

## Article

# Dual-Scale Modelling of the Vacuum Drying Process for Transformer Cellulose-Based Insulation

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## Abstract

The vacuum drying of cellulose-based insulation is an essential step in the transformer manufacturing process, typically consisting of both heat and vacuum application. The moisture inside cellulose insulation during this process is transferred by various transport mechanisms, some of which are affected by the insulation's temperature. Moreover, the conditions within the vacuum chamber are generally transient and highly dynamic, depending on the employed process control strategy, and may include various phenomena, such as gas expansion during pump-down and radiative heat transfer. From a modelling perspective, these factors can significantly impact the drying rate by altering the boundary conditions of heat and mass transport equations. To account for such effects, a model that considers the process at both the scale of cellulose insulation and the scale of the vacuum chamber is presented. A simplified drying system with two-point process control is introduced to simulate multiple cases. The results highlight the sensitivity of drying behaviour to both the model parameters and the selected control strategy. A comparison with existing Fickian diffusion models indicates that the proposed model, when properly calibrated, can reliably reproduce drying dynamics and thus provide a powerful tool for optimizing vacuum drying procedures.

**Keywords:** transformer cellulose-based insulation; vacuum drying; dual-scale modelling; radiation heat transfer; pump down; non-equilibrium moisture content



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## 1. Introduction

Cellulose-based insulation impregnated with mineral oil, serving both as a dielectric barrier and a mechanical support, constitutes a critical component of high-voltage equipment such as transformers [1]. It is a well-established fact that the moisture inside transformer insulation increases the aging rate and shortens the service life of apparatus [2,3]. Therefore, prior to the oil impregnation step, transformers undergo a thorough drying process typically involving cyclic heating and vacuum application [4,5].

When it comes to establishing a control strategy for the cellulose insulation drying process, manufacturers frequently rely on experience [5], which is the reason why some recent works have been focused on mathematical modelling of moisture dynamics inside non-impregnated cellulose insulation. Access to a reliable and flexible model of the vacuum drying process can enable its improvement in terms of drying time and energy consumption.

A common approach to describing moisture transport inside cellulose insulation is using Fick's law, in which the diffusion coefficient is determined empirically. Notable work

in this area has been conducted by Du et al. [6], who determined the diffusion coefficient for non-impregnated pressboard. Afterwards, this work was improved upon by García et al. [7] and Villarroel et al. [8], who experimentally determined the moisture diffusion coefficient in transformer paper as a function of temperature, moisture content, and thickness of insulation. Similar methods were more recently used by Wang et al. [9] to additionally allow for the effects of insulation aging on the value of the moisture diffusion coefficient.

While the results of those models are in good agreement with the experimental values, it is physically inconsistent for the diffusion coefficient, as an intrinsic material property, to depend on sample dimensions such as thickness. As noted in [10–12], this is a clear indicator that the drying process cannot be fully described by Fick's law alone. García et al. [13] further showed that this thickness dependence is not a consequence of interlayer spaces or voids, but rather a compensation for inaccuracy of the used diffusion model.

This shortcoming of the Fick-based description of moisture transport was also recognized by Kang et al. [14], who applied a Langmuir model to study moisture dynamics. This model is based on the assumption that the moisture contained within insulation can be separated into free and bound moisture, which are mutually convertible. To extract bound moisture, it first needs to be transformed into free moisture, which is then transported according to Fick's law. The additional parameters of this model are adsorption and desorption coefficients, which are determined by fitting the analytical solution of the Langmuir model to the experimentally obtained drying curves. A comparison between the models showed that the Fickian model significantly underestimates the necessary drying time of pressboard.

On the other hand, Brahami et al. [5] adopted an empirical model to describe the drying of transformer insulation. This model additionally considers degradation of cellulose insulation due to its exposure to oxygen and elevated temperatures, thus providing a mathematical tool for manufacturers to determine the desired compromise between energy consumption and drying time for the process.

All these studies assume a uniform temperature distribution within cellulose insulation. Although heat diffuses through insulation faster than moisture does, there are indications that considerable temperature gradients through cellulose insulation might exist even in later stages of the drying process. They might be caused by local cooling due to the evaporation of water vapor from cellulose fibres [15], or uneven heating [16]. Furthermore, as the drying process consists of series of cycles of heating and vacuum application, transient periods will occur in between the phases, making it unclear on how boundary conditions at the interface between cellulose insulation and the chamber atmosphere should be imposed in those periods. The vacuum chamber atmosphere also interacts with cellulose insulation through the exchange of heat and mass, thus making the whole problem strongly coupled. Accurately capturing this behaviour requires a modelling framework that accounts for both internal dynamics of the cellulose insulation and the surrounding chamber atmosphere. This complexity has previously been addressed in other fields, such as wood vacuum drying [17–20] and pharmaceutical freeze drying [21], where a dual-scale modelling approach has been used. The basic idea is to introduce a computational fluid dynamics (CFD) [22] or zero-dimensional (0D) model at the scale of the dryer, thus extending the existing (macroscale) model of heat and mass transfer through porous media. Such models enable more accurate predictions of the process's energy consumption and drying time, thus supporting its analysis and optimization [23,24].

This study presents a comprehensive mathematical model that describes the vacuum drying process at both the scale of the cellulose insulation and the scale of the vacuum chamber. Implemented in the Python programming language, the model can be employed to simulate various process configurations. Numerical experiments conducted for a sim-

plified drying system indicate a strong influence of key model parameters as well as the selected control strategy. A comparison with existing models confirms the model's potential for calibration (parameter fitting), a prerequisite for its use in analyzing and optimizing the vacuum drying process for transformer cellulose-based insulation.

## 2. Mathematical Model

The mathematical model describing the vacuum drying process of transformer cellulose-based insulation is structured to capture the relevant physical phenomena occurring within the drying system. The following subsections first provide a description of the observed drying system, followed by the formulation of the governing equations, interface conditions between the cellulose and the chamber atmosphere, initial conditions, and numerical solution procedure.

### 2.1. Drying System Description

In this study, a simplified yet sufficiently sophisticated drying system is considered, which is designed to enable control of the two primary process variables: drying temperature and pressure. As illustrated in Figure 1, the system consists of a thermally insulated vacuum chamber with two ports. The first port is connected to a vacuum pump through the suction line, with a throttle valve installed to reduce or completely shut off the pumping speed when necessary. Additionally, to ensure that no significant over-pressure occurs and to enable the venting process at the end of each vacuum phase, a vacuum chamber is equipped with a valve with both breathing and venting functions. Lastly, the heating of the cellulose insulation is achieved with infrared (IR) heaters mounted on the inner walls of the vacuum chamber shell. In addition to radiation, the heat is also being transferred by natural convection since no fan or forced air circulation is employed.

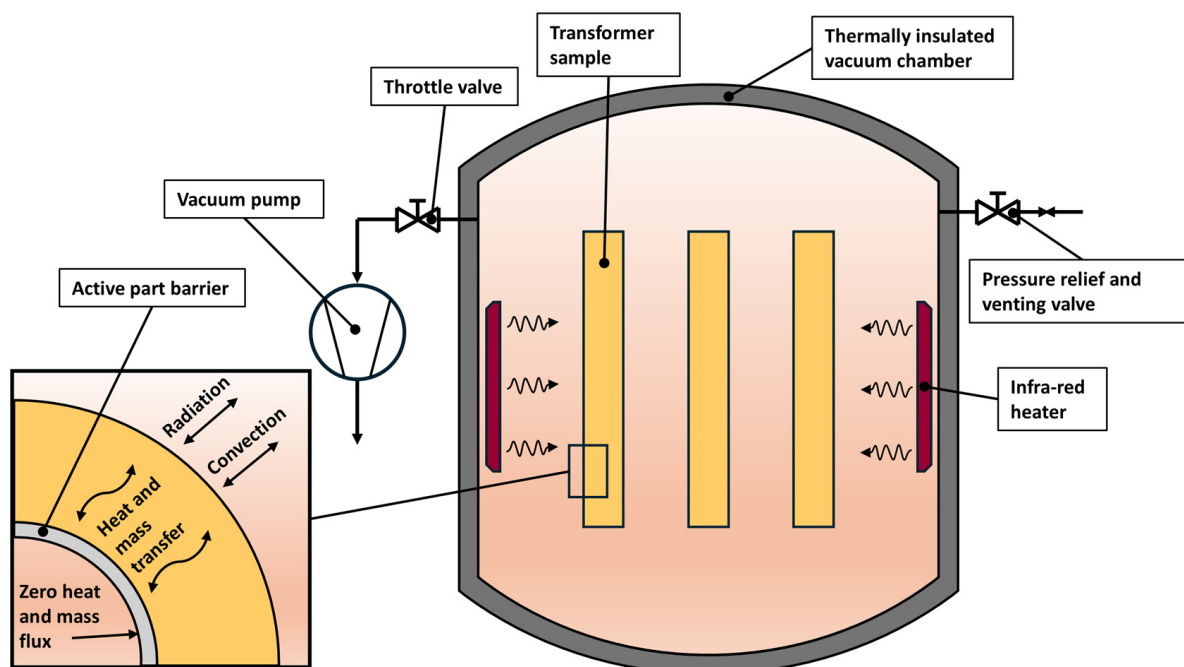


Figure 1. Schematic diagram of the vacuum drying system.

Inside the chamber, transformer samples with a simple geometry are placed. Each sample consists of cellulose insulation paper wrapped around a cylindrical metal core, serving as a reasonable approximation for the actual transformer. In that case, one can assume that the outer side of the sample is completely exposed to the vacuum chamber conditions, and the inner side is thermally insulated and impermeable to mass transfer. The

number, size, and spatial arrangement of the samples can vary depending on the available space inside the vacuum chamber.

## 2.2. Governing Equations at the Scale of the Cellulose Insulation

Unlike previous studies, a different approach is considered in modelling the heat and mass transport through cellulose insulation. Knowing that the transformer insulation is composed of cellulose fibres containing moisture and pores filled with humid air (Figure 2), it is assumed that the moisture inside the fibres is practically immobile and that its transport is achieved in the form of water vapor through the pore space by means of diffusion and convection. Furthermore, a local non-equilibrium is assumed between cellulose fibres and humid air inside the pores. The moisture content of the fibres will tend to reach the equilibrium state, which will then result in local moisture transport between the fibres and humid air inside the pores. That is, moisture inside cellulose fibres will either be absorbed or desorbed, depending on the difference between current moisture content and equilibrium moisture content.

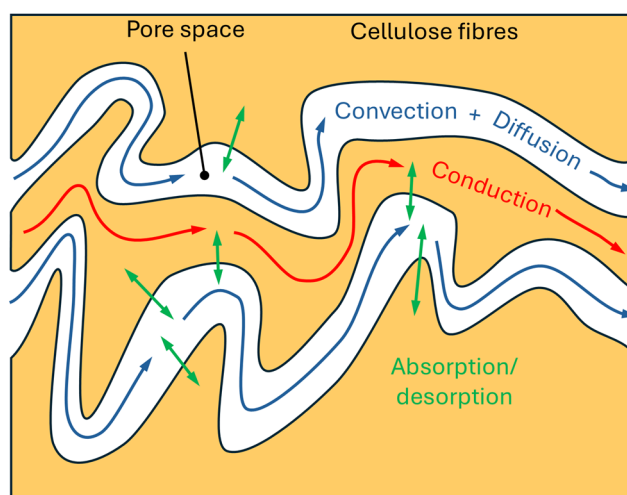


**Figure 2.** Heat and mass transport mechanisms through cellulose insulation.

The cellulose insulation usually has a cylindrical shape with radial dimensions much smaller than height. Additionally, the conditions on the outer and inner boundaries are assumed to be uniform along the perimeter and height, making it reasonable to assume that variations in physical fields in the axial and azimuthal directions can be neglected. In that case, the transport of heat and mass is considered only in radial direction, as shown in Figure 1.

Because the insulation's pores can be filled with dry air and water vapor, mass conservation for both components must be applied. The equation governing dry air is a convection–diffusion type:

$$\varepsilon_p \frac{\partial C_a}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rvC_a) + \frac{1}{r} \frac{\partial}{\partial r} \left( rD_{eq}C_g \frac{\partial y_v}{\partial r} \right) = 0, \quad (1)$$

whereas the water vapor conservation law has an additional source term due to the absorption or desorption of moisture from the cellulose fibres:

$$\varepsilon_p \frac{\partial C_v}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rvC_v) - \frac{1}{r} \frac{\partial}{\partial r} \left( rD_{eq}C_g \frac{\partial y_v}{\partial r} \right) + \frac{\rho_c}{M_v} \frac{\partial X}{\partial t} = 0. \quad (2)$$

In Equations (1) and (2),  $C_a$  [kmol/m<sup>3</sup>] and  $C_v$  [kmol/m<sup>3</sup>] are molar densities of air and water vapor, respectively. Both water vapor and dry air are assumed to behave like ideal gases, so following relations hold:

$$p = p_v + p_a = C_g R_m T, \quad (3)$$

$$C_g = C_v + C_a, \quad (4)$$

$$y_v = \frac{C_v}{C_g} = \frac{p_v}{p}, \quad (5)$$

where  $p$  is the total pressure of the gaseous phase, with  $p_v$  and  $p_a$  being partial pressures of water vapor and dry air, respectively.

The equivalent binary diffusivity  $D_{eq}$  in Equations (1) and (2) includes the Knudsen effect that could arise during vacuum phases and is given by the following relation [20]:

$$\frac{1}{D_{eq}} = \frac{1}{D_{ef}} + \frac{1}{D_K}, \quad (6)$$

where  $D_{ef}$  is an effective binary diffusivity of water vapor and dry air mixture, expressed as a function of absolute temperature and pressure [25]:

$$D_{ef} = \frac{\varepsilon_p}{\tau_p} 1.8947775 \cdot 10^{-5} \frac{T^{2.072}}{p}, \quad (7)$$

and  $D_K$  [m<sup>2</sup>/s] is the Knudsen diffusivity, which should be empirically determined during the model calibration [20].

The velocity of the gaseous phase is calculated according to Darcy's law:

$$v = -\frac{k_{ef}}{\mu_g} \frac{\partial p}{\partial r}, \quad (8)$$

where  $K_{ef}$  [m<sup>2</sup>] is an effective permeability coefficient of the cellulose insulation obtained with Klinkenberg correction to account for the slip effects at low pressures [26]:

$$k_{ef} = k_0 \left( 1 + \frac{b}{p} \right). \quad (9)$$

In Equation (9),  $k_0$  [m<sup>2</sup>] is an absolute (or sometimes called intrinsic) permeability that is measured at large pressures and  $b$  [Pa] is a Klinkenberg correction factor that increases with the decrease in absolute permeability  $k_0$ .

Moisture inside the cellulose fibres can be absorbed or desorbed depending on the equilibrium moisture content which it tends to reach. The mass conservation law for moisture contained in the cellulose fibres has the following form:

$$\frac{\partial X}{\partial t} = K(X_{eq} - X), \quad (10)$$

where  $K$  [1/s] is an internal overall mass transfer coefficient whose value depends on the resistance of the moisture transport from the inner parts of the cellulose fibres to the pore–fibre interface. It also strongly depends on the total pore–fibre interface surface area per unit of volume of the insulation. Accurately calculating  $K$  is not an easy task, so a simplified approach is undertaken in this work, assuming a constant value that should be empirically determined.

The dry-basis equilibrium moisture content  $X_{eq}$  in Equation (10) is a function of temperature and water activity of the surrounding media. An analytical form with temperature and water vapor partial pressure as independent variables was previously given by Fessler et al. [27] and later corrected by Du et al. [28]. This expression, although accurate for low moisture contents, might give non-physical results for water activities closer to unity. Przybylek et al. [29] calculated the parameters of the GAB (Guggenheim, Anderson, and de Boer) sorption isotherm model based on the experimental results for Kraft paper samples with different aging degrees. This model gives accurate results in a wider range of relative humidities (Figure 3) and was used in calculating the equilibrium moisture content in this study.

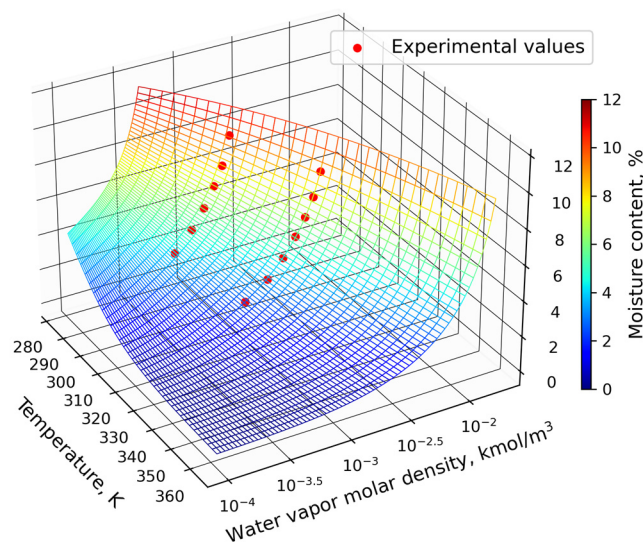


**Figure 3.** GAB model equilibrium moisture content of the new Kraft paper (wireframe) and experimental values of moisture content (dots) [29].

Since water activity was not chosen as a dependent variable of the model, it is expressed in terms of molar density and temperature. This follows from the definition of water activity and the assumption that dry air and water vapor behave like ideal gases.

$$a_w(C_v, T) = \frac{p_v}{p_{sat}(T)} = \frac{C_v R_m T}{p_{sat}(T)}. \quad (11)$$

In the above equation,  $p_{sat}(T)$  is the water vapor saturation pressure that can easily be calculated for a known temperature [30].

Equilibrium moisture content according to the GAB model is calculated from the following expressions:

$$X_{eq}(C_v, T) = \frac{X_m \cdot C(T) \cdot k \cdot a_w(C_v, T)}{[1 - k \cdot a_w(C_v, T)] \cdot [1 + k \cdot a_w(C_v, T) \cdot (C(T) - 1)]}, \quad (12)$$

$$C(T) = C_0 \cdot \exp \left[ \frac{Q}{R_m} \left( \frac{1}{T} - \frac{1}{T_{ref}} \right) \right]. \quad (13)$$

For a more detailed explanation of the GAB model and the physical meaning behind parameters  $X_m$ ,  $k$ ,  $C_0$ ,  $Q$ , and  $T_{ref}$ , one can refer to the work of Staudt et al. [31]. The values of the above parameters used for this model are taken from [29] and are given in Table A1 in Appendix A.

To close the system of the governing equations at the scale of the cellulose insulation, conservation of energy is applied. Unlike moisture concentration, a local thermal equilib-

rium between cellulose fibres and the gaseous phase within the pores is assumed. The effective specific heat of the insulation is calculated using the specific heat of solid cellulose fibres alone, since the contributions from humid air and moisture (due to its low content) are considered negligible. The resulting energy equation is

$$\varepsilon_f \rho_f c_f \frac{\partial T}{\partial t} - \lambda_{ef} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) - \Delta h_{evap} \varepsilon_f \rho_f \frac{\partial X}{\partial t} = 0, \quad (14)$$

where  $\lambda_{ef}$  [W/(m·K)] is the effective thermal conductivity of the cellulose insulation and  $\Delta h_{evap}$  [J/kg] is the specific heat of water evaporation.

### 2.3. Governing Equations at the Scale of the Vacuum Chamber

The governing equations at the scale of the vacuum chamber are derived under the assumption that temperature, pressure, and molar densities of water vapor and dry air are uniformly distributed fields within the available volume  $V$  (m<sup>3</sup>) inside the vacuum chamber. Moreover, the temperature fields of the chamber walls and IR heaters are also considered uniform and dependent only on the time variable.

The gaseous phase (water vapor and dry air) can enter the chamber through the venting valve and leave the chamber through the pressure relief valve or during the pump-down process. At the interface between the transformer cellulose insulation and vacuum chamber atmosphere, the molar flow rate of the gaseous phase can be directed either inward or outward. The conservation laws are thus expressed accordingly:

$$V \frac{dC_{v,ch}}{dt} = \dot{n}_{vent,v} - \dot{n}_{relief,v} - \dot{n}_{pump,v} + \dot{n}_{t,v}, \quad (15)$$

$$V \frac{dC_{a,ch}}{dt} = \dot{n}_{vent,a} - \dot{n}_{relief,a} - \dot{n}_{pump,a} + \dot{n}_{t,a}, \quad (16)$$

After expanding and rearranging the terms, the following equations are obtained:

$$\frac{dC_{v,ch}}{dt} = \frac{C_{v,amb} Q_{vent} - C_{v,ch} Q_{relief} - C_{v,ch} Q_{pump} + A_t \sum_{i=1}^{N_t} J_{v,t,i}}{V}, \quad (17)$$

$$\frac{dC_{a,ch}}{dt} = \frac{C_{a,amb} Q_{vent} - C_{a,ch} Q_{relief} - C_{a,ch} Q_{pump} + A_t \sum_{i=1}^{N_t} J_{a,t,i}}{V}. \quad (18)$$

In the above equations,  $Q_{vent}$  and  $Q_{relief}$  are volume flow rates through venting valve and pressure relief valve, respectively. Since a single valve is used for both venting and pressure relief, volume flow rates  $Q_{vent}$  and  $Q_{relief}$  are mutually excluding (only one at the time can have non-zero value). In view of that,  $Q_{vent}$  and  $Q_{relief}$  are calculated as follows:

$$Q_{vent} = \begin{cases} K_v \sqrt{p_{amb} - p_{ch}}, & p_{amb} > p_{ch} \\ 0, & p_{amb} \leq p_{ch} \end{cases}, \quad (19)$$

$$Q_{relief} = \begin{cases} K_v \sqrt{p_{ch} - p_{amb}}, & p_{ch} > p_{amb} \\ 0, & p_{ch} \leq p_{amb} \end{cases}, \quad (20)$$

where  $p_{amb}$  [Pa] and  $p_{ch}$  [Pa] are ambient and chamber pressures, respectively. During the venting process, a choked flow might occur; however, since venting is relatively rapid part of the process in comparison to other phases, the choked flow phenomenon is not considered in this work.

The pumping speed at the outlet port of the vacuum chamber (effective pumping speed),  $Q_{pump}$  [m<sup>3</sup>/s], is generally different from the pumping speed at the inlet of the vacuum pump. The inlet pumping speed  $Q_{pump,in}$  can be found in the manufacturer's datasheet as a function of the pressure at the vacuum pump inlet  $p_{in}$ . To calculate the effective pumping speed, the throughput continuity equation under the assumption of isothermal flow in the suction line is used:

$$Q_{pump} = \frac{p_{in} \cdot Q_{pump,in}(p_{in})}{p_{ch}}. \quad (21)$$

To obtain the pressure at the inlet of the vacuum pump, the effective pumping speed from Equation (21) is substituted into the equation for the conductance of the suction line:

$$C_s = \frac{p_{in} \cdot Q_{pump,in}(p_{in})}{p_{ch} - p_{in}}. \quad (22)$$

For a given conductance  $C_s$  [m<sup>3</sup>/s] and the pressure inside the vacuum chamber  $p_{ch}$ , Equation (22) is solved for the pressure at the inlet  $p_{in}$ . This pressure is then substituted into Equation (21) to obtain the effective pumping speed  $Q_{pump}$ , which is then used in the governing equations. More details regarding the calculation of the effective pumping speed can be found in [32].

Energy conservation laws for dry air and water vapor can be incorporated into a single equation, since both species are at the same temperature  $T_{ch}$ . Accumulation of the internal energy of the open system is due to the convective heat flow rate  $\dot{Q}$  and the enthalpy flows through the boundary:

$$V \frac{d[(C_{m,v,v}C_{v,ch} + C_{m,v,a}C_{a,ch})T_{ch}]}{dt} = \dot{Q} + \dot{H}_{in} - \dot{H}_{out}. \quad (23)$$

The enthalpy difference can be written as

$$\begin{aligned} \dot{H}_{in} - \dot{H}_{out} = & Q_{vent}C_{g,amb}C_{m,p,g}T_{amb} - (Q_{relief} + Q_{pump})C_{g,ch}C_{m,p,g}T_{ch} \\ & + A_t \sum_{i=1}^{N_t} J_{v,t,i}C_{m,p,v}T_{s,i} + A_t \sum_{i=1}^{N_t} J_{a,t,i}C_{m,p,a}T_{s,i}, \end{aligned} \quad (24)$$

where  $T_{s,i}$  is cellulose insulation surface temperature of the  $i$ -th transformer.

Atmosphere in the chamber can convectively exchange heat with chamber walls, IR heaters, and cellulose insulation of the transformers. The term  $\dot{Q}$  in Equation (23) can therefore be written as

$$\dot{Q} = A_t \sum_{i=1}^{N_t} h_{t,i}(T_{s,i} - T_{ch}) + A_h h_h(T_h - T_{ch}) + A_w h_w(T_w - T_{ch}), \quad (25)$$

where  $h_{t,i}$ ,  $h_h$ , and  $h_w$  are convective heat transfer coefficients at the interface between chamber atmosphere and other components: transformer cellulose insulation, IR heaters, and chamber walls.

After combining Equations (23)–(25) and rearranging, a final form of the energy equation is obtained:

$$\begin{aligned}
\frac{dT_{ch}}{dt} = & \frac{A_t \sum_{i=1}^{N_t} h_{t,i} (T_{s,i} - T_{ch}) + A_h h_h (T_h - T_{ch}) + A_w h_w (T_w - T_{ch})}{V (C_{m,v,v} C_{v,ch} + C_{m,v,a} C_{a,ch})} \\
& + \frac{Q_{vent} C_{g,amb}}{V (C_{v,ch} + C_{a,ch})} (T_{amb} \gamma_g - T_{ch}) \\
& + \frac{Q_{pump} + Q_{relief}}{V} T_{ch} (1 - \gamma_g) \\
& + \frac{C_{m,v,v} A_t \sum_{i=1}^{N_t} J_{v,t,i} (T_{s,i} \gamma_v - T_{ch})}{V (C_{m,v,v} C_{v,ch} + C_{m,v,a} C_{a,ch})} \\
& + \frac{C_{m,v,a} A_t \sum_{i=1}^{N_t} J_{a,t,i} (T_{s,i} \gamma_a - T_{ch})}{(C_{m,v,v} C_{v,ch} + C_{m,v,a} C_{a,ch})}.
\end{aligned} \quad (26)$$

where  $C_{g,amb}$  [kmol/m<sup>3</sup>] is molar density of the ambient air, and  $\gamma_g$  is specific heat ratio of the gaseous phase inside the vacuum chamber.

An energy equation for the IR heaters, derived in a similar fashion, reads

$$\frac{dT_h}{dt} = \frac{1}{m_h c_h} (A_h h_h (T_{ch} - T_h) + \dot{Q}_{rad,h} + \dot{Q}_{el}), \quad (27)$$

where  $\dot{Q}_{rad,h}$  [W] and  $\dot{Q}_{el}$  [W] are the radiative heat flow rate from the heater surface and input electrical power, respectively.

A vacuum chamber wall energy equation reads

$$\frac{dT_w}{dt} = \frac{1}{m_w c_w} \left( A_w h_w (T_{ch} - T_w) + A_w \frac{\lambda_{ins}}{\delta_{ins}} (T_{amb} - T_w) + \dot{Q}_{rad,w} \right), \quad (28)$$

where  $\dot{Q}_{rad,w}$  is radiative heat flow rate at the wall surface,  $\lambda_{ins}$  is the thermal conductivity of the vacuum chamber thermal insulation, and  $\delta_{ins}$  is the thermal insulation thickness.

### 2.3.1. Convective Heat Transfer Coefficients Calculation

It would be incorrect to assume that convective heat transfer coefficients remain constant during the vacuum drying process, since the rarified atmosphere at low pressures causes them to diminish. To calculate the heat transfer coefficients at low pressures, a model from Gonzales et al. [33], which provides the relation between Nusselt and Rayleigh numbers for natural convection along a vertical plate, is used:

$$Nu = 0.737 \left[ Ra^{1/4} + 5.725 \cdot Ra^{0.019} \right], \quad (29)$$

where  $Nu$  and  $Ra$  are dimensionless quantities defined as

$$Nu = \frac{h_j H_j}{\lambda_g}, \quad (30)$$

$$Ra = Pr \cdot Gr = \frac{\mu_g c_{p,g}}{\lambda_g} (\rho_g - \rho_{g,j}) \rho_{g,j} \frac{g H_j^3}{\mu_{g,j}^2}. \quad (31)$$

where  $H_j$  is a height of the  $j$ -th surface. Every surface is considered to behave like a vertical plate, even the transformer samples (vertical cylinders) since their height is much larger than their outer diameter. The properties with  $j$  in the index are evaluated for the temperature of the  $j$ -th surface.

Equation (29) is valid for Rayleigh numbers in the range from  $10^{-2}$  to  $10^5$ , a condition that is satisfied throughout this type of drying process.

### 2.3.2. Radiative Heat Flow Rates Calculation

Under the assumption that all surfaces in the system are grey and diffuse, the balance of radiative heat flow rates can be expressed as [34]

$$\sum_{j=1}^{N_{surf}} \left[ \left( \frac{\delta_{kj}}{\varepsilon_j} - F_{k-j} \frac{1 - \varepsilon_j}{\varepsilon_j} \right) \frac{\dot{Q}_{rad,j}}{A_j} \right] = \sum_{j=1}^{N_{surf}} \left[ F_{k-j} \sigma (T_k^4 - T_j^4) \right], \quad k = 1, 2, \dots, N_{surf} \quad (32)$$

where  $N_{surf}$  is the number of surfaces that exchange heat by radiation,  $\varepsilon_j$  is the total emissivity of the  $j$ -th surface,  $F_{k-j}$  is the view factor from surface  $k$  to surface  $j$ ,  $A_j$  is the surface area of the  $j$ -th surface, and  $\delta_{kj}$  is the Kronecker delta operator that is equal to unity when indices  $k$  and  $j$  are the same and zero otherwise.

Equation (32), written down for all the surfaces, results in a linear system of equations, which is then solved for radiative heat flow vector  $\dot{Q}_{rad,j}$ .

### 2.4. The Interface Between Transformer Cellulose Insulation and Vacuum Chamber Atmosphere

To close the system of governing equations, the interface between transformer cellulose insulation and the chamber atmosphere must be modelled.

It is assumed that the partial pressures of water vapor and dry air at the interface are equal to those in the vacuum chamber atmosphere. Therefore, for all transformer samples, the following equations hold:

$$p_{v,i}|_{r=r_{out}} = p_{v,ch}, \quad (33)$$

$$p_{a,i}|_{r=r_{out}} = p_{a,ch}, \quad (34)$$

However, since every transformer sample can have a different temperature field, the following Dirichlet boundary conditions are used for the  $i$ -th sample:

$$C_{v,i}|_{r=r_{out}} = \frac{C_{v,ch} R_m T_{ch}}{R_m T_i|_{r=r_{out}}}, \quad (35)$$

$$C_{a,i}|_{r=r_{out}} = \frac{C_{a,ch} R_m T_{ch}}{R_m T_i|_{r=r_{out}}}. \quad (36)$$

From Equations (33) and (34), it follows that the total pressure at the interface is equal to the pressure in the vacuum chamber:

$$p_i|_{r=r_{out}} = p_{ch}, \quad (37)$$

On the inner radius, zero molar flux boundary conditions are imposed:

$$\left( v_i C_{v,i} - C_{g,i} D_{ef,i} \frac{dy_{v,i}}{dr} \right) \Big|_{r=r_{in}} = 0, \quad (38)$$

$$\left( v_i C_{a,i} + C_{g,i} D_{ef,i} \frac{dy_{v,i}}{dr} \right) \Big|_{r=r_{in}} = 0, \quad (39)$$

The boundary condition for the energy equation (14) is a Robin type and includes both convective and radiative heat fluxes:

$$-\lambda_{ef} \frac{\partial T_i}{\partial r} \Big|_{r=r_{out}} = h_{t,i} (T_i|_{r=r_{out}} - T_{ch}) - \frac{\dot{Q}_{rad,t,i}}{A_t}, \quad (40)$$

where  $\dot{Q}_{rad,t,i}$  is a radiative heat flow rate at the  $i$ -th transformer surface, which can either be transferred from the surface (negative) or onto the surface (positive).

On the inner radius, zero heat flux is imposed:

$$-\lambda_{ef} \frac{\partial T_i}{\partial r} \bigg|_{r=r_{in}} = 0, \quad (41)$$

For given fields of molar densities and temperature, molar fluxes of water vapor and air at the interface between cellulose insulation and vacuum chamber atmosphere can be calculated:

$$J_{v,t,i} = \left( v_i C_{v,i} - C_{g,i} D_{ef,i} \frac{dy_{v,i}}{dr} \right) \bigg|_{r=r_{out}}, \quad (42)$$

$$J_{a,t,i} = \left( v_i C_{a,i} + C_{g,i} D_{ef,i} \frac{dy_{v,i}}{dr} \right) \bigg|_{r=r_{out}}. \quad (43)$$

These fluxes are then substituted into the governing Equations (17), (18), and (26).

## 2.5. Initial Conditions

Initially, all components of the vacuum drying system as well as the cellulose insulation are assumed to be at ambient temperature. The gaseous phase inside the pores of the cellulose insulation and the atmosphere of the vacuum chamber are under ambient pressure and ambient relative humidity. In that case, the initial moisture content of the cellulose insulation corresponds to the equilibrium moisture content for a given state of ambient air. The initial conditions are as follows:

$$T_0 = 293.15 \text{ K}, \quad (44)$$

$$p_0 = 100000 \text{ Pa}, \quad (45)$$

$$a_{w,0} = 0.6, \quad (46)$$

$$C_{v,0} = \frac{a_{w,0} p_{sat}(T_0)}{R_m T_0} \approx 5.7566 \cdot 10^{-4} \frac{\text{kmol}}{\text{m}^3} \quad (47)$$

$$C_{a,0} = \frac{p_0}{R_m T_0} - C_{v,0} \approx 4.045223 \cdot 10^{-2} \frac{\text{kmol}}{\text{m}^3}, \quad (48)$$

$$X_0 = X_{eq}(C_{v,0}, T_0) \approx 0.081465 = 8.1465 \% \quad (49)$$

## 2.6. Numerical Solution Procedure

The governing Equations (1), (2), (10) and (14) at the scale of cellulose insulation and governing Equations (17), (18) and (26)–(28) at the scale of vacuum chamber, along with all the boundary conditions and constitutive relations, form a nonlinear coupled PDE-ODE system.

One way to decouple the system and find the solution would be to solve the ODE system with guessed values of cellulose insulation fields. The ODE solutions can then be used to define necessary boundary conditions for which the PDE system is solved and the better guess for cellulose insulation fields is obtained. This is the iterative procedure that is repeated until convergence is achieved. In this study, however, our need to solve a coupled PDE-ODE system was addressed using an alternative approach.

Firstly, all equations were scaled so that the variables of the model had an order of magnitude around unity or less. Then, spatial derivatives in governing Equations (1), (2) and (14) and in interface Equations (38)–(43) were discretized using a second-order finite difference scheme. This leads to a large system of ODEs, which is then solved using the LSODA method implemented in the SciPy package for the Python programming language [35]. This method automatically switches between Adams (non-stiff part of the solution) and implicit BDF (stiff part of the solution) methods. Relative and absolute

tolerances were set to  $10^{-5}$  and  $10^{-7}$ , respectively. A similar methodology for solving PDE-ODE systems was investigated by Filipov et al. [36] so readers interested in further details are referred to their work.

### 3. Results and Discussion

Numerical simulations for all studied cases, including the test case, drying case, and comparative scenarios, were carried out using the physical properties for non-impregnated Kraft paper, reflecting its role as a standard transformer cellulose-based insulation material. All parameter values and functional relations are listed in Tables A1–A4 given in Appendix A. The exceptions are two parameters: the absolute permeability  $k_0$ , and the internal overall mass transfer coefficient  $K$ , for which a series of values were used to study their influence on moisture dynamics. Additionally, for the purpose of comparison with Fickian diffusion models, certain parameters were adjusted to ensure a meaningful evaluation.

The vacuum dryer dimensions correspond to those of a laboratory-scale vacuum chamber used at our institution. Likewise, the choice of vacuum pump and its characteristic pumping speed correspond to a commercially available unit (*Leybold VD 65*) currently in use.

In addition to specifying parameter values and their interrelations, solving the governing equations requires prior computation of the view factors. This was carried out firstly by generating a rudimentary geometry of the surfaces that exchange heat by radiation, namely, the IR heaters, transformer samples, and vacuum chamber walls, as shown in Figure 4. The generated mesh is a prerequisite for the computation of view factors; however, it is not necessary to calculate them for all cells since reciprocity and summation rules can be applied [34].

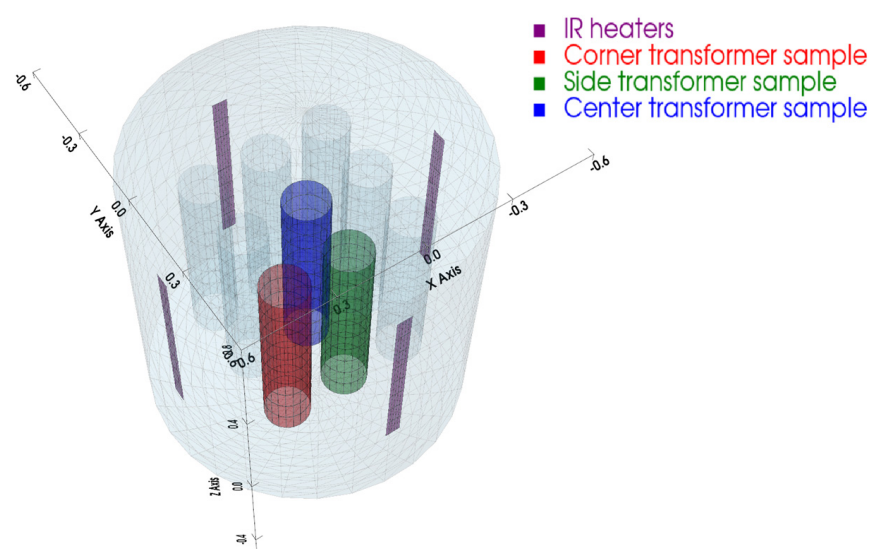


**Figure 4.** Rudimentary geometry of the IR heaters, transformer samples, and vacuum chamber generated with PyVista [37] package (used in all subsequent simulations).

Furthermore, due to the spatial layout of the transformer samples, three representative ones—corner, side, and center transformer samples—can be identified out of the total of nine (as highlighted in Figure 4). Due to the unique radiative exposure of these samples, limiting view factor calculations to just these three significantly reduces computational effort.

The calculation of view factors was performed with pyviewfactor [38], a Python package that utilizes PyVista [37] functionalities, such as ray tracing, which was used for checking the shadowing of the cells. The calculated view factors subsequently used in Equation (32) are listed in Table 1.

**Table 1.** Calculated values of the view factors. Rows refer to the value of the view factor from the surface marked in the index towards a surface marked on the top of the corresponding column.

| View Factor/<br>Surface | IR Heaters | Vacuum<br>Chamber<br>Walls | Corner<br>Transformer<br>Sample | Side<br>Transformer<br>Sample | Center<br>Transformer<br>Sample |
|-------------------------|------------|----------------------------|---------------------------------|-------------------------------|---------------------------------|
| $F_{\text{heaters}-i}$  | 0.0000     | 0.4102                     | 0.2604                          | 0.3295                        | 0.0000                          |
| $F_{\text{walls}-i}$    | 0.0098     | 0.7060                     | 0.1533                          | 0.1125                        | 0.0183                          |
| $F_{\text{corner t}-i}$ | 0.0271     | 0.6667                     | 0.0000                          | 0.2406                        | 0.0657                          |
| $F_{\text{side t}-i}$   | 0.0343     | 0.4892                     | 0.2406                          | 0.1313                        | 0.1046                          |
| $F_{\text{center t}-i}$ | 0.0000     | 0.3189                     | 0.2626                          | 0.4185                        | 0.0000                          |

The drying process described by this model can be controlled by on/off switching of three main components:

- IR heaters (when off,  $\dot{Q}_{el} = 0$  W);
- Vacuum pump (when off,  $C_s = 0$  m<sup>3</sup>/s);
- Venting/pressure relief valve (when off,  $K_v = 0$ ).

Depending on the component type and timing of the switching, the process can discretely be controlled in numerous ways.

### 3.1. Test Case Results

To facilitate the analysis of the results in terms of spatio-temporal physical fields, an initial short-duration test case was simulated. The main component states and their corresponding durations are listed in Table 2 and as such provide enough information to define the test case process. The states of the components are defined in such a way that all the commonly encountered conditions, such as IR heating, vacuum pump-down, and venting, are encompassed in this numerical simulation. The test case therefore consists of a 2.5 h IR heating period with the pressure relief valve open to the atmosphere, followed by 2.5 h of vacuum pump-down with heaters turned off, ending with a 2.5 h venting period during which the heaters remain off.

**Table 2.** Operational states of the main components and their durations governing the simulated test case.

| Order | Component Status |             |                                   | Duration |
|-------|------------------|-------------|-----------------------------------|----------|
|       | IR Heaters       | Vacuum Pump | Venting/<br>Pressure Relief Valve |          |
| 1     | on               | off         | on                                | 2.5 h    |
| 2     | off              | on          | off                               | 2.5 h    |
| 3     | off              | off         | on                                | 2.5 h    |

As heaters are activated, the temperature of cellulose insulation increases. This reduces the equilibrium moisture content of the cellulose fibres according to Equation (12). Since cellulose fibres tend to reach a lower moisture content state, desorption of moisture occurs, consequently making the pores of the cellulose insulation more saturated with water vapor, as can be seen in Figure 5a. The developed gradient of the water vapor molar fraction acts as a driving force for equimolar diffusion of dry air and water vapor within the pores. The diffusive flux during the heat-up period, however, is not unidirectional; it changes direction at a certain radius, which is the consequence of water vapor molar ratio maxima at that point. This is not surprising; since heat is transferred to the outer surface only, the molar density of water vapor is highest near the surface and lower at the very interface. A similar pattern can be observed in the pressure field (Figure 6b), where an under-pressure region forms near the internal surface. This behaviour, characteristic of convective porous media drying [39], could result from the combined effect of dry air diffusing outwards and absorption of water vapor that diffuses inwards, toward the parts that are still unheated.

The temporal variations in physical fields for corner and side transformer samples are nearly identical, which is expected given the relatively similar values of the view factors from IR heaters to these samples. In contrast, the center transformer sample is completely obstructed by the surrounding samples, so the radiative flux reaching its surface originates from neighbouring samples and not directly from the heaters. This manifests as a slower increase in temperature (Figure 5c) and slower decrease in moisture content (Figure 5d). Moreover, the moisture content of the center transformer even shows a slight increase during the heating period near the outer surface, which concerns the variation in molar density in the vacuum chamber atmosphere.

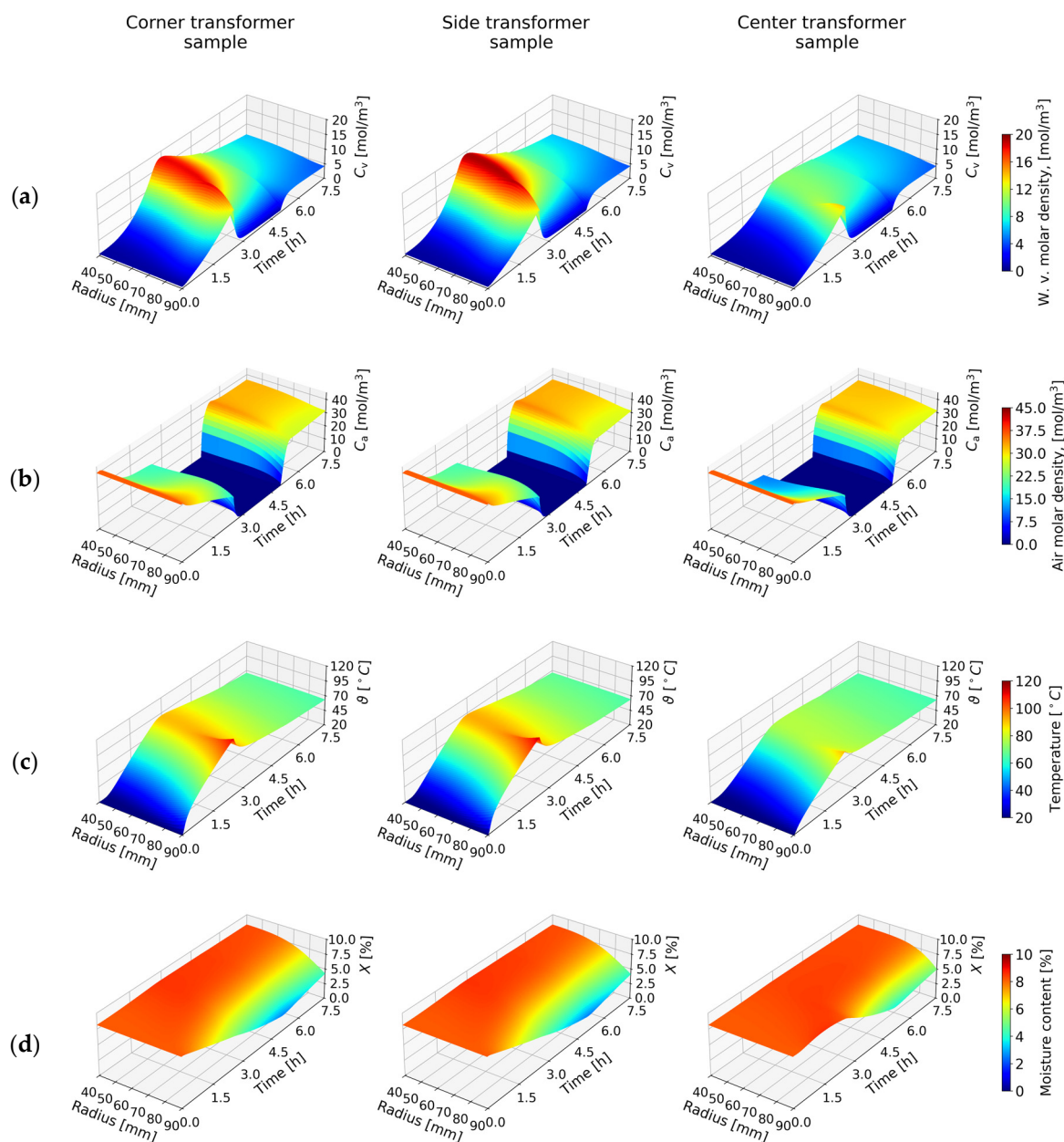


**Figure 5.** Temporal variations in physical fields for each characteristic transformer sample ( $k_0 = 10^{-16} \text{ m}^2$ ,  $K = 10^{-4} \text{ 1/s}$ ): (a) water vapor molar density, (b) dry air molar density, (c) temperature, and (d) moisture content.

Water vapor is convectively and diffusively transferred at the surface of corner and side transformer samples to the vacuum chamber atmosphere according to the interface (Equation (42)). Furthermore, both dry air and water vapor exit the chamber through

the venting/over-pressure valve due to the rising pressure. This results in an increase in water vapor and a decrease in dry air molar density, as shown in Figure 7a. The elevated water vapor molar fraction and lower surface temperature of the center transformer sample correspond to a higher equilibrium moisture content, causing a slight absorption of water vapor at the surface of the center transformer sample.

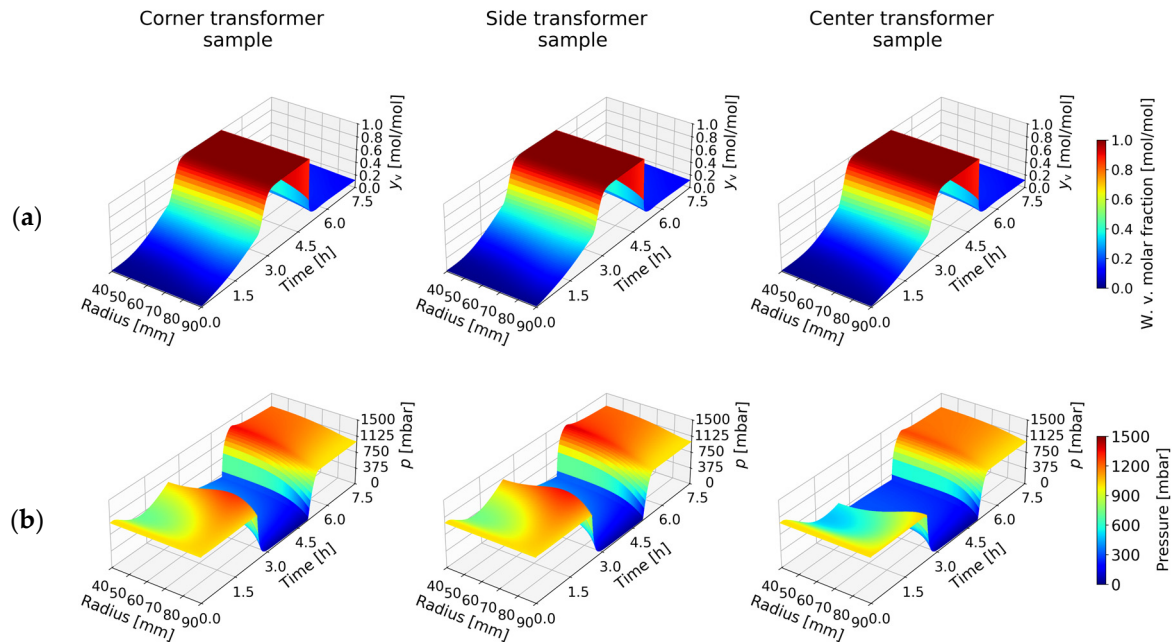


**Figure 6.** Temporal variations in derived physical fields for each characteristic transformer sample ( $k_0 = 10^{-16} \text{ m}^2$ ,  $K = 10^{-4} \text{ 1/s}$ ): (a) water vapor molar fraction, (b) absolute pressure.

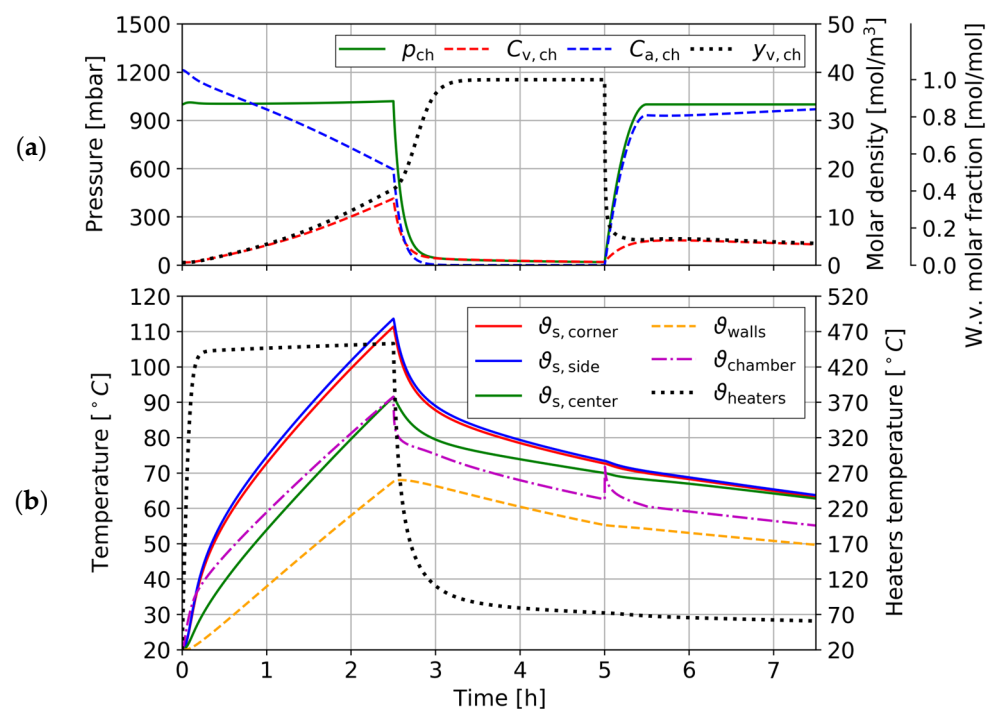


**Figure 7.** Time variations in the variables at the scale of the vacuum chamber ( $k_0 = 10^{-16} \text{ m}^2$ ,  $K = 10^{-3} \text{ 1/s}$ ): (a) molar densities, pressure, and water vapor molar fraction of the vacuum chamber atmosphere, (b) surface temperatures of the characteristic transformer samples, vacuum chamber walls' temperature, and IR heaters' temperature (different scale on the right side).

As the vacuum pump-down period starts, the dry air, both within insulation pores and the vacuum chamber, is rapidly evacuated. Simultaneously, the molar fraction of water vapor increases to unity, a behaviour consistent with previous findings in [40]. In contrast, the water vapor molar density remains non-zero due to continued desorption from the cellulose fibres (Figure 5a). As a result, a pressure gradient develops along the thickness of the cellulose insulation (Figure 6b), which gives rise to more intensive convective transport and consequently to an increase in the drying rate. The diffusive flux in this period is zero, since the water vapor molar fraction is uniform and equal to unity (Figure 6a).

During the vacuum chamber venting period, dry air diffuses into the cellulose insulation, leading to an increase in its molar density. The elevated dry air concentration combined with a still relatively high temperature causes desorption of moisture in the inner layers and consequently an increase in pressure shortly after opening the venting valve.

Figure 7b shows temporal variations in surface temperatures inside the vacuum chamber. Among the transformer sample surfaces, the side transformer sample surface exhibits the most rapid increase in temperature, which is attributed to the highest view factor from the heaters (Table 1). A similar trend is observed for the corner sample surface, though its temperature is slightly lower due to a lower value of the view factor. On the other hand, a center transformer sample surface shows a significantly slower increase in temperature and additionally maintains lower temperatures throughout the entire heating period. This behaviour is clearly a result of the surrounding samples obstructing the center one from direct exposure to the heaters.

These results suggest that the surface temperature of the sample most exposed to the IR heaters (in this case, the side transformer sample) is a suitable candidate for use as a control variable in terms of temperature control. The choice of the highest temperature is due to the upper temperature limit imposed to prevent rapid degradation of cellulose insulation.

Initially, the vacuum chamber atmosphere heats up primarily due to convective heat transfer from the surfaces within the chamber. A sudden drop is observed when the pump-down period starts, which can be attributed to the expansion of the gaseous phase. The reverse effect is seen during venting, when ambient air rushes in and compresses the gaseous phase, sharply increasing temperature. These temperature dynamics are in accordance with those reported in [39] during charging and discharging processes.

Lastly, the vacuum chamber walls exhibit the slowest temperature increase due to their relatively high heat capacity.

### 3.2. Impact of the $k_0$ and $K$ Parameters

The parameters whose impact is investigated are the absolute permeability  $k_0$  and the internal overall mass transfer coefficient  $K$ . All other parameters are either held constant or can be calculated from the relations, both of which are provided in Table A1 in Appendix A. Moreover, the states of the main components for all simulations are the same as in the test case (Table 2). Lastly, given the small difference in physical fields between the corner and side transformer samples, only the center and side samples are observed in this analysis.

To evaluate the influence of the above-mentioned parameters, the average moisture content of the cellulose insulation was computed using the following equation:

$$X_{avg}(t) = \frac{1}{r_{out} - r_{in}} \int_{r_{in}}^{r_{out}} X(r, t) dr. \quad (50)$$

Figure 8a illustrates the influence of the internal overall mass transfer coefficient  $K$  on the evolution of the average moisture content during the test case process. As  $K$  increases, the characteristic time for equilibration decreases, causing local absorption and desorption

processes to become significantly faster than diffusive and convective transport. In that case, a local equilibrium can be assumed to occur instantly, allowing the moisture content field to be computed directly using Equation (12) for given fields of water vapor molar density and temperature. This convergence of drying curves with an increase in  $K$  was also reported in [41]. In contrast, lower values of  $K$  lead to more inert moisture dynamics, characterized by reduced drying rates.

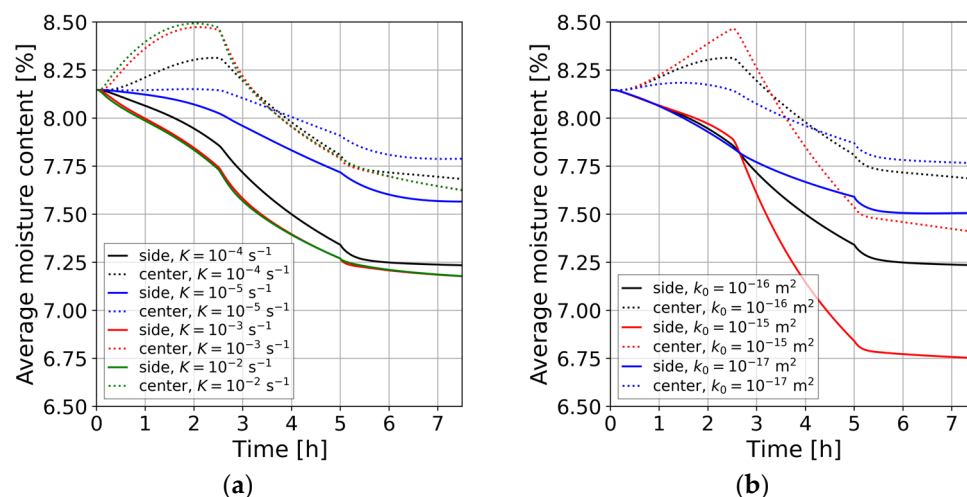


**Figure 8.** Time variations in the average moisture content of the cellulose insulation for the test case process: (a) influence of the overall internal mass transfer coefficient  $K$  (with  $k_0 = 10^{-16} \text{ m}^2$ ) and (b) influence of the absolute permeability  $k_0$  (with  $K = 10^{-4} \text{ 1/s}$ ).

The influence of absolute permeability  $k_0$  is illustrated in Figure 8b. As shown, higher permeability results in an increased drying rate during the vacuum period, which is a direct consequence of enhanced convective transport, consistent with Darcy's law (Equation (8)). Conversely, a lower value of  $k_0$  results in a lower drying rate in the vacuum period. It is also observed that the value of  $k_0$  has a negligible effect on the drying rate during the heating period, at least while pressure gradients remain low. However, as pressure gradients develop, convective transport becomes more prominent, thereby increasing the sensitivity of the results to the  $k_0$  value.

Although both  $k_0$  and  $K$  significantly influence the drying rate, their values are not easily determined through theoretical means, as they strongly depend on the insulation's microstructure. The most time-efficient approach would therefore be to estimate these parameters through model calibration, by comparing the simulation results with experimental data.

The Knudsen diffusivity parameter  $D_K$  was also examined as a part of this analysis and it was found to have no significant influence on the drying rate. This observation is consistent with the model formulation. Specifically, Equation (6) indicates that the contribution of  $D_K$  becomes relevant at low pressures. However, under vacuum conditions, the molar fraction of water vapor approaches unity and becomes uniform across the domain, eliminating concentration gradients and, consequently, diffusive transport.

### 3.3. Drying Case Results

To demonstrate the influence of the process control strategy on the moisture transport, the drying case was defined to resemble a process with conditions typically met in the transformer manufacturing procedure, consisting of a sequence of alternating heating and vacuum phases [42,43]. To maintain the specified temperature and pressure levels throughout the process, temperature and pressure two-point (on/off) control was implemented

in the simulation. The operational states of the main components and their respective durations are summarized in Table 3.

**Table 3.** Operational states of the main components and their durations governing the simulated drying case.

| Order | Controlled Variable | Component Status |             |                                      | Duration <sup>1</sup> |
|-------|---------------------|------------------|-------------|--------------------------------------|-----------------------|
|       |                     | IR Heaters       | Vacuum Pump | Venting/<br>Pressure<br>Relief Valve |                       |
| 1     | $130 \pm 1$ °C      | on/off           | off         | on                                   | 10 h                  |
| 2     | $25 \pm 0.5$ kPa    | off              | on/off      | off                                  | 2 h                   |
| 3     | $130 \pm 1$ °C      | on/off           | off         | on                                   | 10 h                  |
| 4     | $15 \pm 0.5$ kPa    | off              | on/off      | off                                  | 2 h                   |
| 5     | $130 \pm 1$ °C      | on/off           | off         | on                                   | 10 h                  |
| 6     | $10 \pm 0.5$ kPa    | off              | on/off      | off                                  | 2 h                   |
| 7     | $130 \pm 1$ °C      | on/off           | off         | on                                   | 10 h                  |
| 8     | $5 \pm 0.5$ kPa     | off              | on/off      | off                                  | 2 h                   |
| 9     | $130 \pm 1$ °C      | on/off           | off         | on                                   | 10 h                  |
| 10    | min. pressure       | off              | on          | off                                  | 24 h                  |

<sup>1</sup> transient period durations are not included.

To ensure that cellulose insulation did not exceed the upper temperature limit of 130 °C, the controlled temperature was chosen to be the surface temperature of the side transformer sample, as explained in the previous section. Figure 8 presents the temporal variations in the surface temperatures and the innermost-layer temperatures of the corresponding transformer samples. The same figure also displays the variation in pressure of the chamber atmosphere over time. Significant temperature gradients across the thickness of the cellulose insulation are evident. Their negative signature is characteristic of one-sided heating and is unfavourable in terms of drying rate [12].

Furthermore, the obstructed center sample remains at a somewhat lower temperature throughout the entire drying process, a condition that is expected to extend the overall drying time. The observed decrease in temperature of the transformer samples during the vacuum phases results from radiative cooling due to the turned-off IR heaters as well as cooling due to the water vapor evaporation. This is evident from Figure 9, where each successive vacuum phase is characterized by a smaller drop in surface temperatures, indicating a decline in evaporative cooling as the drying rate diminishes over time.

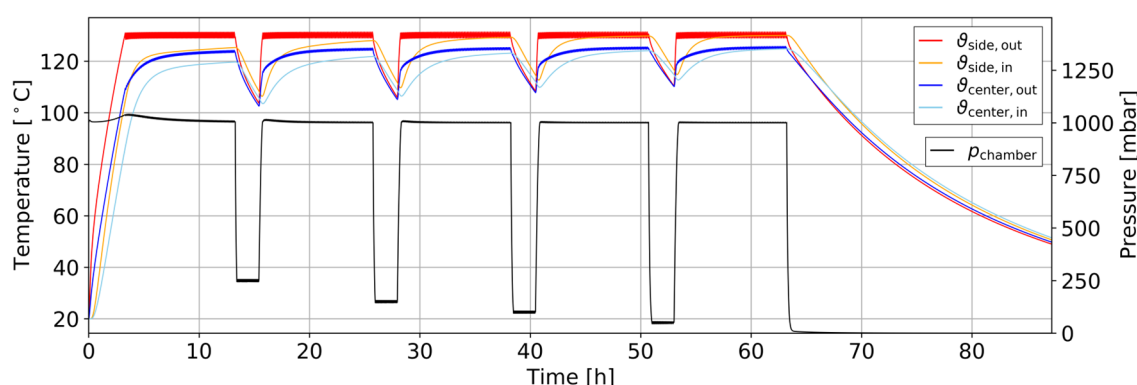


**Figure 9.** Time variations in the side and center transformer sample surface and innermost layer temperatures, along with the variation in pressure of the vacuum chamber atmosphere ( $k_0 = 10^{-16}$  m<sup>2</sup> and  $K = 10^{-4}$  1/s).

Convective and radiative heat fluxes at the surface of the side and center transformer samples are illustrated in Figure 10. The results demonstrate that convective heat flux diminishes during the vacuum phases, consistent with the reduction in the convective heat transfer coefficient at low Rayleigh numbers, as described by Equation (29) and reported

in [44]. For all transformer samples, convective heat flux remains generally negative due to the lower temperature of the vacuum chamber atmosphere relative to the sample surfaces. Peaks in convective heat flux appearing at the onset of each vacuum phase result from the rapid cooling of the vacuum chamber atmosphere caused by its expansion during pump-down.

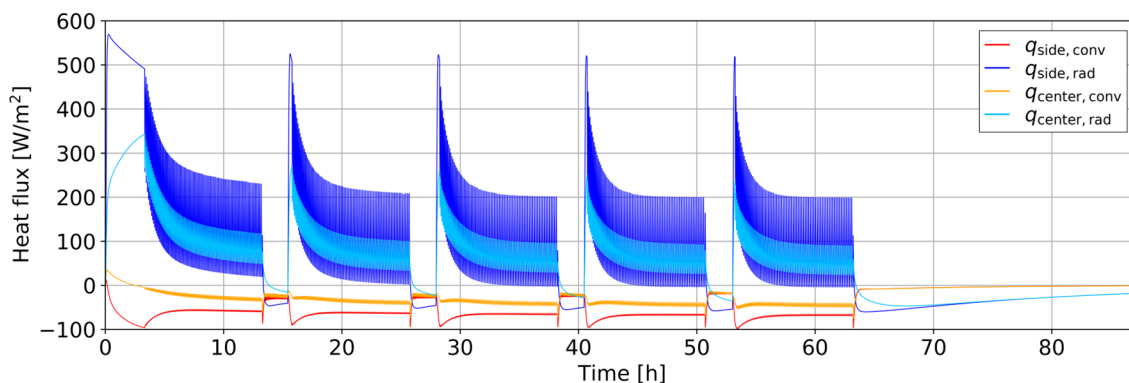


**Figure 10.** Time variations in the convective and radiative heat fluxes at the surface of the side and center transformer samples ( $k_0 = 10^{-16} \text{ m}^2$  and  $K = 10^{-4} \text{ 1/s}$ ).

On the other hand, radiative heat flux exhibits periodic oscillations originating from the two-point temperature control achieved by changing the operating states of the heaters and thus the emitted radiative flux. The magnitude of these oscillations reflects variations in the temperature difference between the radiating surfaces. The side transformer sample, being directly exposed to IR heaters whose temperature responds rapidly and over a wide range to switching, experiences pronounced radiative heat flux oscillations. In comparison, the center transformer sample exchanges radiative heat only with surrounding samples of similar temperature, resulting in much smaller variations.

Figure 11a illustrates a molar flux at the surface of the side and center transformers throughout the drying process. Initially, diffusion dominates, with diffusive flux reaching a peak before gradually decreasing. This behaviour results from the rising water vapor molar fraction in the chamber, which eventually causes the diffusive flux to vanish entirely. Shortly after the process begins, as insulation continues to heat up and water desorbs from the cellulose fibres into the pore space, pressure gradients develop across the insulation thickness, initiating convective molar transport. The convective flux reaches its peak after the diffusive flux and then declines as the pressure field becomes more uniform.

To extract water vapor from the pore space of the insulation, at some point, the chamber pressure must be reduced to regain an increase in convective molar flux. As these gradients again start to diminish, the chamber is vented and temperature control is reactivated. At the beginning of the next heating phase, the molar fraction of the water vapor is no longer uniformly distributed since ambient air is introduced, which then results in a subsequent increase in diffusive flux.

Periods of increased molar flux correspond to higher drying rate intervals, as shown in the drying rate time evolution plot for the side and center transformer samples, in Figure 11b. For both transformer samples, each successive phase exhibits lower drying rate values. Interestingly, though, the center transformer sample showing a lower drying rate at the beginning of the process eventually surpasses the side sample. This behaviour can be attributed to its higher moisture content throughout the process, which makes it more responsive to subsequent heating and vacuum phases.

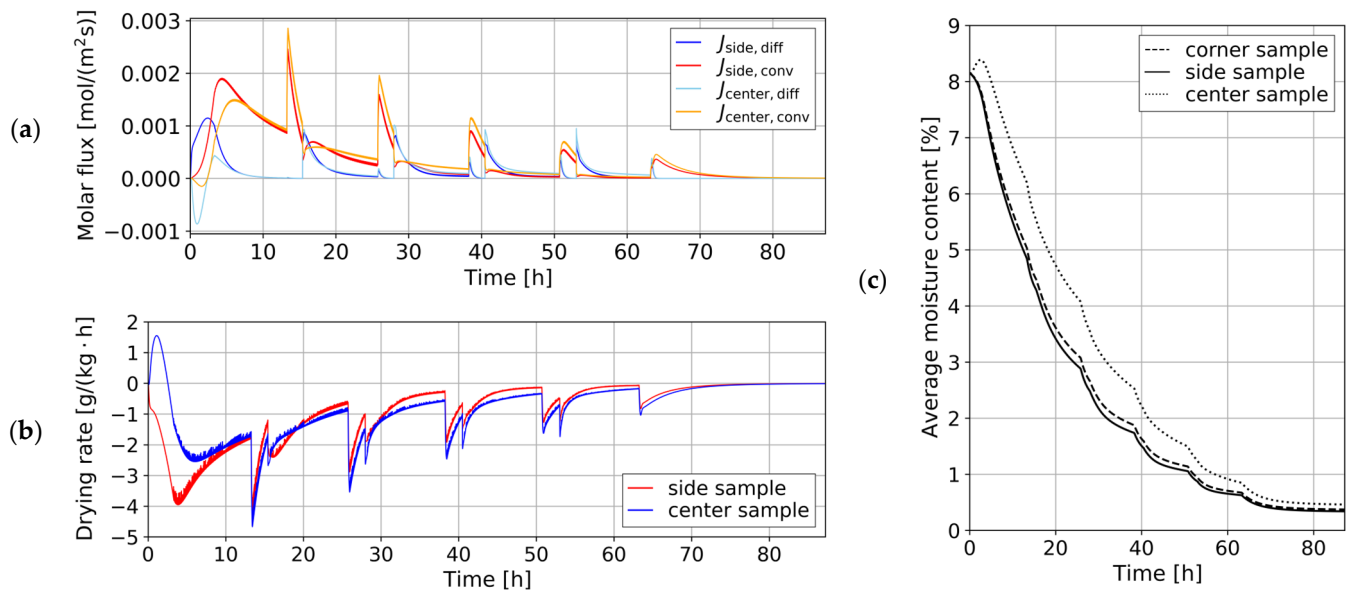


**Figure 11.** Time variations ( $k_0 = 10^{-16} \text{ m}^2$  and  $K = 10^{-4} \text{ 1/s}$ ) of the following quantities: (a) convective and diffusive molar fluxes at the surfaces of the side and center transformer samples, (b) drying rates of the side and center transformer samples, (c) drying curves of the corner, side, and center transformer samples.

The drying curves of all three transformer samples are shown in Figure 11c. The side and corner samples, both exposed to similar radiative conditions, exhibit nearly identical drying behaviour. In contrast, the center transformer sample initially experiences a slight increase in moisture content. This is attributed to a rapid increase in the water vapor molar fraction of the vacuum chamber atmosphere (primarily due to moisture desorption from the surrounding samples) and comparatively slow heating of the center sample. As the center sample heats up, its drying rate changes the signature; however, its moisture content remains elevated throughout the entire process. This observation clearly suggests that the overall drying duration is dictated by the moisture dynamics of the radiatively obstructed regions and cold spots within the cellulose insulation. These challenges are among the reasons why alternative drying technologies, such as low-frequency current or DC heating [15,45] and vapor phase drying [42], are often employed in industrial practice.

The drying case results presented should be interpreted qualitatively, as the underlying model has not been calibrated with respect to key parameter values. Nevertheless, the simulation results suggest that, by modifying the controlled setpoints, as well as the timing and duration of heating and vacuum phases, it is possible to influence the drying rate trend and, consequently, the total drying time. As the model accounts for the process at the vacuum chamber scale, it also enables estimation of the total energy consumption of the IR heaters. This allows for process optimization with respect to drying time, energy consumption, or a trade-off between the two.

### 3.4. Comparison with Existing Fickian Models

The formulated dual-scale model provides insights into the influence of key model parameters and the selected process control strategy. Although the predicted drying times lack quantitative accuracy due to the absence of model calibration, it is nevertheless important to demonstrate the model's potential for validation by verifying its ability to realistically simulate physical behaviour in terms of drying times and moisture content levels. To assess this capability, the model was compared with existing Fickian diffusion models reported in the literature.

The moisture transport description by Fick's second law reads

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial x} \left( D \frac{\partial X}{\partial x} \right), \quad (51)$$

where  $D$  is the effective moisture diffusivity. In this study, three diffusivity expressions from the literature were considered. The expression used by Foss, reported in [14], and Du [6] is dependent on the local moisture content  $X$  [%] and (assumed uniform) temperature of the insulation  $T$  [K]:

$$D(X, T) = D_0 \exp \left[ kX + E_a \left( \frac{1}{T_{ref}} - \frac{1}{T} \right) \right], \quad (52)$$

where  $D_0$ ,  $k$ , and  $E_a$  are parameters obtained by fitting simulated drying curves to experimental values. The values calculated by Foss and Du for non-impregnated Kraft paper are given in Table 4.

**Table 4.** Values of the parameters in expression for effective moisture diffusivity for non-impregnated Kraft paper.

| Author | $D_0$ [m <sup>2</sup> /s] | $k$ [-] | $E_a$ [K] | $T_{ref}$ [K] |
|--------|---------------------------|---------|-----------|---------------|
| Foss   | $2.62 \cdot 10^{-11}$     | 0.5     | 8140      | 298           |
| Du     | $2.25 \cdot 10^{-11}$     | 0.1955  | 8834      | 298           |

Garcia et al. [13] further improved upon this model by accounting for the insulation thickness  $l$  [mm], yielding the following expression:

$$D(l, X, T) = 3.1786 \cdot l^{-3.665} \exp \left[ 0.32458 \cdot X - \frac{8241.6 \cdot l^{-0.254}}{T} \right]. \quad (53)$$

In all cases, steady-state boundary conditions were imposed:

$$\left. \frac{\partial X}{\partial x} \right|_{r=r_{in}} = 0, \quad (54)$$

$$X|_{r=r_{out}} = X_{eq}, \quad (55)$$

where  $X_{eq}$  is the equilibrium moisture content that can be determined from Equations (11) and (12) for a given partial water vapor pressure  $p_v$  and temperature of the insulation  $T$ .

Since the dual-scale model captures transient behaviour at the vacuum chamber scale, special attention must be paid when configuring the simulations. To keep the transient periods as short as possible for a given drying system, the suction line conductance  $C_s$ , was set to 0.1 m<sup>3</sup>/s, enabling a rapid decrease to ultimate pressure. Likewise, the total power of the IR heaters  $\dot{Q}_{el}$  was increased to 12 kW to accelerate the heating of the insulation. Lastly, the insulation thickness was reduced to 10 mm while keeping the outer diameter constant (with inner diameter adjusted accordingly), allowing reuse of the view factors listed in Table 1. Adjusting the thickness to be small relative to the radius is also important because Fickian models assume a uniform insulation temperature and simplified plate-like geometry.

For an unbiased comparison, the insulation temperature and boundary moisture content in the Fick-based simulations were prescribed a priori and not extracted from the dual-scale model results. This is justified, as the temperature of the side sample remains closely around the fixed value due to the implemented two-point control. Furthermore, once the vacuum pump is activated, the molar fraction of water vapor rapidly approaches unity, making the water vapor partial pressure equal to the total chamber pressure, a boundary condition also used in [14]. For the Leybold VD65 pump used in this study,

the ultimate pressure is below 0.01 mbar, resulting in an equilibrium moisture content in Equation (55) practically equal to zero.

The side transformer sample (Figure 4) serves as a basis for comparison as its temperature is used in an implemented two-point control. Throughout the comparison, its surface temperature is maintained near a fixed value while pressure is reduced to the pump's minimum. This enables imposing a fixed insulation temperature and zero moisture content boundary condition in the Fickian diffusion model simulations.

Figure 12 illustrates the comparison between dual-scale and Fickian models. As discussed in Section 3.2, the internal overall mass transfer coefficient  $K$  and the absolute permeability  $k_0$  have the strongest influence on moisture dynamics. Two limiting dual-scale cases were simulated: one representing faster drying ( $k_0 = 10^{-14} \text{ m}^2$ ,  $K = 10^{-2} \text{ 1/s}$ ) and the other slower drying ( $k_0 = 10^{-16} \text{ m}^2$ ,  $K = 10^{-4} \text{ 1/s}$ ). Fickian model drying curves fall within the shaded area encompassed by the bounding drying curves in Figure 12 across all studied temperatures. This indicates that the dual-scale model is physically consistent and has a capacity to reproduce real results. Notably, two values of  $k_0$  used in simulations fall within reasonable ranges for paper materials, as supported by permeability measurements in [46].

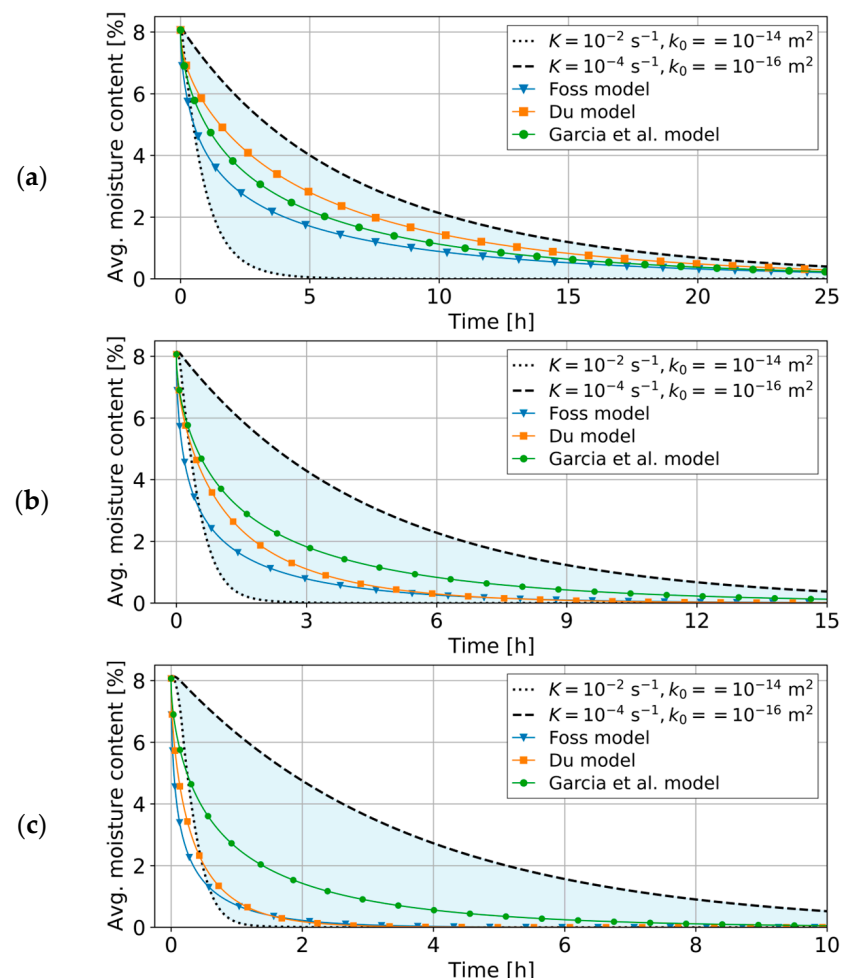


**Figure 12.** Drying simulations of side transformer sample with 10 mm thick insulation. Results of the dual-scale model with different values of insulation parameters  $k_0$  and  $K$  are compared to Fickian models with different effective diffusivities reported in [6,13,14]: (a) pump-down to ultimate pressure, two-point temperature control:  $70 \pm 1 \text{ }^\circ\text{C}$ , (b) pump-down to ultimate pressure, two-point temperature control:  $90 \pm 1 \text{ }^\circ\text{C}$ , (c) pump-down to ultimate pressure, two-point temperature control:  $110 \pm 1 \text{ }^\circ\text{C}$ .

Importantly, Figure 12 reveals significant discrepancies between the Fickian models themselves, a point raised earlier by Garcia et al. [47]. The model with thickness-dependent diffusivity consistently follows the trend of the bounding curves across the studied temperature range, suggesting that it captures the overall drying behaviour more accurately than the other Fickian models. Its position within the shaded area remains relatively stable, unlike the Foss and Du models, which increasingly deviate towards the lower bound at higher temperatures (Figure 12c). This improved agreement could stem from the fact that the thickness-dependent model indirectly accounts for additional transport mechanisms that are not captured by moisture- and temperature-dependent diffusivity formulations alone [13].

It is observed that all Fickian models exhibit noticeable deviations at the beginning of the drying process under elevated temperatures. This can be attributed to the fact that, unlike the presented dual-scale model, existing Fickian models require externally imposed boundary conditions and cannot reproduce transient boundary behaviour on their own. At elevated temperatures, the characteristic times of vacuum chamber-scale transients become comparable to those of the Fickian models, making the steady-state assumptions increasingly inaccurate for early stages of drying.

#### 4. Conclusions

This study has presented a detailed mathematical model of the vacuum drying process for transformer cellulose-based insulation, accounting for coupled heat and mass transfer phenomena at both the cellulose insulation scale and the vacuum chamber scale. Although simulations were carried out for a laboratory-scale dryer with a simplified geometry, implementing real dimensions and geometry is largely a matter of parameter adjustment. The presented model could also be adapted to alternative heating methods, such as hot air circulation, by modifying the constitutive relations used to compute the convective heat transfer coefficient, or drying with internal heating, by modifying the boundary conditions at cellulose insulation interface.

By incorporating convective heat transport at reduced pressures and radiative heat transfer with view factor computation, the model results have highlighted the influence of the spatial layout of transformer samples, an important practical consideration when setting up the drying process.

Unlike previous studies discussed in the Section 1, the results of this model demonstrate the important role of combining heating and vacuum phases in the drying procedure. While this strategy is already common in the transformer manufacturing industry, the present study provides a clear physical explanation for its effectiveness.

To verify the proposed model, drying curves simulated using two distinct parameter sets within their physically reasonable bounds were compared against those from existing, validated numerical models. The drying curves predicted by the established models were found to fall within the band of the drying curves produced by the proposed model, indicating that, despite the lack of experimental calibration, the model can reproduce realistic drying dynamics.

A major limitation of this model remains in the uncertainty of key cellulose insulation properties, such as absolute permeability, internal overall mass transfer coefficient, and sorption isotherm data. Addressing these parameter uncertainties, exploring their possible dependence on temperature and moisture, and identifying additional potentially relevant moisture transport mechanisms are crucial tasks. These objectives can be accomplished by comparing the model with data obtained from transient experiments, an essential step planned for our future work.

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## Abbreviations and Nomenclature

The following abbreviations and symbols are used in this manuscript:

|               |                                                                             |
|---------------|-----------------------------------------------------------------------------|
| GAB           | Guggenheim, Anderson, and de Boer                                           |
| 0D            | Zero-dimensional                                                            |
| CFD           | Computational fluid dynamics                                                |
| IR            | Infrared                                                                    |
| ODE           | Ordinary differential equation                                              |
| PDE           | Partial differential equation                                               |
| Latin symbols |                                                                             |
| $A_h$         | IR heater surface area, $m^2$                                               |
| $A_t$         | Transformer cellulose insulation outer surface area, $m^2$                  |
| $a_w$         | Water activity, -                                                           |
| $A_w$         | Vacuum chamber wall surface area, $m^2$                                     |
| $b$           | Klinkenberg parameter, Pa                                                   |
| $C_{a,amb}$   | Ambient dry air molar density, $kmol/m^3$                                   |
| $C_{a,ch}$    | Dry air molar density of the vacuum chamber atmosphere, $kmol/m^3$          |
| $C_{g,amb}$   | Ambient molar density, $kmol/m^3$                                           |
| $C_{g,ch}$    | Molar density of the vacuum chamber atmosphere, $kmol/m^3$                  |
| $C_{m,p,a}$   | Dry air molar heat capacity at constant pressure, $J/(kmol \cdot K)$        |
| $C_{m,p,g}$   | Gaseous-phase molar heat capacity at constant pressure, $J/(kmol \cdot K)$  |
| $C_{m,p,v}$   | Water vapor molar heat capacity at constant pressure, $J/(kmol \cdot K)$    |
| $C_{m,v,a}$   | Dry air molar heat capacity at constant volume, $J/(kmol \cdot K)$          |
| $C_{m,v,v}$   | Water vapor molar heat capacity at constant volume, $J/(kmol \cdot K)$      |
| $c_{p,g}$     | Gaseous-phase specific heat capacity at constant pressure, $J/(kg \cdot K)$ |
| $C_{v,amb}$   | Ambient water vapor molar density, $kmol/m^3$                               |
| $C_{v,ch}$    | Water vapor molar density of the vacuum chamber atmosphere, $kmol/m^3$      |
| $C_a$         | Dry air molar density, $kmol/m^3$                                           |
| $c_f$         | Cellulose fibre specific heat capacity, $J/(kg \cdot K)$                    |
| $C_g$         | Gaseous-phase molar density, $kmol/m^3$                                     |
| $c_h$         | IR heater specific heat capacity, $J/(kg \cdot K)$                          |
| $C_s$         | Suction line conductance, $m^3/s$                                           |
| $C_v$         | Water vapor molar density, $kmol/m^3$                                       |
| $c_w$         | Vacuum chamber wall specific heat capacity, $J/(kg \cdot K)$                |
| $D$           | Effective moisture diffusivity, $m^2/s$                                     |
| $D_{ef}$      | Effective binary diffusivity (water vapor and dry air), $m^2/s$             |
| $D_{eq}$      | Equivalent binary diffusivity (water vapor and dry air), $m^2/s$            |
| $D_K$         | Knudsen diffusivity parameter, $m^2/s$                                      |
| $F$           | View factor, -                                                              |
| $Gr$          | Grashof number, -                                                           |
| $\dot{H}$     | Enthalpy flow rate, W                                                       |
| $h_{t,i}$     | $i$ -th transformer convective heat transfer coefficient, $W/(m^2 \cdot K)$ |
| $h_{ch}$      | Vacuum chamber wall convective heat transfer coefficient, $W/(m^2 \cdot K)$ |
| $h_h$         | IR heater convective heat transfer coefficient, $W/(m^2 \cdot K)$           |

|                     |                                                                                       |
|---------------------|---------------------------------------------------------------------------------------|
| $J_{a,t,i}$         | $i$ -th transformer outer surface molar flux of dry air, kmol/(m <sup>2</sup> ·s)     |
| $J_{v,t,i}$         | $i$ -th transformer outer surface molar flux of water vapor, kmol/(m <sup>2</sup> ·s) |
| $J_{conv}$          | Convective molar flux, kmol/(m <sup>2</sup> ·s)                                       |
| $J_{diff}$          | Diffusive molar flux, kmol/(m <sup>2</sup> ·s)                                        |
| $K$                 | Internal overall mass transfer coefficient, 1/s                                       |
| $k_0$               | Absolute permeability of the cellulose insulation, m <sup>2</sup>                     |
| $k_{ef}$            | Effective permeability of the cellulose insulation, m <sup>2</sup>                    |
| $K_v$               | Venting/pressure relief valve flow coefficient, m <sup>3</sup> /s                     |
| $l$                 | Insulation thickness, mm                                                              |
| $m_h$               | Mass of the IR heaters, kg                                                            |
| $M_v$               | Water vapor molar mass, 18 kg/kmol                                                    |
| $m_w$               | Mass of the vacuum chamber walls, kg                                                  |
| $\dot{n}$           | Molar flow rate, kmol/s                                                               |
| $Nu$                | Nusselt number, -                                                                     |
| $p$                 | Pressure of the gaseous phase, Pa                                                     |
| $p_{a,ch}$          | Dry air partial pressure in the vacuum chamber atmosphere, Pa                         |
| $p_{v,ch}$          | Water vapor partial pressure in the vacuum chamber atmosphere, Pa                     |
| $p_a$               | Dry air partial pressure, Pa                                                          |
| $p_{amb}$           | Ambient pressure, Pa                                                                  |
| $p_{ch}$            | Pressure of the vacuum chamber atmosphere, Pa                                         |
| $p_{in}$            | Pressure at the inlet of the vacuum pump, Pa                                          |
| $p_{sat}$           | Water vapor saturation pressure, Pa                                                   |
| $p_v$               | Water vapor partial pressure, Pa                                                      |
| $Pr$                | Prandtl number, -                                                                     |
| $\dot{Q}$           | Convective heat flow rate to vacuum chamber atmosphere, W                             |
| $\dot{Q}_{rad,h}$   | Radiative heat flow rate from IR heater surface, W                                    |
| $\dot{Q}_{rad,t,i}$ | Radiative heat flow rate from the surface of the $i$ -th transformer, W               |
| $\dot{Q}_{rad,w}$   | Radiative heat flow rate from vacuum chamber wall surface, W                          |
| $\dot{Q}_{el}$      | IR heater input electrical power, W                                                   |
| $\dot{Q}_w$         | Radiative heat flow from the surface of vacuum chamber walls, W                       |
| $Q_{pump, in}$      | Vacuum pump inlet pumping speed, m <sup>3</sup> /s                                    |
| $q_{conv}$          | Convective heat flux, W/m <sup>2</sup>                                                |
| $Q_{pump}$          | Vacuum pump effective pumping speed, m <sup>3</sup> /s                                |
| $q_{rad}$           | Radiative heat flux, W/m <sup>2</sup>                                                 |
| $Q_{relief}$        | Volume flow rate through pressure relief valve, m <sup>3</sup> /s                     |
| $Q_{vent}$          | Volume flow rate through venting valve, m <sup>3</sup> /s                             |
| $r$                 | Radius of the cellulose insulation, m                                                 |
| $r_{in}$            | Inner radius, m                                                                       |
| $R_m$               | Universal gas constant, 8314.4 J/(kmol·K)                                             |
| $r_{out}$           | Outer radius, m                                                                       |
| $Ra$                | Rayleigh number, -                                                                    |
| $t$                 | Time variable, s                                                                      |
| $T$                 | Cellulose insulation temperature, K                                                   |
| $T_{s,i}$           | $i$ -th transformer cellulose insulation outer surface temperature, K                 |
| $T_{amb}$           | Ambient temperature, K                                                                |
| $T_{ch}$            | Vacuum chamber atmosphere temperature, K                                              |
| $T_h$               | IR heater temperature, K                                                              |
| $T_w$               | Vacuum chamber wall temperature, K                                                    |
| $V$                 | Vacuum chamber atmosphere volume, m <sup>3</sup>                                      |
| $v$                 | Darcy's velocity of gaseous phase, m/s                                                |

|                   |                                                                                     |
|-------------------|-------------------------------------------------------------------------------------|
| $X$               | Dry-basis moisture content of the cellulose fibres, kg/kg                           |
| $X_{avg}$         | Dry-basis average moisture content of the cellulose fibres, kg/kg                   |
| $X_{eq}$          | Dry-basis equilibrium moisture content of the cellulose fibres, kg/kg               |
| $y_v$             | Water vapor molar fraction, kmol/kmol                                               |
| Greek symbols     |                                                                                     |
| $\gamma_a$        | Dry air specific heat ratio, -                                                      |
| $\gamma_g$        | Gaseous-phase specific heat ratio, -                                                |
| $\gamma_v$        | Water vapor specific heat ratio, -                                                  |
| $\delta_{ins}$    | Vacuum chamber thermal insulation thickness, m                                      |
| $\Delta h_{evap}$ | Specific evaporation heat, J/kg                                                     |
| $\varepsilon_f$   | Fibre volume fraction, -                                                            |
| $\varepsilon_j$   | $j$ -th surface emissivity, -                                                       |
| $\varepsilon_p$   | Porosity of the cellulose insulation, -                                             |
| $\vartheta$       | Relative temperature, °C                                                            |
| $\lambda_{ef}$    | Cellulose insulation effective thermal conductivity, W/(m·K)                        |
| $\lambda_f$       | Cellulose fibre thermal conductivity, W/(m·K)                                       |
| $\lambda_{ins}$   | Vacuum chamber thermal insulation thermal conductivity, W/(m·K)                     |
| $\mu_g$           | Gaseous-phase dynamic viscosity, Pa·s                                               |
| $\rho_c$          | Cellulose insulation bulk density, kg/m <sup>3</sup>                                |
| $\rho_f$          | Cellulose fibre density, kg/m <sup>3</sup>                                          |
| $\rho_g$          | Gaseous-phase density, kg/m <sup>3</sup>                                            |
| $\sigma$          | Stefan–Boltzmann constant, $5.67 \cdot 10^{-8}$ W/(m <sup>2</sup> ·K <sup>4</sup> ) |
| $\tau_f$          | Cellulose fibre tortuosity, -                                                       |
| $\tau_p$          | Pore space tortuosity, -                                                            |

## Appendix A

**Table A1.** Physical properties of cellulose insulation.

| Parameter                               | Value/Relation                                | Unit              |
|-----------------------------------------|-----------------------------------------------|-------------------|
| Fibre density, $\rho_f$                 | 1550 [48]                                     | kg/m <sup>3</sup> |
| Bulk density, $\rho_c$                  | 1000 (typical value for Weidmann Kraft paper) | kg/m <sup>3</sup> |
| Fibre specific heat capacity, $c_f$     | 1340 [48]                                     | J/(kg·K)          |
| Fibre thermal conductivity, $\lambda_f$ | 0.335 [48]                                    | W/(m·K)           |
| Porosity, $\varepsilon_p$               | $\varepsilon_p = 1 - \rho_c / \rho_f$         | -                 |
| Fibre volume fraction, $\varepsilon_f$  | $\varepsilon_f = 1 - \varepsilon_p$           | -                 |
| Pore tortuosity, $\tau_p$               | $\tau_p = 1 - \ln(\varepsilon_p) / 2$ [49]    | -                 |
| Fibre tortuosity, $\tau_f$              | $\tau_f = 1 - \ln(\varepsilon_f) / 2$ [49]    | -                 |
| Klinkenberg parameter, $b$              | $b = 0.15 \cdot k_0^{-0.37}$ [50]             | Pa                |
| Knudsen diffusivity parameter, $D_K$    | $10^{-5}$                                     | m <sup>2</sup> /s |
| Emissivity, $\varepsilon_t$             | 0.9                                           | -                 |
| GAB isotherm parameter, $X_m$           | 0.05128 [29]                                  | kg/kg             |
| GAB isotherm parameter, $k$             | 0.716 [29]                                    | -                 |
| GAB isotherm parameter, $C_0$           | 6.1446 [29]                                   | -                 |
| GAB isotherm parameter, $T_{ref}$       | 323.15 [29]                                   | K                 |
| GAB isotherm parameter, $Q$             | 19,319.76 [29]                                | kJ/kmol           |

**Table A2.** Physical properties of other components.

| Parameter                                                            | Value/Relation        | Unit     |
|----------------------------------------------------------------------|-----------------------|----------|
| Vacuum chamber wall specific heat capacity, $c_w$                    | 461 (stainless steel) | J/(kg·K) |
| Vacuum chamber wall emissivity, $\varepsilon_w$                      | 0.1                   | -        |
| Vacuum chamber wall thermal insulation conductivity, $\lambda_{ins}$ | 0.04                  | W/(m·K)  |
| IR heater specific heat capacity, $c_h$                              | 800 (ceramics)        | J/(kg·K) |
| IR heater emissivity, $\varepsilon_h$                                | 0.95                  | -        |

**Table A3.** Process parameters, dimensions, and masses of the components.

| Parameter                                                   | Value/Relation                                                                         | Unit              |
|-------------------------------------------------------------|----------------------------------------------------------------------------------------|-------------------|
| Transformer inner radius, $r_{in}$                          | 40                                                                                     | mm                |
| Transformer cellulose insulation thickness, $\delta_c$      | 50                                                                                     | mm                |
| Transformer outer radius, $r_{out}$                         | $r_{out} = r_{in} + \delta_c$                                                          | mm                |
| Transformer height, $H_t$                                   | 0.8                                                                                    | m                 |
| Transformer surface area, $A_t$                             | $A_t = 2 \cdot r_{out} \cdot \pi \cdot H_t$                                            | m <sup>2</sup>    |
| Number of transformers in the vacuum chamber, $N_t$         | 9                                                                                      | -                 |
| Spacing between transformers                                | 0.08                                                                                   | m                 |
| Vacuum chamber wall mass, $m_w$                             | 370.389                                                                                | kg                |
| Vacuum chamber atmosphere volume, $V$                       | 1.7567                                                                                 | m <sup>3</sup>    |
| Vacuum chamber wall surface area, $A_w$                     | 7.86388                                                                                | m <sup>2</sup>    |
| Vacuum chamber shell height, $H_w$                          | 1.21                                                                                   | m                 |
| Vacuum chamber thermal insulation thickness, $\delta_{ins}$ | 32                                                                                     | mm                |
| IR heater width, $W_h$                                      | 62.5                                                                                   | mm                |
| IR heater height, $H_h$                                     | 0.75                                                                                   | m                 |
| IR heater number, $N_h$                                     | 4                                                                                      | -                 |
| IR heater total mass, $m_h$                                 | 2.7                                                                                    | kg                |
| IR heater total electrical power, $\dot{Q}_{el}$            | 3000                                                                                   | W                 |
| Vacuum pump nominal pumping speed                           | 65 ( <i>Leybold VD65</i> )                                                             | m <sup>3</sup> /h |
| Vacuum pump inlet pumping speed, $Q_{pump, in}$             | Linearly interpolated from the data for <i>Leybold VD65</i> (gas ballast valve closed) | m <sup>3</sup> /s |
| Suction line conductance, $C_s$                             | 0.01                                                                                   | m <sup>3</sup> /s |
| Venting/pressure relief valve flow coefficient, $K_v$       | 0.00005                                                                                | -                 |

**Table A4.** Dry air and water vapor properties, thermodynamical relations, and constants.

| Parameter                                                     | Value/Relation                        | Unit                                |
|---------------------------------------------------------------|---------------------------------------|-------------------------------------|
| Dry air molar mass, $M_a$                                     | 28.96                                 | kg/kmol                             |
| Water vapor molar mass, $M_v$                                 | 18                                    | kg/kmol                             |
| Gaseous-phase molar mass, $M_g$                               | $M_g = M_a + (M_v - M_a)y_v$          | kg/kmol                             |
| Universal gas constant, $R_m$                                 | 8314.4                                | J/(kmol·K)                          |
| Stefan–Boltzmann constant, $\sigma$                           | $5.6703 \cdot 10^{-8}$                | W/(m <sup>2</sup> ·K <sup>4</sup> ) |
| Water specific evaporation heat, $\Delta h_{evap}$            | $2.5 \cdot 10^6$                      | J/kg                                |
| Dry air constant-pressure specific heat capacity *, $c_{p,a}$ | $c_{p,a} = 0.1455 \cdot T + 964$ [51] | J/(kg·K)                            |

Table A4. Cont.

| Parameter                                                           | Value/Relation                                                                                         | Unit              |
|---------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|-------------------|
| Water vapor constant pressure specific heat capacity *, $c_{p,v}$   | $c_{p,v} = 0.48 \cdot T + 1727$ [51]                                                                   | J/(kg·K)          |
| Gaseous phase constant-pressure specific heat capacity *, $c_{p,g}$ | $c_{p,g} = c_{p,v} \frac{y_v M_v}{M_g} + c_{p,a} \frac{(1-y_v)M_a}{M_g}$                               | J/(kg·K)          |
| Dry air constant-pressure molar heat capacity, $C_{m,p,a}$          | $C_{m,p,a} = c_{p,a} M_a$                                                                              | J/(kmol·K)        |
| Water vapor constant-pressure molar heat capacity, $C_{m,p,v}$      | $C_{m,p,v} = c_{p,v} M_v$                                                                              | J/(kmol·K)        |
| Gaseous-phase constant-pressure molar heat capacity, $C_{m,p,g}$    | $C_{m,p,g} = c_{p,g} M_g$                                                                              | J/(kmol·K)        |
| Water vapor dynamic viscosity, $\mu_v$                              | $\mu_v = 3.43 \cdot 10^{-8} \cdot T - 5.19045 \cdot 10^{-7}$ [51]                                      | Pa·s              |
| Dry air dynamic viscosity, $\mu_a$                                  | $\mu_a = 2.22 \cdot 10^{-11} \cdot T^2 - 3.517214 \cdot 10^{-8} \cdot T + 5.906367 \cdot 10^{-6}$ [51] | Pa·s              |
| Gaseous-phase dynamic viscosity, $\mu_g$                            | $\mu_g = \mu_a + (\mu_v - \mu_a)y_v$                                                                   | Pa·s              |
| Water vapor saturation pressure, $p_{v, sat}$                       | Calculated for given temperature according to IAPWS 95 [30]                                            | Pa                |
| Gaseous-phase density *, $\rho_g$                                   | $\rho_g = \frac{p}{R_m T} [M_a + (M_v - M_a)y_v]$                                                      | kg/m <sup>3</sup> |
| Water vapor thermal conductivity *, $\lambda_v$                     | $\lambda_v = 94.7 \cdot 10^{-6} \cdot T - 9.7 \cdot 10^{-3}$ [51]                                      | W/(m·K)           |
| Dry air thermal conductivity *, $\lambda_a$                         | $\lambda_a = 65 \cdot 10^{-6} \cdot T + 6.7 \cdot 10^{-3}$ [51]                                        | W/(m·K)           |
| Water vapor thermal conductivity *, $\lambda_v$                     | $\lambda_g = \lambda_a + (\lambda_v - \lambda_a)y_v$                                                   | W/(m·K)           |

\* Used only in convective heat transfer coefficient calculation.

## References

- Zhang, D.; Zhai, X.; Wang, S.; Li, X.; Xu, P.; Zhao, H.; Zhang, G.J. Study on Mechanical and Electrical Properties Improvement of Insulating Paper Modified by Cellulose Nanocrystals. *J. Appl. Phys.* **2024**, *136*, 125108. [\[CrossRef\]](#)
- Liang, Z.; Fang, Y.; Cheng, H.; Sun, Y.; Li, B.; Li, K.; Zhao, W.; Sun, Z.; Zhang, Y. Innovative Transformer Life Assessment Considering Moisture and Oil Circulation. *Energies* **2024**, *17*, 429. [\[CrossRef\]](#)
- Cong, H.; Quan, S.; Hu, X.; Zhang, X.; Li, Q.; Xu, M. Microscopic Mechanism Effect of Oxygen and Moisture on Pyrolysis of Transformer Insulating Paper. *IEEE Trans. Dielectr. Electr. Insul.* **2025**, *32*, 1008–1016. [\[CrossRef\]](#)
- Tazhibayev, A.; Amitov, Y.; Arynov, N.; Shingissov, N.; Kural, A. Experimental Investigation and Evaluation of Drying Methods for Solid Insulation in Transformers: A Comparative Analysis. *Results Eng.* **2024**, *23*, 102470. [\[CrossRef\]](#)
- Brahmi, Y.; Betie, A.; Meghnefi, F.; Fofana, I.; Yeo, Z. Development of a Comprehensive Model for Drying Optimization and Moisture Management in Power Transformer Manufacturing. *Energies* **2025**, *18*, 789. [\[CrossRef\]](#)
- Du, Y. Measurement and Modeling of Moisture Diffusion Processes in Transformer Insulation Using Interdigital Dielectrometry Sensors. Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 1999.
- García, D.F.; García, B.; Burgos, J.C.; García-Hernando, N. Determination of Moisture Diffusion Coefficient in Transformer Paper Using Thermogravimetric Analysis. *Int. J. Heat Mass Transf.* **2012**, *55*, 1066–1075. [\[CrossRef\]](#)
- Villarreal, R.; Garcia, D.; Garcia, B.; Burgos, J. Diffusion Coefficient in Transformer Pressboard Insulation Part 1: Non Impregnated Pressboard. *IEEE Trans. Dielectr. Electr. Insul.* **2014**, *21*, 360–368. [\[CrossRef\]](#)
- Wang, D.; Zhou, L.; Wang, L.; Guo, L.; Liao, W. Modified Expression of Moisture Diffusion Factor for Non-Oil-Immersed Insulation Paper. *IEEE Access* **2019**, *7*, 41315–41323. [\[CrossRef\]](#)
- Krabbenhoft, K.; Damkilde, L. A Model for Non-Fickian Moisture Transfer in Wood. *Mater. Struct.* **2004**, *37*, 615–622. [\[CrossRef\]](#)
- Perre, P. The Proper Use of Mass Diffusion Equation in Drying Modelling: From Simple Configurations to Non-Fickian Behaviours. In Proceedings of the 19th International Drying Symposium (IDS 2014), Lyon, France, 24–27 August 2014.
- Perré, P.; Rémond, R.; Almeida, G.; Augusto, P.; Turner, I. State-of-the-Art in the Mechanistic Modeling of the Drying of Solids: A Review of 40 Years of Progress and Perspectives. *Dry. Technol.* **2023**, *41*, 817–842. [\[CrossRef\]](#)

13. Garcia, D.F.; Villarroel, R.D.; Garcia, B.; Burgos, J.C. Effect of the Thickness on the Water Mobility Inside Transformer Cellulosic Insulation. *IEEE Trans. Power Deliv.* **2016**, *31*, 955–962. [\[CrossRef\]](#)
14. Kang, M.; Yang, L.; Zhao, Y.; Wang, K.; He, Y. Diffusion Behavior of Free Water and Bound Water in Insulation Paper Based on Langmuir Model. *Cellulose* **2024**, *31*, 3275–3287. [\[CrossRef\]](#)
15. Przybyłek, P. A New Concept of Applying Methanol to Dry Cellulose Insulation at the Stage of Manufacturing a Transformer. *Energies* **2018**, *11*, 1658. [\[CrossRef\]](#)
16. Siddiqui, T.; Pattiwar, J.T.; Paranjape, A.P. Statistical Analysis of the Influence of Various Temperatures on the Drying Time of Transformer Insulation in Vacuum Drying Process. In Proceedings of the 4th International Conference for Convergence in Technology (I2CT), Mangalore, India, 27–28 October 2018.
17. Perré, P.; Rémond, R.; Colin, J.; Mougel, E.; Almeida, G. Energy Consumption in the Convective Drying of Timber Analyzed by a Multiscale Computational Model. *Dry. Technol.* **2012**, *30*, 1136–1146. [\[CrossRef\]](#)
18. Torres, S.S.; Jomaa, W.; Puiggali, J.R.; Avramidis, S. Multiphysics Modeling of Vacuum Drying of Wood. *Appl. Math. Model.* **2011**, *35*, 5006–5016. [\[CrossRef\]](#)
19. Zadin, V.; Kasemägi, H.; Valdna, V.; Vigonski, S.; Veske, M.; Aabloo, A. Application of Multiphysics and Multiscale Simulations to Optimize Industrial Wood Drying Kilns. *Appl. Math. Comput.* **2015**, *267*, 465–475. [\[CrossRef\]](#)
20. Jomaa, W.; Baixeras, O. Discontinuous Vacuum Drying of Oak Wood: Modelling and Experimental Investigations. *Dry. Technol.* **1997**, *15*, 2129–2144. [\[CrossRef\]](#)
21. Rasetto, V.; Marchisio, D.L.; Fissore, D.; Barresi, A.A. On the Use of a Dual-Scale Model to Improve Understanding of a Pharmaceutical Freeze-Drying Process. *J. Pharm. Sci.* **2010**, *99*, 4337–4350. [\[CrossRef\]](#)
22. Zhang, X.; Gan, H.; Xue, H.; Jiang, Q. Numerical Simulation of an Experimental Study on Structure Optimization for Compartment Dryers. *Trans. FAMENA* **2024**, *48*, 95–110. [\[CrossRef\]](#)
23. Stéphan, A.; Perré, P.; Simo-Tagne, M.; L’Hostis, C.; Rémond, R. A Model-Based Advanced Control System to Cope with Energy Fluctuations in Industrial Wood Drying. *Dry. Technol.* **2025**, *43*, 1418–1428. [\[CrossRef\]](#)
24. Simo-Tagne, M.; Stéphan, A.; Perré, P.; L’Hostis, C.; Rémond, R. On-Line Evaluation of Moisture Transfer Properties for a Feed-Forward Control of a Wood Drying Kiln with a Mechanistic Model. *Dry. Technol.* **2024**, *42*, 2348–2361. [\[CrossRef\]](#)
25. Lienhard, J.H., IV; Lienhard, J.H., V. *A Heat Transfer Textbook*, 6th ed.; Phlogiston Press: Cambridge, MA, USA, 2024.
26. Ashrafi Moghadam, A.; Chalaturnyk, R. Expansion of the Klinkenberg’s Slippage Equation to Low Permeability Porous Media. *Int. J. Coal Geol.* **2014**, *123*, 2–9. [\[CrossRef\]](#)
27. Fessler, W.A.; Rouse, T.O.; McNutt, W.J.; Compton, O.R. A Refined Mathematical Model for Prediction of Bubble Evolution in Transformers. *IEEE Trans. Power Deliv.* **1989**, *4*, 391–404. [\[CrossRef\]](#)
28. Du, Y.; Zahn, M.; Lesieutre, B.C.; Mamishev, A.V.; Lindgren, S.R. Moisture Equilibrium in Transformer Paper-Oil Systems. *IEEE Electr. Insul. Mag.* **1999**, *15*, 11–20. [\[CrossRef\]](#)
29. Przybyłek, P. Water Saturation Limit of Insulating Liquids and Hygroscopicity of Cellulose in Aspect of Moisture Determination in Oil-Paper Insulation. *IEEE Trans. Dielectr. Electr. Insul.* **2016**, *23*, 1886–1893. [\[CrossRef\]](#)
30. Wagner, W.; Pruß, A. The IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use. *J. Phys. Chem. Ref. Data* **2002**, *31*, 387–535. [\[CrossRef\]](#)
31. Staudt, P.B.; Tessaro, I.C.; Marczak, L.D.F.; Soares, R.d.P.; Cardozo, N.S.M. A New Method for Predicting Sorption Isotherms at Different Temperatures: Extension to the GAB Model. *J. Food Eng.* **2013**, *118*, 247–255. [\[CrossRef\]](#)
32. Zhao, J. Thermodynamics and Particle Formation during Vacuum Pump Down. Ph.D. Thesis, University of Minnesota, Minneapolis, MN, USA, 1990.
33. González-Bárcena, D.; Martínez-Figueira, N.; Fernández-Soler, A.; Torralbo, I.; Bayón, M.; Piqueras, J.; Pérez-Grande, I. Experimental Correlation of Natural Convection in Low Rayleigh Atmospheres for Vertical Plates and Comparison between CFD and Lumped Parameter Analysis. *Int. J. Heat Mass Transf.* **2024**, *222*, 125140. [\[CrossRef\]](#)
34. Howell, J.R.; Pinar Mengüç, M.; Daun, K.; Siegel, R. *Thermal Radiation Heat Transfer*, 7th ed.; Taylor & Francis Group, LLC: Boca Raton, FL, USA, 2021; ISBN 978-0-367-34707-9.
35. Virtanen, P.; Gommers, R.; Oliphant, T.E.; Haberland, M.; Reddy, T.; Cournapeau, D.; Burovski, E.; Peterson, P.; Weckesser, W.; Bright, J.; et al. SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python. *Nat. Methods* **2020**, *17*, 261–272. [\[CrossRef\]](#)
36. Filipov, S.M.; Hristov, J.; Avdzhieva, A.; Faragó, I. A Coupled PDE-ODE Model for Nonlinear Transient Heat Transfer with Convection Heating at the Boundary: Numerical Solution by Implicit Time Discretization and Sequential Decoupling. *Axioms* **2023**, *12*, 323. [\[CrossRef\]](#)
37. Sullivan, C.; Kaszynski, A. PyVista: 3D Plotting and Mesh Analysis through a Streamlined Interface for the Visualization Toolkit (VTK). *J. Open Source Softw.* **2019**, *4*, 1450. [\[CrossRef\]](#)
38. Bogdan, M.; Walther, E.; Alecian, M.; Chapon, M. Calcul Des Facteurs de Forme Entre Polygones-Application à La Thermique Urbaine et Aux Études de Confort. In Proceedings of the IBPSA, Châlons-en-Champagne, France, 19–20 May 2022.

39. Pakowski, Z.; Adamski, R. Formation of Underpressure in an Apple Cylinder during Convective Drying. *Dry. Technol.* **2012**, *30*, 1238–1246. [\[CrossRef\]](#)
40. Erriguible, A.; Bernada, P.; Couture, F.; Roques, M.A. Simulation of Vacuum Drying by Coupling Models. *Chem. Eng. Process. Process Intensif.* **2007**, *46*, 1274–1285. [\[CrossRef\]](#)
41. Zhang, Z.; Kong, N. Nonequilibrium Thermal Dynamic Modeling of Porous Medium Vacuum Drying Process. *Math. Probl. Eng.* **2012**, *2012*, 347598. [\[CrossRef\]](#)
42. Oksana, M.; Alexander, D. Comparative Analysis of Transformer Solid Insulation Drying Methods. In Proceedings of the 2022 IEEE Electrical Insulation Conference, Knoxville, TN, USA, 19–23 June 2022; pp. 385–389. [\[CrossRef\]](#)
43. Przybyłek, P.; Gielniak, J. The Use of Methanol Vapour for Effective Drying of Cellulose Insulation. *Energies* **2023**, *16*, 4465. [\[CrossRef\]](#)
44. Liu, W.; Wang, J.; Li, Y.; Zhu, Z.; Qie, D.; Ding, L. Natural Convection Heat Transfer at Reduced Pressures. *Exp. Heat Transf.* **2019**, *32*, 14–24. [\[CrossRef\]](#)
45. Siddiqui, M.T. Cycle Time Reduction Using Intermittent DC Internal Heating in Vapour Phase Drying of Cellulose-Based Insulation Used in Transformers. *Aust. J. Electr. Electron. Eng.* **2024**, *22*, 431–436. [\[CrossRef\]](#)
46. Rasi, M. Permeability Properties of Paper Materials. Ph.D. Thesis, University of Jyväskylä, Jyväskylä, Finland, 2013.
47. García, D.F.; García, B.; Burgos, J.C. A Review of Moisture Diffusion Coefficients in Transformer Solid Insulation-Part 1: Coefficients for Paper and Pressboard. *IEEE Electr. Insul. Mag.* **2013**, *29*, 46–54. [\[CrossRef\]](#)
48. Nilsson, J.; Stenström, S. Modelling of Heat Transfer in Hot Pressing and Impulse Drying of Paper. *Dry. Technol.* **2001**, *19*, 2469–2485. [\[CrossRef\]](#)
49. Foss, W.R.; Bronkhorst, C.A.; Bennett, K.A. Simultaneous Heat and Mass Transport in Paper Sheets during Moisture Sorption from Humid Air. *Int. J. Heat Mass Transf.* **2003**, *46*, 2875–2886. [\[CrossRef\]](#)
50. Chen, X.; Liu, Y.; Zhang, R.; Zhu, H.; Li, F.; Yang, D.; Jiao, Y. Radio Frequency Drying Behavior in Porous Media: A Case Study of Potato Cube with Computer Modeling. *Foods* **2022**, *11*, 3279. [\[CrossRef\]](#)
51. Alexandersson, M.; Ristinmaa, M. Coupled Heat, Mass and Momentum Transport in Swelling Cellulose Based Materials with Application to Retorting of Paperboard Packages. *Appl. Math. Model.* **2021**, *92*, 848–883. [\[CrossRef\]](#)

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