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Determination of Reference Density from Measured Density and Temperature Data: A Diesel Fuel Case Study

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Abstract

Accurate density measurement is essential in oil and liquid petroleum product handling, particularly in mass and volumetric flow measurement, loss control, and quality assurance processes. Reference density values are commonly determined using standard API tables, which provide a standardized procedure for correcting measured density to reference conditions. The proposed method is evaluated using experimental data acquired during a diesel fuel transfer operation at the "Ilinden"–Makpetrol Skopje terminal in North Macedonia. Density and temperature were monitored using a Micro Motion ELITE Coriolis flow meter. Based on the collected data, the reference density was calculated using the proposed formulation and compared with values obtained from API reference tables. The results show a difference between the reference density values obtained using the two approaches. The proposed approach provides an alternative means of estimating reference density from measured process data. Further validation using independent reference measurements and additional petroleum products under different operating conditions is required to establish its accuracy and applicability.

Keywords: reference density; Coriolis meter; oil and liquid petroleum derivatives

1. Introduction

Measurement of density is required in many applications within the oil and liquid petroleum derivatives industry, including the measurement of mass and volumetric flow, detection and quality control, as well as determination of concentration.

The control of all products stored in a warehouse, including crude oil and liquid petroleum derivatives, requires accurate quantification—either by volume at standard temperature, by mass, by weight, or by package—from the point of receipt to the point of dispatch. This process necessitates the use of appropriate equipment, precise measurement practices, a proper calculation procedure, and an established monitoring methodology.

Measurement of storage tank contents for crude oil and petroleum derivatives involves determining the liquid level and temperature within the tank and may also include density measurement when the mass of the stored product is to be calculated. In cases where the total quantity loaded into road tankers is determined based on tank gauging, losses may occur because of inaccurate measurement.

In the process of identifying and quantifying losses, the most critical activity is measurement, which can be performed in two principal ways [1,2]:

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(a) Static measurement (gauging systems)—conducted in tanks or other types of storage equipment, whose primary function is to determine and monitor the quantity of stored product.
(b) Dynamic measurement (metering systems)—primarily used during the transfer of product into or out of a tank, typically to or from a road tanker or railcar.

In this paper, the focus is placed on dynamic measurement. In dynamic measurement, quantification is performed using flow meters, whose accuracy must remain within ±0.1% of the flow rate, with repeatability of ±0.025% at the specified flow rate over a minimum period of six months, without the need for frequent recalibration [1,2].

Historically, positive displacement meters were almost the only devices capable of meeting these requirements. However, with the development and industrial application of Coriolis-based mass flow meters, whose performance characteristics significantly exceed the specified criteria, very high measurement accuracy can now be achieved in loading and unloading systems for stored products [3,4]. Furthermore, these meters enable simultaneous high-precision measurement of instantaneous volumetric flow rate, mass flow rate, and real-time fluid density, which is of critical importance for monitoring and control of crude oil and liquid petroleum derivatives in storage facilities [5].

Density is used in the mass flow equation [6,7]:

$$q_m = q_V \cdot \rho \quad (1)$$

where

q_m—mass flow rate (kg/s);
q_V—volume flow rate (m^3/s);
ρ—fluid density (kg/m^3) under the given operating conditions.

It is essential that the volumetric flow and the density of the fluid are determined under the same conditions, i.e., at the same fluid temperature and pressure, for the above equation to be valid:

$$q_V = q_V(t,p);$$
$$\rho = \rho(t,p)$$

Accordingly, the mass flow is defined by:

$$q_m = q_V(t,p) \cdot \rho(t,p) \quad (2)$$

Coriolis flow meters are widely recognised as highly accurate instruments for fluid flow measurement, primarily due to their capability to directly measure mass flow rate. Their operating principle is based on the Coriolis effect, whereby a phase shift in vibrating measurement tubes occurs as a consequence of fluid motion and is proportional to the mass flow rate [8]. In addition to mass flow measurement, Coriolis meters can simultaneously determine fluid density and temperature, enabling precise calculation of volumetric flow without external compensation [9]. Owing to their high precision, repeatability, and independence from fluid properties such as viscosity and pressure, Coriolis flow meters have found extensive application in industrial processes requiring reliable and traceable flow measurements [10]. Furthermore, recent studies highlight their role as transfer standards in metrological applications, confirming their suitability for high-accuracy calibration and verification tasks [11].

Density must be measured when determining the Volume Correction Factor (VCF), during procedures for calculating volume flow rate at standardized temperature, or when determining product mass based on its measured volume, or volume based on its measured mass.

The ASTM D1250 standard defines an iterative method for correcting the density and volume of petroleum liquids to reference conditions. Since the thermal expansion coefficient of hydrocarbons depends on the reference density, and the reference density itself is unknown a priori, an iterative numerical procedure is required. The method successively updates the reference density and thermal expansion coefficient until convergence within a prescribed tolerance is achieved. This procedure enables accurate determination of the VCF, which is essential for fiscal and custody transfer measurements in petroleum production, transportation, and storage systems. Owing to its high accuracy and international standardisation, the ASTM D1250 algorithm is widely implemented in modern flow computers and measurement software [12–14].

When density is used to determine the mass of a petroleum product, the required measurement accuracy depends on the target uncertainty of the calculated mass and the contribution of density measurement to the overall uncertainty. This requirement may differ from that applicable to density measurements used for determining the VCF.

The aim of this study is to investigate an alternative approach for determining the reference density of petroleum products from measured density and temperature data and to compare the results with those obtained using the standard ASTM D1250 procedure, based on experimental data collected during the unloading of diesel fuel.

2. Physical Quantities—Volume and Density of Liquids

The storage of products at oil and petroleum derivative depots primarily requires quantitative measurement throughout the processes of receipt, delivery, and storage [2,15,16]. The losses that occur in tanks are classified into storage losses and working losses. Working losses occur due to the filling and discharging process [17].

The basic physical quantities used to express the amount of product, i.e., the quantity of oil and petroleum derivatives, are volume, mass, and weight. The volume flow rate of a fluid (q_V), as a state variable, depends on the fluid's temperature (t) and pressure (p). Every state variable, including volume, has the characteristic that the total change in this quantity depends only on the initial and final states of the process, and not on the way the process is carried out.

Therefore, a state quantity, i.e., volume, is a function of the state of a physical system, not a function of the process within that system, which is characterized by a process quantity. This characteristic of the state variable of a fluid, volume flow rate, is [2]:

$$\mathrm{d}q_V = \left(\frac{\partial q_V}{\partial p}\right)_t \cdot \mathrm{d}p + \left(\frac{\partial q_V}{\partial t}\right)_p \cdot \mathrm{d}t \tag{3}$$

The total change in the volume flow rate of a fluid depends on the total changes in other state variables, namely temperature and pressure (dt and dp):

$$\frac{\mathrm{d}q_V}{q_V} = \frac{1}{q_V} \cdot \left(\frac{\partial q_V}{\partial p}\right)_t \cdot \mathrm{d}p + \frac{1}{q_V} \cdot \left(\frac{\partial q_V}{\partial t}\right)_p \cdot \mathrm{d}t \tag{4}$$

If we define α as the volumetric coefficient of thermal expansion, i.e.,

$$\alpha = \frac{1}{q_V}\left(\frac{\partial q_V}{\partial t}\right)_p$$

And if we define z as the volumetric (isothermal) compressibility coefficient, i.e.,

$$z = -\frac{1}{q_V} \cdot \left(\frac{\partial q_V}{\partial p}\right)_t$$

Then, the relative change in the fluid volume flow rate is given by:

$$\frac{\mathrm{d}q_{\mathrm{V}}}{q_{\mathrm{V}}} = \alpha(p,t)\cdot \mathrm{d}t - \mathrm{z}(p,t)\cdot \mathrm{d}p \tag{5}$$

where

α—volumetric coefficient of thermal expansion (K^{-1});

z—isothermal compressibility coefficient (Pa^{-1});

dt—change in temperature;

dp—change in pressure.

The volumetric coefficient of thermal expansion and the volumetric (isothermal) compressibility coefficient depend exclusively on the type and nature of the fluid; in other words, they are among the fundamental characteristics of each fluid.

Compared to other liquids, for example, water, crude oil and liquid petroleum products have significantly higher coefficients of thermal expansion α and compressibility z. This is the main reason the influence of temperature and pressure on the measured volume of oil or liquid petroleum products is considered important. It should also be emphasized that the effect of temperature on oil and liquid petroleum products is many times greater than the effect of pressure on volume.

As already mentioned, density is a key physical quantity in the system for controlling process losses in the oil and petroleum derivative industry, as well as an important parameter in determining the quantity and verifying the quality of stored products. The density data obtained from measurements is used [12]:

(a) As an input parameter in the process of determining the VCF which is required to calculate the volume flow rate at reference conditions, $q_{\mathrm{V}}(t_{\mathrm{R}})$. A high level of accuracy in density measurement is not required for determining the VCF,

(b) As an input parameter for converting standard volume flow rate into weight, or mass, of oil and petroleum derivative. A high level of accuracy in density measurement is required. $\Delta\rho_{\mathrm{m}} \leq 0.2\ [\mathrm{kg/m^3}]$, $\overline{\varepsilon_{\rho}} \leq 0.025\ [\%]$,

(c) As an input parameter for quality control of petroleum derivatives, i.e., as a control parameter to determine whether the product quality conforms to the specified quality standards. A high level of accuracy in density measurement is also required. $\Delta\rho_{m} \leq 0.2\ [\mathrm{kg/m^3}]$, $\overline{\varepsilon_{\rho}} \leq 0.025\ [\%]$.

The VCF is a coefficient used to convert the measured volume flow rate equivalent volume under reference conditions, at temperature $\mathrm{t_R}$ and pressure $\mathrm{p_R}$.

The VCF represents the ratio between the volume flow rate of a liquid at the reference temperature and the volume flow rate of the same liquid at the actual (measured) temperature.

Based on the above definition, it can be stated that:

$$q_{\mathrm{V}}\ (p_R,\ t_R)\ = F_{\mathrm{CV}}(p,\ t)\cdot q_{\mathrm{V}}(p,\ t) \Longrightarrow\ \mathrm{VCF(p,\ t)} = \frac{q_{\mathrm{V}}\ (p_{\mathrm{R}},\ t_R)}{q_{\mathrm{V}}(p,\ t)} = \frac{q_{\mathrm{V_R}}}{q_{\mathrm{V}}} \tag{6}$$

Starting from the law of conservation of mass flow rate in a closed physical system,

$$q_{\mathrm{m_R}} = q_{\mathrm{V_R}}(p_{\mathrm{R}},\ t_R\ )\cdot \rho_{\mathrm{R}}\ (p_{\mathrm{R}},\ t_R\ ) = q_{\mathrm{V}}(t,p)\cdot \rho(t,p) = q_{\mathrm{m}} \tag{7}$$

the VCF is obtained:

$$F_{\mathrm{CV}}(t,p) = \frac{q_{\mathrm{V}}\ (p_{\mathrm{R}},\ t_R\ )}{q_{\mathrm{V}}(p,\ t)} = \frac{q_{\mathrm{V_R}}}{q_{\mathrm{V}}} = \frac{\rho(t,p)}{\rho_{\mathrm{R}}\ (p_{\mathrm{R}},\ t_R\ )} = \frac{\rho}{\rho_{\mathrm{R}}} \tag{8}$$

Taking into account the previously stated fact regarding the influence of temperature which is many times greater than the effect of pressure on the volume of stored crude oil and liquid petroleum products it follows that:

$$F_{CV}(t,p) \Longrightarrow F_{CV}(t) = \frac{q_V\,(p_R\,)}{q_V(p,\,t)} = \frac{q_{V_R}}{q_V} = \frac{\rho(t,p)}{\rho_R\,(p_R\,)} = \frac{\rho}{\rho_R} \tag{9}$$

where

$q_{V_R}\,(t_R) = q_{V_R}$—volume flow rate under standard (reference) conditions, i.e., reference volume flow rate at the reference temperature.

$t_R = 15$ °C—reference temperature.

$q_V(t) = q_V$—volume flow rate at the current temperature of the quantified liquid petroleum derivative.

t—current temperature of the liquid petroleum derivative.

F_{CV}—the Volumetric Correction Factor (VCF).

It is obtained that the volumetric correction factor is a function of temperature:

$$F_{CV}(t) = \exp\left[-\int_{t_R}^{t} \alpha(t)\mathrm{d}t\right] \tag{10}$$

that is, a function of the volumetric thermal expansion coefficient, which represents a fundamental physical property of the given fluid.

In order to obtain the temperature-dependent function of the VCF which is used to determine the value of the VCF at a given temperature and to calculate the volume flow rate of a liquid under reference conditions, it is necessary to precisely know the temperature dependence of the thermal expansion coefficient $\alpha(t)$ of the given liquid. Furthermore, the use of a single temperature-dependent function of the thermal expansion coefficient is only applicable for a specifically defined liquid, i.e., only for the liquid to which that coefficient corresponds.

However, based on multi-year research conducted by the American Petroleum Institute [11,12], it is recommended that the VCF be calculated using an approximate equation, which, with a certain degree of accuracy, guarantees its values for precisely defined and tested groups of oil and petroleum derivative of consistent quality and known physicochemical properties. The API tables (ASTM D1250) are standardized tables used in the petroleum industry to convert the volume and density of petroleum products from one temperature to another—typically to the standard reference temperature of 15 °C [12,15].

The recommended equation, as proposed by [11–13], is:

$$F_{CV}(t) = \frac{\rho(t)}{\rho(t_R = 15\ ^\circ\mathrm{C})} = \exp[-\alpha(t_R)\cdot(t - t_R)\cdot(1 + 0.8\cdot\alpha(t_R)\cdot(t - t_R))] \tag{11}$$

where the volumetric thermal expansion coefficient at the reference temperature is determined using the following formula:

$$\alpha(t_R) = \alpha_R = \frac{K_0 + K_1\cdot\rho_R}{\rho_R^2} \tag{12}$$

K_0, K_1—constants are defined for each group of petroleum derivatives, classified according to the reference density, $\rho_R = \rho(t_R)$.

The values of these constants are taken from tables and are based on laboratory testing [11,12,14], where

(a) For crude oil with a reference density value within the range $\rho_R \in [770.5\text{–}787.5]\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$, the volumetric thermal expansion coefficient at the reference temperature, is taken as:

$$\alpha(t_R) = \alpha_R = \frac{K_0}{\rho_R^2} + \frac{K_1}{\rho_R},\ \mathrm{K_0} = -0.00336312\left[\frac{(\mathrm{kg/m^3})^2}{^\circ\mathrm{C}}\right],\ \mathrm{K_1} = 2680.3206\left[\frac{(\mathrm{kg/m^3})}{^\circ\mathrm{C}}\right]$$

(b) For motor gasoline with a reference density value within the range $\rho_R \in [600.0–770.4]\left[\frac{kg}{m^3}\right]$ the volumetric thermal expansion coefficient at the reference temperature, is taken as:

$$\alpha(t_R) = \alpha_R = \frac{K_0}{\rho_R^2} + \frac{K_1}{\rho_R},\ K_0 = 346.4228\left[\frac{(kg/m^3)^2}{°C}\right],\ K_1 = 0.4388\left[\frac{(kg/m^3)}{°C}\right]$$

(c) For kerosene and Jet A-1, whose reference density values fall within the range $\rho_R \in [787.6–838.5]\left[\frac{kg}{m^3}\right]$ the volumetric thermal expansion coefficient at the reference temperature, is taken as:

$$\alpha(t_R) = \alpha_R = \frac{K_0}{\rho_R^2},\ K_0 = 594.5418\left[\frac{(kg/m^3)^2}{°C}\right]$$

(d) For fuel oil, whose reference density value falls within the range $\rho_R \in [838.6–1200.0]\left[\frac{kg}{m^3}\right]$ the volumetric thermal expansion coefficient at the reference temperature, is taken as:

$$\alpha(t_R) = \alpha_R = \frac{K_0}{\rho_R^2} + \frac{K_1}{\rho_R},\ K_0 = 186.9696\left[\frac{(kg/m^3)^2}{°C}\right],\ K_1 = 0.4862\left[\frac{(kg/m^3)}{°C}\right]$$

(e) For lubricating oils, whose reference density values fall within the range $\rho_R \in [750.0–1164.0]\left[\frac{kg}{m^3}\right]$ the volumetric thermal expansion coefficient at the reference temperature, α_{ref}, is taken as:

$$\alpha(t_R) = \alpha_R = \frac{K_1}{\rho_R},\ K_1 = 0.6278\left[\frac{(kg/m^3)}{°C}\right]$$

Based on the above, we can draw the following conclusions when it comes to calculating the VCF:

- In order to obtain the necessary temperature-dependent function of the volume correction factor on the basis of which the value of the volume correction factor for a given temperature is determined, and on the basis of which the volume of the liquid is recalculated for the reference conditions, it is necessary to know exactly the temperature dependence function of the volume coefficient of thermal expansion $\alpha(t)$ of the given liquid.
- Using one function of the temperature dependence of the volume coefficient of thermal expansion is possible only for a precisely determined liquid and for the one to which that coefficient belongs.
- It can be observed that the recommended Equation (11), when using approximate values of the volumetric thermal expansion coefficient, determined from the proposed constants, and the known exact density, of the fluid at reference conditions, yields an approximate value of the VCF. This calculated VCF deviates from the exact value, indicating the effect of approximation on the final result.

The recommended Equations (11) and (12) actually constitute a system of two equations with two unknowns $[(\rho_R) \wedge (\alpha_R)]$ which can be easily observed:

$$\begin{gathered}\frac{\rho(t)}{\rho_R} = exp(-\alpha_R(t - t_R)(1 + 0.8{\cdot}\alpha_R(t - t_R))\\ \alpha_R = \frac{K_0 + K_1\rho_R}{\rho_R^2}\end{gathered} \quad (13)$$

where the known quantities in this system are the measured physical variables, density (ρ) and temperature (t), whereas ρ_R denotes the reference density, which is the unknown parameter.

This system of Equation (13) cannot be solved explicitly for either of the unknowns, ρ_R or α_R; therefore, an iterative approximation method is employed. This involves a step-by-step solution through repeated calculations.

An initial estimate of the reference density is assumed as $\rho_{R_0} = \rho(t)$, based on which the first value of the volumetric thermal expansion coefficient, α_{R_0}, is calculated. Using the obtained α_{R_0}, a new approximate value of the reference density, ρ_{R_1}, is determined, from which the first value of the VCF is subsequently calculated:

$$F_{CV_1} = \frac{\rho(t)}{\rho_{R_1}} \tag{14}$$

Based on the value of ρ_{R1}, a new value of α_{R1} is calculated, which in turn determines a new value of ρ_{R2}, and consequently a new value of the volumetric correction factor F_{CV_2}.

This iterative process continues until the difference between successive results becomes smaller than a predefined tolerance value, i.e.,:

$$\left|\rho_{R_{i+1}} - \rho_{R_i}\right| \leq \varepsilon \tag{15}$$

where ε represents the density deviation error.

As the density is a key parameter in the loss control system, and considering the importance and application of the reference density in the quality control process of the measured liquid, where high accuracy is required, in contrast to the statement mentioned at the beginning of this chapter [12–14] that high accuracy is not required for the determination of the volume correction factor for the density measurement of a given petroleum liquid derivative, there is a great need for further research regarding the possibility of determining the value of the reference density (ρ_R) without applying the existing API tables from [12]. The possibility of determining the value of the reference density (ρ_R) without applying the existing API tables will be discussed below.

3. Application of the Coriolis Meter for Automatic Measurement of Reference Density

Coriolis-based mass flow meters are modern measuring instruments that enable simultaneous measurement of mass flow and fluid density with high accuracy [4,18]. In the oil and petroleum products industry, their application is particularly important because they allow automatic determination of reference density (ρ_R), i.e., the fluid density recalculated to the reference temperature (usually temperature of 15 °C).

As discussed earlier, determining fluid density using the previously presented Equation (13) requires several known parameters. In the following section, the volumetric thermal expansion coefficient $\alpha(t)$ and the reference density (ρ_R) of liquid petroleum products will be determined.

During this investigation, a high-accuracy Coriolis meter, namely the Micro Motion ELITE sensor, was employed for density measurement [19]. All meters in the ELITE series have the same declared measurement accuracies, namely a maximum flow measurement error of 0.1% of the reading and a maximum density measurement error of 0.5 kg/m^3. The meter used in this study was the CMF300 model. The temperature sensor is an external device and is a Class B Pt100 resistance temperature detector (RTD).

The Micro Motion ELITE Coriolis flow meter, developed by Emerson, is a high-precision multivariable measurement instrument widely applied for mass flow, volumetric flow, density, and temperature measurement of liquids, gases, and slurries. The meter provides exceptional accuracy, typically ±0.10% of rate for liquid mass flow and ±0.25% for gas mass flow, with density measurement accuracy on the order of ±0.0005 g/cm^3, making it suitable for both process control and custody transfer applications. The ELITE series covers nominal pipe sizes from DN2 to DN300 and operates reliably under extreme pressure and temperature conditions. Its construction from robust materials, such as 316L stainless steel and nickel alloy C-22, ensures high resistance to aggressive media and harsh environmental conditions.

A distinguishing feature of the ELITE meters is Smart Meter Verification (SMV), an advanced diagnostic function that enables in-situ verification of meter performance, thereby ensuring long-term measurement integrity without interrupting operation. The instrument exhibits outstanding immunity to external field effects, including pressure and temperature variations as well as mechanical vibrations, guaranteeing stable and repeatable measurements. The integrated densitometer offers very high accuracy, with density measurement uncertainty of ±0.0002 g/cm^3 (±0.2 kg/m^3) and repeatability of ±0.0001 g/cm^3 (±0.1 kg/m^3). Owing to its high sensitivity to density variations, reliability, and absence of moving parts, the Micro Motion ELITE Coriolis meter is regarded as an industry-leading solution for high-accuracy flow and density measurements in complex industrial and custody transfer systems [20,21].

The basic definition of the coefficient of thermal expansion is:

$$\alpha = \alpha(t) \equiv \frac{1}{q_V(t)} \frac{\mathrm{d}q_V(t)}{\mathrm{d}t} = -\frac{1}{\rho(t)} \frac{\mathrm{d}\rho(t)}{\mathrm{d}t} \tag{16}$$

Or expressed via the fluid density at a given temperature (t):

$$\rho(t) = \rho(t_R)\exp\left[-\int_{t_R}^{t} \alpha(t)\mathrm{d}t\right] \tag{17}$$

Or via the reference fluid density at a reference temperature (t_R):

$$\rho(t_R) = \rho(t)\exp\left[\int_{t_R}^{t} \alpha(t)\mathrm{d}t\right] \tag{18}$$

The coefficient of thermal expansion α is a measure of the change in density and volume with temperature. It is the fundamental and only necessary parameter for calculating reference values of density and volume.

In order to calculate $\int_{t_R}^{t} \alpha(t)\mathrm{d}t$, it is necessary to know the functional dependence of the coefficient of thermal expansion on temperature, which is essentially one of the most important properties, i.e., a characteristic of the given fluid. This property depends on the intrinsic nature of its physical structure.

Under the first assumption, starting from the fact that for a given temperature the coefficient of thermal expansion is the same (i.e., there is no hysteresis), we further assume that it remains constant and independent of temperature for a small incremental change in temperature.

Under the above assumption, the solution of the differential equation is obtained by the mathematical operation of integration:

$$\frac{d\rho(t)}{\rho(t)} = -\alpha(t)\mathrm{d}t \tag{19}$$

is obtained by the mathematical operation of integration

$$\int_{t_1}^{t_2} \frac{\mathrm{d}\rho(t)}{\rho(t)} = -\int_{t_1}^{t_2} \alpha(t)\mathrm{d}t \tag{20}$$

and under the condition that $\Delta t = t_1 - t_2 \to 0$, we have:

$$\alpha = \frac{1}{t_1 - t_2}\ln\left[\frac{\rho(t_2)}{\rho(t_1)}\right] \tag{21}$$

The temperature of the fluid is measured with a thermometer with an accuracy of $\pm\Delta t(\varepsilon)$ while the density of the fluid is measured with an accuracy of $\pm\Delta\rho(\varepsilon)$, which means that the thermometer reading is $t_1 \pm \Delta t(\varepsilon)$ and $t_2 \pm \Delta t(\varepsilon)$, while the reading of the density meter is $\rho(t_1) \pm \Delta\rho(\varepsilon)$ and $\rho(t_2) \pm \Delta\rho(\varepsilon)$.

The measured value of the coefficient of thermal expansion becomes:

$$\alpha = \frac{1}{t_1 \pm \Delta t(\varepsilon) - t_2 \mp \Delta t(\varepsilon)} \ln\left[\frac{\rho(t_2) \pm \Delta\rho(\varepsilon)}{\rho(t_1) \pm \Delta\rho(\varepsilon)}\right] = \frac{1}{\Delta t} \ln\left[1 \pm \frac{\Delta\rho(\Delta t)}{\rho(t_1)} \pm \frac{\Delta\rho(\varepsilon)}{\rho(t_1)}\right] \quad (22)$$

Now, a second assumption will be introduced. If it is satisfied that $\Delta\rho(\Delta t) \gg \Delta\rho(\varepsilon)$ then we have

$$\alpha = \frac{1}{\Delta t} ln\left[1 \pm \frac{\Delta\rho(\Delta t)}{\rho(t_1)}\right] \quad (23)$$

In other words, for the value of the coefficient of thermal expansion at temperature (t_2), which has changed by $\Delta t = t_1 - t_2$, the following formula applies:

$$\alpha(t_2) = \frac{1}{t_1 - t_2} \ln\left[1 \pm \frac{\Delta\rho(t_1 - t_2)}{\rho(t_1)}\right], \quad (24)$$

If we measure the density of a liquid at temperature (t), which is $(\rho(t))$, and the liquid's temperature changes by (Δt), the density will change by $(\Delta\rho(\Delta t))$ accordingly, so now we have that the coefficient of thermal expansion of the given liquid is:

$$\alpha = \alpha(t) = \frac{1}{\Delta t} \ln\left[1 + \frac{\Delta\rho(\Delta t)}{\rho(t)}\right] \quad (25)$$

The density of the liquid at the reference temperature (t_R) is determined based on the equation:

$$\rho(\mathrm{t_R}) \equiv \rho(t)\exp[\alpha(t) \cdot (t - \mathrm{t_R})] \quad (26)$$

then,

$$\rho(\mathrm{t_R}) = \rho(t)\exp\left[\left(\frac{t - \mathrm{t_R}}{\Delta t}\right) \cdot \ln\left(1 + \frac{\Delta\rho(\Delta t)}{\rho(t)}\right)\right] = \rho(t)\exp\left[\ln\left(1 + \frac{\Delta\rho(\Delta t)}{\rho(t)}\right)^{\left(\frac{t-\mathrm{t_R}}{\Delta t}\right)}\right] \quad (27)$$

we obtain the formula for the density at the reference temperature $\rho(\mathrm{t_R})$:

$$\rho(\mathrm{t_R}) = \rho(t) \times \left[1 + \frac{\Delta\rho(\Delta t)}{\rho(t)}\right]^{\left(\frac{t-\mathrm{t_R}}{\Delta t}\right)} \quad (28)$$

A Coriolis meter, at a precisely defined moment in time and with a specified measurement frequency, measures the fluid density, while a temperature sensor simultaneously measures the temperature of the fluid within the measuring tube. During a single operating process, such as filling or emptying a tank, the measurement system provides a large number of paired measured values $[\rho(t); t]$.

These data pairs contain, among other information, the thermal expansion characteristics of the fluid. In other words, based on these data, it is also possible to determine the average density of the given fluid at the reference temperature, expressed as:

$$\overline{\rho(\mathrm{t_R})} = \frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} \rho(\mathrm{t_i}) \cdot \left[\frac{\rho(\mathrm{t_{i+1}})}{\rho(\mathrm{t_i})}\right]^{\left(\frac{\mathrm{t_i} - \mathrm{t_R}}{\mathrm{t_i} - \mathrm{t_{i+1}}}\right)} \quad (29)$$

If the measured fluid passes through the meter at a variable flow rate, it is necessary to determine the average reference density as a mass-flow-weighted quantity. Owing to the ability of the Coriolis meter to simultaneously measure the instantaneous mass flow rate q_i using an independent flow sensor, the averaging procedure can be performed based on the mass flow rate. Thus, the final expression is obtained as:

$$\overline{\rho}(\mathrm{t_R}) = \frac{1}{\sum_{i=1}^{n} q_i} \times \sum_{i=1}^{n} q_{\mathrm{m_i}} \cdot \rho(t_i) \cdot \left[\frac{\rho(t_{i+1})}{\rho(t_i)}\right]^{\left(\frac{t_i - \mathrm{t_R}}{t_i - t_{i+1}}\right)} \quad (30)$$

where

$q_{m_i} = \frac{dm_i}{d\tau} \cdot \tau_i$ is the mass flow rate of the fluid over the time interval (τ_i).

t—current temperature of the liquid petroleum product.

ρ_t—current density of the liquid petroleum product at the current temperature.

When the fluid flow rate through the meter is constant or equal to zero ($q_i = q \vee 0$), the averaged reference density value becomes equal to the arithmetic mean: $\overline{\rho}(t_R) = \overline{\rho(t_R)}$.

In this study, the analysis is based on the data and measurement results obtained during the diesel fuel transfer process at the "Ilinden"–Makpetrol Skopje terminal in North Macedonia. The data were collected during the unloading of a road tanker carrying diesel fuel (D2) on 20 September 2006 at the Ilinden depot of Makpetrol Skopje. The unloading process started at 11:22:47 and ended at 11:48:38, resulting in a total unloading and measurement duration of 1548 s.

During the measurements at the Ilinden–Makpetrol terminal in Skopje, North Macedonia, a total of 1548 data points were recorded for each measured parameter using a Micro Motion ELITE Coriolis mass flow meter. The recorded parameters included mass flow rate, temperature, and density, corresponding to the total measurement duration of 1548 s.

In addition, in accordance with the methodology presented in [12], the Coriolis meter also provided the reference density, i.e., the density at 15 °C, as defined by the API petroleum measurement tables.

For all 1548 measurement points, reference density values were calculated using the proposed Equation (30) and subsequently compared with the corresponding reference density values obtained from the API tables [12]. The comparison revealed the presence of significant deviations between the calculated and tabulated reference density values.

Given that the interval between two successive measurements was $\Delta\tau = 1$ s, a total of $n = 1548$ ordered pairs $[\rho(t); t]$ were obtained. Using these data and the proposed Equation (25), the averaged reference density for the fuel derivative was calculated to be approximately 839 kg/m^3. The results indicated a density of approximately 839 kg/m^3 at an average temperature of 23 °C, with the corresponding mean reference density at 15 °C, as defined by the API tables.

Figure 1 presents a diagram illustrating the distribution of measured density values, along with the density at 15 °C (i.e., the reference density), for two cases. In the first case, the distributions correspond to reference density values obtained from the API tables, whereas in the second case, the reference density values at 15 °C were calculated using the proposed Equation (30).

Figure 1 compares the density values obtained during the unloading of D2 diesel fuel. The blue line represents the reference density calculated using the API tables, the magenta line represents the reference density calculated using the proposed Equation (30), and the yellow line shows the measured density at the actual fluid temperature. This comparison illustrates the differences between the reference density values obtained using the two calculation methods and the measured density at the operating temperature.

During the overflowing of the tank with oil derivative, the current volume flow rate was poured out $q_V(t_{SR}) = 29{,}750$ L/s $= 29.75$ m^3/s at a medium temperature $t_{SR} = \bar{t} = 23$ °C), which corresponds to the mean calculated mass flow rate: $\overline{q_m} = q_V(t_{SR}) \cdot \rho(t_{SR}) = \overline{q_V} \cdot \overline{\rho} = 24{,}841.25$ kg/s.

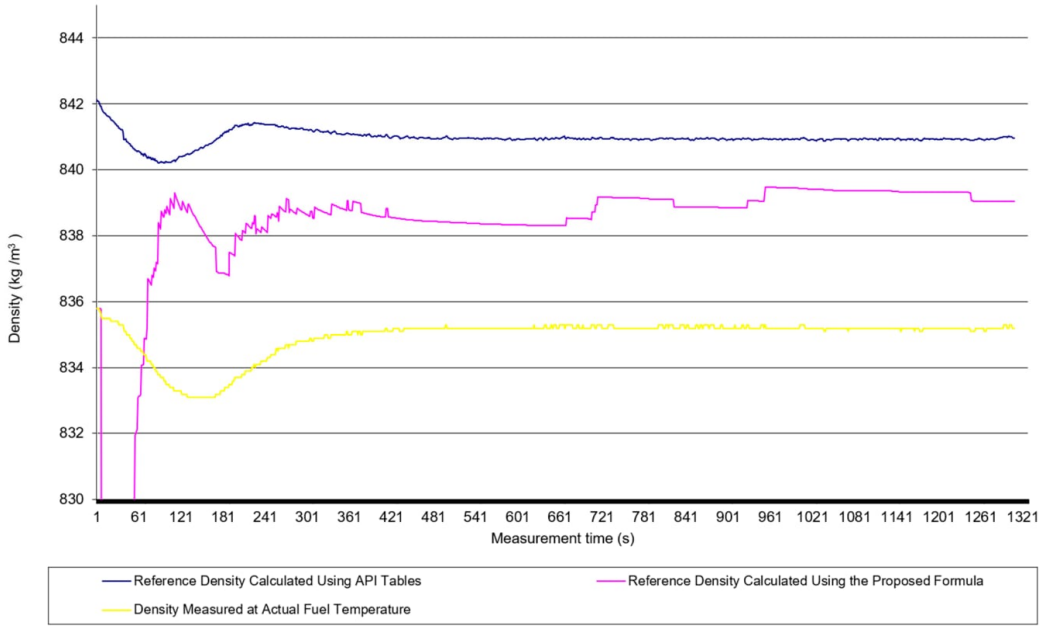


Figure 1. The distribution of measured reference density, calculated reference density and density at 15 °C.

Following the procedure for calculating the VCF as recommended by the standards, presented in Equation (11), and based on this, the obtained reference volume flow rate, i.e., the deviation of the reference volume flow rate calculated using the reference density determined from Equation (30), we obtain the following results:

(a) If the value is taken as the correct reference *density* $\rho_R = \rho(t_{\mathrm{R}}) = \rho(15\ ^\circ\mathrm{C})$ obtained on the basis of the tables from ASTM D 1250-80 starting from the measured density values $\rho(t_{\mathrm{SR}})$ and temperature values t_{SR}, we have:

$$F_{\mathrm{CV}}(\mathrm{t_{SR}}) = \exp[-\alpha_{\mathrm{R}}\Delta t(1 + 0.8{\cdot}\alpha_{\mathrm{R}}\Delta t)] = 0.993247$$

The reference volume flow rate is: $q_{\mathrm{V}R} = F_{\mathrm{CV}}(t_{\mathrm{SR}}) \times q_{\mathrm{V}}(t_{\mathrm{SR}}) = 29{,}549.1\ \mathrm{L/s} = 29.5491\ \mathrm{m^3/s}$.

(b) If we take the value measured based on the proposed formula (30) as the reference density $\rho_R^F = \rho^F(t_{\mathrm{R}})$ we have:

$$\rho_R^F = 839\ \mathrm{kg/m^3},\ \Delta t = 8\ ^\circ\mathrm{C},\ \alpha_{\mathrm{R}} = 8.45111 \times 10^{-4}\ 1/^\circ\mathrm{C},\ {F_{\mathrm{CV}}}^{\mathrm{F}}(t_{SR}) = 0.993226$$

So the reference volume flow rate is calculated using this coefficient:

$$q_{\mathrm{V}R}^F = {F_{\mathrm{CV}}}^{\mathrm{F}}(t_{\mathrm{SR}}) \times q_{\mathrm{V}}(t_{\mathrm{SR}}) = 29{,}548.5\ \mathrm{L/s} = 29.5485\ \mathrm{m^3/s}$$

The deviation of the reference volume flow rate, calculated using the reference density determined on the basis of the proposed formula is:

$$\Delta q_{\mathrm{V_R}} = q_{\mathrm{V}R} - q_{\mathrm{V}R}^F \cong 0.6\ \mathrm{L/s},\ \frac{\Delta q_{\mathrm{V_R}}}{q_{\mathrm{V}R}} \cdot 100\% = \varepsilon(q_{\mathrm{V}R}) = 0.002\%$$

This example demonstrates that there is a significant deviation when the reference density, i.e., the density at 15 °C, is obtained using the API tables compared to the reference density values calculated using the proposed formula (30).

It can be concluded that applying the described calculation procedure, using the recommended equation with approximate values of the volumetric thermal expansion coefficient (α_{R}) derived from the proposed constants (K_0, K_1) in the API tables [12] and

the fluid density (ρ_R) at reference conditions, also obtained from the API tables, results in a volume correction factor $(F_{CV}(t))_{API}$ that is only an approximate value and deviates from the exact value.

4. Conclusions

Measurement of density is essential in many industrial applications, including the transfer, accounting, and quality control of liquid fuels. This study investigates the determination of reference density using measured density and temperature data and examines the differences between the proposed formulation and the API-table-based method.

The experimental analysis was conducted during a D2 diesel fuel transfer operation at the "Ilinden"–Makpetrol Skopje terminal in North Macedonia. Density and temperature data were recorded using a Micro Motion ELITE Coriolis flowmeter and a temperature sensor. Based on these measurements, the reference density was calculated using the proposed formula and compared with the values obtained using the API tables in accordance with ASTM D1250-80.

For each of the 1548 measurement samples, the reference density was calculated using the proposed formula. The comparison revealed differences between the reference density values obtained using the two approaches under the investigated operating conditions. However, since the experimental investigation was limited to a single D2 diesel fuel transfer operation and no independent reference standard was used, the observed differences cannot be interpreted as evidence that the proposed method is more accurate than the API-table-based method.

The proposed formulation provides an alternative approach to determining reference density based on measured density and temperature data. Unlike the conventional approach described in this study, which uses coefficients associated with specified groups of petroleum products, the proposed formulation does not require the use of the constants (K_0, K_1) in the same manner. This feature may be particularly relevant in cases where the required coefficients are unavailable, provided that the proposed formulation is applicable to the liquid and measurement conditions considered.

The findings of this study are limited to the investigated D2 diesel fuel and the specific transfer conditions. Further research involving independent reference measurements, different diesel fuel batches, and a wider range of operating conditions is required to assess the measurement uncertainty, accuracy, and broader applicability of the proposed method.

Given the importance of density determination in fuel transfer, loss control, and quality assurance, the proposed formulation merits further investigation. Its potential advantages should be assessed through additional experimental validation and a quantitative uncertainty analysis.

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