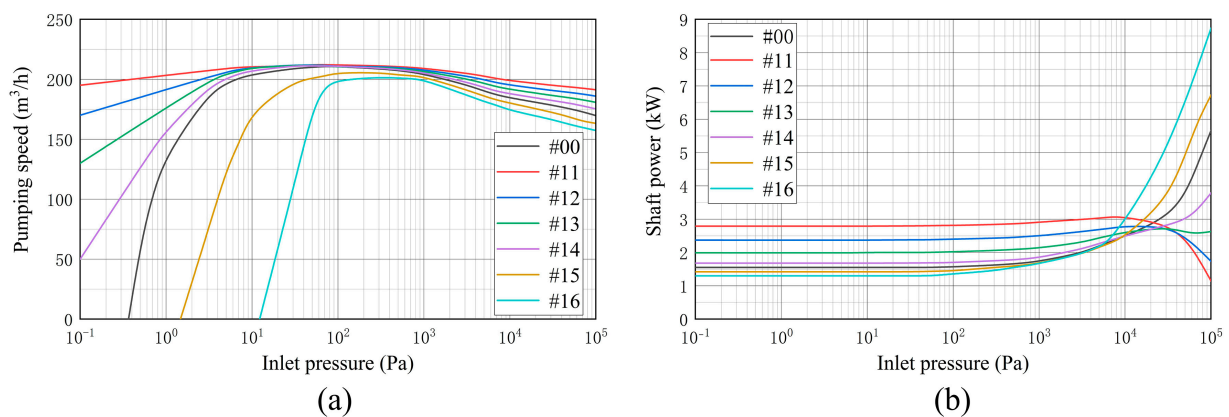


Figure 13. The geometric characteristics of different variable-radius designs for new rotors: (a) working chamber volume; (b) leakage channel area A_1 , A_2 , A_3 , A_4 ; (c) three-dimensional model.

Table 2. Design parameters of different variable-radius designs for new rotors.

	L (mm)	N	P (mm)	A (mm)	R_1 (mm)	α_c (°)	ν
#00	350	7	65–35	100.5	72–57	2.5	4.6
#11	350	7	65–35	100.5	72–69	0.5	1.9
#12	350	7	65–35	100.5	72–66	1	2.2
#13	350	7	65–35	100.5	72–63	1.5	2.7
#14	350	7	65–35	100.5	72–60	2	3.4
#15	350	7	65–35	100.5	72–54	3	6.9
#16	350	7	65–35	100.5	72–51	3.5	15.7

As shown in Figure 14a, the pumping speed decreases with increasing α_c . This decline is attributed primarily to the elevated internal volume ratio ν , which amplifies the pressure differential between adjacent working chambers, thereby exacerbating leakage. Under high-vacuum conditions, this effect becomes more pronounced, significantly influencing the ultimate vacuum. It demonstrates that, all other conditions being equal, as the cone angle increases, the sealing performance deteriorates and the pumping speed decreases.

**Figure 14.** The performance of different variable-radius designs for new rotors: (a) pumping speed curve; (b) shaft power curve.

The internal volume ratio ν significantly influences the trend of shaft power, with the main changes detailed in Figure 14b. Specifically, as the α_c increases, leading to a higher ν , the average shaft power is reduced. For instance, the average shaft power of Rotor #11 is 9.3% lower than that of Rotor #16, suggesting that an increase in ν contributes to energy savings.

4.2. Effect of Variable Pitch on Rotor Performance

As shown in Table 3, under identical conditions, only the pitch variation pattern was adjusted.

As shown in Figure 15, the variable-pitch design in new rotors most predominantly affects the radial clearance leakage area A_2 . A smaller pitch reduces A_2 , with a more pronounced reduction observed near the discharge side; at the discharge position, the A_2 of Rotor #21 is 44% lower than that of Rotor #24. Consequently, enhancing ν by pitch reduction is constrained by the deterioration in sealing performance and the structural limitations associated with tooth width reduction. Conversely, the variable-pitch design has a minor impact on other leakage paths (A_1 , A_3 , and A_4).

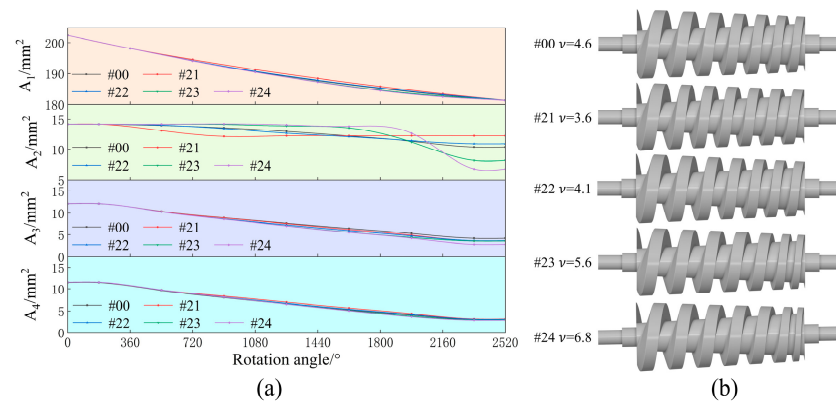


Figure 15. The geometric characteristics of different variable-pitch designs for new rotors: (a) leakage channel area A_1 , A_2 , A_3 , A_4 ; (b) three-dimensional model.

As shown in Figure 16a, under low-vacuum conditions, the pumping speed is observed to increase progressively with the internal volume ratio ν . For instance, at 1×10^4 Pa, increasing ν from 3.6 (Rotor #21) to 6.8 (Rotor #24) results in a rise of the volumetric flow rate from $176.71 \text{ m}^3/\text{h}$ to $184.64 \text{ m}^3/\text{h}$, a slight increase of 4.3%. As shown in Figure 16b, In the region where the inlet pressure is below 1×10^4 Pa, the shaft power decreases significantly with increasing ν . For instance, at an inlet pressure of 1×10^3 Pa, as ν rises from 3.6 to 6.8, the shaft power drops from 1.75 kW to 1.43 kW, representing an 18.3% decrease. However, at inlet pressures above 1×10^4 Pa, rotors with higher ν values require greater shaft power, a phenomenon primarily attributed to the substantial influence of the internal volume ratio on the gas compression process.

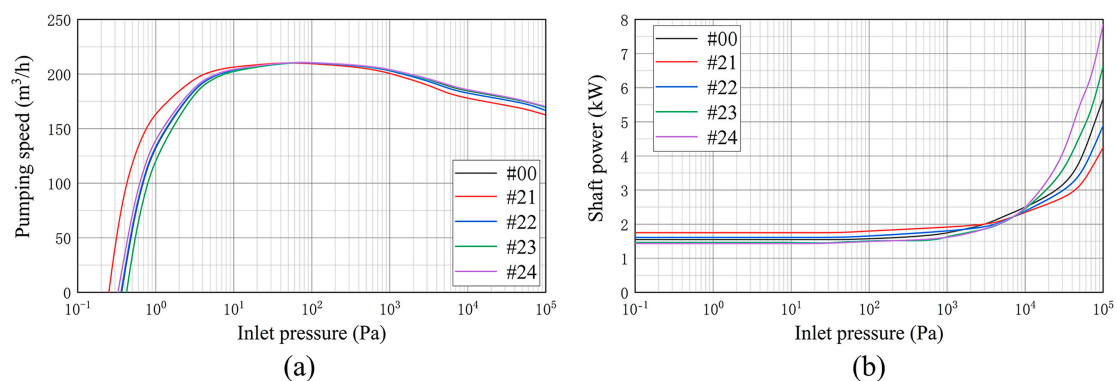


Figure 16. The performance of different variable-pitch designs for new rotors: (a) pumping speed curve; (b) shaft power curve.

Table 3. Design parameters of different variable-pitch designs for new rotors.

	L (mm)	N	P (mm)	A (mm)	R_1 (in—out)	α_c (°)	ν
#00	350	7	65–35	100.5	72–57	2.5	4.6
#21	350	7	65–45	100.5	72–57	2.5	3.6
#22	350	7	65–40	100.5	72–57	2.5	4.1
#23	350	7	65–30	100.5	72–57	2.5	5.6
#24	350	7	65–25	100.5	72–57	2.5	6.8

4.3. Internal Working Situation

Using the proposed thermodynamic calculation model, the working state of the new Rotor #00 was simulated, and the pressure distribution in the working chamber under

different inlet pressures was obtained, as shown in Figure 17. The pressure increases progressively from the suction side to the discharge side. At inlet pressures below 10 kPa, all chamber pressures remain below the outlet pressure (101,325 kPa), whereas above 10 kPa, the maximum pressure exceeds the outlet pressure. The gas eventually mixes with backflow at the discharge port and reaches the outlet pressure at the discharge side. The temperature distribution is governed by the cooling conditions; notably, the use of a water-cooled pump chamber leads to substantial suppression of temperature rise.

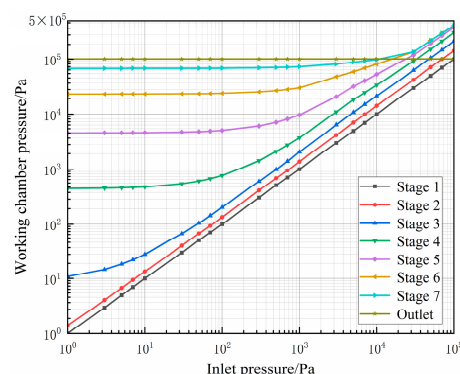


Figure 17. The pressure distribution in the working chamber under different inlet pressures.

4.4. Effect of Clearance on Pump Performance

As shown in Figure 18, the pumping speed decreases significantly with increased clearance, primarily due to aggravated leakage. The overall decrease in shaft power is primarily attributed to reduced useful work from diminished effective gas mass flow and to attenuated mechanical friction and viscous shear between the rotors. It is noteworthy that due to the higher internal compression ratio of the new Rotor #00, the temperature and pressure on the discharge side are correspondingly greater, leading to more severe thermal deformation. Consequently, a larger operational clearance is generally required for variable cross-section rotors compared to conventional uniform cross-section rotors in practical design.

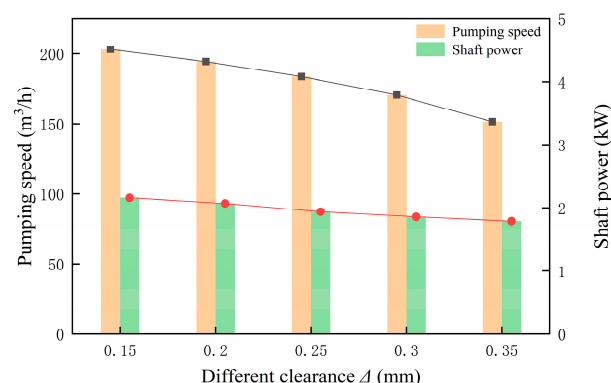


Figure 18. The pumping speed and shaft power under different clearance conditions.

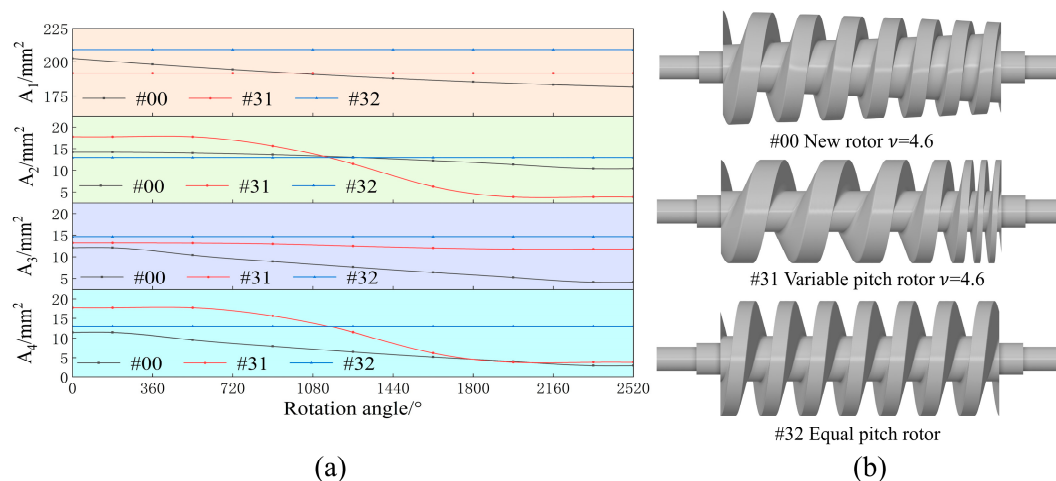
4.5. Comparison with Traditional Rotors

As shown in Table 4, a comparative analysis was conducted between the newly proposed variable-radius and variable-pitch rotor (#00) and the traditional rotors: the variable-pitch rotor (#31) and the equal-pitch rotor (#32). The table clearly demonstrates that while all three rotors achieve similar maximum pumping speed, the proposed rotor (#00) offers a superior combination of high internal volume ratio, robust structural design (wider tooth at discharge side), and significantly lower shaft power.

Table 4. Comparison of three types of screw rotors.

	#00	#31	#32
L (mm)	350	350	350
N	7	7	7
P (mm)	65–35	80–17	50
A (mm)	100.5	100.5	100.5
R_1 (in – out)	72–57	65	76
α_c (°)	2.5	0	0
Nominal clearance (mm)	0.25	0.25	0.25
ν	4.6	4.6	1
Min. tooth width at discharge (mm)	18	6	22
Maximum pumping speed (m ³ /h)	210.8	209.6	211.55
Avg. shaft power reduction vs. #32	52.9%	54.6%	100%
Sealing performance	Excellent	Good	Moderate
Structural integrity	High	Low	High

Specifically, as shown in Figure 19, to achieve the same ν , Rotor #00 demonstrates a relatively gradual pitch reduction, whereas Rotor #31 undergoes an abrupt pitch decrease from 80 mm to 17 mm, resulting in a tooth width reduction from 29.56 mm to 6.42 mm (a 78.3% decrease). As visualized in the 3D model in Figure 19b, this extreme thinning at the discharge side leads to insufficient structural strength and manufacturing challenges, while the high temperature and pressure in this region aggravate thermal deformation and clearance leakage. These issues severely constrain the development of high-performance rotors. In contrast, the variable-radius and variable-pitch rotor effectively addresses this problem by incorporating variable cross-section compression, which enhances ν while avoiding excessive tooth width reduction at the discharge side.

**Figure 19.** The geometric characteristics of three types of screw rotors: (a) leakage channel area A_1 , A_2 , A_3 , A_4 ; (b) three-dimensional model.

Regarding the total leakage area, Rotor #31 exhibits a 3.6% larger value than Rotor #00, indicating that the new rotor (#00) has superior sealing performance. This result is consistent with that shown in Figure 20a: the pumping speed of Rotor #00 is generally slightly higher than that of Rotor #31, and its ultimate vacuum performance is also superior. Although the conventional equal-pitch Rotor #32 has a 12% larger total leakage area than Rotor #00, it demonstrates a higher pumping speed. This is primarily because no internal gas compression occurs in Rotor #32, resulting in an absence of pressure differences between chambers and a consequent significant reduction in pressure-driven clearance leakage.

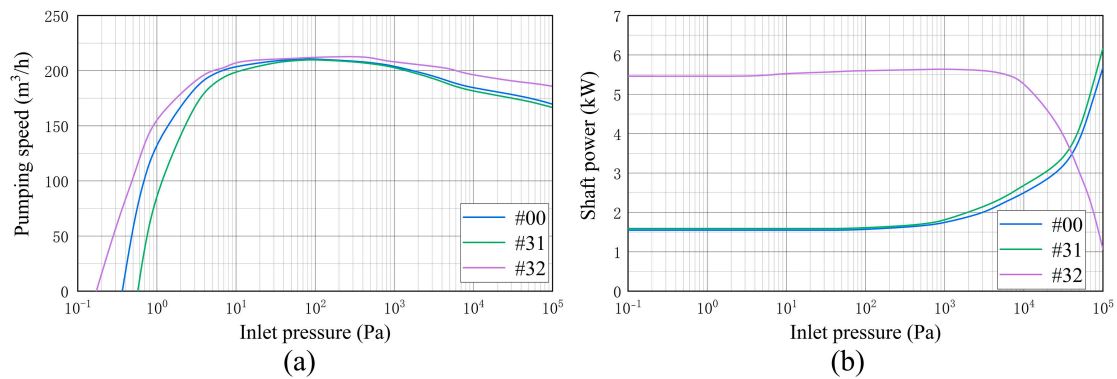


Figure 20. The performance of three types of screw rotors: (a) pumping speed curve; (b) shaft power curve.

As shown in Figure 20b, the shaft power of Rotor #32 is considerably higher than that of the rotors with internal compression (#00 and #31), with an average increase of 52.9%. This is mainly due to the lack of internal compression in Rotor #32, leading to substantially higher discharge power consumption. Rotors #00 and #31 show similar shaft power trends, but #00 exhibits marginally lower overall shaft power. This improvement stems from the more uniform distribution of the working chamber volume in Rotor #00, which optimizes the internal pressure distribution, avoids large pressure differentials, and thereby enhances sealing and reduces shaft power.

5. Conclusions

This study first establishes a parametric modeling and design methodology for variable-radius and variable-pitch rotors for screw vacuum pumps. A dedicated thermodynamic computational model is then developed. Using this model, the influence of structural parameters on performance is systematically analyzed. Furthermore, a comparative study is conducted against conventional rotor configurations. The main conclusions are as follows:

- (1) The proposed rotor profile comprises seven fully smooth segments, effectively eliminating stress concentration points typical in conventional profiles. A conical screw rotor is generated through the synergistic integration of variable-radius and variable-pitch design methodology. Corresponding clearance design methods are provided, resulting in optimized clearance distribution.
- (2) A clearance leakage model incorporating four types of leakage channels was developed. Based on this, a thermodynamic computational model was constructed and validated experimentally.
- (3) The variable-radius and variable-pitch rotor achieves a dual internal compression mechanism, with the variable-radius contribution exceeding that of the variable-pitch. As the rotor cone angle increases, the internal volume ratio ν rises significantly, leading to lower shaft power. However, sealing performance deteriorates, leading to a decrease in pumping speed.
- (4) The shaft power of the rotor with internal compression is significantly lower than that of the equal-pitch rotor, with the average shaft power decreasing by 52.9%. The new rotor slightly outperforms the conventional variable-pitch rotor in both sealing performance and power consumption. Furthermore, it achieves a higher internal volume ratio ν , which enhances internal compression efficiency, improves structural compactness, avoids excessively small tooth width at the discharge side, and demonstrates promising potential for broader application scenarios.

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Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Nomenclature

R_1	Radius of the addendum circular arc (mm)	A	Center distance (mm)
R_2	Radius of the pitch circle (mm)	A_i	Clearance area (mm ²)
R_3	Radius of the dedendum circular arc (mm)	L_i	Clearance length (mm)
R_4	Radius of the back circular arc (mm)	h_i	Clearance depth (mm)
R_5	Radius of the tip circular arc (mm)	α	Involute initial angle (rad)
R_b	Radius of the generating circle (mm)	β	Divergence angle (rad)
ν	The internal volume ratio	γ	Curve EF offset Angle (rad)
Δ_1	Tooth top clearance width (mm)	θ	The offset Angle of arc BC (rad)
Δ_2	Radial clearance width (mm)	L	Rotor length (mm)
Δ_3	Tooth clearance width (mm)	N	Spiral turns
Δ_4	Tooth side clearance width (mm)	α_c	Spiral turns (°)
B	Tooth width (mm)	p	Pitch (mm)

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