

Article

Linear Algebra-Based Internal Model Control Strategies for Non-Minimum Phase Systems: Design and Evaluation

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Abstract

This paper addresses the challenge of trajectory tracking in non-minimum-phase systems, which are known for their limitations in performance and stability within process control. The primary objective is to evaluate the feasibility of using linear-algebra-based control strategies to achieve precise tracking in such systems. The primary hypothesis is that internal model-based compensators can transform non-minimum-phase behavior into equivalent minimum-phase dynamics, thereby enabling the application of linear algebra techniques for controller design. To validate this approach, both simulation and experimental tests are conducted, first with a Continuous Stirred Tank Reactor (CSTR) model and then with the TCLab educational platform. The results show that the proposed method effectively achieves robust trajectory tracking, even in the presence of external disturbances and sensor noise. The primary contribution of this work is to demonstrate that internal model-based compensation enables the application of linear control methods to a class of systems that are typically considered challenging to control. This not only simplifies the design process but also enhances control performance, highlighting the practical relevance and applicability of the approach for real-world non-minimum-phase systems processes.

Keywords: linear algebra-based control; non-minimum phase systems; iinoya compensator; smith predictor; trajectory tracking



Academic Editors: Weicun Zhang and Quanmin Zhu

Received: 4 July 2025

Revised: 24 August 2025

Accepted: 29 August 2025

Published: 15 September 2025

Citation: Insuasti, S.; Gómez-Guerra, G.; Scaglia, G.; Camacho, O. Linear Algebra-Based Internal Model Control Strategies for Non-Minimum Phase Systems: Design and Evaluation. *Processes* **2025**, *13*, 2942. <https://doi.org/10.3390/pr13092942>

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1. Introduction

In chemical and biochemical process industries, achieving precise trajectory tracking in systems such as bioreactors and chemical reactors is critical for ensuring product quality, process efficiency, and safety [1,2]. These systems are often subject to time-varying operating conditions, including temperature, pH, substrate concentration, and dissolved oxygen, that must be controlled to follow predefined trajectories for optimal yield [3]. However, many of these processes exhibit non-minimum-phase (NMP) characteristics due to transport delays, measurement lags, or intrinsic process dynamics. These properties present significant challenges for real-time control design.

The PID (proportional-integral-derivative) controller is among the most widely used control strategies in industrial and embedded systems because of its simplicity, ease of implementation, and overall performance. However, despite its popularity, it has several limitations when used for complex trajectory tracking tasks in autonomous systems. One

major drawback is its dependence on precise manual or heuristic tuning of the gains, such as K_p , K_i , and K_d . Without automated optimization, this tuning process can be time-consuming and often yields suboptimal results. Additionally, traditional PID control does not consider nonlinear dynamics or geometric constraints, leading to poor performance on curved or rapidly changing trajectories [4,5].

Moreover, conventional PID control fails to account for nonlinear dynamics, geometric constraints, and higher-order behavior commonly found in chemical reactors and mechanical systems. This often results in poor performance when tracking curved or rapidly changing trajectories. In addition, PID controllers are inherently non-adaptive, making them sensitive to variations in system conditions such as temperature, friction, or velocity. These limitations highlight the need for more advanced control strategies—such as metaheuristic-tuned, adaptive, or model-based PID frameworks—that can overcome the rigidity and tuning sensitivity of traditional designs [6,7].

A particularly challenging case arises with non-minimum-phase systems, which inherently limit the applicability of exact inversion techniques. The presence of non-minimum-phase zeros renders the inverse system unstable, often leading to unbounded or diverging control actions. From a state-space perspective, these systems may also exhibit non-stabilizability, as the unstable modes associated with such zeros cannot be effectively regulated by available inputs [8–10].

Trajectory tracking for non-minimum phase (NMP) systems remains a significant challenge due to the unstable zero dynamics that prevent straightforward application of inverse-based control techniques. Recent strategies have addressed this issue by either modifying the reference output or introducing robust and learning-based architectures. In [11], a cascaded reinforcement learning framework combining feedback linearization and actor–critic algorithms was proposed, where a symmetry-based learning rule stabilizes the internal dynamics while ensuring accurate tracking. In the context of underactuated multibody systems, [12] employed servo-constraints to derive feedforward control laws that are integrated with funnel control feedback, bypassing the need for Byrnes–Isidori normal form. The method in [13] tackles the NMP problem via output redefinition, adjusting the system output through a tunable displacement parameter to guarantee internal stability while preserving trajectory fidelity. In parallel, stable inversion methods have emerged as a viable solution for achieving accurate feedforward tracking in non-minimum phase systems, where direct model inversion typically results in unbounded control inputs. Classical approaches depend on non-causal control actions and require an infinite preview horizon to achieve exact tracking, which limits their practical implementation. More recent strategies employing time-lifting techniques overcome these limitations by enabling finite-time tracking without the need for infinite preview. Originally formulated for linear time-invariant systems, these methods have been successfully extended to multivariable linear periodic time-varying systems through the use of lifted system representations, thus providing a structured framework for bounded-input tracking control in more general settings [14].

To address these challenges, linear-algebra-based control strategies have been developed. In this work, the term refers to state-space internal model control formulations that employ linear transformations to compensate for non-minimum-phase systems, following the approach described in [15]. By treating the system as if it were minimum-phase [15], the method enables a systematic controller design process that facilitates trajectory tracking [3,15]. The proposed control algorithm, unlike many of the recently developed approaches, does not introduce additional implementation complexity. Instead, it maintains a comparable computational burden while offering a simpler design rationale. The core idea relies on a model transformation in which the original non-minimum phase zero is replaced with a minimum-phase counterpart. As a result, the system achieves

trajectory tracking with a delay that is directly related to the zero's location. When the reference signal is available in advance, this delay can be compensated by a time shift, leading to an asymptotically zero tracking error.

Several researchers have investigated alternative solutions to address this limitation, including approximate model inversion, preview-based feedforward control, zero-phase error tracking control (ZPETC), and modified internal model control (IMC) architectures [16,17]. These strategies often succeed in reducing tracking errors under specific conditions, but come at the cost of increased sensitivity to model mismatch, implementation complexity, or limited robustness to disturbances, a particularly important concern in biochemical processes, where system dynamics are nonlinear and uncertain.

This article addresses the problem of trajectory tracking in non-minimum-phase systems, a well-known challenge in process control due to the inherent limitations these systems impose on performance and stability. The primary objective of this work is to evaluate the feasibility of applying linear-algebra-based control strategies to achieve effective trajectory tracking in such systems. Previous studies [18] have proposed predictor-based schemes to compensate non-minimum-phase dynamics, showing that such transformations enable the use of linear controllers; however, these efforts have largely been limited to simulation and highlighted the need to extend the approach to other nonlinear models and real implementations. Building on this foundation, we contribute by exploring alternative compensation strategies and comparing three distinct state-space representations, each of which yields controllers with different advantages. Moreover, we validate the proposed controllers not only through simulation with a Continuous Stirred Tank Reactor (CSTR) model but also through experimental implementation on the TCLab educational platform.

The primary contribution of this study lies in demonstrating how internal model-based compensation enables the use of linear algebraic methods for controller synthesis in the context of non-minimum-phase processes. This approach not only simplifies the control design but also enhances tracking accuracy in the presence of external disturbances and measurement noise.

The remainder of this paper is organized as follows. Section 2 presents fundamental theoretical concepts. Section 3 describes the process models and control problem formulation. Section 4 introduces the proposed control approach and its implementation. Section 5 presents simulation results and comparative analysis. Finally, Section 6 concludes the paper and discusses future research directions.

2. Fundamentals of Linear Algebra-Based Control

This section introduces the key concepts behind the linear algebra-based control method. It starts with formulating the control problem for nonlinear systems, focusing on the structure of the mathematical model and the assumptions needed for control feasibility. Then, it covers the basics of control design, emphasizing the importance of reference tracking, sacrificed variables, and deriving the control law using linear algebra techniques. Readers interested should read [15].

2.1. Problem Statement

A mathematical model of the plant and the trajectory to be followed is assumed to be given. The general state-space model is expressed as:

$$\dot{x}(t) = F(x(t), u(t), d(t), t) \quad (1)$$

$$y(t) = H(x(t), u(t), t) \quad (2)$$

where $x \in \mathbb{R}^r$ denotes the system state, $u \in \mathbb{R}^m$ the input, $y \in \mathbb{R}^p$ the system output, and $d \in \mathbb{R}^r$ an external disturbance. In our study, we make the following simplifying assumptions:

Assumption 1. *The model is affine in the control.*

Assumption 2. *The model is minimum-phase.*

Assumption 3. *The model is time-invariant.*

Assumption 4. *The state is measurable.*

Assumption 5. *The model is exact and there are no disturbances.*

Under these assumptions, the model becomes:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \quad (3)$$

$$y(t) = x(t) \quad (4)$$

The assumptions are clarified as follows: Assumption 5 will be revised to explicitly consider uncertainties and disturbances. Assumption 4 can be relaxed by incorporating state observers or dynamic output feedback. Assumption 3 may also be lifted if the parameter variation law is known, although this increases the complexity of the control design. Assumption 1 is relatively mild, as it is typically satisfied by most practical systems. Finally, Assumption 2 is essential to allow model cancellation in the analytical solution.

To fully describe the tracking problem, a reference trajectory must be provided. This is known as the path tracking problem. We assume that the motion planner supplies a feasible trajectory—that is, one that can be followed by applying an appropriate control input. Therefore, we add the following assumption:

Assumption 6. *The reference and its derivatives are known.*

Often, the desired trajectory is defined only for a subset of the state variables (typically positions and/or velocities), while the remaining states are unspecified, leaving room for control flexibility.

Several methods in the literature are available to tackle this problem. Some are relatively simple and based on linear approximations of the plant; however, they often need additional adaptation or compensation to handle complex dynamics effectively. Other approaches rely on detailed nonlinear models, which can more accurately represent the system but typically demand high computational resources, making them less suitable for real-time implementation.

The approach leverages the flexibility in the evolution of certain state variables and formulates the control problem in an algebraic framework, yielding a computationally efficient controller.

2.2. Control Design

Let us partition the state vector into two parts: the tracked variables $\xi(t) \in \mathbb{R}^{r_1}$ and the remaining variables $z(t) \in \mathbb{R}^{r-r_1}$, which we call sacrificed variables. Typically, the number of tracked variables (r_1) matches the number of independent control inputs (m), and the number of sacrificed variables depends on the plant model.

The system can then be expressed as:

$$\begin{bmatrix} \dot{\xi}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} f_{\xi}(\xi(t), z(t)) \\ f_z(\xi(t), z(t)) \end{bmatrix} + \begin{bmatrix} g_{\xi}(\xi(t), z(t)) \\ g_z(\xi(t), z(t)) \end{bmatrix} u(t) \quad (5)$$

Given the reference $\xi_{ref}(t)$ from the motion planner, and assuming a smooth approach to the desired values, we define:

$$\begin{bmatrix} \dot{\xi}(t) \\ \dot{z}(t) \end{bmatrix} = \begin{bmatrix} \dot{\xi}_{ref}(t) + K_{\xi}(\xi_{ref}(t) - \xi(t)) \\ \dot{z}_{ref}(t) + K_z(z_{ref}(t) - z(t)) \end{bmatrix} \quad (6)$$

where K_{ξ} , K_z are diagonal matrices containing the control parameters.

Combining this with the model yields:

$$\begin{bmatrix} \dot{\xi}_{ref}(t) + K_{\xi}(\xi_{ref}(t) - \xi(t)) - f_{\xi}(\xi(t), z(t)) \\ \dot{z}_{ref}(t) + K_z(z_{ref}(t) - z(t)) - f_z(\xi(t), z(t)) \end{bmatrix} = \begin{bmatrix} g_{\xi}(\xi(t), z(t)) \\ g_z(\xi(t), z(t)) \end{bmatrix} u(t) \quad (7)$$

$$b(t) = A(t)u(t) \quad (8)$$

where $A(t)$ is a known $r \times m$ -dimensional matrix, and $b(t)$ is an r -dimensional vector, some of whose entries are partially unknown, specifically $z_{ref}(t)$ and $\dot{z}_{ref}(t)$.

To find a solution for $u(t)$ in Equation (8), it is required that $b(t)$ be a linear combination of the column vectors of $A(t)$. This condition determines the possible values for the reference of some sacrificed variables, $z_{ref}(t)$. Also, it implies a modification of the first row of Equation (6), so that the first row of Equation (7) satisfies the required condition.

Once $b(t)$ and $A(t)$ are defined, the control input can be obtained by solving Equation (8) using the least squares solution:

$$u(t) = A^{\dagger}(t)b(t) \quad (9)$$

where $A^{\dagger}(t)$ denotes the pseudoinverse matrix of $A(t)$.

3. Inverse Response Model Identification

There are several methods for modeling inverse response systems, which have been proven to be accurate [19–24]. In this paper, the systems are identified using an approach proposed by Alfaro and Balaguer [25]. This approach yields a system representation consisting of two first-order systems connected in series, which form the transfer function given in (10). It has already been proven that this system can effectively reproduce the behavior of a non-minimum-phase system [5]. A detailed explanation of the methodology used to identify the inverse response system using the react curve can be found in [25]. However, I will provide a brief introduction for rapid application.

$$G(s) = \frac{Y_1(s)}{U(s)} = \frac{K(-\eta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (10)$$

First, the static gain K is determined from the system response input change Δu and the output change Δy_1 .

$$K = \frac{\Delta y_1}{\Delta u} \quad (11)$$

Then, 3 points of the curve must be taken, considering the y value as a percentage of the overall output change (Δy_1), one at the inverse peak with coordinates (t_p, y_p) , and the other two at arbitrary values of the curve. Balaguer suggests to take the values corresponding of 47% of the final output (t_{47}, y_{47}) and the other one at 90% of the final output ($t_{90},$

y_{90}), which allow to make a further simplification. The first pole can be determined by Equation (14).

$$\tau_1 = \frac{t_{90} - t_{47}}{\ln\left(\frac{y_{47\%}-1}{y_{90\%}-1}\right)} \quad (12)$$

where, $y_{x\%}$ is defined by $y_x/\Delta y_1$. Then, the right half plane zero was calculated by (13).

$$\eta = \tau_1 \left(1 - \frac{1 + |y_{p\%}|}{e^{-t'_p}}\right) \quad (13)$$

where, t' is given by t/τ_1 . To determine τ_2 , the following equation suggested by Balaguer [25] can be used, considering some terms n and m that establish a relationship between τ_2 and η and are given as an approximated function of an adimensional parameter $b = \frac{\eta}{\tau_1}$:

$$\tau_2 = \tau_1 \left(\frac{t'_{47} - n}{m}\right) \quad (14)$$

This result of τ_2 can be further tuned to improve the overall approximation.

4. Compensator Schemes for Inverse Response Systems

Non-minimum-phase systems are characterized by the presence of one or more zeros located in the right-half of the complex plane (RHP). These right-half-plane (RHP) zeros inherently reduce the stability margins and impose fundamental limitations on achievable closed-loop performance. In particular, they constrain the controller bandwidth and lead to undesirable inverse response behaviors, which typically occur when the zero is simple, the most common case in practical systems, where the output initially deviates in the opposite direction of the desired steady-state value. In industrial applications, such dynamics often result in degraded system performance, especially in terms of product quality and response time, as the system cannot react promptly or predictably to control actions.

This work proposes the control of non-minimum-phase systems using linear algebra-based control laws that must follow a variable trajectory. It has been proven that Linear-algebra-based controllers can effectively track variable trajectories using kinematic or dynamic models. However, these methods face significant challenges when applied to non-minimum-phase systems, as enforcing exact trajectory tracking typically demands control actions that diverge or become unbounded. This behavior arises from the presence of right-half-plane zeros, which impose fundamental limitations on the system's response and significantly constrain closed-loop performance.

Therefore, this work proposes two different methods to manage the non-minimum-phase fraction of the system, allowing the control law to operate on its minimum-phase component.

Both strategies involve the use of internal models that, through block algebra, compensate for the non-minimum-phase component, commonly referred to as inverse response, so that the controller acts upon a minimum-phase-like behavior. The first strategy involves introducing a transfer function that enables the application of a Padé approximation, converting the non-minimum-phase component of the system into a delay, which is then managed by a Smith Predictor, effectively canceling out the non-minimum-phase behavior. The second internal model analyzed consists of an Inoya-Alpeter compensator, also known as Inoya, which integrates an internal transfer function designed to cancel out the influence of the RHP zero effectively.

This section describes and clarifies the design of both internal model strategies proposed in this work, and presents the system block diagram with the internal models integrated within the feedback loop.

4.1. Smith Predictor Scheme

The second internal model proposed is an interpretation of the Smith predictor for inverse response systems. This control strategy was originally designed to handle systems with a significant time delay within the feedback loop. The handling of a long delay has been a challenge, as it can cause instability in the controller and affect loop performance, resulting in a downgrade in product quality [26,27].

This control scheme involves a model of the process, which separates the delay component from the original plant dynamics. The predictor then uses this model to generate an estimate of the system output without delay, enabling the controller to act on an approximation of the plant output without waiting for the delayed feedback signal [26].

By removing the delay component, the Smith predictor can enhance the process's performance with faster and more stable responses. However, since the predictor relies on a plant model, the success of this control strategy depends on the accuracy of the model used in the internal loop.

Since the Smith predictor allows the controller to operate without considering the delay component of the system, the objective of the strategy proposed in this work is to convert the non-minimum-phase component of the analyzed system into a time delay that the predictor can compensate for.

To achieve this goal, the system transfer function is multiplied by $(\eta s + 1)/(\eta s + 1)$ and rewritten to have the form shown in (15).

$$G(s) = \frac{K(\eta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \frac{(-\eta s + 1)}{(\eta s + 1)} \quad (15)$$

From the right factor in Equation (15), a Padé approximation was made to modify the inverse response system in a time delay system. The invertible part represents a minimum phase system without delay to be used for design purposes, as shown in (16).

$$G(s) \approx \frac{K(\eta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-2\eta s} \quad (16)$$

From the transfer function presented in (16), the block diagram of the Smith predictor was constructed. Figure 1 shows the block diagram of the internal model proposed in this work.

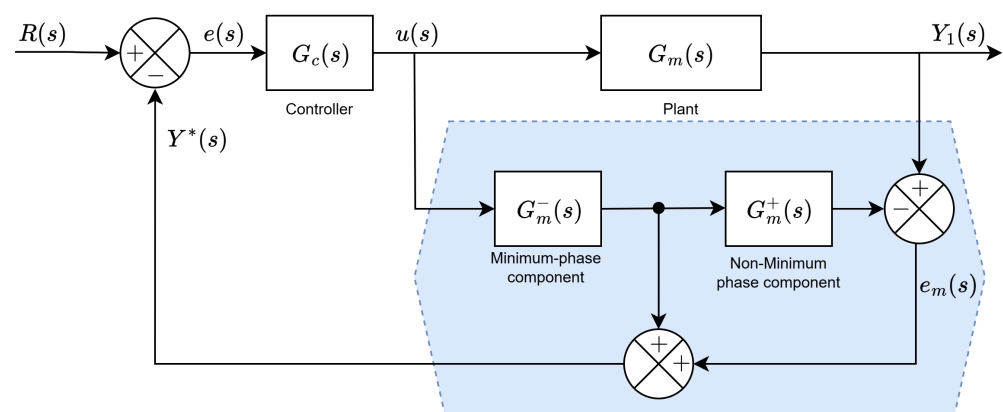


Figure 1. Smith Predictor block diagram scheme.

With,

$$G_m(s) = G_m^-(s)G_m^+(s) \quad (17)$$

where,

$$G_m^-(s) = \frac{K(\eta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)}; \quad G_m^+(s) = e^{-2\eta s} \quad (18)$$

and the $G_m^-(s)$ is used for design, as it represents a minimum-phase system, and the controller now monitors it for control purposes.

4.2. Iinoya-Alpeter Compensator

The dead-time compensation technique of the Smith Predictor has been extended by Iinoya and Alpeter [20,28] to regulate processes that present inverse response [29]. A linear system with inverse response can be represented as follows:

$$G(s) = G_0(s)(1 - \eta s) \quad (19)$$

where $G(s)$ is the plant model with an inverse response, $G_0(s)$ is the plant model without the right-half-plane (RHP) zero, and $(1 - \eta s)$ represents the RHP zero.

Figure 2 shows the structure of the Iinoya-Alpeter (I&A) compensator used to achieve this. This compensator employs an Internal Model structure to transform the original non-minimum-phase system into an equivalent minimum-phase system. Specifically, the (I&A) compensator mitigates the adverse effects caused by the right-half-plane (RHP) zero by introducing a carefully designed internal transfer function within the feedback loop. This internal function effectively cancels or shifts the RHP zero to the left-half-plane (LHP), thereby enabling more favorable control conditions and improving the overall system stability and performance.

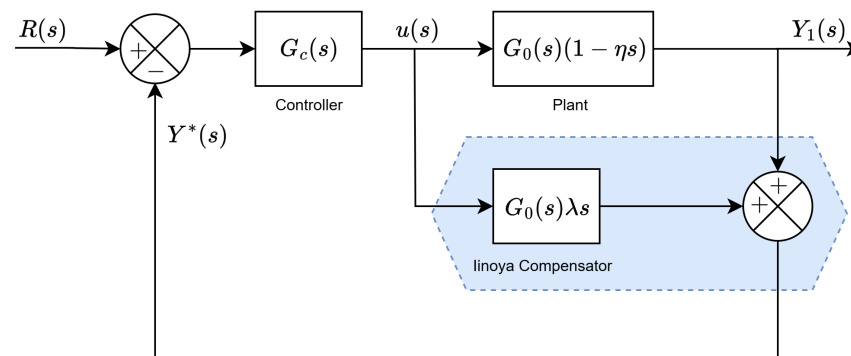


Figure 2. Iinoya-Alpeter compensator block diagram.

From the scheme, it can be seen that after the sum is obtained, the following:

$$G^*(s) = G_0(s)[1 + (\lambda - \eta)s] \quad (20)$$

The transfer function $G^*(s)$ represents the overall output response of the compensated process and model with respect to the controller output $u(s)$. The parameter λ is a tuning parameter in the I&A compensator. When $\lambda > \eta$, the original right-half-plane (RHP) zero is effectively moved to the left-half-plane (LHP), which helps reduce the negative effects typically caused by inverse response behavior.

An adequate tuning of the parameter λ must be performed. Choosing $\lambda = \eta$ eliminates the RHP zero. If $\lambda > \eta$, the response of $G^*(s)$ will be faster than that of $G(s)$.

Therefore, by choosing $\lambda = 2\eta$, the compensation strategy, based on the structure defined in Equation (10), ensures that the controller interacts with a compensated system exhibiting a minimum phase behavior, as characterized by Equation (21).

$$G^*(s) = \frac{K(\eta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (21)$$

5. Controller Design

This section focuses on the design of the controller. First, we consider the general system model using the two internal model structures presented in the previous section. In both cases, the non-minimum-phase systems are converted to a minimum phase with a zero. Once the model is selected, it is transformed into various equivalent state-space representations of the same system, enabling a comparison of the advantages and disadvantages of designing a control law from different system representations. However, in all representations, the system's dynamics remain unchanged. Nonetheless, the control law will be adjusted based on the chosen state variables for the state-space model. The first representation defines the states directly as the output and its derivative, which is the most natural or “canonical” choice. However, it can be sensitive to noise when derivatives are estimated from measurements. The second representation redefines the second state as a linear combination of the output and its derivative, improving numerical conditioning and mitigating sensitivity to derivative noise. The third representation incorporates the input explicitly into the state definition, producing a controllable canonical form that facilitates controller design, but at the expense of making the state more sensitive to input measurement errors.

Each control law will then be implemented for both options: the I&A compensator and the Smith predictor structure. Three different controllers are proposed. In this work, the controllers were tested in two different plants. The first plant is a Continuous Stirred Tank Reactor (CSTR) in a simulated environment. For this case, the continuous approach of the controller design was used. The second plant used is a Temperature Control Laboratory (TCLab), a laboratory equipment used to perform real-time temperature control experiments. For this plant, since the sensor measurement is discrete, the discrete approach of the control law was used.

5.1. General Model for Design Purposes

In the previous section, the inverse response system is converted into a minimum-phase system, considering the advantages of internal model structures for design purposes. Since both $G^*(s)$ and $G_m^-(s)$ has the same transfer function.

$$G^*(s) = G_m^-(s) = \frac{K(\eta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)}; \quad (22)$$

If we consider that $Y(s) = Y^*(s) = Y_m^-(s)$, and rearranging the transfer function of the system, it can be presented as follows: (23).

$$Y(s) \left(s^2 + \frac{\tau_1 + \tau_2}{\tau_1 \cdot \tau_2} s + \frac{1}{\tau_1 \cdot \tau_2} \right) = U(s) \left(\frac{K \cdot \eta}{\tau_1 \cdot \tau_2} s + \frac{K}{\tau_1 \cdot \tau_2} \right) \quad (23)$$

From Equation (23), the inverse Laplace transform was calculated, resulting in the equation shown in (24).

$$\ddot{y}(t) + \frac{\tau_1 + \tau_2}{\tau_1 \cdot \tau_2} \cdot \dot{y}(t) + \frac{1}{\tau_1 \cdot \tau_2} \cdot y(t) = \frac{K \cdot \eta}{\tau_1 \cdot \tau_2} \cdot \dot{u}(t) + \frac{K}{\tau_1 \cdot \tau_2} \cdot u(t) \quad (24)$$

To simplify Equation (24) and to make controller design more manageable, the differential equation constants are defined as described in (25).

$$\begin{aligned} a_1 &= \frac{\tau_1 + \tau_2}{\tau_1 \cdot \tau_2} & b_1 &= \frac{K \cdot \eta}{\tau_1 \cdot \tau_2} \\ a_0 &= \frac{1}{\tau_1 \cdot \tau_2} & b_0 &= \frac{K}{\tau_1 \cdot \tau_2} \end{aligned} \quad (25)$$

Thus, the differential equation that describes the system's dynamics is presented in (26).

$$\ddot{y}(t) + a_1 \cdot \dot{y}(t) + a_0 \cdot y(t) = b_1 \cdot \dot{u}(t) + b_0 \cdot u(t) \quad (26)$$

5.2. Controller 1

The first controller work was designed by expressing the differential Equation in (26) as a state-space system, given by the state variables described in (27).

$$\begin{aligned} x_1 &= y(t) \\ x_2 &= \dot{y}(t) \end{aligned} \quad (27)$$

Likewise, a new control action is defined according to (28) to facilitate the formulation of the controller.

$$b_1 \cdot \dot{u}(t) + b_0 \cdot u(t) = z(t) \quad (28)$$

In this way, the representation of the space state system with the chosen state variables is presented in (29).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot z(t) \quad (29)$$

5.2.1. Continuous Approach

The steps to compute the control action using the LABC technique are listed below [15]:

- Formulate the control action computation problem as the solution of a system of linear equations of the form $Az = b$.

$$\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_A \cdot z(t) = \underbrace{\begin{bmatrix} \dot{x}_{1d} - x_2 \\ \dot{x}_2 - a_0 \cdot x_1 - a_1 \cdot x_2 \end{bmatrix}}_b \quad (30)$$

where $\dot{x}_{1d}$ denotes the desired value of $\dot{x}_1$, aimed at driving the tracking error to zero. The desired tracking error dynamics is given by Equation (31), where $x_{1,\text{ref}}$ represents the desired value of the variable x_1 :

$$(\dot{x}_{1,\text{ref}} - \dot{x}_1) + k_1(x_{1,\text{ref}} - x_1) = 0, \quad k_1 > 0 \quad (31)$$

Accordingly, $\dot{x}_{1d}$ is expressed as in Equation (38):

$$\dot{x}_{1d} = \dot{x}_{1,\text{ref}} + k_1(x_{1,\text{ref}} - x_1) \quad (32)$$

- Determine the conditions under which the resulting system of equations admits an exact solution.

To satisfy this requirement, the rank of matrix A must be equal to the rank of the augmented matrix $[A \ b]$. This condition is fulfilled when x_2 equals $\dot{x}_{1d}$. This particular value of x_2 , denoted as x_{2ez} , is given by Equation (33):

$$x_{2ez} = \dot{x}_{1,\text{ref}} + k_1(x_{1,\text{ref}} - x_1) \quad (33)$$

- Solve the system of equations, using x_{2ez} as the reference for x_2 and proceeding analogously to the case of variable x_1 (Equations (31) and (38)).

$$z(t) = \dot{x}_{2ez} + k_2 \cdot (x_{2ez} - x_2) + a_1 \cdot x_2 + a_0 \cdot x_1, k_2 > 0 \quad (34)$$

It is important to note that both k_1 and k_2 must be strictly positive to ensure convergence of the state variables toward the reference trajectory.

Finally, substituting from Equations (28) and (34) yields:

$$u(t) = \int \frac{\overbrace{\dot{x}_{2ez} + k_2 \cdot (x_{2ez} - x_2) + a_1 \cdot x_2 + a_0 \cdot x_1}^{z(t)} - b_0 \cdot u}{b_1} \cdot dt \quad (35)$$

5.2.2. Discrete Approach

The discrete implementation of the proposed controller requires a sampling period T_0 to compute the control action at each time step. The initial step involves rewriting and discretizing the continuous-time system defined in Equation (30). The resulting discrete-time state-space representation is given in Equation (36):

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \underbrace{z(nT_0)}_{z_n} = \begin{bmatrix} \frac{x_{1d,n+1} - x_{1n}}{T_0} - x_{2n} \\ \frac{x_{2n+1} - x_{2n}}{T_0} + a_1 \cdot x_{2n} + a_0 \cdot x_{1n} \end{bmatrix} \quad (36)$$

Here, n denotes the current discrete-time sample index associated with the variables used in the controller design. Following a procedure analogous to the continuous-time case, a desired error evolution is imposed—see Equation (37)—to drive the tracking error to zero gradually. As indicated in Equation (37), this requirement implies that the gain k_1 must lie in the interval $[0, 1)$:

$$\underbrace{x_{1\text{ref},n+1} - x_{1n+1}}_{e_{1n+1}} - k_1 \cdot \underbrace{(x_{1\text{ref},n} - x_{1n})}_{e_{1n}} = 0 \quad (37)$$

$$e_{1n+1} = k_1 \cdot e_{1n}, \quad 0 \leq k_1 < 1$$

Accordingly, the desired value for x_{1n+1} is defined by:

$$x_{1d,n+1} = x_{1\text{ref},n+1} - k_1 \cdot (x_{1\text{ref},n} - x_{1n}) \quad (38)$$

Analogous to the continuous-time case, the next step is to determine the conditions under which the discrete system described in Equation (36) admits an exact solution. This leads to defining the desired value for x_{2n} as shown in Equation (39):

$$x_{2ezn} = \frac{x_{1\text{ref},n+1} - k_1 \cdot (x_{1\text{ref},n} - x_{1n}) - x_{1n}}{T_0} \quad (39)$$

This value serves as the target trajectory for the second state variable. Following a similar approach to that of the continuous-time case, the variable z_n can be computed as:

$$z_n = \frac{x_{2ezn+1} - k_2 \cdot (x_{2ezn} - x_{2n}) - x_{2n}}{T_0} + a_1 \cdot x_{2n} + a_0 \cdot x_{1n}, \quad 0 \leq k_2 < 1 \quad (40)$$

Remark 1. In Equation (40), the evaluation of z_n requires the value of x_{2ezn+1} . This can be estimated using either $x_{2ezn+1} \approx x_{2ezn}$ or a first-order extrapolation such as $x_{2ezn+1} \approx 2x_{2ezn} - x_{2ezn-1}$, as discussed in Sardella et al. (2019) [3].

In this formulation, $x_{1ref,n}$ denotes the discrete-time reference trajectory for the first state variable. The auxiliary signal x_{2ezn} acts as a reference to guide the second state variable x_2 .

The tuning parameters k_1 and k_2 determine the aggressiveness of the control law in the discrete-time domain. For stable and accurate tracking, both gains must be strictly within the interval $[0, 1)$. Lower values of k_1 and k_2 produce more aggressive responses.

Finally, the control input is obtained by applying a discrete-time integration to the derivative expression associated with the second subsystem, as defined in Equation (28). The resulting control law implemented in the experimental setup is expressed in Equation (41):

$$u_n = u_{n-1} + \frac{1}{b_1} \cdot \left(\frac{x_{2e,n+1} - k_2 \cdot (x_{2e,n} - x_{2n} - x_{2n})}{T_0} + a_1 \cdot x_{2n} + a_0 \cdot x_{1n} - b_0 \cdot u_{n-1} \right) \quad (41)$$

5.3. Controller 2

Similarly to the procedure adopted in Equation (28), a new control input $z(t)$ is introduced. Based on the alternative state-space formulation defined in Equation (42), the system dynamics can be represented in state-space form as shown in Equation (43).

$$\begin{aligned} x_1(t) &= y(t), \\ x_2(t) &= \dot{y}(t) + a_1 y(t), \end{aligned} \quad (42)$$

This representation leads to the following linear time-invariant state-space model:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -a_1 & 1 \\ -a_0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} z(t), \quad (43)$$

where the control law is now designed using the newly defined state variables $x_1(t)$ and $x_2(t)$. This formulation provides a structurally distinct approach compared to the one previously discussed.

5.3.1. Continuous Approach

Following the same procedure outlined in Section 5.2.1, which presents the controller design based on Linear Algebra-Based Control (LABC), the variable x_{2ez} is defined as in Equation (44). Subsequently, the control input $z(t)$ is computed using Equation (45), and the final control action $u(t)$ is obtained from Equation (46).

$$x_{2ez} = \dot{x}_{1ref} + k_1 \cdot (x_{1ref} - x_1) + a_1 \cdot x_1; k_1 > 0 \quad (44)$$

Here, x_{1ref} denotes the reference trajectory that the system is intended to track, while k_1 represents the proportional control gain associated with the first state variable.

$$z(t) = \dot{x}_{2ez} + k_2 \cdot (x_{2ez} - x_2) + a_0 \cdot x_1; k_2 > 0 \quad (45)$$

Analogously to k_1 , the parameter k_2 represents the proportional gain associated with the second state variable.

$$u(t) = \int \frac{\dot{x}_{2ez} + k_2 \cdot (x_{2ez} - x_2) + a_0 \cdot x_1 - b_0 \cdot u}{b_1} dt \quad (46)$$

5.3.2. Discrete Approach

Consistent with the previous section, Equation (43) is reformulated and discretized to facilitate the design of the discrete-time implementation of the proposed controller, as detailed in (47).

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot z_n = \begin{bmatrix} \frac{x_{1n+1} - x_{1n}}{T_0} + a_1 \cdot x_{1n} - x_{2n} \\ \frac{x_{2n+1} - x_{2n}}{T_0} + a_0 \cdot x_{1n} \end{bmatrix} \quad (47)$$

Analogous to the previous controller, a design variable was introduced to enable the plant to track a time-varying reference, ensuring the system of equations admits a unique solution. This design variable is defined in (48).

$$x_{2ezn} = \frac{x_{1refn+1} - k_1 \cdot (x_{1refn} - x_{1n}) - x_{1n}}{T_0} + a_1 \cdot x_{1n} \quad (48)$$

In this context, x_{1refn} denotes the discrete-time reference signal, T_0 represents the sampling period, and n indicates the current sample index of each variable. The design variable is consequently treated as a reference trajectory for the state variable x_2 , resulting in the reformulation of the second equation in the system (47) as presented in (49).

$$z_n = \frac{x_{2ezn+1} - k_2 \cdot (x_{2ezn} - x_{2n} - x_{2n})}{T_0} + a_0 \cdot x_{1n} \quad (49)$$

The constants k_1 and k_2 define the control gains that regulate the magnitude and responsiveness of the controller action. In the discrete-time implementation, these gains are constrained to lie within the interval $[0, 1)$ to ensure stability and appropriate dynamic behavior. Similar to the continuous control law, the discrete-time derivative of the controller action is computed as specified in Equation (50).

$$u_n = u_{n-1} + \frac{1}{b_1} \cdot \left(\frac{x_{2ezn+1} - k_2 \cdot (x_{2ezn} - x_{2n} - x_{2n})}{T_0} + a_0 \cdot x_{1n} - b_0 \cdot u_{n-1} \right) \quad (50)$$

5.4. Controller 3

Similarly, to develop the third control law proposed in this study, the state-space representation of the differential equation in (26) is reformulated to investigate the impact of an alternative state-space formulation on controller performance. For this control law, the state variables are selected as specified in (51).

$$\begin{aligned} x_1 &= y(t) \\ x_2 &= \dot{y}(t) + a_1 \cdot y(t) - b_1 \cdot u(t) \end{aligned} \quad (51)$$

Using the defined state variables, the state-space representation of the system is formulated as shown in (52).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -a_1 & 1 \\ -a_0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_0 \end{bmatrix} \cdot u(t) \quad (52)$$

5.4.1. Continuous Approach

The control law is derived from the first equation of the state-space system presented in (52), which is reformulated as shown in (53).

$$b_1 \cdot u(t) = \dot{x}_1 + a_1 \cdot x_1 - x_2 \quad (53)$$

To ensure the system tracks the reference trajectory and admits a unique solution, $\dot{x}_1$ is substituted by $\dot{x}_{1ref} + k_1 \cdot (x_{1ref} - x_1)$. Consequently, the control law derived from this state-space representation is expressed in (54).

$$u(t) = \frac{\dot{x}_{1ref} + k_1 \cdot (x_{1ref} - x_1) + a_1 \cdot x_1 - x_2}{b_1} \quad (54)$$

It is important to note that this control law lacks an integral component within the controller action, in contrast to the control laws presented in Sections 5.2 and 5.3.

5.4.2. Discrete Approach

For the discrete implementation, Equation (52) is reformulated and discretized. The resulting discrete-time system is presented in (55).

$$\begin{bmatrix} b_1 \\ b_0 \end{bmatrix} \cdot u_n = \begin{bmatrix} \frac{x_{1n+1} - x_{1n}}{T_0} + a_1 \cdot x_{1n} - x_{2n} \\ \frac{x_{2n+1} - x_{2n}}{T_0} + a_0 \cdot x_{1n} \end{bmatrix} \quad (55)$$

The first equation was utilized to compute the controller action. The term x_{1n+1} was substituted with the reference trajectory and the corresponding tracking error to enable the system to follow the desired reference. Consequently, the discrete controller action is formulated as shown in (56).

$$u_n = \frac{1}{b_1} \cdot \left(\frac{x_{1refn+1} - k_1 \cdot (x_{1refn} - x_{1n}) - x_{1n}}{T_0} + a_1 \cdot x_{1n} - x_{2n} \right) \quad (56)$$

Here, k_1 denotes the control gain that regulates the magnitude of the controller action, n represents the current discrete-time index for each variable, and x_{1refn} specifies the reference trajectory that the plant is required to track.

6. Results

This section presents the evaluation of the proposed control laws, which exploit internal model structures to enable the application of a linear-algebra-based methodology for controlling processes with inverse response. Two systems were considered. The first is a Continuous Stirred Tank Reactor (CSTR), a nonlinear process extensively studied in the literature and frequently used for developing and validating identification methods for inverse-response systems [25]. The second is the Temperature Control Laboratory (TCLab), a real-time experimental platform that emulates non-minimum-phase behavior by adding, in series, an inversion module, a transfer function with a right-half-plane zero, as described in [30,31], enabling real-time assessment under inverse-response conditions.

The selection of these systems is intentional: their well-characterized and documented dynamics make them adequate benchmarks for rigorous, reproducible, and comparable evaluation. The CSTR serves as a reference model for validating diverse control and identification strategies [29,32,33], while the TCLab, despite its compact size, has demonstrated remarkable versatility in advanced applications such as controller evaluation, system identification, and artificial intelligence [34–37]. This combination ensures that the pro-

posed methods are tested in both simulated and real-time experimental environments, strengthening the robustness and applicability of the results.

The three proposed control laws were implemented within both internal model structures described in Section 4, the Smith predictor and the Iinoya–Alpeter compensator, and evaluated in terms of trajectory tracking accuracy and disturbance rejection capability. In addition, a frequency response analysis was performed to determine the bandwidth over which each control law remains effective. This was carried out via a Bode analysis, in which the reference input frequency was varied and the corresponding closed-loop output was examined, following the methodology in [31].

The remainder of this section is organized as follows: first, the simulation results for the CSTR system are presented; second, the experimental results obtained from the TCLab setup are discussed; and finally, the frequency response analysis for both systems is reported. It is worth noting that the time spans shown in the tracking results differ between the CSTR and TCLab cases, since each plant exhibits distinct dynamic response times. The chosen time horizons were selected to provide a clear visualization of the oscillatory behavior and the ability of each system to track the reference.

6.1. Assessment Methods for Controller Approaches

This section details the performance metrics employed to evaluate the proposed controllers. Both the process output and the control effort are analyzed to enable a comprehensive assessment of the various design alternatives. Furthermore, a Bode analysis is conducted to characterize the frequency response, thereby providing insight into each controller's ability to manage dynamic effects across a range of frequencies.

6.1.1. Performance Indicators

To evaluate the performance of the controllers in trajectory tracking tasks, three performance indicators are considered:

1. Integral Square Error (ISE)

Quantifies how closely the system output follows the reference trajectory [8].

$$ISE = \int e(t)^2 dt \quad (57)$$

2. Integral Square Controller Output (ISCO)

Measures the total energy consumed by the final control element [38].

$$ISCO = \int u(t)^2 dt \quad (58)$$

3. Integral Square Derivative Controller Output (ISDCO)

Assesses the abruptness of the control action, which can lead to premature wear or fatigue of the actuator [38].

$$ISDCO = \int \left(\frac{du(t)}{dt} \right)^2 dt \quad (59)$$

6.1.2. Bode Analysis

This analysis aims to determine the range of reference frequencies at which the controller performs optimally. To achieve this, we focus on the magnitude graph, which illustrates how the system output responds to a variable-frequency input, measured in decibels (dB). By exploring this relationship, we gain valuable insight into the dynamic behavior of the system, allowing us to evaluate and improve control performance under

varying conditions. The magnitude plot, calculated using Equation (60), serves as a key tool in this process.

$$|H(s)| = 20 \log_{10} \left| \frac{O(s)}{I(s)} \right| \quad (60)$$

In the frequency domain, $O(s)$ denotes the system's normalized output and $I(s)$ represents the variable-frequency normalized reference input, as shown in Figure 3. Analysis of the system's transfer function reveals that when the magnitude response approaches 0 dB (i.e., a gain of unity), the output effectively tracks the reference. This condition signifies minimal steady-state error and is indicative of high-fidelity tracking performance, reflecting an optimally tuned controller.

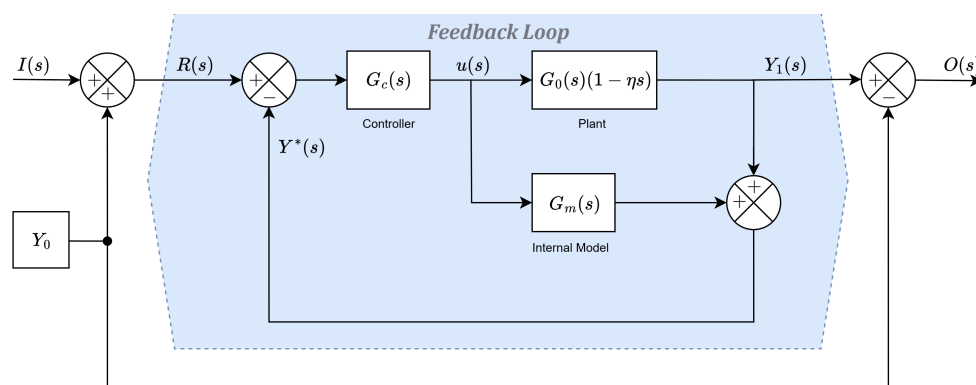


Figure 3. BODE Analysis Block Diagram.

To enable this analysis, mean-centering of the reference signal is required. Since the original reference is not zero-centered, it must be shifted toward the mean of its oscillation. This shifting value is denoted as Y_0 , and is subtracted from the reference signal to ensure proper linearization.

The linearization is carried out in Simulink by designating $I(s)$ as the Input Perturbation and $O(s)$ as the Open Loop Output. The resulting linearized model is then used to generate the Bode magnitude and phase plots.

6.2. Control Strategies in a Simulated Continuous Stirred Tank Reactor (CSTR)

The first application of this study is a simulated an isothermal Continuous Stirred Tank Reactor (CSTR). This chemical reactor is categorized as an inverse response system because of a Van der Vusse reaction. This system has already been analyzed in [25,31], however, this study proposes three novel control laws using Linear Algebra to calculate the controller action, and two different methodologies to compensate for the non-minimum-phase component of the system that to the best of our knowledge have not been proposed before.

The reactor is used to modify the concentration of the output chemical by regulating the valve that adjusts the flow of the reactant. The chemical concentration of the reactor product is defined as C_B , and the reactant flow rate, the controlled variable, as F_r . Figure 4 shows the CSTR system diagram used during the tests of this work [25].

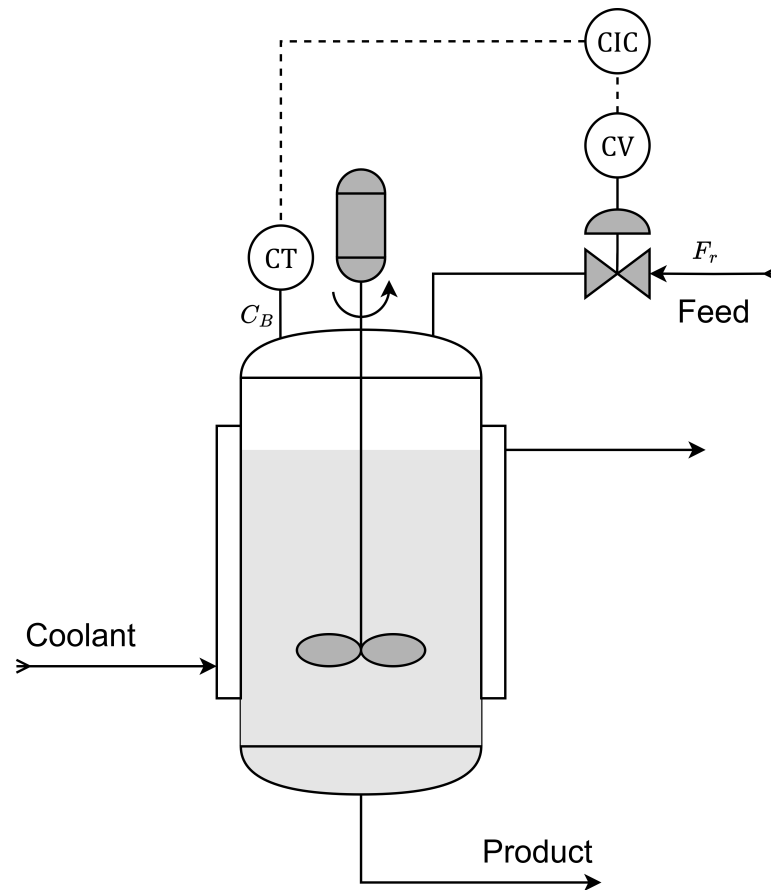


Figure 4. Continuous Stirred Tank Reactor Diagram

6.2.1. Non-Linear Model

The CSTR produces an exothermic reaction with a mathematical model described in [25]. To determine the mathematical model, the following assumptions were considered:

- Heat and density capacities of the reactants are constant.
- The heat loss in coolant jacket is considered negligible.
- The reaction heat and volume remain constant.
- The reaction and reacted material are uniformly mixed.

For the purpose of linearization, according to [25], it is assumed that reference variations remain within $\pm 10\%$ of the nominal operating point, due to the system's non-linearity.

The chemical reaction that occurs in the CSTR is described in (61).



Here, $A \xrightarrow{K_1} B$ describes the exothermic reaction. Considering the mass balance on reactants A and B, the system dynamics are expressed as (62).

$$\begin{aligned} \frac{dC_A(t)}{dt} &= \frac{F_r(t)}{V} [C_{Ai} - C_A(t)] - K_1 C_A(t) - K_3 C_A^2(t) \\ \frac{dC_B(t)}{dt} &= -\frac{F_r(t)}{V} C_B(t) + K_1 C_A(t) = K_2 C_B(t) \end{aligned} \quad (62)$$

The sensor-transmitter element takes the form described in (63) and the relationship between the reactant flow rate and the process input that controls the valve is given by (64).

$$y(t)\% = \left(\frac{100}{1.5714} \right) C_B(t) \quad (63)$$

$$F_r(t) = \left(\frac{634.1719}{100} \right) u(t)\% \quad (64)$$

The variables used to describe the dynamic model of the CSTR are described as follows:

F_r : flow through the reactor.

V : reactor volume that will remain constant during operation.

C_A : the concentration of A in the reactor (mol L^{-1}).

C_B : the concentration of B in the reactor (mol L^{-1}).

C_{A0} : the concentration of A in steady state (mol L^{-1}).

C_{B0} : the concentration of B in steady state (mol L^{-1}).

$K_i (i = 1, 2, 3)$: the reaction rate constants for the three reactions.

$y(t)\%$: transmitter signal.

$u(t)\%$: process input.

Finally, Table 1 presents the steady state values and the initial conditions of each variable in the CSTR. The concentration range of the output chemical C_B is selected from 0 to $1.5714 \text{ mol} \cdot \text{L}^{-1}$ and the control valve capacity is selected with a maximum flow rate of $634.1719 \text{ L} \cdot \text{min}^{-1}$, according to [39]. This way, the sensor measurement signal and valve control signal ($y(t)\%$ and $u(t)\%$) are expressed in percentage.

Table 1. Operation values of the continuous stirred reactor tank [25].

Model Parameters	Value
K_1	$\left(\frac{5}{6}\right) \text{ min}^{-1}$
K_2	$\left(\frac{5}{3}\right) \text{ min}^{-1}$
K_3	$\left(\frac{1}{6}\right) \text{ L} \cdot \text{mol}^{-1} \cdot \text{min}^{-1}$
C_{Ai}	$10 \text{ mol} \cdot \text{L}^{-1}$
V	700 L
C_{A0}	$2.9175 \text{ mol} \cdot \text{L}^{-1}$
C_{B0}	$1.1 \text{ mol} \cdot \text{L}^{-1}$
$u_{0\%}$	60%

6.2.2. Inverse Response System Identification

The CSTR system was characterized as a second-order inverse response system, in order to match the system transfer function, shown in (10), from which the proposed control laws are based. The identification methodology was the one proposed by Alfaro and Balaguer [25] and described in Section 3.

For the identification process, a step input change was applied to the system with a step change of 10% at time zero. With the system response to the step input, the identified transfer function was obtained and is reported in (65).

$$G(s) = \frac{0.32 \cdot (-0.356 \cdot s + 1)}{(0.35 \cdot s + 1)(0.483 \cdot s + 1)} \quad (65)$$

Figure 5 shows the comparison of the identified model in (65) with the CSTR response. The model dynamics closely resembles the system response, with some minor errors that are not substantial enough to affect the controller design.

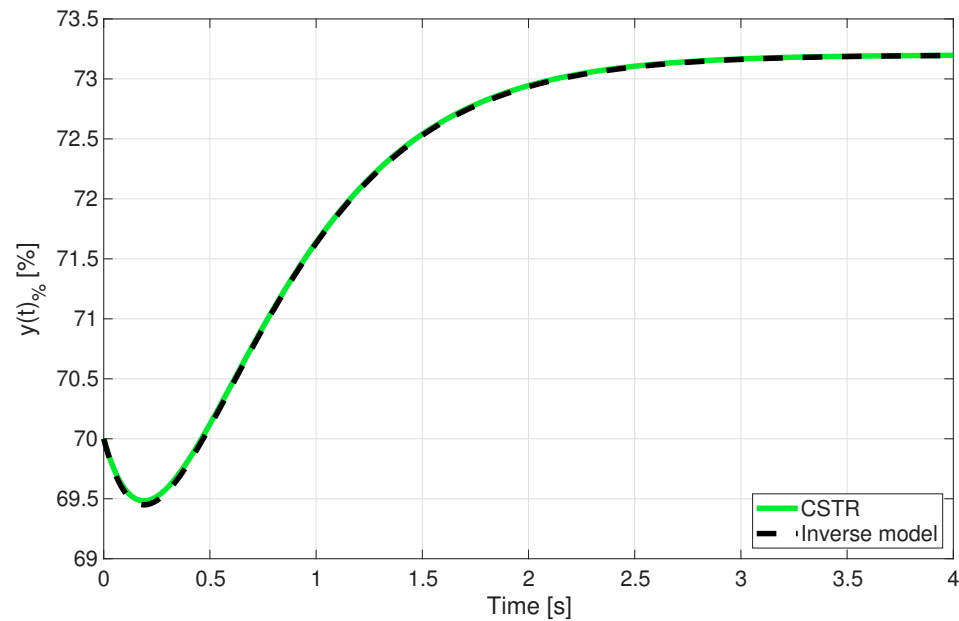


Figure 5. CSTR system identification.

6.2.3. Simulation Results

The CSTR system was tested using six different control laws, one with I&A Compensator and the other one with Smith Predictor for each of the three controller design approaches.

The simulation was performed using Simulink in MATLAB 2024a version. The software ran on an Acer Predator PH315-53 laptop (Acer Inc., New Taipei City, Taiwan) equipped with an Intel (R) Core (TM) i7-8750H @ 2.20 GHz processor (Intel Corporation, Santa Clara, CA, USA).

For the first two seconds of the test, the reference concentration was held constant at 60%. After this initial phase, it transitioned to follow the combined sinusoidal pattern described by Equation (66).

$$R_C(t) = \begin{cases} 60 & \text{if } t \leq 2 \\ 60 + 5 \cdot \sin(0.1(t - 2)) + 2.05(t - 2) \cdot \sin(0.4(t - 2)) & \text{if } t > 2 \end{cases} \quad (66)$$

Figure 6 presents the response of the CSTR system under the six control strategies for the reference trajectory defined in (66) with the gains presented in Table 2.

Table 2. CSTR tuned gains for continuous controllers.

Controller Scheme	k_1	k_2
Controller 1 Smith	50	50
Controller 1 Iinoya	70	70
Controller 2 Smith	5	5
Controller 2 Iinoya	50	50
Controller 3 Smith	1	NA
Controller 3 Iinoya	200	NA

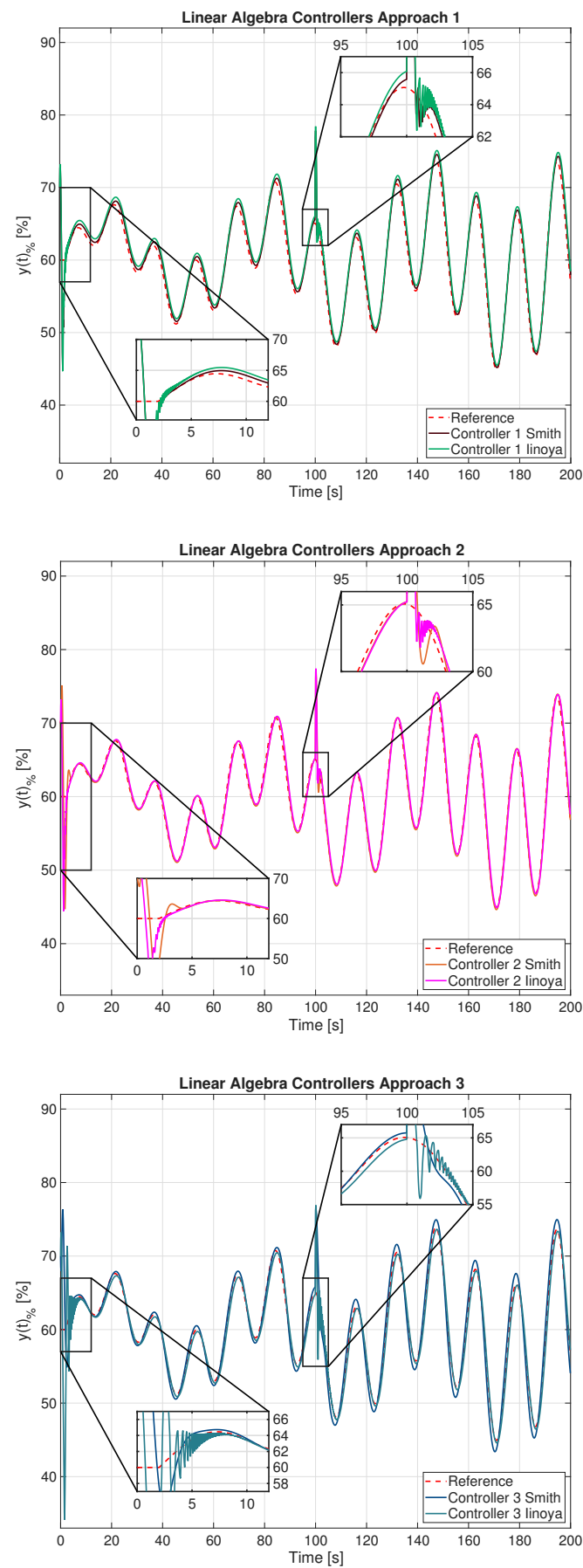


Figure 6. CSTR Reference Tracking Test Results for different controllers designed.

As shown in Figure 6, all controllers successfully follow the reference trajectory, although their initial responses exhibit significant underdamping. This effect is most pronounced in Controller 3 using the I&A approach. Regarding disturbance rejection, a disturbance of magnitude 10% was introduced at 100 s of the experiment, all controllers manage to correct the perturbation within 5 s. However, Controller 3 with the I&A method takes the longest to stabilize.

Figure 7 demonstrates that all controllers exhibit an abrupt initial control action, both in response to trajectory tracking and disturbance rejection. These control signals quickly reach the saturation limits of the actuator, with Controller 3 using the I&A method showing the most noticeable saturation behavior. Additionally, when rejecting disturbances while simultaneously tracking the trajectory, the amplitude of the control signal oscillations increases, which eventually causes the system to temporarily reach its saturation limit, compromising its ability to follow the trajectory accurately.

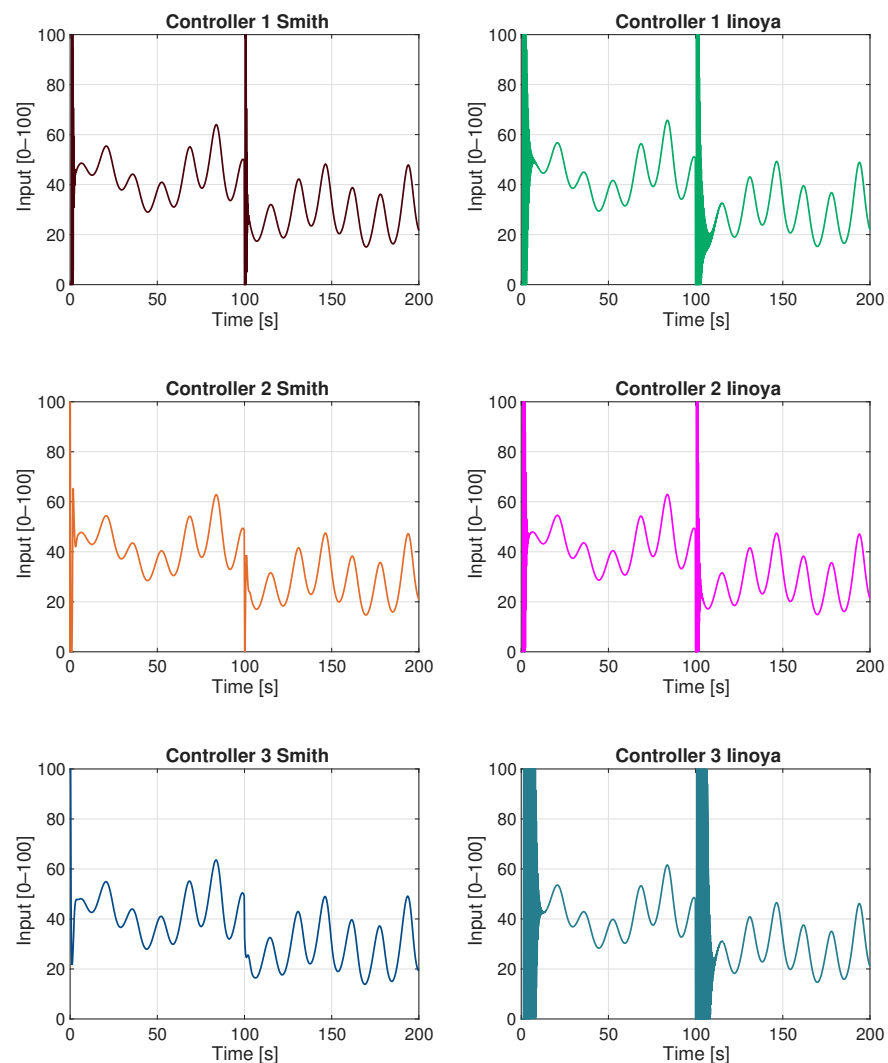


Figure 7. CSTR Reference Tracking Controller Action for different controllers designed.

To compare the controllers more comprehensively, a Spider Plot, Figure 8, was generated using the three selected metrics (ISE, ISCO, and ISDCO) as axes. In this visualization, a smaller triangular area generally indicates better overall tracking performance, considering the three indicators jointly. Regarding ISCO, all controllers perform similarly, suggesting comparable energy consumption. However, for ISDCO, Controller 3 with the I&A approach performs considerably worse, reflecting more abrupt control actions that could, in indus-

trial scenarios, accelerate actuator wear or reduce its operational lifespan. In terms of ISE, tracking accuracy varies more significantly across controllers, with Controllers 1 and 2 using the Smith Predictor exhibiting the smallest errors. Controller 2 with the I&A approach achieves a favorable balance between low ISE and slightly lower ISCO, although this comes at the cost of higher ISDCO. This highlights a clear trade-off between tracking accuracy and control smoothness, which must be considered in applications where actuator durability is critical.

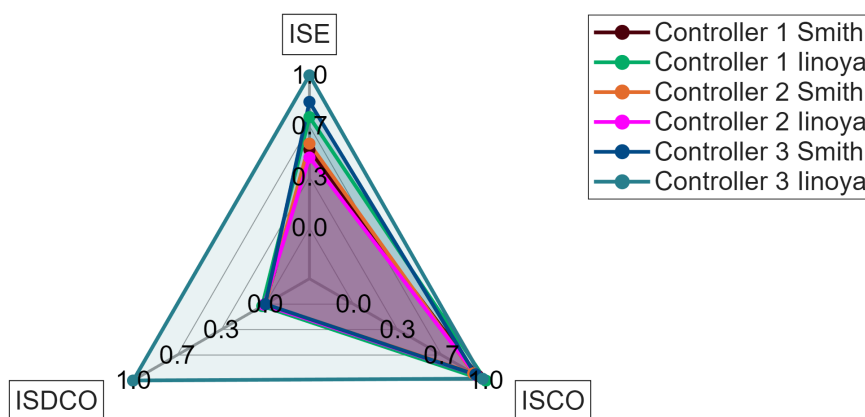


Figure 8. CSTR System: Performance Comparison.

6.2.4. Frequency Analysis

The results for both the magnitude and phase diagrams shown in Figures 9 and 10 were made using the continuous controller designs.

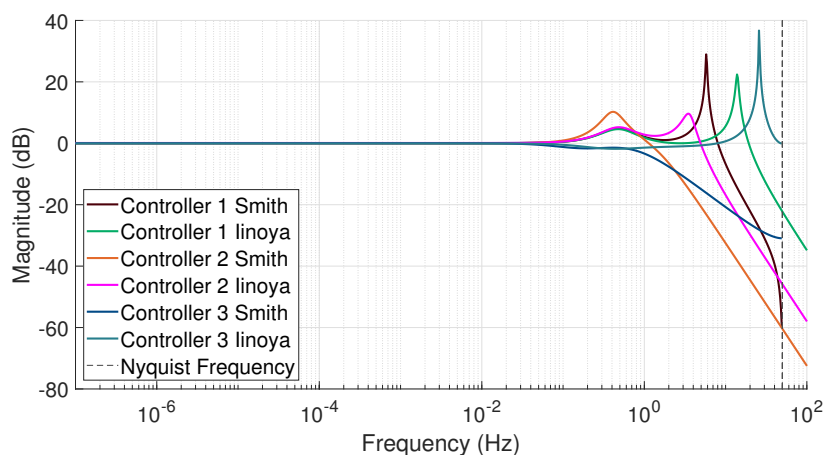


Figure 9. CSTR System Magnitude Bode diagram for variable frequency reference.

Figure 9 demonstrates that the controllers can replicate the reference signal with a near 1:1 magnitude ratio when the input frequency is below 0.01 Hz. However, as the frequency increases, differences in controller performance become evident. Controller 3 (Iinoya) stands out by maintaining accurate reference tracking up to frequencies as high as 10 Hz, outperforming the others in high-frequency response, albeit at the expense of pronounced peaks at specific frequencies, which are indicative of potential undersampling or resonance effects. In contrast, Controller 3 (Smith) does not exhibit such peaks, suggesting a more overdamped response that prioritizes stability over rapid tracking.

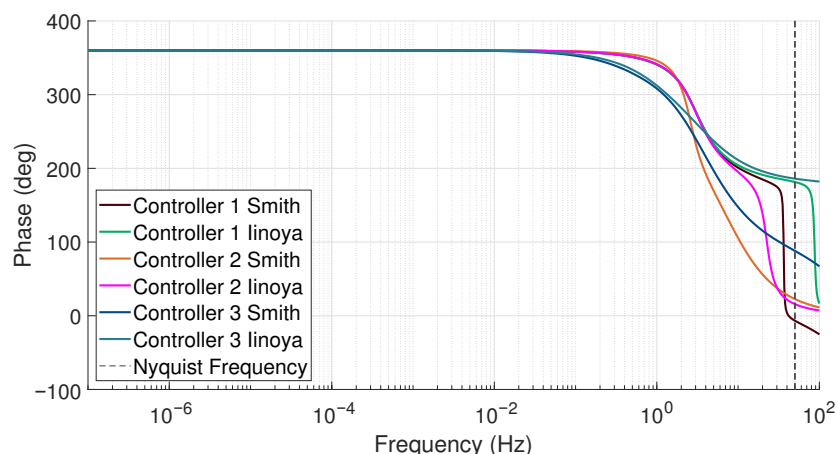


Figure 10. CSTR System Phase Bode diagram for variable frequency reference.

Regarding the phase plot shown in Figure 10, all controllers maintain accurate alignment with the input signal, exhibiting a consistent 360° phase relationship up to a frequency of 0.01 Hz. However, beyond this point, the effects of system delay become more evident, with phase shifts reaching up to 360° at 10 Hz. Among the controllers, Controller 3 (I&A) displays the smallest phase lag, indicating a faster response to high-frequency signals. Nevertheless, this rapid responsiveness comes at the cost of an undersampled and erratic control action, which compromises overall stability as shown in Figures 6 and 9.

The high-frequency magnitude peaks reported for Controller 3 (Iinoya) in the CSTR Bode plot (Figure 9) do not indicate closed-loop instability. We show that Controller 3 (I&A) extends accurate tracking to higher frequencies. Still, exhibits pronounced peaks at specific bands (Figure 9), a behavior we already interpreted as undersampling/resonance-like effects due to the controller structure and implementation details. Time-domain results (Figures 6 and 7) remain bounded—no divergent oscillations or growing amplitudes are observed—while the performance radar (Figure 8) reveals a trade-off: improved high-frequency tracking is accompanied by larger control variation (higher ISDCO), which is typical of amplified high-frequency sensitivity rather than instability. These observations are now stated explicitly in the revised text—impact on robustness. Although stability is preserved in the time domain, the magnitude peaking at high frequencies implies reduced robustness margins to measure noise and unmodeled fast dynamics. This is consistent with the increased ISDCO of Controller 3 (I&A) (Figure 8) and with the more aggressive control action visible in Figure 7. We have added a short discussion and practical mitigations: (i) introduce mild high-frequency roll-off (or a light low-pass on the control signal), (ii) ensure proper anti-aliasing on measurements, and (iii) if needed, apply a narrow notch at identified resonant lines. The high-frequency peaks in Figure 9 for Controller 3 (I&A) reflect loop-shaping/discretization trade-offs rather than instability, and we summarize the robustness implications with pointers to Figures 6–8.

6.3. Experimental Evaluation on the Temperature Control Laboratory (TCLab) Platform

In this part, we present the experimental results for the reference tracks. The Temperature Control Laboratory (TCLab) is used to evaluate the performance of the proposed controller. The TCLab is a portable, pocket-sized laboratory designed for testing different control system design applications [37]. The experiments utilized the Simulink version of MATLAB2024a and Python 3.10 version.

The TCLab consists of two heaters, two temperature sensors, a Leonardo Arduino board, and a power supply to power the heaters, as shown in Figure 11.

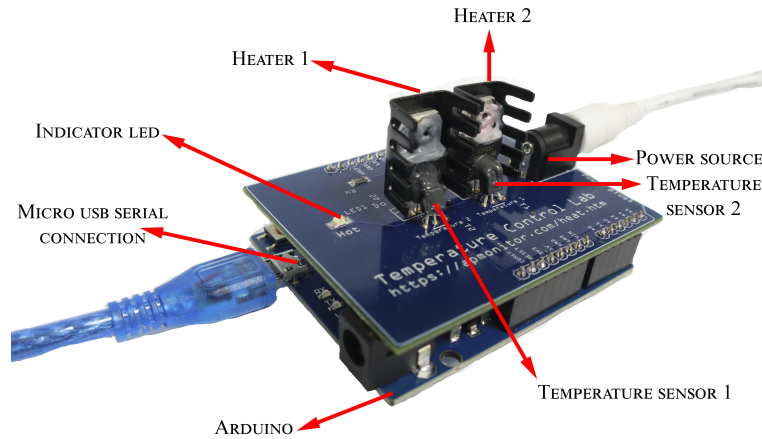


Figure 11. Temperature Control Laboratory (TCLab).

The control maintains the outlet temperature at a desired reference by adjusting the power output of the heater [34], where the thermal energy produced by the heater is transferred to the temperature sensor through conduction, radiation and convection. This work uses only the single-input and single-output (SISO) configurations of TCLab.

6.3.1. Inverse Response Module (Inversion Module)

The TCLab is a laboratory device designed to perform real-time temperature control experiments. In its original design, the system does not produce an inverse response and it has been proven that it can be accurately modeled as a FOPDT system of the form in (67) [30,34,40,41].

$$G_p(s) = \frac{K}{\tau_p \cdot s + 1} \cdot e^{-t_0 \cdot s} \quad (67)$$

However, since this study focuses on control strategies of inverse response systems, the TCLab was transformed into a non-minimum phase system by integrating an inversion module in series with the original plant. The design of the inversion module follows the methodology proposed in [30]. Since the original temperature system can be modeled as an FOPDT, which contains one pole and a time delay in its transfer function, the inversion module incorporated in series needs to have a second pole and a zero in the right half-plane as shown in (68) [30].

$$G_I(s) = \frac{1 - \eta \cdot s}{\tau_I \cdot s + 1} \quad (68)$$

For the overall system to have a non-minimum phase behavior, the inversion module needs to satisfy the following conditions to guarantee a stable inverse response.

- The integrated zero must be in the right half plane, which means that η must be greater than 0.
- The integrated pole must keep the system stable, which means that τ_I must be greater than 0.
- If the RHP zero is placed closer to the origin, the inverse response will be stronger.
- A faster added pole sharpens the initial slope opposite of the steady-state response.

To not affect the dynamics of the original system, the added pole must be faster than the dominant pole of the plant. If the pole is too slow, closer to the dominant pole, it can cause the effect of the inverse response to last longer and affect the original systems dynamics [42,43].

Similarly, the RHP zero must not be too close to the origin, since it generates a stronger and long-lasting inverse response, which results in a more difficult system to control [42,43].

The RHP zero is therefore selected to guarantee that it is faster than the dominant pole, to confine the inverse response to the early transient response and then allow the dominant pole to take over the system dynamics ($\eta < \tau_p$).

An experiment was performed in the TCLab to determine the First Order Plus Dead Time (FOPDT) model of the plant. First, the temperature was stabilized with an input of 40% and at 1000 seconds a step input of an additional 20% is introduced. The FOPDT model is determined with the equations in (69), following the procedure proposed by Alfaro, described in [44].

$$\begin{aligned} K &= \frac{\Delta y}{\Delta u} \\ \tau_p &= -0.91 \cdot t_{25\%} + 0.91 \cdot t_{75\%} \\ t_0 &= 1.262 \cdot t_{25\%} - 0.262 \cdot t_{75\%} \end{aligned} \quad (69)$$

where Δy is the variation in the system output, Δu is the variation in the system input, $t_{25\%}$ is the time it takes for the output to reach 25% of the steady-state value and $t_{75\%}$ is the time it takes for the output to reach 75% of the steady-state value.

The identified FOPDT system is given by (70).

$$G_p(s) = \frac{0.75}{163.3 \cdot s + 1} \cdot e^{-23.5 \cdot s} \quad (70)$$

The TCLab system response to a 20% step change and the identified FOPDT are presented on Figure 12. It is observed that the identified transfer function is able to replicate the system dynamics efficiently.

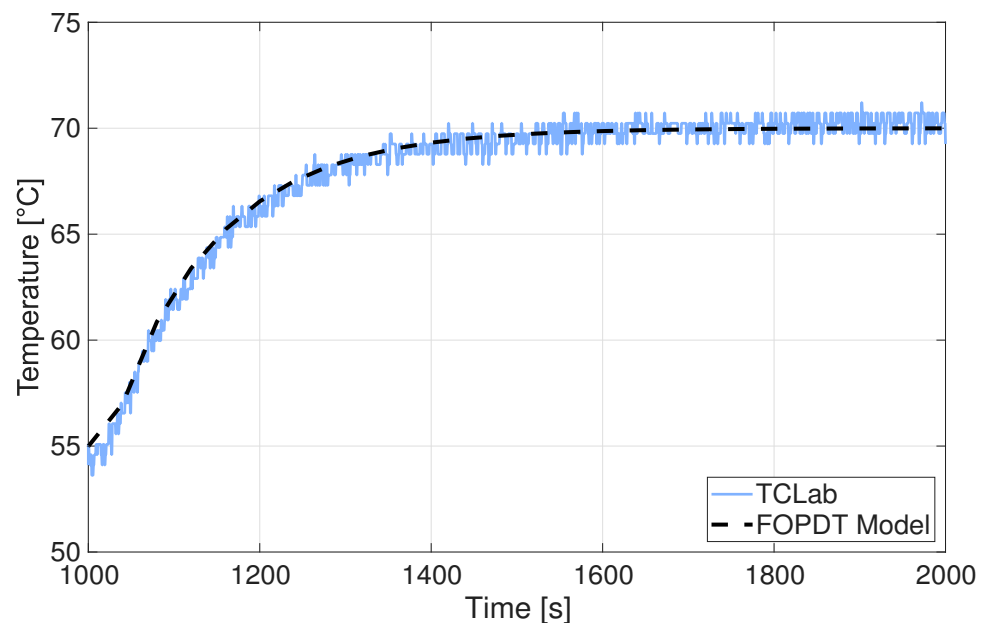


Figure 12. Temperature system identification as an FOPDT.

Taking into consideration the requirements described in this section, the inverse module added in series to the original plant was manually tuned to produce an inverse response with a 5 °C temperature drop for a 20% step input. The dynamics of the inverse system were then validated first through simulation, using the FOPDT model as the plant, and experimentally, using the TCLab in series with the inversion module.

As a result, the transfer function of the inversion module used during the experiments is presented in (71).

$$G_I(s) = \frac{1 - 140s}{60s + 1} \quad (71)$$

Thus, the modified temperature plan block diagram is as shown in Figure 13.

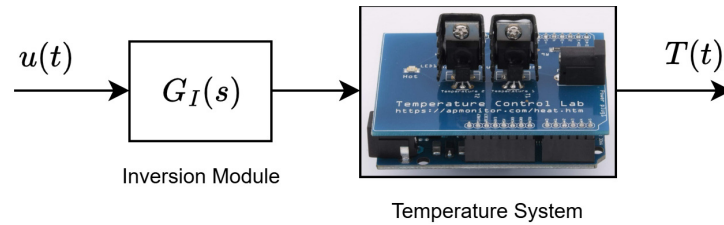


Figure 13. Temperature system block diagram.

To validate the dynamics of the non-minimum phase system, the experiment performed for system identification was repeated using the plant described in Figure 13. The results are shown in Figure 14. At the beginning of the experiment, the temperature was stabilized at 53 °C, and when the step input was applied at 1000 s, a temperature drop of 5 °C was observed, reaching a minimum of 48 °C.

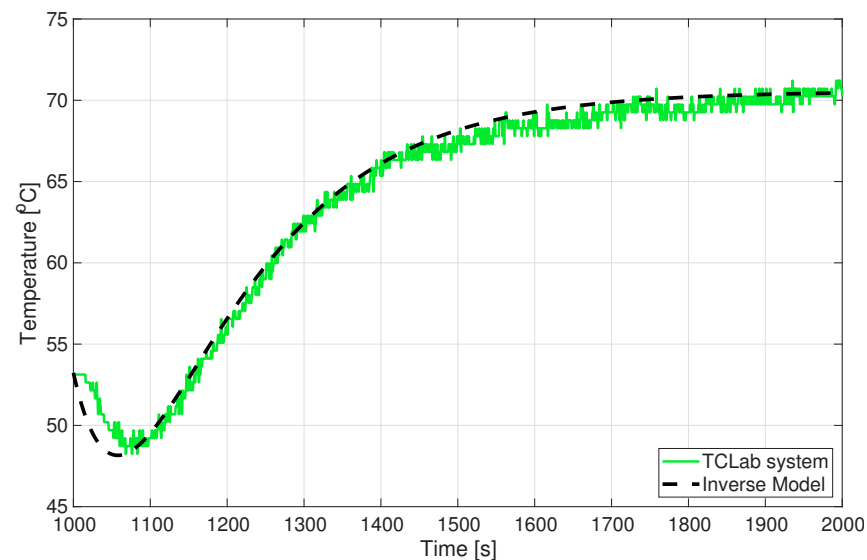


Figure 14. Temperature system identification.

6.3.2. Inverse Response System Identification

The system described in Figure 13 was characterized as a second-order inverse response system, to get a model of the form of (10).

The model is obtained following the methodology described in Section 3. A step input with a magnitude of 20% was given to the system in order to get a step response and identify the system. The identified transfer function of the system is shown in (72).

$$G(s) = \frac{0.75 \cdot (-136.9 \cdot s + 1)}{(146.96 \cdot s + 1)(61.72 \cdot s + 1)} \quad (72)$$

It is noteworthy that the TCLab system lacks a cooling mechanism, which causes that the temperature drop in the real system is slower than the model. Nonetheless, Figure 14 shows that the model is able to replicate the dynamics of the modified real system with a minimal error that does not affect the controller performance.

6.3.3. Experimental Results

The TCLab was tested using six different control laws, one with I&A Compensator and the other one with Smith Predictor for each of the three controller design approaches, as done with the CSTR.

The experiment was carried out in a open space without affectation of sunlight in an environment of 23 °C to 25 °C. Each experiment lasted 3 h, and multiple attempts regarding the tune of their gains were made. The best results for each controller are presented in this section.

For the first 300 s of the experiment, the reference temperature was held constant at 50 °C. Following this period, it transitioned to the dynamic trajectory defined by Equation (73). The frequency of this signal was intentionally kept low to account for the system's natural cooling behavior, as the plant lacks an active cooling mechanism.

$$R_T(t) = \begin{cases} 50 & \text{if } t \leq 300 \\ 50 + 5 \cdot \sin\left(\frac{1}{500}(t - 300)\right) & \text{if } t > 300 \end{cases} \quad (73)$$

Figure 15 presents the response of the TCLab under the six control strategies for the reference trajectory defined in (73) with the gains presented in Table 3.

Table 3. TCLab tuned gains for discrete controllers.

Controller Scheme	k_1	k_2
Controller 1 Smith	0.01	0.2
Controller 1 Iinoya	0.85	0.9
Controller 2 Smith	0.92	0.9
Controller 2 Iinoya	0.8	0.8
Controller 3 Smith	0.4	NA
Controller 3 Iinoya	0.3	NA

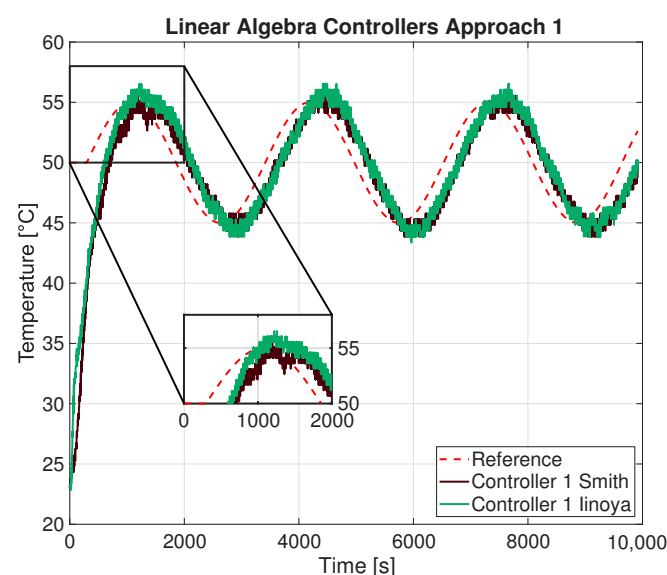


Figure 15. Cont.

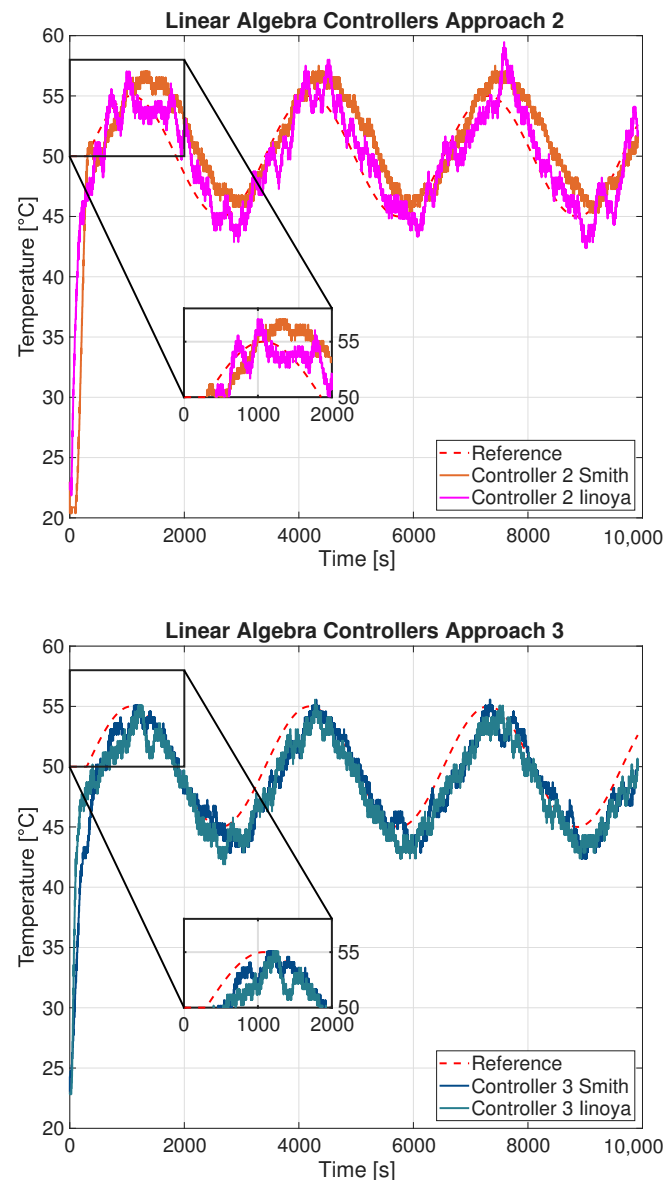


Figure 15. Temperature Reference Tracking Test Results for different controllers designed.

As shown in Figure 15, although the controllers generally track the reference adequately, there is a noticeable delay of approximately 300 s between the system output and the reference signal. This lag is attributed to the system's inverse response, which induces a delay that is significantly more pronounced than in the CSTR setup, where such behavior was negligible.

Additionally, the controllers exhibit varying degrees of sensitivity to measurement noise. To quantify the measurement noise of the TCLab temperature sensor, the system was stabilized at 39 °C and the fluctuations around this setpoint were analyzed. After subtracting the mean operating temperature, the noise signal exhibited an average value of 0.2584 °C, indicating a small residual bias. The standard deviation was calculated as 0.3651 °C, which represents the typical amplitude of the noise around the mean. Correspondingly, the variance was obtained as 0.1333 (°C)², providing a measure of the noise power. From these results, the signal-to-noise ratio (SNR) was computed as 38.81 dB, which indicates that the sensor exhibits a relatively low noise compared to the nominal signal level.

Controllers 2 and 3 (in both control approaches) are heavily affected by sensor noise, while the controllers based on Approach 1 present more stable behavior. Controller 2,

which attempts to compensate more aggressively for the delay, amplifies high-frequency noise as a side effect. This is particularly evident around second 7700, where the system's response resembles a disturbance, despite no actual perturbation occurring in the controlled environment, highlighting that the apparent anomaly results from noise amplification by the controller.

Figure 16 reveals that the noise amplification in Controller 3 using the I&A approach is so pronounced that the control signal nearly saturates continuously. This behavior makes the controller heavily reliant on small sampling intervals to maintain control effectiveness. Other controllers, except Controller 1 with the Smith Predictor, also display erratic behavior, albeit to a lesser extent. Notably, Controller 1 with the Smith approach maintains a relatively smooth control action, demonstrating its superior capability to filter out high-frequency noise that adversely affects the other control strategies.

As with the CSTR system, a Spider Plot is used to compare controller performance across the three metrics (ISE, ISCO, and ISDCO). The visualization in Figure 17 highlights that each control strategy presents distinct strengths and weaknesses, reflecting inherent trade-offs between tracking accuracy, energy efficiency, and smoothness of control action. Regarding ISE, Controller 2 with the I&A approach and Controller 3 with the same method achieve the lowest tracking errors. Controller 2 also attains the lowest ISCO, indicating efficient energy usage, whereas Controller 3 records the highest ISDCO, suggesting a highly aggressive control signal that, in industrial applications, could accelerate actuator wear. Although Controller 2 with I&A does not achieve the best ISDCO (this distinction belongs to Controller 1 with the Smith Predictor, as previously observed in the control action plots), it maintains a favorable balance between low tracking error and moderate energy consumption. This balance may be advantageous in scenarios where tracking accuracy is prioritized over actuator longevity, but the trade-off should be carefully evaluated for each application.

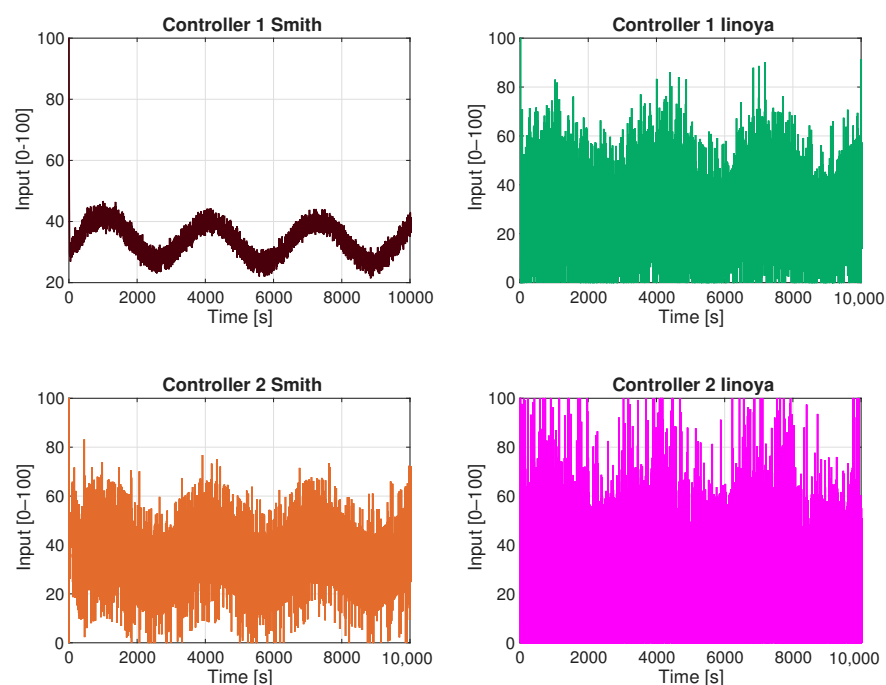


Figure 16. Cont.

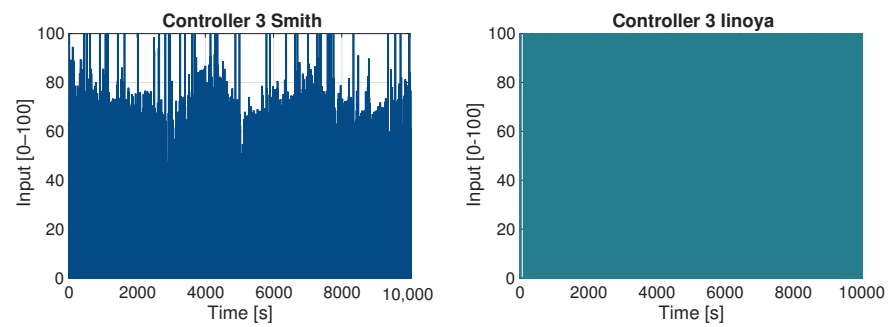


Figure 16. Temperature Reference Tracking Controller Action for different controllers designed.

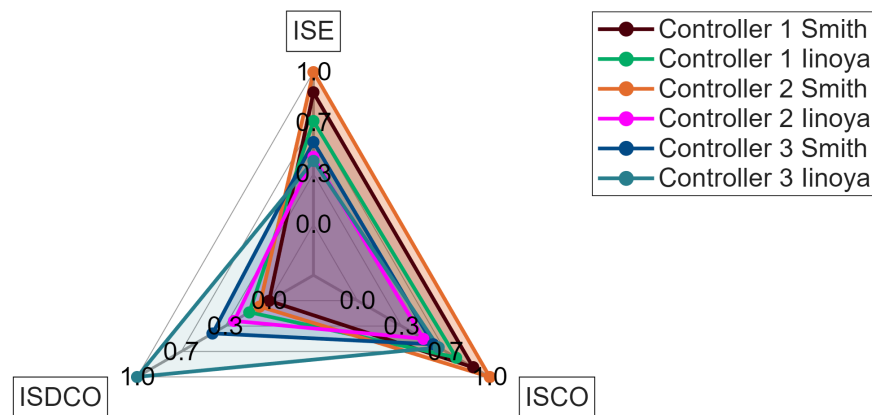


Figure 17. Temperature System: Performance Comparison.

6.3.4. Frequency Analysis

The results for both the magnitude and phase diagrams shown in Figures 18 and 19 were obtained using the discrete controller designs. In both plots, the magnitude and phase responses are displayed only up to a vertical dotted line, which represents the Nyquist frequency. This frequency corresponds to half the sampling rate of the discrete system and defines the maximum frequency that can be accurately described without introducing aliasing. Beyond this point, frequency components cannot be correctly interpreted. They may lead to distortions in the system's response, making it essential to limit the analysis to frequencies below the Nyquist limit.

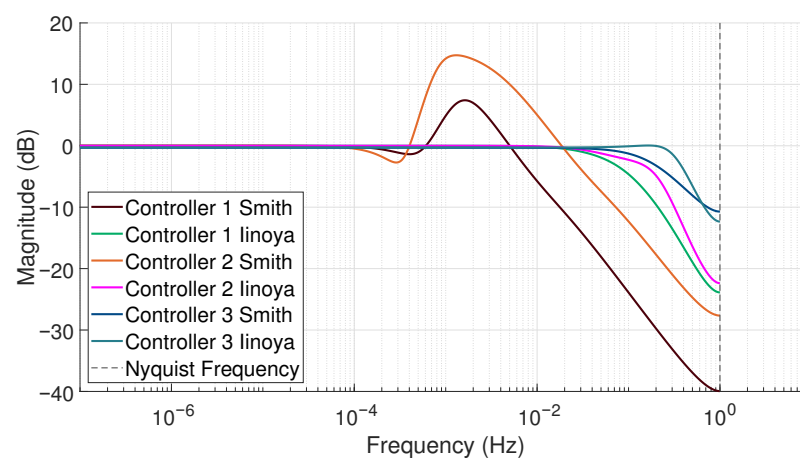


Figure 18. Temperature System Magnitude Bode diagram for variable frequency reference.

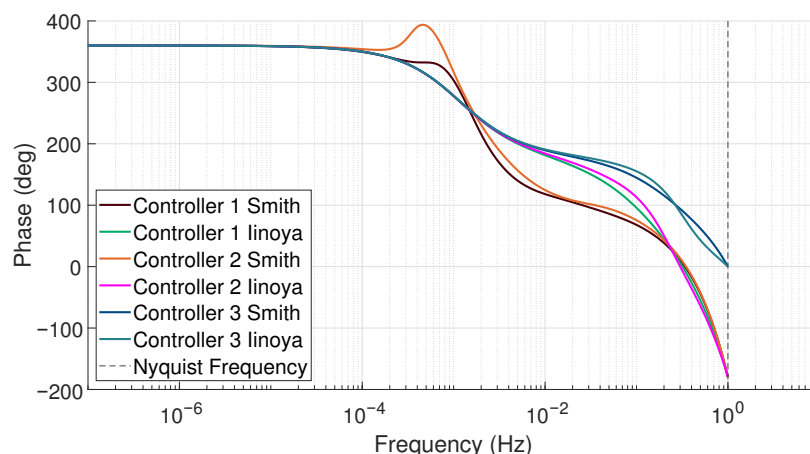


Figure 19. Temperature System Phase Bode diagram for variable frequency reference.

Similar to the CSTR magnitude plot, Figure 18 shows that all controllers maintain a 1:1 magnitude ratio with the reference signal at low frequencies, up to approximately 10^{-4} Hz. However, Controller 1 Smith and Controller 2 Smith exhibit signs of undersampling, evident in the pronounced overshoot around 10^{-3} Hz. Among the tested controllers, Controller 3 (I&A) maintains the most accurate magnitude tracking at higher frequencies. However, this performance comes at the cost of a significantly aggressive control effort, as illustrated in Figure 16.

Regarding the phase plot in Figure 19, all controllers can closely follow the input signal up to a frequency of approximately 10^{-4} Hz. As the frequency increases, a progressively larger phase lag is observed, indicating a growing delay between the input and output signals. Furthermore, values around 180° represent a major lag in the system's response relative to the reference signal. In this case, the negative phase shift indicates that the system is more than one cycle delayed from the reference signal, which could also be a sign of potential system instability.

The phase lag observed beyond 0.01 Hz in the TCLab Bode phase (Figure 17) arises primarily from the inversion module placed in series with the plant and, secondarily, from tuning choices that favor faster tracking. The module adds a pole-zero structure with a right-half-plane zero (Equation (68)), which effectively increases the apparent delay of the compensated plant; as frequency grows, this manifests as a progressively larger negative phase. The effect is accentuated by TCLab's slow thermal dynamics and the absence of active cooling, as noted in the experimental section. We have clarified this mechanism in the revision and referenced the inversion module design and identification steps. We explicitly attribute the TCLab phase lag (Figure 17) to the inversion module (Equation 68) and controller tuning, and we mention the Nyquist-limit marker in the plots as a reminder that the sampling rate bounds analysis. These additions make clear that the observed Bode peaks are a by-product of controller/implementation trade-offs under the (I&A) structure, not a sign of instability, and that the TCLab phase behavior is consistent with the introduced inversion module and the system's thermal time constants.

7. Conclusions

This work has presented and validated a novel control strategy for trajectory tracking in non-minimum-phase systems, leveraging internal model-based compensation to enable the use of linear algebra techniques in controller design. Through both simulation and experimental studies on a CSTR model and the TCLab platform, the proposed approach demonstrated the ability to transform non-minimum-phase dynamics into equivalent minimum-phase behavior, thereby facilitating controller design and achieving precise

tracking even in the presence of disturbances and sensor noise. The findings confirm that internal model-based compensation not only simplifies the controller design process but also improves tracking performance in systems traditionally considered difficult to control. Building upon this, recent work has demonstrated that linear-algebra-based trajectory tracking strategies, originally developed for minimum-phase systems, can be effectively extended to non-minimum-phase systems. This extension enables accurate tracking of time-varying references without requiring excessively complex control structures. The proposed approach maintains the simplicity of traditional implementations by incorporating two well-established compensators, I&A and Smith predictors, which are widely recognized and routinely used by control engineers and students alike. Their familiarity and industrial adoption make the method particularly suitable for practical deployment in process control applications, offering a robust and accessible solution to the challenges posed by non-minimum-phase dynamics.

Future work will focus on extending this approach to multi-variable non-minimum-phase systems, exploring adaptive and online-tuning schemes to improve performance under time-varying conditions, and assessing its applicability to larger-scale industrial processes with significant nonlinearities and delays.

Author Contributions: Conceptualization, S.I., G.G.-G., G.S. and O.C.; methodology, S.I., G.G.-G., G.S. and O.C.; validation, G.G.-G. and S.I.; investigation, S.I., G.G.-G., G.S. and O.C.; writing—original draft preparation, S.I., G.G.-G., G.S. and O.C.; writing—review and editing, S.I., G.G.-G., G.S. and O.C.; supervision, G.S. and O.C. All authors have read and agreed to the published version of the manuscript.

Funding: Universidad San Francisco de Quito supported this work through the Poli-Grants Program under Grant 33603.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflict of interest.

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