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Total Model-Free Robust Control of Non-Affine Nonlinear Systems with Discontinuous Inputs

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Abstract: Taking the plant as a total uncertainty in a black box with measurable inputs and attainable outputs, this paper presents a constructive control design of agnostic nonlinear dynamic systems with discontinuous input (such as hard nonlinearities in the forms of dead zones, friction, and backlashes). This study expands the model-free sliding mode control (MFSMC), based on the Lyapunov differential inequality, to a total model-free robust control (TMFRC) for this class of piecewise systems, which does not use extra adaptive online data fitting modelling to deal with plant uncertainties and input discontinuities. The associated properties are analysed to justify the constraints and provide assurance for system stability analysis. Numerical examples in control of a non-affine nonlinear plant with three hard nonlinear inputs—a dead zone, Coulomb and viscous friction, and backlash—are used to test the feasibility of the TMFRC. Furthermore, real experimental tests on a permanent magnet synchronous motor (PMSM) are also given to showcase the control's applicability and offer guidance for implementation.

Keywords: nonlinear systems; agnostic systems; model-free robust control; dead zone; viscous friction; backlash

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1. Introduction

Non-smooth dynamics are common in engineering systems. These include hard nonlinearities like dead zones, saturation, friction, and backlash. They often appear when advanced structure and functionality are intentionally added. They also occur naturally and unavoidably in hardware. Such hardware is widely used in mechatronic systems, hydraulic and piezoelectric actuators, energy transmission, and other electromechanical devices [1].

With such high-demand applications, effort has been widely and intensively devoted to expanding the control techniques to nonlinear systems with these hard non-smooth characteristics. It is agreed that these dynamics should be properly coped with, otherwise they may degrade or limit specified control performance, since directly using the existing smooth control approaches is problematic [2].

For motion control system designs, the critical challenge is to deal with discontinuous inputs. In this line, tremendous model-based control approaches [1–4] have been developed. As an example, for dynamic plants with a preceded dead zone, vast studies have been

conducted, including the dead zone model inversion-based compensation [5,6] to eliminate the dead zone effect, and online learning-based dead zone compensation via adaptive control [7], neural network [8], and fuzzy logic systems [9]. Nevertheless, control designs that do not construct the dead zone inverse in [10] still need to adaptively update dynamic models, and other representative approaches also need the bounds of the slope and the break points of the dead zone [11] in designing the control systems. Similar approaches have been proposed for control with other discontinuous inputs, such as saturation [1,12], friction [13–15], and hysteresis [16,17]. It is obvious that the model-based approach requires extra effort to handle the modelling of such input discontinuities, even though it has been matured to some extent with the developed approaches. Nevertheless, for the dynamic control systems with discontinuous inputs, such as dead zones, friction, and backlash, most existing publications have treated each discontinuous input separately [18–21]. This analysis provokes a fundamental question if the extra model request for designing the controllers can be removed in dealing with the discontinuous inputs in a general and unified scenario, so that the control system can be designed as the smooth control system.

On the other hand, to deal with unknown plant models in control designs, adaptive control (AC) [22,23], active disturbance rejection control (ADRC) [24–26], and, more recently, model-free universal control (U-control) have been proposed [27–30]. Among these methods, it should be noted that AC needs to estimate the unknown model or controller parameters by data-driven recursive online learning, so it is generally a data-driven model-based approach. ADRC needs to estimate a lumped uncertainty in the model via an extended state observer, while U-control forms a closed-loop asymptotic dynamic inversion using a mode-free sliding mode control (MFSMC) derived from a Lyapunov differential inequality. Thus, U-control does not need such an online updated model, as it treats the agnostic plant as a whole uncertainty, like in a black box with measurable inputs and attainable outputs. Therefore, it is a type of data-driven model-free approach. However, U-control has not been explored for nonlinear systems with discontinuous inputs.

This study is not intended to provide a comprehensive review of existing approaches. Instead, it takes a view from a particular angle—whether or not models are used as references in control system design—in order to justify the motivation for developing a model-free control approach. In principle, designing a control system involves plant/problem knowledge and the controller/solution within a configurated feedback framework. When using plant knowledge to design control systems, three paradigms have generally been adopted: physical models, pointwise data-driven models, and black-box models covering the full range of plant knowledge (the model spectrum). A comprehensive understanding of model-based and model-free control system design can be expressed as follows: $aD_{mb} + bD_{mf}$, $a + b = 1$, $a = 1$ for model-based, $0 < a < 1$ for nominal model-based, and $a = 0$ for model-free. The factor a represents the trust in the model's fidelity [31]. Obviously, the total model-free approach is more challenging due to the lack of available knowledge for design, even though it offers advantages in simplicity and robustness. This is why total model-free approaches have not been properly developed—due to some of the challenges presented in the study.

Based on the above discussions, this paper aims to provide a generic solution for controlling a class of agnostic nonlinear systems with discontinuous inputs by using the model-free U-Control approach—that is, taking the plant as a total uncertainty in a black box. The MFSMC will be expanded to a total model-free robust control (TMFRC) to stabilise this class of discontinuous systems to achieve nonlinear dynamic inversion, which does not use extra adaptive online modelling/control or model inverse-based cancellation to deal with the input discontinuities.

The major contributions of this study are listed below.

(1) Establishing a total model-free robust control by taking plant and discontinuous inputs as a whole uncertainty for the control. Therefore, the approach is applicable to many discontinuous inputs, e.g., dead zone, friction, or backlash, without knowing the nonlinear input function.
(2) Extending the MFSMC to piecewise continuous system control. A control Lyapunov function (CLF) for non-affine control systems is created [32] using a sliding mode manifold (SMM) for an unknown model plant. Moreover, a sensitivity function is defined to address its robustness, which is a complement to the model-based sensitivity function.
(3) Providing comprehensive simulation demonstrations and real lab tests to verify the feasibility and efficiency of the TMFRC in controlling unknown nonlinear plants with hard nonlinear inputs.

The remaining sections are organised as follows: Section 2 provides preliminaries and problem formulation; Section 3 presents the detailed design of the TMFRC and provides its key properties for application assurance; Section 4 provides both simulations and experiments for the systems with unknown nonlinear dynamics cascaded with a dead zone, Coulomb friction, and backlash; and Section 5 summarises the work.

2. Problem Statement and Preliminaries

2.1. Dynamic System with Input Discontinuities

Consider a general single-input single-output (SISO) n-th order non-affine nonlinear system as

$$P : y^{(n)} = f\left(y^{(0\sim n-1)}, u(v), d\right) \tag{1}$$

where the triplet $\{y \in \mathbb{R}, v \in \mathbb{R}, d \in \mathbb{R}\}$ denotes the system output, input, and external disturbance, respectively, $y^{(0\sim n-1)} = [y, \cdots, y^{(n-1)}]^T \in \mathbb{R}^n$ is the output vector, and u is a discontinuous input function. $f : v \to u \to y$ is an unknown function including a piecewise continuously differentiable mapping $v \to u$ of class $\mathcal{C}n$, which fulfils the Lipschitz condition $|f(x_1) - f(x_2)| \leq L|x_1 - x_2|$, $\mathrm{L} \in \mathbb{R}^+$.

The input function $u(v)$ is used to describe discontinuous nonlinearities, such as dead zones, saturation, friction, and backlash, which widely appear in motion control systems [33]. To further elaborate the description of hard nonlinearities, write $u(v)$ as a general piecewise continuous function with accountable discontinuity points

$$u(v) : v \xrightarrow{\phi} u, \phi \in \begin{pmatrix} (b_0, b_1) & (b_1, b_2) & \cdots & (b_{j-1}, b_j) \end{pmatrix} \tag{2}$$

with $b_i \in \mathbb{R}, i = [0, j] \in \mathbb{R}^+$ the discontinuous points, where a piecewise function changes its formula and values at the points with side (left and right) limits. Therefore, it gives

$$u = \phi(v) = \begin{cases} \phi(v \to -b_i), \forall v < b_i \\ \phi(v \to +b_i), \forall v \geq b_i \end{cases} \tag{3}$$

The pair of two conjunction discontinuity points (b_{i-1}, b_i) specifies the piecewise continuous intervals, and $\phi \in (b_{i-1}, b_i)$ is continuously differentiable from class $\mathbb{C}^n$, fulfilling $||\phi(b_{i-1})| - |\phi(b_i)|| \leq \Phi$ for any $||b_{i-1}| - |b_i|| \leq B$ with $B, \Phi \in \mathbb{R}^+$. Consequently, system (1) can be expressed as

$$P : y^{(n)} = f\left(y^{(0\sim n-1)}, u([v_0, \cdots, v_j]), d\right) \tag{4}$$

Dead Zone Dynamics: The dead zone denotes the most popular discontinuous input dynamics encountered in actuators [1], which impose a non-sensitivity zone for small

excitation inputs around the origin on the input/output plane. The dead zone function $u(v)$ is shown in Figure 1 and expressed as follows:

$$u(t) = D(v(t)) = \begin{cases} -mb_r & \forall v(t) \geq b_r \\ -mv(t) & \forall b_l < v(t) < b_r \\ -mb_l & \forall v(t) \leq b_l \end{cases} \quad (5)$$

where $m = m_r = m_l$ is the dead zone slope, and b_l, b_r are the boundary parameters for specifying the dead zone width. It is clear that $|D(v)| \leq \gamma$ with the upper bound $\gamma = \max\{m_{\max} b_{r_\max}, m_{\max}(-b_{l_\max})\}$.

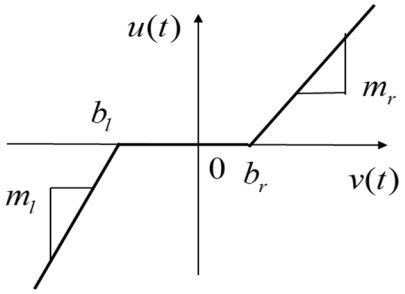


Figure 1. Dead zone function.

As explained in the introduction, the existing predominant control approaches [34] are model-based, including (1) the model-based cancellation (inversion) of the dead zone function [35], and (2) adaptive dead zone compensation [10].

Friction Dynamics: Friction has been observed in almost all mechanical systems [15], inducing unfavourable phenomena such as discontinuous nonlinear dynamics, constant oscillation, and jagged behaviour at low speed in motion control systems [13]. The friction function $u(v)$ is shown in Figure 2 and expressed as follows:

$$u(t) = F(v(t)) = \text{sgn}(v(t))(m|v(t)| + b) \quad (6)$$

where m is the viscous gain and b is the offset of the Coulomb friction.

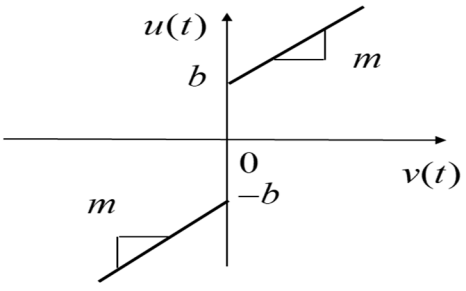


Figure 2. Coulomb and viscous friction function.

It is noted that there are two predominant model-based compensation approaches [13]: (1) the cancellation (inversion) of friction function [36] and (2) adaptive model-based friction compensation [37,38].

Backlash Dynamics: The backlash function is also known as the Krasnosel'skii–Pokrovskii (KP) model, which describes a special type of hysteresis [17]. Backlash characteristics appear widely in electromagnetic-relay units, high-precision servo systems, and

robots, which makes dynamic systems non-smooth and nonlinear [1]. The discontinuous backlash function $u(v)$ is shown in Figure 3 and expressed as

$$u(t) = B(v(t)) = \begin{cases} m(v(t) - b_r) & \forall \dot{v}(t) > 0 \cap u(t) = m(v(t) - b_r) \\ m(v(t) + b_l) & \forall \dot{v}(t) < 0 \cap u(t) = m(v(t) + b_l) \\ u(t_-) & otherwise \end{cases} \quad (7)$$

where m is the slope of the lines, b_l, b_r are the backlash boundary parameters, and $u(t_-)$ denotes that no change occurs at $u(t)$.

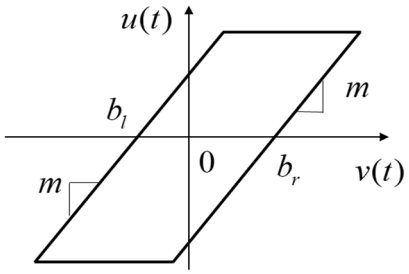


Figure 3. Backlash function.

Like coping with the other hard nonlinear inputs, the main streams of compensation for backlash include the following: (1) model-based cancellation (inversion) schemes [39], and (2) adaptive model-based compensation [16].

Remark 1. *It should be noted that most existing control methods (including inversion compensation and adaptive compensation) for nonlinear systems (1) with hard nonlinearities (e.g., dead zones (5), friction (6) and backlash (7)) presume model information of such discontinuous nonlinear inputs, which, in turn, imposes nontrivial modelling efforts.*

2.2. Control Problem Statement

Assumption 1. *The plant P in (1) is stabilisable on certain specified compact sets $\mathfrak{D} \subseteq \mathbb{R}^n$ with $P(0) = 0$ with the appropriate choice of feedback control mechanisms, which implies that (1) the plant is completely observable/controllable, where the observability/controllability matrices are non-singular. This makes the designed control system realisable with the appropriate signals and control actions, and (2) the plant could be non-minimum phase, instable zero dynamics, and/or open-loop unstable.*

The control objective for a prespecified trajectory x_d ($x_d \in \Omega_d \subset \mathbb{R}^n$ with Ω_d a compact set) in the closed-loop system is to guarantee all the signals are bounded and to ensure that the output vector x tracks the trajectory x_d, such that $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (x(t) - x_d(t)) = 0$. For a prespecified desired state vector x_d, design the TMFRC with the MFSMC to convert the full state feedback error vector $e = x - x_d$ into a single variable sliding mode function $e \to \sigma$ ($\mathbb{R}^n \to \mathbb{R}$) and then the MFSMC to generate the control input v.

Remark 2. *When f is known, it can be modelled by both polynomial and state space equations. The corresponding canonical state-space model can be derived by letting $x = y^{(0\sim n-1)} = [x_1 = y \quad x_2 = \dot{y} \quad \cdots \quad x_{n-1} = y^{(n-1)}]^T \in \mathbb{R}^n$ and $\dot{x}_n = f(x, u, d)$ [28]. If the full state vector x is not measurable, some model-free state observers [27,40] can be adopted with the measured output and control input.*

Remark 3. *Unlike existing control methods for nonlinear systems with hard nonlinearities (see Remark 1), the new TMFRC approach is different. It treats the whole plant as an uncertainty. This allows it to achieve the same control performance. It does not require cancellation of the hard nonlinear input. It also avoids the need for adaptive online model updating. Figure 4 shows the control system configuration. Plant (1), which satisfies Assumption 1, has two parts. It includes a dynamic block P preceded by a hard nonlinearity (HN). These two blocks are connected in series. Together, they form the dotted block, treated as a black box from input v to state vector x.*

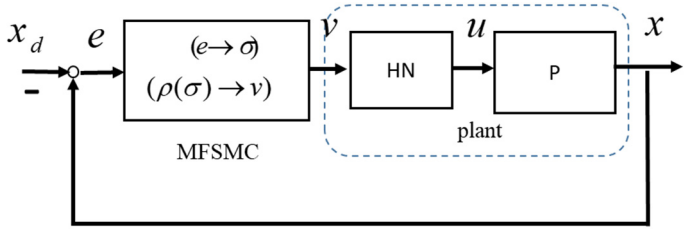


Figure 4. TMFRC system.

3. TMFRC of Nonlinear Systems with Discontinuous Input

3.1. Sliding Mode Control (SMC) with Continuous Inputs

A common approach to robust control design is stabilising a nominal system with bounded uncertainties. This helps reduce the impact of internal parameter variations and external disturbances [2]. The nominal system is often represented by a control input affine model for control system design. In contrast, model-free design can be simpler. It also offers strong robustness against parameter variations and external disturbances [27,29,41]. Therefore, this study uses sliding mode control (SMC) to handle the unknown system (1). The goal is to improve the simplicity in design and tuning while enhancing the robustness.

Considering system (1) with continuous input (i.e., $v = u$), a generalised model-free SMC in the form of the equivalent control has been derived as in [27],

$$\begin{aligned} &\sigma = \mu(e),\ \mathbb{R}^n \to \mathbb{R} \\ &\dot{\sigma} = v = u = \begin{cases} u_{sw} = -k_g \mathrm{sgn}(\sigma) & \forall |\sigma| > \delta \\ u_{eq} = \rho(\sigma)|_{\dot{\sigma}\sigma=0 \cup \dot{\sigma}\sigma<0} & \forall |\sigma| \le \delta \end{cases}, \mathbb{R}^m \to \mathbb{R} \end{aligned} \tag{8}$$

where the doublet $\{\sigma \in \mathbb{R}, \dot{\sigma} \in \mathbb{R}\}$ denotes the sliding mode manifold and its derivative, and $e = x - x_d$ is the tracking error, where $x_d = \begin{bmatrix} x_d & \dot{x}_d & \cdots & x_d^{(n-1)} \end{bmatrix}^T \subset \mathbb{R}^n$ is the desired trajectory to be tracked. The sliding mode manifold $\mu(e) = 0$ is a stable Hurwitz polynomial, and $\delta \in \mathbb{R}^+$ is the sliding mode boundary to the sliding mode $\sigma(e) = 0$ with L^2 norm, $\delta = \|\sigma(e)\|_2$. For the control formation $v = \rho(f|\dot{\sigma} = 0), \mathbb{R}^n \to \mathbb{R}$, is for the model-based SMC (MBSMC) and $v = \dot{\sigma} = \rho(\sigma|\dot{\sigma}\sigma < 0),\ \mathbb{R}^n \to \mathbb{R}$ is for the model-free SMC (MFSMC). The existence of such MFSMC has been proved and applied to model unknown nonlinear dynamic inversion and decoupling control [29].

Remark 4. *The main difference between MBSMC and MFSMC is how they determine the SMC. The MBSMC solves a Lyapunov differential equality, which is locally negative semi-definite. It includes the nominal equation and uncertainty bounds. The MFSMC solves a Lyapunov differential inequality, which is locally negative definite. Model-based methods (solving equalities) give unique or limited controller outputs. Model-free methods (solving inequalities) result in infinite possible controller outputs.*

3.2. Model-Free Sliding Mode Control (MFSMC) with Discontinuous Inputs

MFSMC is the kernel of the control system configuration, and the design procedure is shown below.

(1) Assign a linear sliding mode manifold as

$$\sigma(e) = \left(\frac{d}{dt} + c\right)^{n-1} e,\ \mathbb{R}^n \to \mathbb{R} \tag{9}$$

where the strictly positive constant $c \in \mathbb{R}^+$ denotes the slope (the rate of the exponential convergence) of the sliding manifold, which makes the sliding mode manifold a Hurwitz stable polynomial [33]. Then, the sliding mode $\sigma(e) = 0$ specifies a time-varying hyperplane to make the tracking error trajectory converge to zero $e = x - x_d \to 0_{n*1}$.

(2) The model-free equivalent controller is designed by

$$\dot{\sigma} = v = \begin{cases} v_{sw} = -k_g \mathrm{sgn}(\sigma) & \forall |\sigma| > \delta \\ v_{eq} = \rho(\sigma) & \forall |\sigma| \le \delta \end{cases} \tag{10}$$

where the constant gain $k_g \in R_+$ in the switching control v_{sw} and $\rho(\sigma)$ in the equivalent control v_{eq} are designed to satisfy the inequality $\dot{V} + \alpha V \le 0$ or $\dot{V} = \dot{\sigma}\sigma < 0, (\alpha = 0)$ in which $\alpha \in \mathbb{R}^+$ is the exponential convergent rate for the Lyapunov differential equation, and $V = \sigma^2/2$, $\dot{V} = \dot{\sigma}\sigma$ are the quadratic Lyapunov function and its derivative, respectively.

3.3. Existence of TMFRC

Definition of a class Kappa functions $\mathcal{K}$: For a real valued function $\rho(\sigma), \sigma \in \mathbb{R}$, it is known as monotonically decreasing while $\rho(\sigma_1) \ge \rho(\sigma_2), \forall \sigma_1 \le \sigma_2 \in \mathbb{R}$, and as strictly increasing for $\rho(\sigma_1) > \rho(\sigma_2), \forall \sigma_1 < \sigma_2 \in \mathbb{R}$.

Assumption 2. *The non-affine nonlinear plant P in (1) can be approximate by affine plant* $P : y^{(n)} = f\left(y^{(0\sim n-1)}, v, d\right) = f_a\left(y^{(0\sim n-1)}\right) + v + d$ *within the specified compact set* $\mathfrak{D} \subseteq \mathbb{R}^n$ *with* $P(0) = 0$ *through appropriate choice of the approximation techniques, such as the coordinate transformation* [42].

Theorem 1. *(Existence of TMFRC for piecewise systems): For plant (1) with piecewise input dynamics as* $y^{(n)} = f\left(y^{(0\sim n-1)}, v(v_i), d\right), i = [1, j] \in \mathbb{Z}^+$, *assign the full state feedback sliding mode manifold* σ. *A control Lyapunov function of the sliding mode manifold* $V(\sigma) = \sigma^2/2 > 0$, $V(0) = 0$ *is adopted for designing the TMFRC that makes the Lyapunov differential inequality* $\dot{V} = \dot{\sigma}\sigma < 0$, $\dot{V}(0) = 0$ *hold. Regarding the existence of such model-free controllers* $v = \rho(\sigma)$, *that is,* $\exists v(v_i) = \rho(\sigma) | (\dot{\sigma}\sigma < 0), \forall V \ne 0, i = [1, j] \in \mathbb{Z}^+$, *under the conditions of (1),* $\rho(\sigma)$ *is a strictly decreasing* $\mathcal{K}$ *function for the stabilisation direction and (2)* $\left|\rho(\sigma)\right| > \sup(|f_a| + |\dot{\sigma}\backslash y^{(n)}| + |d|)$. *for the control strength, where* $\sup(|f_a| + |\dot{\sigma}\backslash y^{(n)}| + |d|)$ *is the upper bound associated with the plant* $y^{(n)} = f\left(y^{(0\sim n-1)}, u(v), d\right) = f_a\left(y^{(0\sim n-1)}\right) + v + d$.

Proof. Assume a finite number of $\mathbb{C}^1$ piecewise Lyapunov functions $V = \{V_1, \cdots, V_j\} \in \begin{pmatrix} (b_0, b_1) & (b_1, b_2) & \cdots & (b_{j-1}, b_j) \end{pmatrix}$ exist. To prove the first condition, from $v \in \dot{V} < 0$, $\forall V \ne 0$, $[v_1, \cdots v_j] \in v$, replace $V = \sigma^2/2$ and $\dot{V} = \dot{\sigma}\sigma$. To assign $v = \dot{\sigma} = \rho(\sigma)$, the condition becomes $\rho(\sigma)\sigma < 0$; therefore, $\exists v = \rho(\sigma) \in \mathbb{R}$, satisfying the condition. For example, let $\rho(\sigma) = -k\sigma, k \in \mathbb{R}^+$, such that $\rho(\sigma)\sigma = -k\sigma^2 < 0$ holds.

To prove the second condition, consider the derivative of the given sliding mode manifold and Assumption 2 by

$$\begin{aligned}\dot{\sigma} &= y^{(n)} + \dot{\sigma}\backslash y^{(n)} \\ &= f\big|_{f=y^{(n)}} + \dot{\sigma}\backslash y^{(n)} \\ &= f_a\big|_{f=y^{(n)}} + \dot{\sigma}\backslash y^{(n)} + d + v \\ &= f_a\big|_{f=y^{(n)}} + \dot{\sigma}\backslash y^{(n)} + d + \rho(\sigma)\end{aligned} \tag{11}$$

Let $|\rho(\sigma)| > \sup(|f_a| + |\dot{\sigma}\backslash y^{(n)}| + |d|)$. and substitute into (11) to yield

$$\dot{\sigma} = f_a\Big|_{f=y^{(n)}} + \dot{\sigma}\backslash y^{(n)} + d + \rho(\sigma) = \rho_1(\sigma) = v \tag{12}$$

where $\rho_1(\sigma)$ is still a strictly decreasing monotone function because the condition $|\rho(\sigma)| > \sup(|f_a| + |\dot{\sigma}\backslash y^{(n)}| + |d|)$. holds. Therefore, the derivative of the control Lyapunov function $\dot{V} = \rho_1(\sigma)\sigma < 0$, such that the Lyapunov differential inequality is satisfied. □

Remark 5. *Regarding the type of convergency for the above control, it is characterised by the sliding mode manifold in either linear dynamics (asymptotic convergence) or fractional power nonlinear dynamics (finite time convergence). Moreover, as there are no plant model structure or parameters used for proving the existence of such control, it is a total model-free control.*

Remark 6. *Compared to adaptive control, the proposed model-free control is simpler. Adaptive control estimates plant model parameters online for designing control systems. In contrast, the model-free control does not need a known plant model. It is data-driven and only requires the plant's input and output signals to design control systems.*

3.4. Implementation of Controller $v = \rho(\sigma)$

It is noted that solutions for solving equality are limited and otherwise are unlimited for solving inequality. Similarly, in control system design, model-based design is used to solve nominal model equality equations, and the solutions are limited and optimal. For the model-free SMC, by solving differential inequality equations, the solutions for the controller output are infinite for robust stability.

Instead of having a finite number of solutions from $u = \rho(\sigma)|\dot{\sigma} = 0$ with the MB-SMC, there are various options for selecting infinite solutions for the MFSMC controller $v = \rho(\sigma)|\rho(\sigma)\sigma < 0, \forall\sigma \neq 0$, which, because of the differential inequality, provides a range of solutions for this type of controllers. Some examples of both switching control u_{sw} and equivalent control u_{eq} can be found in [28].

3.4.1. Proportional (P) Function

$$\dot{\sigma} = v = \begin{cases} v_{sw} = -k_g \mathrm{sgn}(\sigma) & \forall|\sigma| > \delta \\ v_{eq} = -k_l\sigma & \forall|\sigma| \leq \delta \end{cases} \tag{13}$$

where $k_g > 0$, $k_l > 0$ are the constant gains. To keep the Lyapunov stability, assign $k_g, k_l > \sup(|\hat{f}|)$ to handle the plant uncertainties, where $\sup(|\hat{f}|) = \sup(|f_a| + |\dot{\sigma}\backslash y^{(n)}| + |d|)$ is the total uncertainty bound.

3.4.2. Proportional and Integral (PI) Function

$$\dot{\sigma} = u = \begin{cases} u_{sw} = -k_g \mathrm{sgn}(\sigma) & \forall|\sigma| > \delta \\ u_{eq} = -k_p\sigma - k_i\int \sigma d\tau & \forall|\sigma| \leq \delta \end{cases} \tag{14}$$

where $k_g, k_p, k_i \in \mathbb{R}^+$ are the constant gains. To keep the Lyapunov stability, set up $k_g > \sup(|\hat{f}|)$ and $(k_p + k_i) > \sup(|\hat{f}|), \forall k_p > k_i, \quad |(k_p)\sigma^2| > |(k_i)\sigma \int \sigma d\tau|$.

3.4.3. Monotone Bounded Logistic Function

There are various sigmoid functions, which can be chosen for the continuous MFSMC. Here, we choose one of the typical monotonic functions for the controller as follows:

$$\dot{\sigma} = u = -k_s sigmoid(k_0\sigma) + \frac{k_s}{2}, \tag{15}$$

where $sigmoid(k_0\sigma) = 1/(1 + e^{-k_0\sigma})$, the sign of the gain k_s is for the convergent direction for $k_s > \sup(|\hat{f}|)$, and the value of the width $k_0 \in \mathbb{R}^+$ can be used to adjust the convergent speed along the sliding mode.

3.5. Sensitivity Analysis

This section analyses the robustness of the proposed MFSMC using the sensitivity function so that sensitivity reduction lead to increased robustness in the control systems [43]. For the dynamic system $y^{(n)} = f\left(y^{(0\sim n-1)}, v, d\right)$, as expressed in (1), robustness can be characterised by way of the sensitivity of the closed-loop model (transfer function for linear systems, or a general non-affine function $f(*)$ for nonlinear systems). Let $T(f)$ be a form of control system performance for the plant $f(*)$. A general model-based system sensitivity function, expanded from the linear transfer function formulation [44], is defined as a ratio of the percentage change in the closed-loop performance to the percentage change in the plant

$$S_f^T(mb) = \frac{\partial T(f)/T}{\partial f/f} = \frac{\partial T(f)}{\partial f}\frac{f}{T} \tag{16}$$

where mb is the model-based formulation. The changes with the plant f could be the variation in one or more of the plant parameters, and the performance T with the control system could be the output and state responses.

While a plant is treated as total uncertainty, it is difficult to take the above calculations without analytical models. Alternatively, symbolically define the model unknown system sensitivity function as

$$S_f^T(mf) = \frac{\Delta T(f)}{\Delta f}\frac{f}{T} \tag{17}$$

where mf is for model-free formulation.

For a stabilisable plant $y^{(n)} = f\left(y^{(0\sim n-1)}, v, d\right)$ with $|f|_{\max} \leq F = \sup(|f|) \in \mathbb{R}^+$, from Theorem 1, there exists a control $v = \rho(\sigma)|\rho(\sigma)\sigma < 0, \forall \sigma \neq 0$ for the TMFRC to stabilise the control system and keep the dynamic performance invariant; that is, the variation $\Delta T = 0|_{\Delta f \neq 0}$, where $\Delta T \in S \subset \mathbb{R}^n$ with S a compact set. Consequently, the sensitivity for the model-free control system is determined by $S_f^T(mf) = \frac{\Delta T(f)}{\Delta f}\frac{f}{T} = \frac{0}{\Delta f}\frac{f}{T}\Big|_{\Delta f \neq 0} = 0$, $\forall \Delta T \in S \subset \mathbb{R}^n$, which is called total robustness in the study within the bound of the uncertainty S. For general model-based control systems, as $\Delta T \neq 0|_{\Delta f \neq 0}$, it gives $S_f^T(mb) = \frac{\Delta T(f)}{\Delta f}\frac{f}{T}\Big|_{\Delta f \neq 0} \neq 0$, which is proportional robustness control against the system uncertainty.

Remark 7. *Compared to the model-free PID controller tuned by the trial-and-error method, the proposed MFSMC takes the control system performance invariant within the system boundary, rather like PID maintains the control system performance variation point by point to maintain stability.*

4. Simulation and Experimental Demonstrations

To demonstrate the feasibility and effectiveness of the TRMFC, both simulations and real experiments with three different hard nonlinear inputs are conducted.

4.1. Simulations

The simulated plant dynamic in (1) is governed by an open-loop unstable non-affine nonlinear equation that is expanded from an affine nonlinear benchmark case study in [10]. The example is chosen as a benchmark comparison for open-loop unstable dynamic plus non-affine control inputs, which satisfies Assumption 1 that the TMFRC can stabilise certain types of instable zero dynamics, and/or open-loop unstable dynamics. Obviously, control of non-affine nonlinear dynamics is more challenging in determining control inputs, which commonly approximate NN dynamics by AN in the neighbourhood of the equilibria.

$$\begin{array}{c} P: \ddot{y} = \frac{1-e^{-y}}{1+e^{y}} + (\dot{y}^2 + 2y)\sin\dot{y} + 0.5y\sin 3t + u(v) \\ +0.5u^3(v), y(0) = \begin{bmatrix} -2.5 & 3.5 \end{bmatrix}^T \end{array} \tag{18}$$

where $u(v(t))$ is the function of three hard nonlinearities—the dead zone, Coulomb and viscous friction, and backlash. It should be noted that plant model (18) and the hard nonlinear inputs are only used for simulation rather than being used to design the TMFRC. Therefore, the plant is treated as an uncertain black box in our case.

By assigning $x_1 = y, \dot{x}_1 = \dot{y} = x_2$, the plant (18) can be expressed in the corresponding state space,

$$\begin{array}{l} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1-e^{-x_1}}{1+e^{-x_1}} + (x_2^2 + 2x_1)\sin x_2 + 0.5x_1\sin 3t + u(v) + 0.5u^3(v) \end{array} \tag{19}$$

The control objective is to drive the closed-loop system state vector to track a trajectory $x_d = [2.5\sin t, \quad 2.5\cos t]^T$ properly and to keep the control input $v(t)$ within an acceptable amplitude range and reasonably good dynamic response.

To run the simulations, assign the dead zone parameters in (5) as $b_l = -0.6, b_r = 0.5$ and $m = 1$, the friction parameters in (6) as $m = 2,\ b = 1$, and the backlash parameters in (7) as $m = 1,\ b_l = -1,\ b_r = 1$ and $u(0) = 0$. The proposed TMFRC is designed with a sliding mode manifold and PI controller (14), where the variables $(\sigma, u, \rho, u_{sw}, u_{eq})$ and parameters (k_g, k_p, k_i, δ) are listed in Table 1.

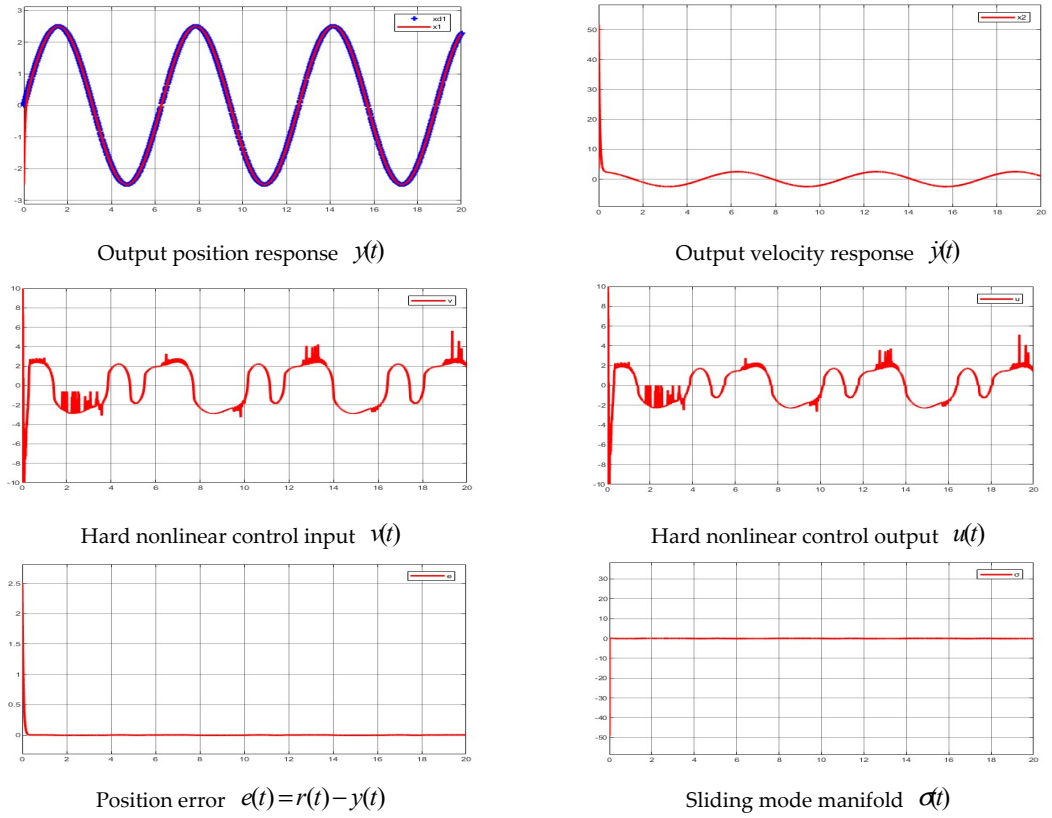
Table 1. TMFRC.

Controller	Parameters
$\sigma(x) = c_1(x - x_d) + (\dot{x} - \dot{x}_d)$	$c_1 = 20$
$\dot{\sigma} = u = \begin{cases} u_{sw} = -k_g \mathrm{sgn}(\sigma) & \forall \lvert\sigma\rvert > \delta \\ u_{eq} = \rho(\sigma) = -k_p\sigma - k_i \int_0^t \sigma d\tau & \forall \lvert\sigma\rvert \le \delta \end{cases}$	$k_g = 50, k_p = 50$ $k_i = 1, \delta = 1$

Control with dead zone input: For comparison with the model-based design approach, the TMFRC is compared to a representative robust adaptive control (RAC) with the same hard nonlinear input [10]. Note the RAC takes affine input $P: y^{(2)} = f\left(y^{(0\sim1)}\right) + bu(v)$, where the sign of the gain b is known, and the non-affine input $0.5u^3(v)$ in (18) cannot be explicitly handled. The RAC uses a model-based design by splitting the plant model into a linear nominal model plus a bounded uncertain part, which uses online adaptive nominal model estimation. However, the TMFRC has no sign request on the gain b, and does not

know the structure of $u(v)$ in its design because the whole plant is treated as a bounded uncertainty. On the other hand, RAC uses trial and error to fine-tune the parameters $k_d, \gamma, \eta, \varepsilon$, which more likely depends on the experience. The TMFRC uses trial and error to fine-tune the parameters k_g, k_p, k_i which have a large range of domains to satisfy the Lyapunov differential inequality and achieve the same output response. Accordingly, the TMFRC indicates flexible robustness in parameter tuning.

Figure 5 shows the simulation performance of the proposed TMFRC, and the simulation plots of RAC can be referred to in [10], which is not provided here due to the length of the paper. It is noted that both algorithms can achieve satisfactory control responses. Moreover, similar conclusions can be drawn up by comparing the TMFRC with other model-based approaches in coping with dead zone inputs [1].



Output position response $y(t)$ Output velocity response $\dot{y}(t)$

Hard nonlinear control input $v(t)$ Hard nonlinear control output $u(t)$

Position error $e(t) = r(t) - y(t)$ Sliding mode manifold $\sigma(t)$

Figure 5. TMFRC of dynamic plant with a dead zone input.

Control with Coulomb friction input: This demonstration is used to validate the control of the non-affine nonlinear plant (18) with Coulomb and viscous friction inputs to test the TMFRC. For a comparison with the model-based approach, consider the adaptive neural network control (ANC) with unknown friction [45]. The ANC is a data-driven model-based approach, including online pointwise modelling of the plant's input-output by a neural network and a model-based adaptive backstepping control. The TMFRC is a data-driven model-free approach, including a sliding mode manifold and a PI controller. Nevertheless, the ANC requires extra effort to train the NN to design the controller and the backstepping approach suffers from difficulty due to higher-order dynamics.

Figure 6 shows the simulation performance of the proposed TMFRC, which indicates that a perfect tracking response can be retained even in the presence of friction, and the sliding mode variable converges to zero.

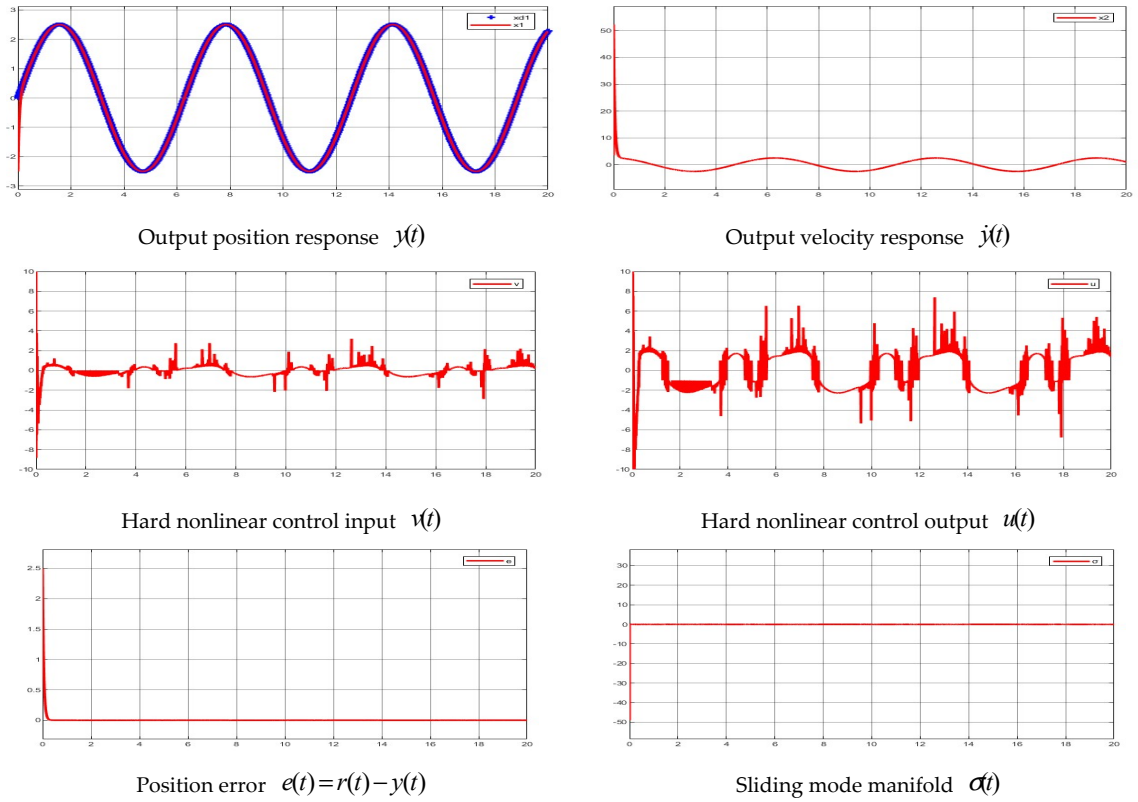


Figure 6. TMFRC of dynamic plant with Coulomb and viscous friction input.

Control with backlash input: The proposed TMFRC is implemented with the configurations given in Table 1. For comparison with model-based design approach, consider the robust adaptive control (RAC) with a backlash input [16]. The RAC deals with a class of plants with a linear nominal model plus nonlinear uncertainties and a backlash input, which assumes a linear nominal model and that the sign of the slope of the backlash is known. Thus, it is a type of data-driven model-based approach, including an online pointwise smooth inversion of the backlash and a model-based adaptive backstepping control. However, the TMFRC considers a general class of nonlinear non-affine dynamic plants with a backlash input and deems the plant as a complete uncertainty from input to output.

The simulation results of the proposed TMFRC are given in Figure 7. Clearly, similar control response can be achieved to those in the other two cases, even with an unknown backlash input.

4.2. Experiments

To showcase the practical applicability of the proposed TMFRC, practical experiments on a servo motion control are also given. Figure 8 gives the schematic diagrams of a servo control system with a permanent magnet synchronous motor (PMSM) with a load, voltage, and power of 1.8 Nm, 250 VAC, and 570 W, respectively. The three-phase inverter used to drive the PMSM is an intelligent power module (IPM) PS21A79. The main control chip on the control board is TMS320F28335 from Texas Instruments, which is connected to a PC with CCS12.0 programming environment. The rotor position is measured by an incremental encoder (resolution: 5000 pulse/r) and the phase current is measured by the TMS320F28335 ADC module. The 300 V DC bus voltage is generated by a DC power supply KPS3005D. The experimental data are sent to the PC by serial communication and processed by the MATLAB (R2024a) software. The parameters of the motor system are listed in Table 2. However, this motor system model is not used in the experiment.

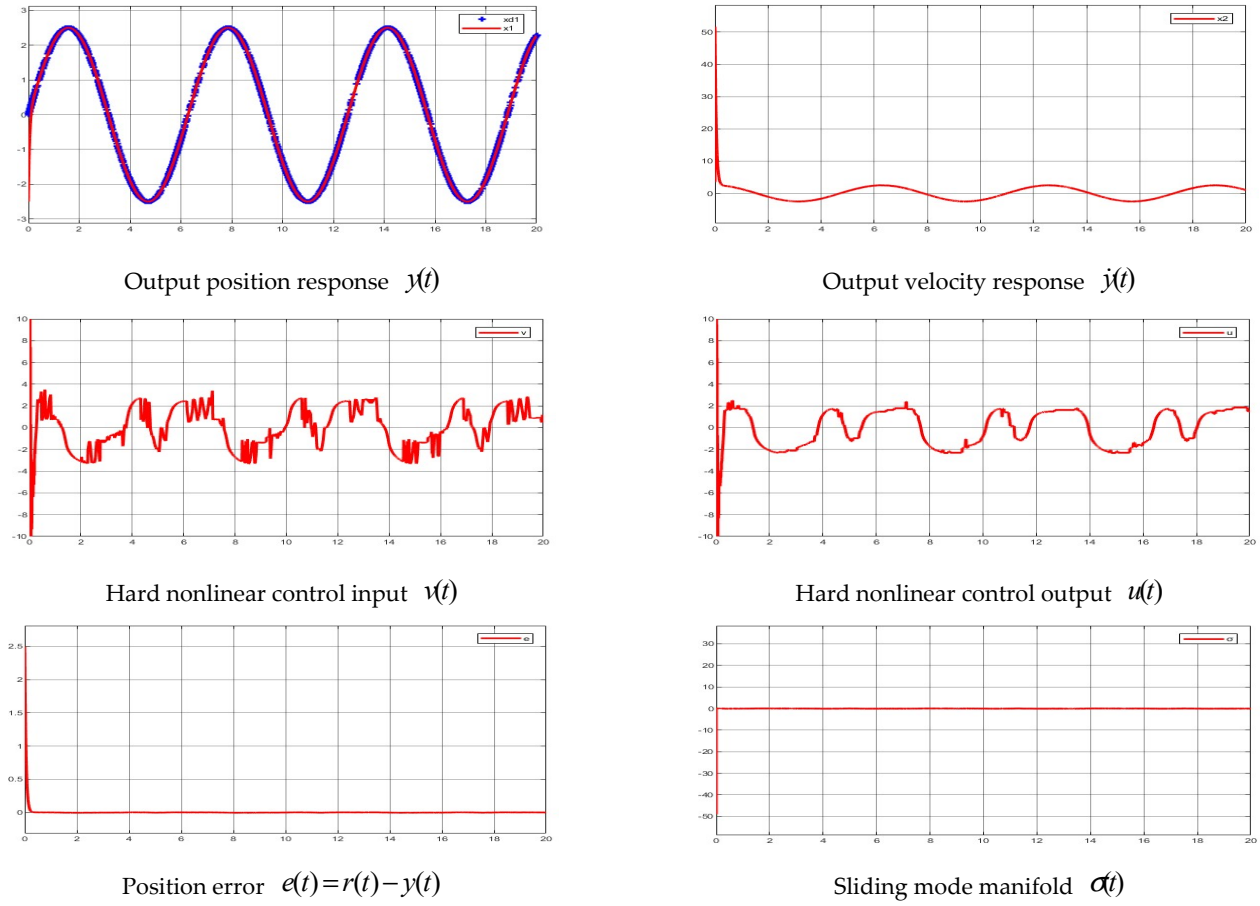


Figure 7. TMFRC of dynamic plant with backlash input.

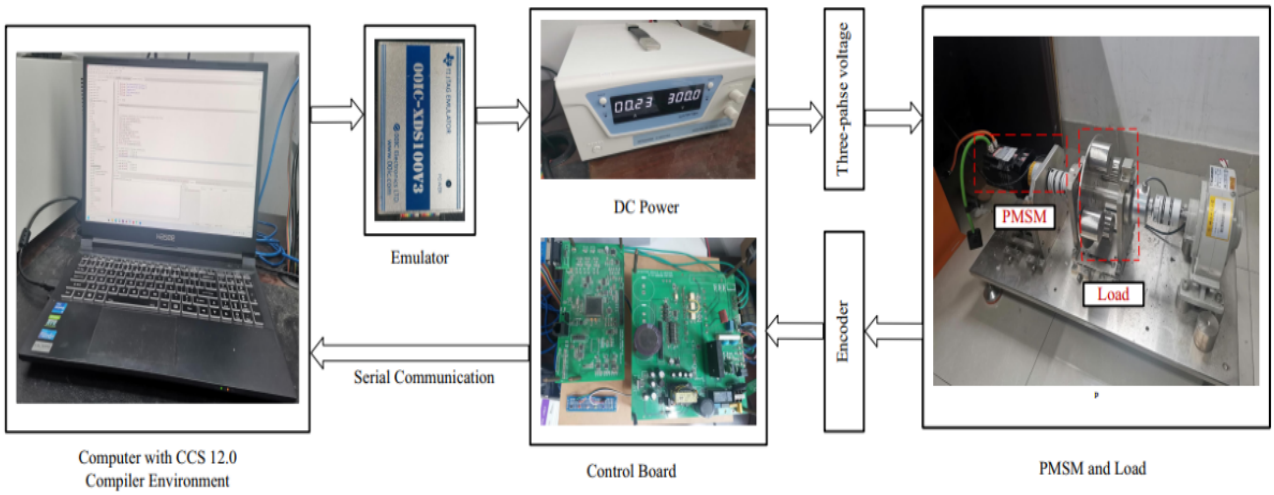


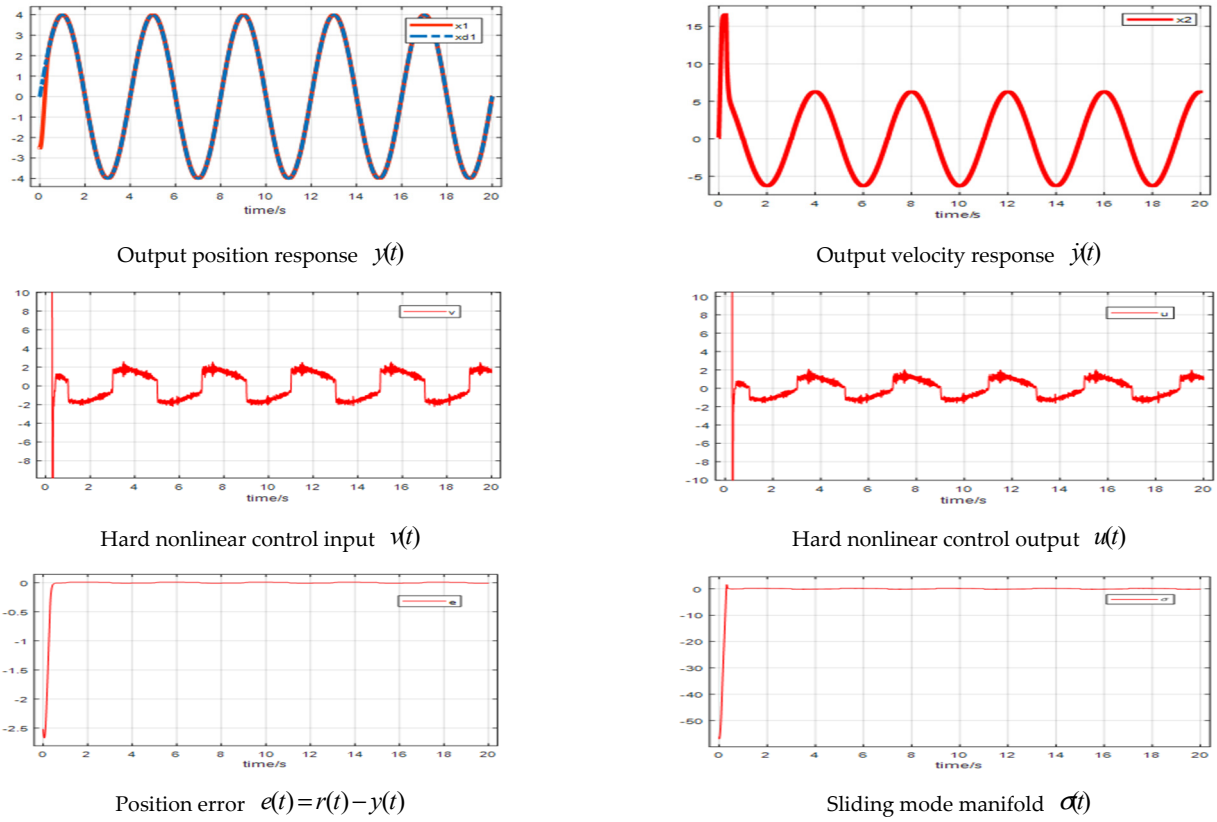
Figure 8. Schematic diagrams of the experimental PMSM servo control system.

Table 2. Parameters of the motor.

Names	Parameters	Values
Moment of Inertia	J (kg m^2)	0.000275
Flux Linkage	ϕ_f(Wb)	0.109
Pole Pairs	n_p	4
Force Coefficient	B	0.0012

For the purpose of verifying the effectiveness of the proposed TMFRC to handle the hard nonlinear input, the dead zone (5), friction (6) and backlash (7), along with the parameters specified in Section IV A, are manually added to the system during experiments. The TMFRC designed with the parameters given in Table 1 is implemented.

Figure 9 shows the real test of the PMSM servo control system for the case with the dead zone input. It can be found that this control can achieve the consistency of the simulated results given in Figure 5. Figure 10 shows the real test for the case with friction dynamics. Once again, Figure 10 shows consistency in the simulation results as shown in Figure 6. Figure 11 shows the real test of the PMSM servo control system with backlash input, where satisfactory motion tracking can also be achieved. In summary of the tests with the PMSM dynamic and the three hard nonlinear inputs, the performance is universally achieved.



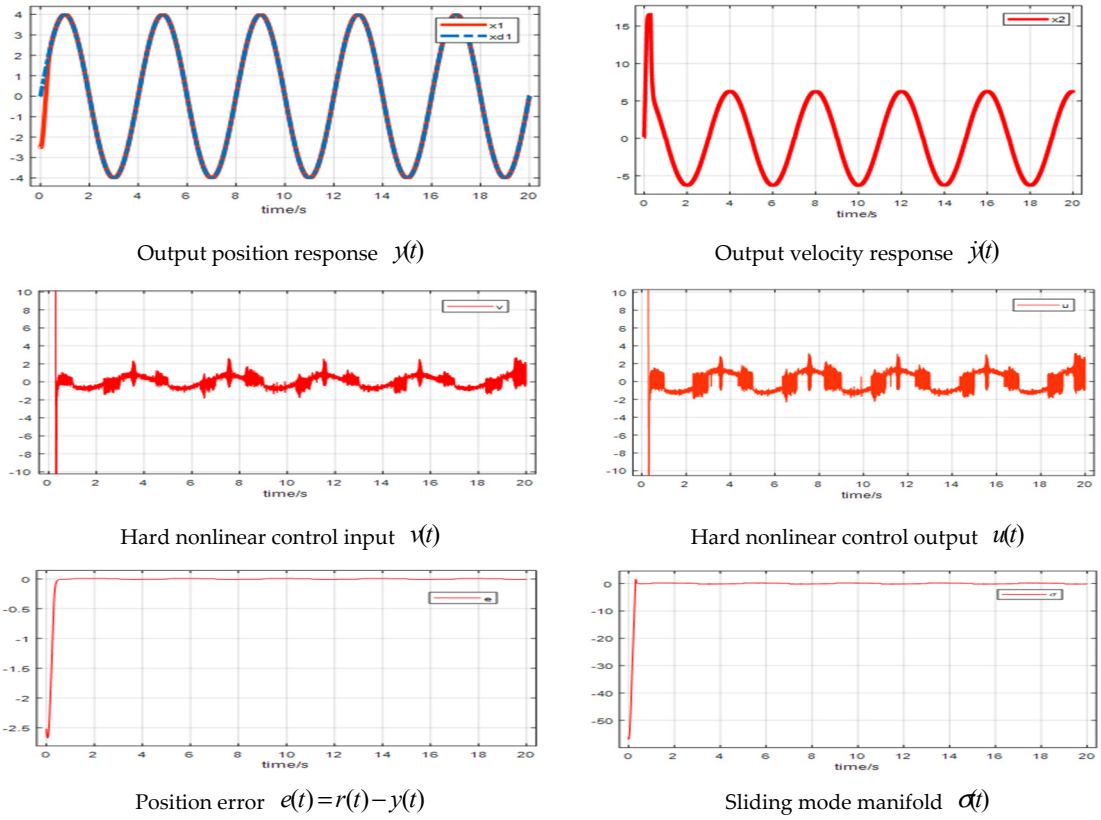
Output position response $y(t)$ Output velocity response $\dot{y}(t)$

Hard nonlinear control input $v(t)$ Hard nonlinear control output $u(t)$

Position error $e(t)=r(t)-y(t)$ Sliding mode manifold $\sigma(t)$

Figure 9. TMFRC of PMSM servo system with dead zone input.

Remark 8. *Further effective research is needed to bridge the gap between theoretical methods and practical implementation, such as control system design and tuning, where the operation mode could take the form of operator ⇔ systems, directly or interactively, to supplement the prevailing conventional operator ⇔ model ⇔ systems. Also, it could relieve the heavy demand on the plant for model accuracy and fidelity.*

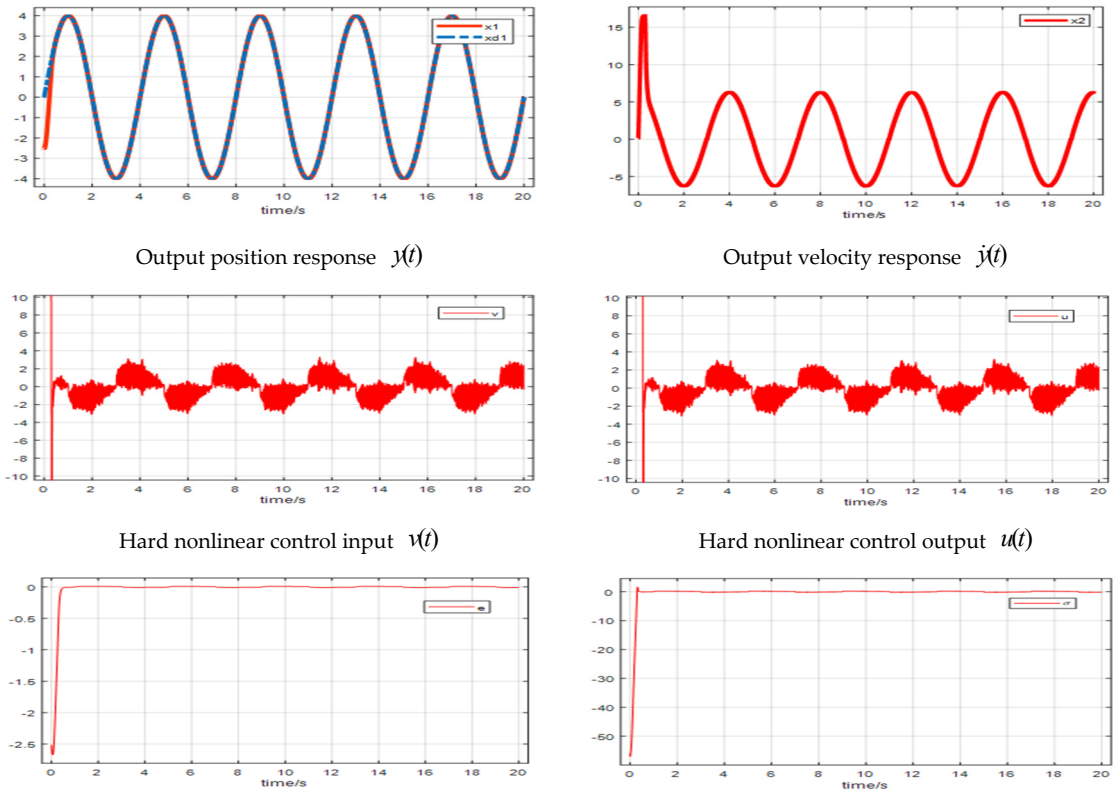


Output position response $y(t)$ Output velocity response $\dot{y}(t)$

Hard nonlinear control input $v(t)$ Hard nonlinear control output $u(t)$

Position error $e(t)=r(t)-y(t)$ Sliding mode manifold $\sigma(t)$

Figure 10. TMFRC of PMSM servo system with Coulomb and viscous friction input.



Output position response $y(t)$ Output velocity response $\dot{y}(t)$

Hard nonlinear control input $v(t)$ Hard nonlinear control output $u(t)$

Position error $e(t)=r(t)-y(t)$ Sliding mode manifold $\sigma(t)$

Figure 11. TMFRC of PMSM servo system with backlash input.

5. Conclusions

This study proposes a model-free robust control method for agnostic nonlinear systems with non-smooth inputs. The control is based on Lyapunov differential inequality. This allows for infinite choices with the same performance. It ensures total robustness and keeps system behaviour stabilisable within set boundaries. This model-free approach complements model-based designs. It also reduces the need for complex system modelling and parameter estimation. Simulations and experiments confirm the control method's effectiveness. They also show that the approach is easy to use and suitable for future applications and development. The proposed method, TMFRC, builds on the recently developed U-control that has shown promising results across various control strategies—model-based, nominal model-based, and model-free [31]. To improve simplicity and robustness in control system design, tuning, and operation, more attention should be given to new model-free methods that can solve well-defined problems typically handled by model-based methods. Some of the future work will include (1) comprehensive bench experiments to validate its applicability and find/remove pitfalls in real system operations, (2) applications in systems requiring finite-time effects, and (3) integrating the Barrier function with the control Lyapunov function to enhance system safety and stability in finite time.

The present study offers new insights and directions for research and real-world use.

Finally, one of the key motivations of this study was to provide a supplementary reference—but not a replacement—to physical model-based and data-driven intelligent approaches.

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