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Optimizing Brown Stock Washing in the Pulp and Paper Industry: A System Dynamics Approach

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Abstract: The process of pulping and papermaking is a complicated, resource-demanding operation that requires energy, water, and chemicals. When not managed properly, the process can also contribute significantly to pollution. The washing process is one critical operation that impacts the process's economics and environmental footprint. Most mills utilize rotary vacuum washers to separate black liquor from pulp, ensuring clean pulp for further processing downstream. Numerous factors influence the efficiency of a brown stock washer, and the washing operation itself is intricate. This study employs the system dynamics modeling approach to examine the critical role of brown stock washing in the pulp and paper industry, emphasizing optimizing process parameters for improved efficiency and sustainability. In the first part of the paper, a single stage of the washer system is modeled by establishing mass balance equations for key streams, including pulp, liquor, and dissolved solids. Within the system dynamics environment, separate models are developed for each stream, allowing for a detailed analysis of their behavior. To enhance modeling efficiency, the brown stock washing process is divided into four distinct operations: dilution, pulp formation, washing, and filtration. Breaking down the process into these operations makes it possible to focus on optimizing each step for improved overall performance. Furthermore, a control strategy is implemented to ensure stability in critical areas such as dilution vat level, discharged pulp consistency, and filtration tank level. In the final phase of the research, a multistage countercurrent brown stock washing line comprising three washers is modeled. Researchers can gain insights into how different components interact and influence overall performance by evaluating various parameters and analyzing the line's efficiency. This comprehensive analysis enables them to identify potential improvements and optimize the washing process for enhanced productivity and quality output. The conclusions drawn from this work offer valuable guidance for optimizing water management practices in the pulp and paper sector, contributing to the industry's sustainability goals and regulatory compliance.

Keywords: brown stock washing; system dynamics; single and multistage process modeling; modeling; wash loss; efficiency

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1. Introduction

The pulp and paper industry is undergoing a significant transformation towards sustainability due to its high resource consumption [1]. The pressing concern of climate change

has been pivotal in driving environmentally conscious technology transfer initiatives [2–4]. The pulp and paper (P&P) industry is one of the largest global water consumers, generating significant volumes of wastewater with high pollutant loads, such as chemical oxygen demand (COD) and suspended solids. Optimizing water management in manufacturing processes is essential for reducing environmental impacts at various stages, such as washer filters, bleaching, liquor clarification, cooling towers, and boilers. Implementing efficient water usage practices helps conserve this precious resource and minimizes wastewater discharge and pollution [5]. Figure 1 illustrates a diagram that depicts various aspects of water usage in a pulp and paper mill. The water, after use, turns into effluent wastewater containing contaminants, which requires treatment before discharge into the environment.

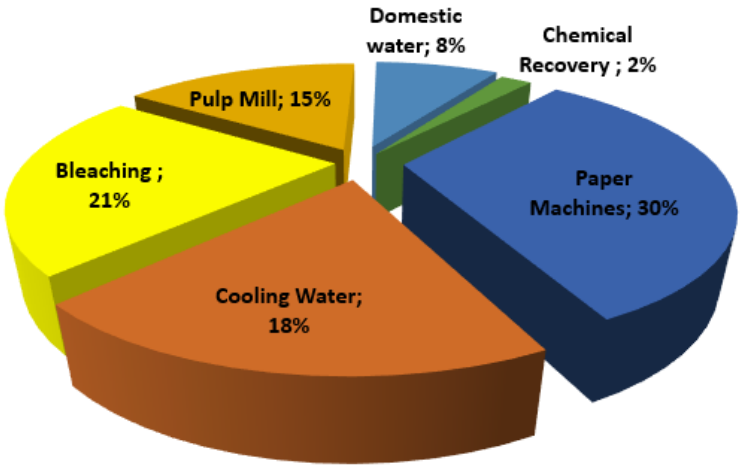


Figure 1. Water consumption in pulp and paper industry.

Many studies focus on minimizing water consumption and wastewater generation as part of sustainable environmental practices in pulp and paper processes [6,7]. The kraft pulping industry is the first known to combine pulp washing with the recovery of materials used and produced in wood cooking [8,9]. The initial motivation behind materials recovery is that the chemicals used in the kraft process are expensive compared to those employed in the sulfite process [10]. A high percentage of cooking chemicals must be recovered and reused for the kraft process to be economically viable. Another benefit of recovering the pulping chemicals and dissolved organics from the pulping process is that recovery significantly reduces the water pollution associated with the kraft cooking process [11]. Efficiency and sustainability are essential in the P&P industry, making improving brown stock washing a key focus area in this paper. Brown stock washing is an integral part of the Kraft process and is vital in enhancing pulp and paper production quality while ensuring environmental responsibility. The brown stock washing process significantly impacts the final product's quality and subsequent manufacturing steps' efficiency [12]. Effective washing ensures the pulp is devoid of impurities, which can otherwise compromise properties such as brightness, strength, and bleachability. Removing these impurities through washing enhances the pulp's quality and optimizes bleaching. By eliminating excess substances, bleaching efficiency improves, reducing the chemical requirement. This minimizes environmental impact and contributes to overall production efficiency and cost-effectiveness. Additionally, thorough brown stock washing is vital in maintaining equipment longevity by preventing buildup and corrosion caused by residual chemicals. It also promotes a cleaner production process, aligning with sustainable practices and regulatory standards within the industry [13]. Numerous other studies have addressed the scientific principles of brown stock washing [14–16].

This study aims to analyze and optimize the brown stock washing process. Modeling and control techniques based upon mass balance equations are employed to minimize wash loss and ensure optimal performance in an individual and multistage brown stock washing line. Several technical publications deal with modeling and optimizing the brown stock process. Several technical publications address the modeling and optimization of the brown stock process. Various mathematical models are explored to predict washer performance, emphasizing balancing accuracy, flexibility, and cost. Kukreja et al. [17] reviewed the complex pulp-washing process, focusing on rotary vacuum washers used to separate black liquor from pulp. Various mathematical models are discussed to predict washer performance, emphasizing the balance between accuracy, flexibility, and cost. The paper also details washing efficiency parameters, such as cake thickness, consistency, velocity, and water usage. In reference [18], Pacheco et al. detailed a calculation method for rotary drum vacuum filters applied in brown stock washing. The process is divided into fundamental steps corresponding to individual unit operations, modeled using mass balance equations. The importance of maintaining the correct displacement ratio, managing dissolved solids, and monitoring feed consistency is also discussed in the paper. The study presented in [19] focuses on the mathematical modeling of pulp washing on rotary drums using material balance equations and adsorption isotherms. The results are compared with previous analytical and numerical studies, showing good agreement. In reference [20], the authors discuss implementing a control strategy for a multistage countercurrent brown stock washing system at a kraft pulp mill. The system was simulated using Simulink to design and test the control strategy. The control strategy improved stability, reduced effluent load, and increased pulp production by 25 tons daily.

A dynamic brown stock washing system simulation was developed in [21] using PAPDYN software to analyze the system's response to various disturbances. The document discusses feedforward and feedback control strategies to manage disturbances in pulp production rate and dissolved solids concentration. The response of the washing system to changes in operating variables and process disturbances is detailed, highlighting the complexity and time required to reach steady-state conditions. A model was developed in [22] to describe the mass transfer process in the wash zone, incorporating system material balances for design and performance calculations. The research presented in this paper aims to minimize wash loss and enhance production efficiency through mass balance equations and advanced modeling techniques. The multistage brown stock washing line controls factors like displacement ratio, dissolved solids, and consistency of feed and discharge materials. System dynamics modeling is employed in the present paper to optimize the brown stock washing process.

2. System Dynamics Modeling for Process Optimization

System dynamics is valuable for comprehending complex systems' evolving interactions [23]. System dynamics are particularly beneficial in industries such as pulp and paper, where numerous processes are interconnected. By creating a system dynamics model, we can visualize how alterations in one component affect the entire system. For instance, research shows that optimizing the dilution ratio in brown stock washing can enhance washing efficiency by up to 20% while reducing chemical carryover to subsequent processes. The modeling technique provides insights into how modifications in individual elements, such as refining brown stock, can influence the overall efficiency and productivity of the mill. System dynamics modeling involves a dynamic feedback loop that captures how various components interact and influence each other within a system. For example, studies have demonstrated that controlling wash liquor flow rates and drum speed can minimize wash loss, leading to a 15% reduction in water usage and a corresponding

decrease in energy consumption during black liquor recovery. By simulating different scenarios and analyzing cause-and-effect relationships, we can make informed decisions to optimize processes, enhance performance, and mitigate risks [24]. The ability to forecast potential outcomes based on changes in parameters or policies makes system dynamics an invaluable tool for strategic planning and operational management in complex industrial settings like pulp and paper manufacturing.

Furthermore, by utilizing historical data and incorporating real-time information into the models, we can continuously improve our understanding of system behavior and make proactive adjustments to achieve desired outcomes. Mills can dynamically adjust operating conditions by monitoring parameters such as pulp feed consistency and dissolved solids concentration to maintain discharge consistency within $\pm 0.5\%$, improving overall process stability. System dynamics not only aids in identifying bottlenecks or inefficiencies but also enables decision-makers to implement targeted interventions that drive sustainable improvements in overall system performance. This holistic approach to analyzing interconnected processes fosters a more adaptive and responsive operational framework [24,25]. System dynamics modeling can be critical in enhancing the brown stock washing process within the pulp and paper industry. This modeling approach delves into various factors such as pulp and liquor flows, pulp feed and discharged consistency, dilution vat, filtration tank level, and dissolved solids to assess the efficiency of the washing process. Through simulations, system dynamics modeling can forecast outcomes, identify potential issues, and propose strategies for process enhancement. By leveraging this methodology, the operations can be finetuned to reduce costs effectively. Studies indicate that optimizing these parameters can reduce chemical losses by 10% to 15%, translating into significant cost savings and improved environmental performance. In addition to optimizing the brown stock washing process, system dynamics modeling can also aid in predicting long-term trends and potential challenges that may arise in the process cycle. This proactive approach enables us to make informed decisions based on data-driven insights, leading to more sustainable and efficient practices. Furthermore, the industry can swiftly adapt to changing market demands and regulatory requirements by continuously refining the system dynamics models based on real-time data and operations feedback. To illustrate, adapting models based on real-time operations feedback allows mills to achieve compliance with environmental standards, such as reducing effluent biochemical oxygen demand (BOD) by 20%. This adaptive capability ensures that the brown stock washing process remains aligned with industry best practices and environmental standards, contributing to overall operational excellence in the pulp and paper sector.

3. Materials and Methods

3.1. Materials

The materials used in this study primarily include the following:

- Pulp: The raw material for the brown stock washing process is sourced from a kraft pulp mill. The pulp is characterized by its initial consistency and composition, which includes cellulose fibers, lignin, hemicellulose, and residual chemicals from the pulping process.
- Wash water: Freshwater used in washing is critical for removing dissolved solids and impurities from the pulp. The quality and temperature of the wash water can significantly affect washing efficiency.
- Chemicals: Various chemicals may be used in the process, including sodium hydroxide and sodium sulfide, which are involved in the pulping process and can influence washing efficiency.

3.2. Processes

The brown stock washing process consists of several key stages, each of which is essential for optimizing pulp quality and minimizing wash loss:

1. Dilution: The pulp is diluted with wash water to facilitate the removal of dissolved solids. The dilution factor (DF) is an important parameter that influences the efficiency of the washing process.
2. Pulp formation: The diluted pulp is formed into a sheet or mat, allowing the pulp to be separated from the wash water.
3. Washing: This stage involves the application of wash water to the pulp to remove residual black liquor and other impurities. The washing process can be conducted in a single-stage or multistage configuration, with countercurrent washing being a preferred method due to its efficiency.
4. Dewatering: After washing, the pulp undergoes dewatering to remove excess water, preparing it for further processing.
5. Filtration: The final stage involves filtering the pulp to ensure the desired consistency before sending it to the next production stage.

3.3. Optimization Method

The optimization of the brown stock washing process is achieved through the application of system dynamics modeling and control strategies. The following methods were employed:

1. Mass balance equations: The core of the optimization process is based on mass balance equations that describe the flow of pulp and liquor through each stage of the washing process. The equations are formulated as follows:
   - Dilution:
   $$PR1 \times MLP1 + L6 = PR2 \times MLP2$$
   - Pulp formation:
   $$PR2 \times MLP2 = PR3 \times MLP3 + L7$$
   - Washing:
   $$PR3 \times MLP3 + WL = PR4 \cdot MLP4 + F$$
   - Dewatering:
   $$PR4 \times MLP4 = PR5 \times MLP5 + L8$$
   - Filtrate tank:
   $$L7 + F + L8 = L9$$
   - Distributor:
   $$L9 = L6 + L10$$

Here, MLPi represents the ratio between the mass of the liquor and the mass of the pulp, and PRi denotes the pulp flow rate measured in oven-dry tonnes/hour.

2. Control strategies: A proportional–integral (PI) control strategy is implemented to regulate the consistency of the discharged pulp. The control system continuously monitors the discharged consistency and adjusts operational variables to maintain optimal performance. The PI controller parameters are tuned to ensure stability and responsiveness to fluctuations in production rates.
3. Simulation and modeling: The entire process is simulated using system dynamics modeling software, which allows for the analysis of the system's response to various disturbances and operational changes. This modeling approach provides insights

into how adjustments in individual components affect overall system efficiency and product quality.

4. Performance metrics: Key performance indicators, such as wash loss, chemical consumption, and energy efficiency, are monitored throughout the process to evaluate the effectiveness of the optimization strategies.

By employing these materials, processes, and optimization methods, this study aims to enhance the efficiency of the brown stock washing process, reduce environmental impact, and achieve cost savings in pulp production.

4. Results

The purpose of brown stock washing is to separate and collect the spent cooking liquor from the cellulose fibers. The black liquor contains solids used to regenerate cooking chemicals and as an energy source in the recovery boiler. The washing system models discussed in this paper focus on rotary vacuum drums, which are widely used in brown stock washing processes. Despite the availability of various systems and equipment combinations to challenge the efficiency of rotary vacuum drums, they remain the most prevalent device for washing pulp. This is evidenced by their ability to process a higher pulp volume than other washer types. The versatility and effectiveness of rotary drum washers make them important in the industry, demonstrating their reliability and efficiency in handling large quantities of material [24]. The brown stock washing plant in a kraft pulp mill plays a crucial role in efficiently removing dissolved organic and soluble inorganic materials from the pulp after the cooking or digestion of wood chips. Its objective is to achieve this using minimal fresh or reused process water. This process has dual benefits for the pulping operation. Firstly, it prepares the pulp for subsequent bleaching treatments. Secondly, it initiates the recovery process by recycling inorganic by-products and capturing the thermal energy from organic components extracted from the wood. Utilizing minimal fresh or recycled process water aims to achieve maximum removal of impurities.

Efficient washing enhances pulping chemical recovery and reduces energy costs associated with recovery evaporation. This leads to a decreased demand for bleach chemicals. It lowers mill discharges of BOD, COD (chemical oxygen demand), and AOX (Adsorbable Organic Halides), contributing to environmental sustainability and cost-effectiveness within the pulp manufacturing process [26,27]. To model the washing system effectively, it is essential to consider individual washers and the entire washing line as a unified unit. Figure 2 overviews the critical variables for modeling a single stage or the complete washing line. The model encompasses all primary operations of the rotary vacuum drum process [13]. While determining the overall washing efficiency of the entire line is crucial, evaluating the individual performance of each washing device is equally significant for operational optimization.

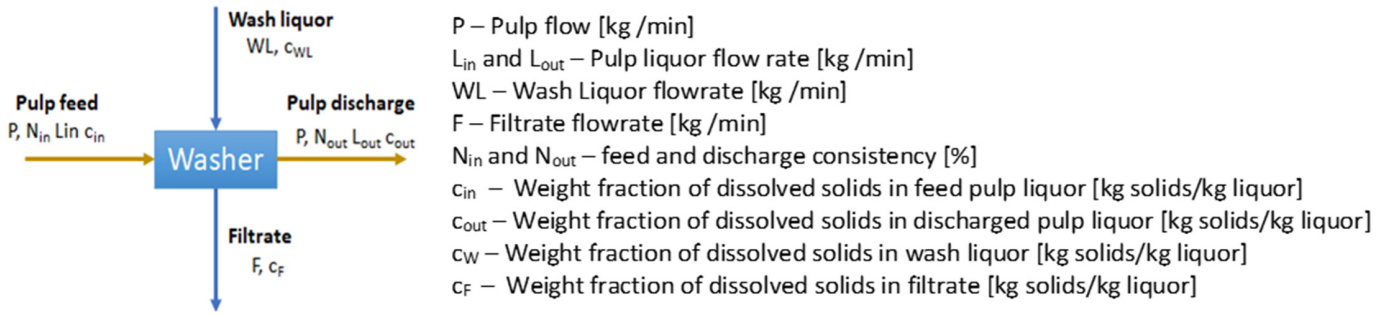


Figure 2. Overview of the key variables for modeling a single-stage washing line.

Liquor consistency is defined as the percentage, by weight, of bone-dry fiber to the total weight of a pulp slurry sample. The higher the consistency, the more solid matter is present

within the same volume of liquid. Understanding and enhancing the brown stock washing plant's efficiency improves pulp quality and promotes sustainability through resource conservation and energy recovery practices. By continuously refining these processes and systems, kraft pulp mills can elevate their operational excellence while minimizing environmental impact [28].

The system dynamics modeling tool STELLA Architect was employed to model the operations of brown stock washing as a dynamic system in an intuitive environment. The system dynamics (SD) approach is based on understanding the complex relationships among different elements within the system. Pioneered by Forrester [29], the SD methodology offers a versatile modeling and simulation framework that effectively addresses dynamic complexities in various problem domains. Its strength lies in comprehending and navigating heightened complexity, particularly from causal influences and uncertainties within a system. By focusing on the interplay between physical flows, information flows, delays, and capacity planning policies, the SD methodology illuminates the dynamics of key variables within complex systems. It aims to identify optimal decisions for expanding existing capacities to enhance long-term system profitability. Central to SD theory are two fundamental concepts: stocks and flows, which, along with feedback loops, capture the intricate dynamics within a system [30,31].

The proposed brown stock washing model endeavors to emulate core processing operations through mass balance equations, emphasizing straightforward parameterization using readily accessible data sources. The schematic representation delineates essential operations and main streams for a rotary drum vacuum system, as illustrated in Figure 3.

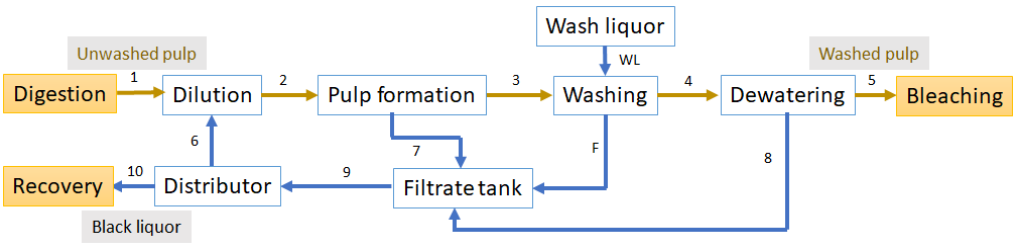


Figure 3. Schematic representation of a rotary drum vacuum system's essential operations and main streams.

The rotary drum vacuum system is a fundamental component in a Kraft pulp mill, playing a critical role in eliminating organic and inorganic materials from pulp after digestion. This sophisticated system involves several intricate unit operations, including dilution, pulp formation, washing, dewatering, filtrate tank, and distribution processes. Each of these operations contributes to the overall efficiency of the rotary drum vacuum system. One notable advantage of this system is its ability to discharge a limited amount of filtrate at high concentrations. Additionally, the countercurrent washing method employed by the system offers reasonable energy efficiency. Limiting the amount of filtrate produced minimizes thermal energy losses with the discharged filtrate. In the operation of vacuum washers within the system, unwashed pulp and washed liquor are introduced as entering streams, while washed pulp and black liquor are extracted as exiting streams. Vacuum washers typically operate at low feed consistencies ranging from 1% to 1.5% and discharge at higher consistencies between 12% and 15% [15]. The process begins with the pulp being diluted and dispersed in an inlet box before overflowing into the vat and being picked up onto the rotating drum for further processing. Efficient brown stock washing is essential for maintaining high-quality pulp production standards by effectively removing impurities and enhancing the overall cleanliness of the pulp. The careful design and operation of the rotary drum vacuum system contribute significantly to achieving these objectives while ensuring optimal

performance and resource utilization within the kraft pulp mill. The mass balance equations used for system dynamics simulation are applied to three process streams:

1. Pulp stream;
2. Liquor and wash water stream;
3. Dissolved solids.

System dynamics modeling enables a detailed analysis of the interplay between physical and information flows within the system by simulating these key process streams of brown stock washing. This approach not only aids in optimizing the process but also enhances the overall efficiency and sustainability of the kraft pulp mill. Integrating system dynamics modeling within the operations of a kraft pulp mill offers a multifaceted approach to understanding and improving system performance.

4.1. Pulp Stream Model

The mass balance equations for the pulp streams may be written as follows:

$$\begin{aligned} PR2 &= (1 - f1)\, PR1 \\ PR3 &= (1 - f2)\, PR2 \\ PR4 &= (1 - f3)\, PR3 \\ PR5 &= (1 - f4)\, PR4 \end{aligned} \quad (1)$$

where PRi is the mass flow rate (production capacity) of the pulp [kg/min], and fi is the pulp loss fraction (%). According to theoretical considerations for drum filtration, the most important parameters for pulp mat formation on the drum are drum speed, effective filtration pressure, pulp slurry temperature, washer inlet consistency, and pulp air content [32]. These variables can be selected to calculate the specific washer surface load and, by this, the washer capacity:

$$G = f(n, Nin, T, \gamma air, \Delta p) \quad (2)$$

where G = specific washer surface load that is the pulp weight per cylindrical drum surface area and day (odt/m^2/day); n = washer drum rotations (min^{-1}); Nin = pulp feed consistency (%); yair = air content in pulp feed (vol. %); T = pulp temperature (°C); Δp = effective filtration pressure (kPa).

The parameter ranges are as follows:

- Washer drum speed n = 1–4 rpm;
- Washer inlet consistency Nin = 1–2.5%;
- Effective filtration pressure Δp = 0.5–1.5 m H_2O;
- Pulp air content yair = 0–5 vol-%;
- Pulp temperature T = 30–90 °C.

The specific surface load is defined as the ratio between the pulp production rate and the surface area of the cylindrical drum. When the desired capacity and specific surface load for a washing application are determined, calculating the appropriate washer surface is straightforward: dividing the capacity by the particular surface load.

4.1.1. VAT Dilution Flow Control

The vat dilution is generally set to achieve a vat consistency of something between 1 and 2% consistency in the vat of the washer, such that the washer drum speed and vat level control can maintain a balance and produce a pulp mat with consistent thickness [32].

4.1.2. Drum Speed Control

The rotational speed of the drum is critical for enhanced washing efficiency. It has to be adjusted according to the vat level or the formation of the mat. A uniform thickness of the pulp mat throughout the washer is advantageous. A mat that is too thick will hold more dirty liquor, as the shower water cannot completely displace the dirty fluid. On the other hand, a pulp mat that is too thin can be challenging to handle because it does not effectively prevent air penetration, leading to the drum sealing over and hindering adequate drainage and liquor displacement [15]. In some washers, drum speed controls the vat level. Higher drum speeds maximize water removal from the drum vat, reducing the vat level and vice versa. Regarding production rate, doubling the cylinder speed (from 1 rpm to 4 rpm) results in a 42% increase in production. The optimal combination of drum speed and VAT consistency is key to enhancing productivity [26]. According to the control strategy presented in this paper, the drum speed control automatically adjusts to maintain the vat level set point. It is directly connected to the vat level control and the vat dilution flow control, operating within strict limits (refer to the liquor stream section below). During washer operation, the drum speed increases or decreases in response to the changes in the vat dilution flow. In the system dynamics simulation platform, the representation of pulp mat transport during operational steps involves first in, first out (FIFO) conveyors that continuously handle and release material at regular intervals (DT). The material moves along the conveyor's length, assumed to be 1 unit, over a specified transit time (Figure 4). Initially, the material is evenly distributed over the conveyor's starting edge, with a DT/transit time length. Subsequently, the material is removed from the other end of the conveyor (at the last DT/transit time), determining the outflow rate. In cases where there are leakages, the quantity of material on the conveyor, including upcoming material for removal, is reduced by an amount corresponding to the leakage rate at that point on the conveyor. With linear leakage, a consistent fraction of material is removed at each interval (DT). For instance, if the leakage fraction is 8% (0.08), 1% of the material would be extracted during each of the eight DTs it spends on the conveyor. The mass flow rate Pi of the pulp for each operation aligns with specific equations (1) that govern these processes. This thorough representation allows a detailed analysis of material flow dynamics within this simulated environment. In the SD model represented in Figure 4, each operation within the pulp stream transports a fixed quantity of pulp mat, which corresponds to the production rate and the fraction loss, along the conveyors to be eventually accumulated in the discharge site represented by dry pulp stock in the SD model. This process is illustrated in Figure 5 for two operations: pulp washing and pulp dewatering.

Beginning with the dynamic model of the pulp stream, the response of a washing stage to variations in pulp production rate and vat consistency has been analyzed. Figure 6 illustrates the drum rotational speed and the production rate response for a 0.5m step change in the VAT level. The drum rotational speed changes from 1.4 to 2.56 rpm, while the production rate changes from 300 kg/min to 563 kg/min with a 0.8s delay.

Maintaining an accurate balance between the quantity of washing water utilized and the desired cleanliness of the pulp is critical in the brown stock washing process. Excessive washing losses can prove to be detrimental for several reasons. Firstly, high washing losses can lead to significant material wastage, impacting the overall efficiency and cost-effectiveness of the production process. Moreover, excessive use of washing water increases water consumption and raises environmental concerns about water usage and disposal [33]. The liquid level is maintained in the vat at a predetermined set point, achieved when there is an equilibrium between the vat's dilution flow and the drum speed, which governs the system's hydraulic capacity. Figure 7 demonstrates how the VAT level is controlled during an operation that experiences a 13% fluctuation in the production rate.

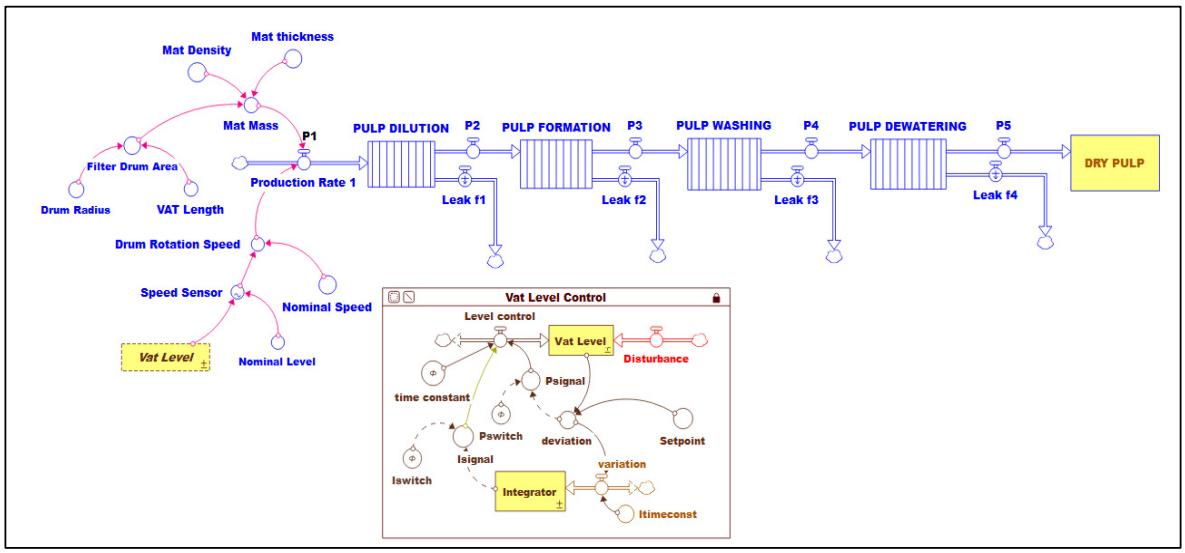


Figure 4. System dynamics model of the pulp stream.

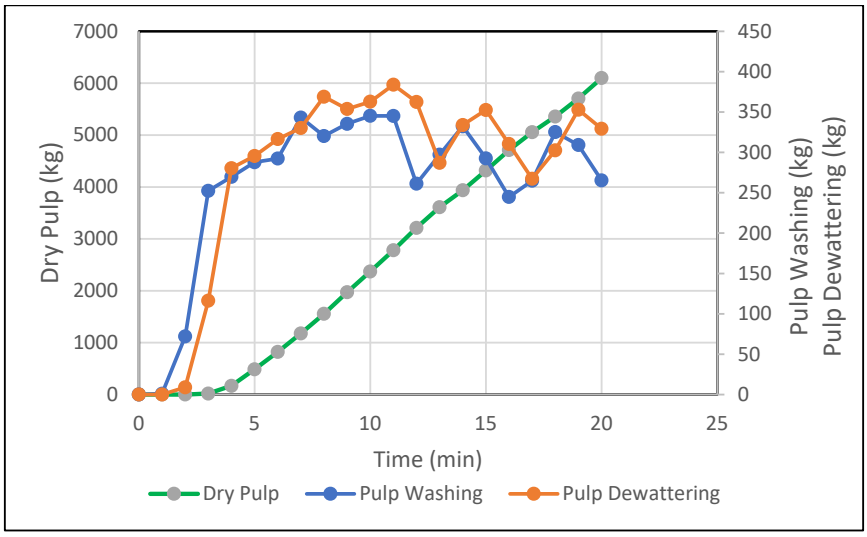


Figure 5. The process of simulating the pulp transport.

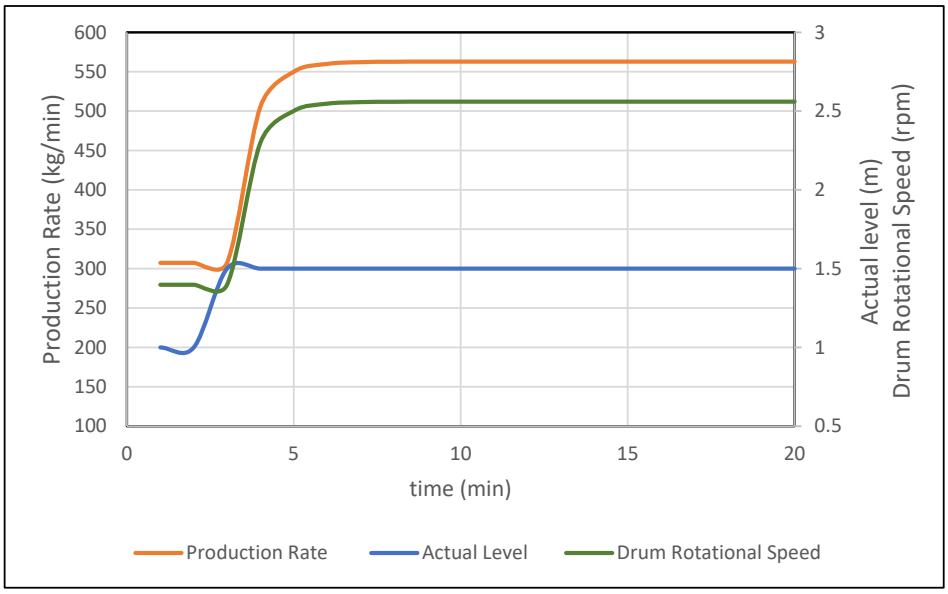


Figure 6. Step response of the drum rotational speed and the production rate.

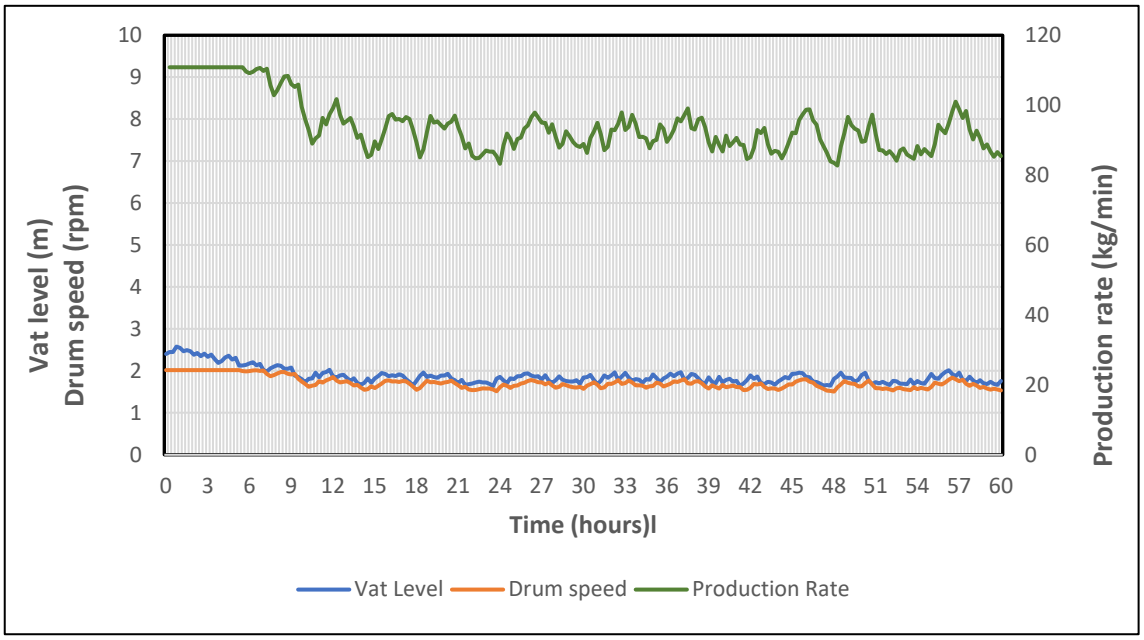


Figure 7. Controlling the vat level and the drum speed during fluctuations in the production rate.

4.2. Liquor Stream Model

Brown stock washing separates the dissolved solids into an aqueous phase called black liquor, which is sent to the evaporators. The fiber is divided into an aqueous suspension as washed pulp. This separation process consists of several stages: dilution, pulp formation, washing, dewatering, filtration, and distribution. For modeling, the mass balance equations for the liquor stream are represented for every stage using notations from Figure 2 and operations described in Figure 7 as follows [18]:

$$\begin{array}{c} \text{Dilution:} \quad P_{R1}M_{LP1} + L_6 = P_{R2}M_{LP2}; \\ \text{Pulp formation} \quad P_{R2}M_{LP2} = P_{R3}M_{LP3} + L_7; \\ \text{Washing} \quad P_{R3}M_{LP3} + WL = P_{R4}M_{LP4} + F; \\ \text{Dewatering} \quad P_{R4}M_{LP4} = P_{R5}M_{LP5} + L_8; \\ \text{Filtrate tank} \quad L_7 + F + L_8 = L_9; \\ \text{Distributor} \quad L_9 = L_6 + L_{10}. \end{array} \tag{3}$$

where M_{LPi} is the ratio between the mass of the liquor, M_L, and the mass of pulp, M_P.

The ratio between the mass of liquor and the unit mass of pulp, M_{LPi}, is calculated using the formula in [26]. If P_{Ri} is the pulp flow rate measured in oven-dry tonnes/hour and L_i is the liquor flow measured in tonnes liquor/hour, then the consistency N_i is

$$N_i = \frac{M_P}{M_L + M_P} \Rightarrow \frac{1}{N_i} = \frac{M_L + M_P}{W_P} \text{ and further} \frac{1}{N_i} = 1 + \frac{M_L}{M_P} \tag{4}$$

then the ratio between the mass of the pulp and the mass of the liquor is

$$M_{LP} = \frac{M_L}{M_P} = \frac{1 - N_i}{N_i} \text{ tonnes pulp/tonnes liquor} \tag{5}$$

The computation of mass balance equations in the SD model begins with the washing operation, where the input and output parameters used in the model simulation are as follows:

- Pulp mass flow rate, P_R = 38.5 odt/h;
- Washer discharge consistency, N_{out} = 13%;
- Washer inlet consistency, N_{in} = 4.2%;

- Dilution factor, DF = 2.0 t odt^{-1}.

Then, using Equations (4) and (5), the liquor flow rate

$$L_3 = P_R\left(\frac{1}{N_{in}} - 1\right) = 38.5\left(\frac{1}{0.042} - 1\right) = 878.16 \text{ t} \quad (6)$$

$$L_4 = P_R\left(\frac{1}{N_{out}} - 1\right) = 38.5\left(\frac{1}{0.13} - 1\right) = 257.65 \text{ th}^{-1} \quad (7)$$

$$WL = L_4 + DF \cdot P_R = 167.30 \cdot 2 = 334.65 \text{ th}^{-1} \quad (8)$$

The flow rate can be calculated from the mass balance equation of the liquor stream of the washer, $L_3 + WL = L_4 + F$.

$$F = L_3 - L_4 + WL = 878.16 - 257.65 + 334.60 = 955.16 \text{ th}^{-1} \quad (9)$$

The remaining parameters in the mass balance equations for the liquor stream can be calculated with these values. The liquor stream of the liquor stream is represented in the SD model in Figure 8. For simulation and control, the pulp washing operations are divided into 4. The pulp enters the dilution phase at 12% consistency and is diluted by recirculated filtrate to a low consistency of 1.2% before entering the pulp formation operation. The pulp liquor enters the washing phase at a 0.8% consistency. After dewatering, the filtrated liquor enters the filtration tank recirculated, while the discharged liquor leaves the system at a 12% discharged consistency.

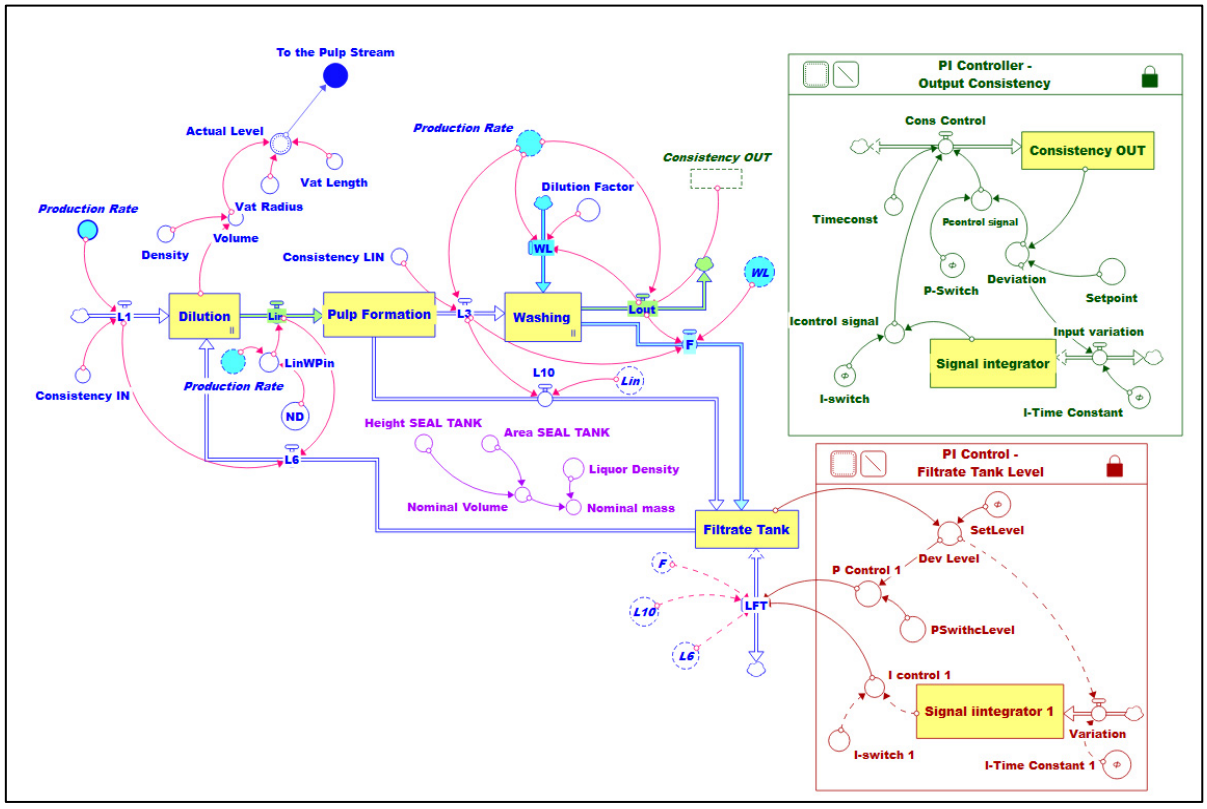


Figure 8. System dynamics model of the liquor stream.

In the pulp stream, implementing a control strategy is important for maintaining a stable level in the filtration tank and ensuring that the discharged consistency meets the required value. One effective method is a proportional–integral control function, which helps stabilize the filtration tank level and maintains a consistent discharged consistency.

This control strategy plays a key role in achieving stable level control for the seal tank, ultimately preventing issues such as foaming and overflowing that can lead to increased effluent load. Furthermore, optimizing the control strategy enhances vacuum stability, improving pulp washing and overall system efficiency. The smoother operation of the brown stock system results in reduced dissolved solids and weak black liquor concentration variability. As a direct outcome, this reduction contributes to lower chemical costs and decreased steam consumption in both the bleach plant and evaporators. Overall, an efficient control strategy addresses immediate operational challenges and has long-term benefits on system performance, cost savings, and environmental impact within the pulp production process. Figure 9 depicts the outcomes of managing the filtration tank in a controlled environment. The response of the filtration tank level to an unexpected random change in production rate is effectively regulated by controlling the outlet stream flow. Fluctuations in production rate impact the flow rate of the filtrate. As more filtrate enters the tank, the outlet stream flow rate increases accordingly, prompting the PI control to regulate the filtrate tank level to match the setpoint. To control the outlet stream's flow rate, proportional and integral control signals are combined, ensuring efficient system management. This integrated approach allows for a more stable and controlled operation of the filtration process, optimizing overall efficiency and performance.

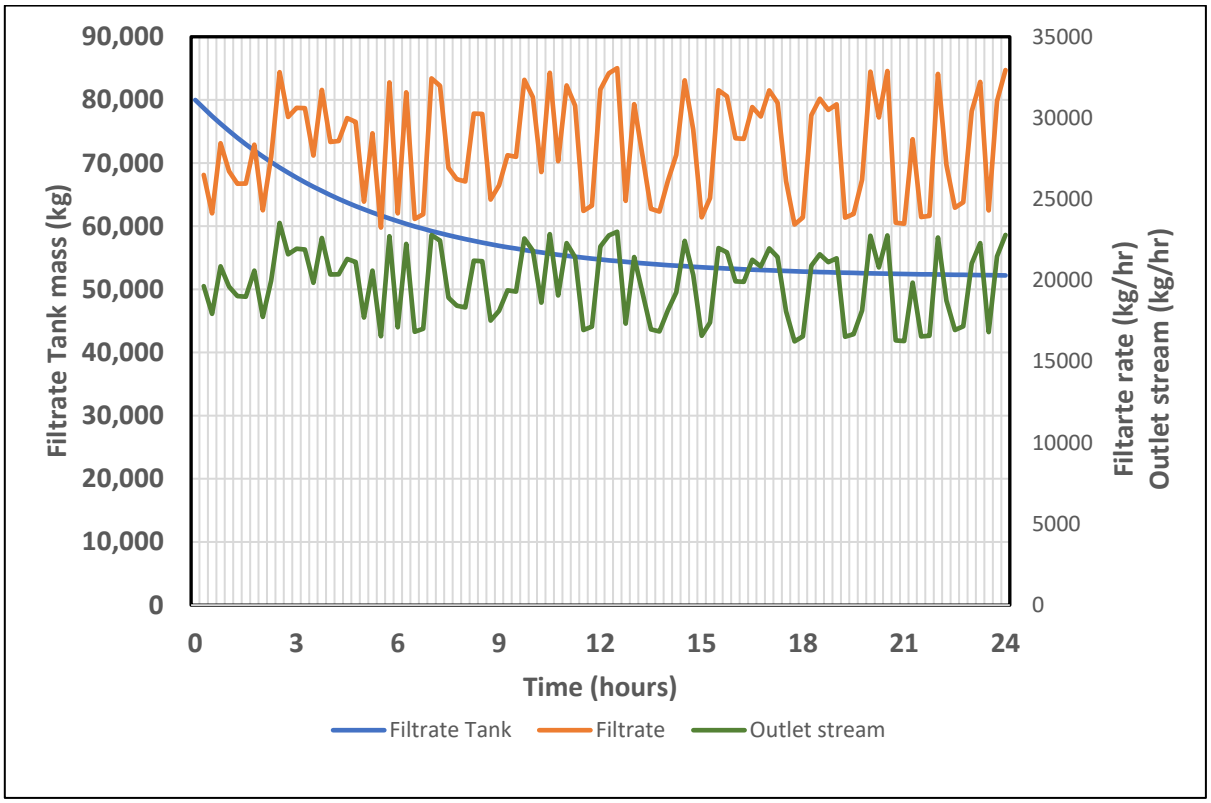


Figure 9. PI control of the filtrate tank level under disturbances of the filtrate rate and of the outlet.

In pulp processing, a PI control function is also utilized to regulate the consistency of discharged pulp. To accurately determine the percentage consistency at the discharge point of each stage, ongoing monitoring of the discharged consistency is imperative. By employing a system dynamics model of the liquor stream, it has been possible to assess how washing stages respond to fluctuations in pulp production rates and discharged consistencies. Furthermore, this model has enabled a comprehensive analysis of the impact of external process disturbances and how adjusting operational variables can influence the overall process efficiency and product quality.

Figure 10 shows the impact of a sudden fluctuation in production rate on the discharged pulp consistency value. At the onset, the discharged consistency is established at 0.32, with a designated setpoint of 0.12. The illustration in the figure delineates the reaction for varying proportional constants, maintaining a constant integral constant value of 0.25 throughout. The response depicted in Figure 11 underscores the vital role of PI control in regulating and stabilizing discharged pulp consistency levels amidst changes in production rates. Examining the graph for different proportional constant values makes it apparent how variations in this parameter can influence the system's response to disturbances. This visual representation aids in comprehending the dynamics of control mechanisms within pulp production processes, highlighting the need for optimal tuning of proportional constants to maintain desired consistency levels efficiently. Furthermore, analyzing how different constants affect the system's behavior provides valuable insights for refining control strategies and enhancing process efficiency.

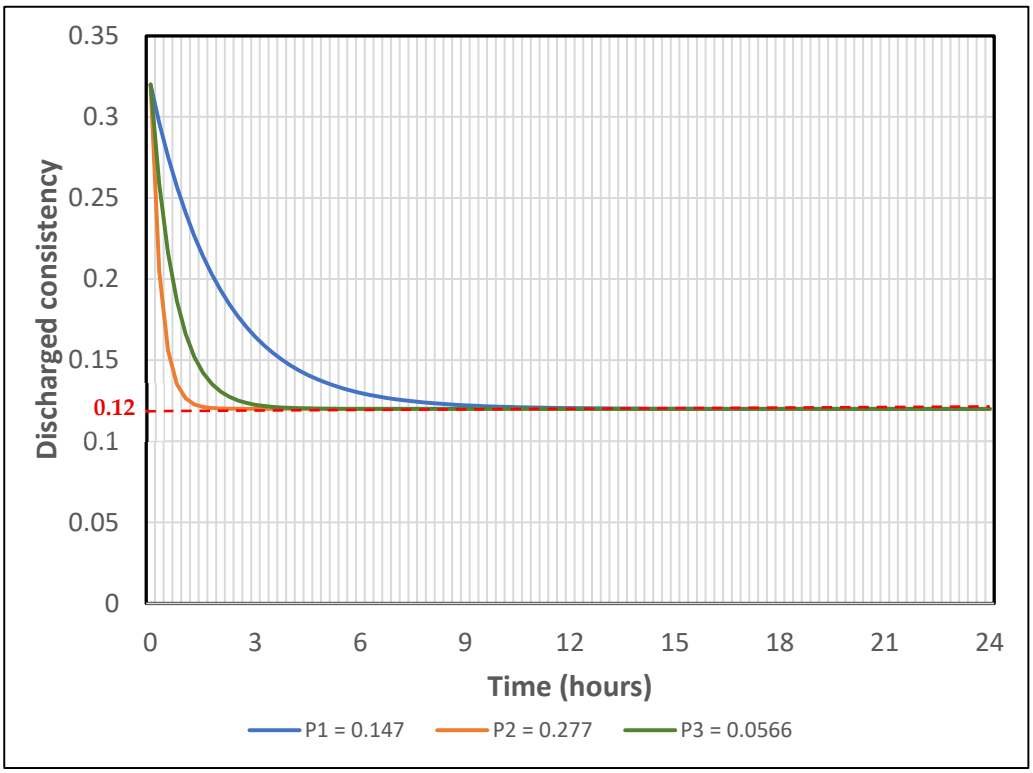


Figure 10. Discharged consistency control with 3 different P-control parameters.

The response of the discharged consistency is introduced back into the system to calculate the discharged liquor to the next stage of the multistage brown stock washing line. When the discharged consistency is not controlled, the discharged liquor constantly increases, and the liquor in the system is not balanced. Figure 12 depicts the reaction of the discharged liquor and production rate flow rates to the variation in the discharged consistency.

The liquor flow rate at the filtrate tank is kept constant, maintaining a steady dilution factor. By controlling the consistency of the pulp as it is discharged from the last washer, it is possible to adjust the liquid vat level set point, the vat dilution flow, and the speed of rotation of the drum to produce the maximum consistency of the drum as influenced by these variables and thus improve the efficiency of the process.

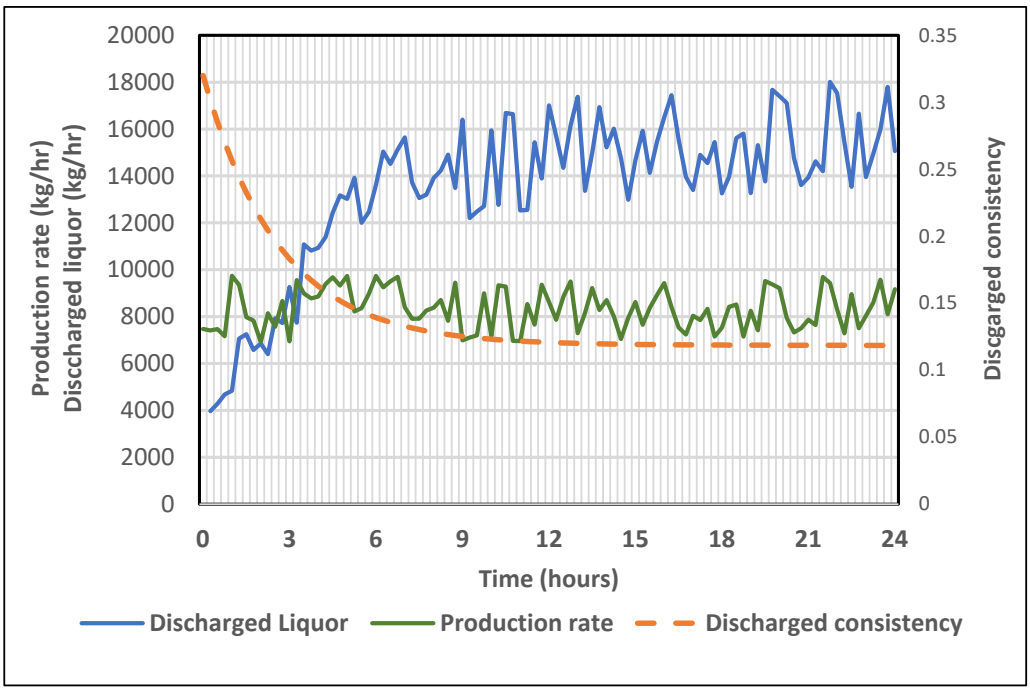


Figure 11. Control of the discharged consistency.

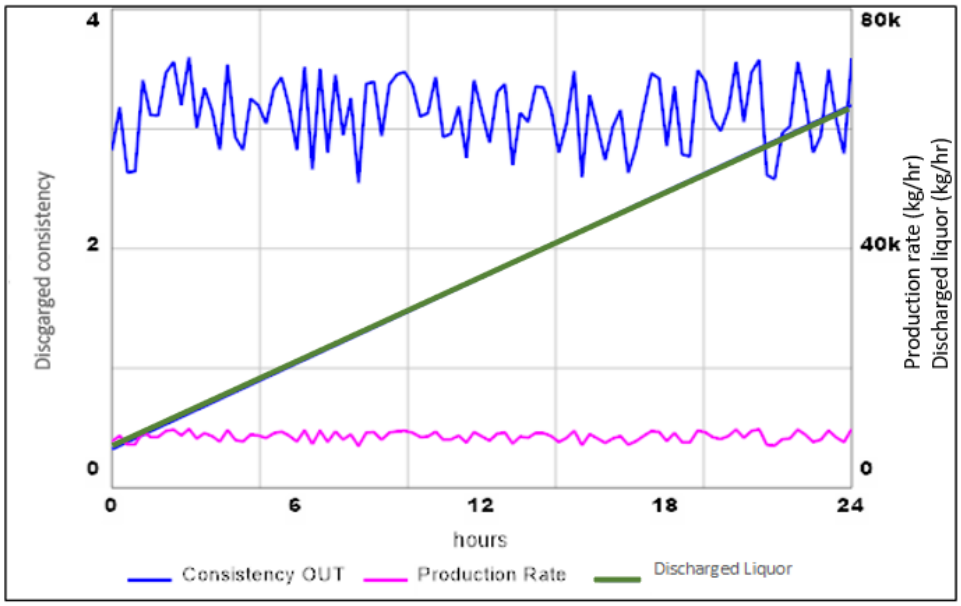


Figure 12. Variation in the discharged liquor for uncontrolled consistency.

4.3. Dissolved Solids Stream Model

The overall mass balance for a soluble substance over all the mixing stages can be written in the form:

$$L_{IN}c_{L,IN} - Fc_F - L_{out}c_{L,out} + WLc_{WL} = 0 \quad (10)$$

where L_{IN}, L_{out} = flow rate of liquor accompanying the pulp (kg min^{-1}); F = flow rate of filtrate (kg s^{-1}); c_{LIN}, c_{LOUT} = concentration of the soluble substance in liquor accompanying the pulp (kg kg^{-1}); WL = flow rate of the wash liquor (kg min^{-1}); c_{WL} = concentration of the soluble substance in the wash liquor (kg kg^{-1}); and c_F = concentration of soluble substance in filtrate (kg kg^{-1}).

Using Equation (10), the concentration of the soluble substance in the filtrate is calculated in the SD model, illustrated in Figure 13, using

$$c_F = \frac{1}{F}(L_{in}c_{in} - L_{out}c_{iout}WL_{cWL}) \tag{11}$$

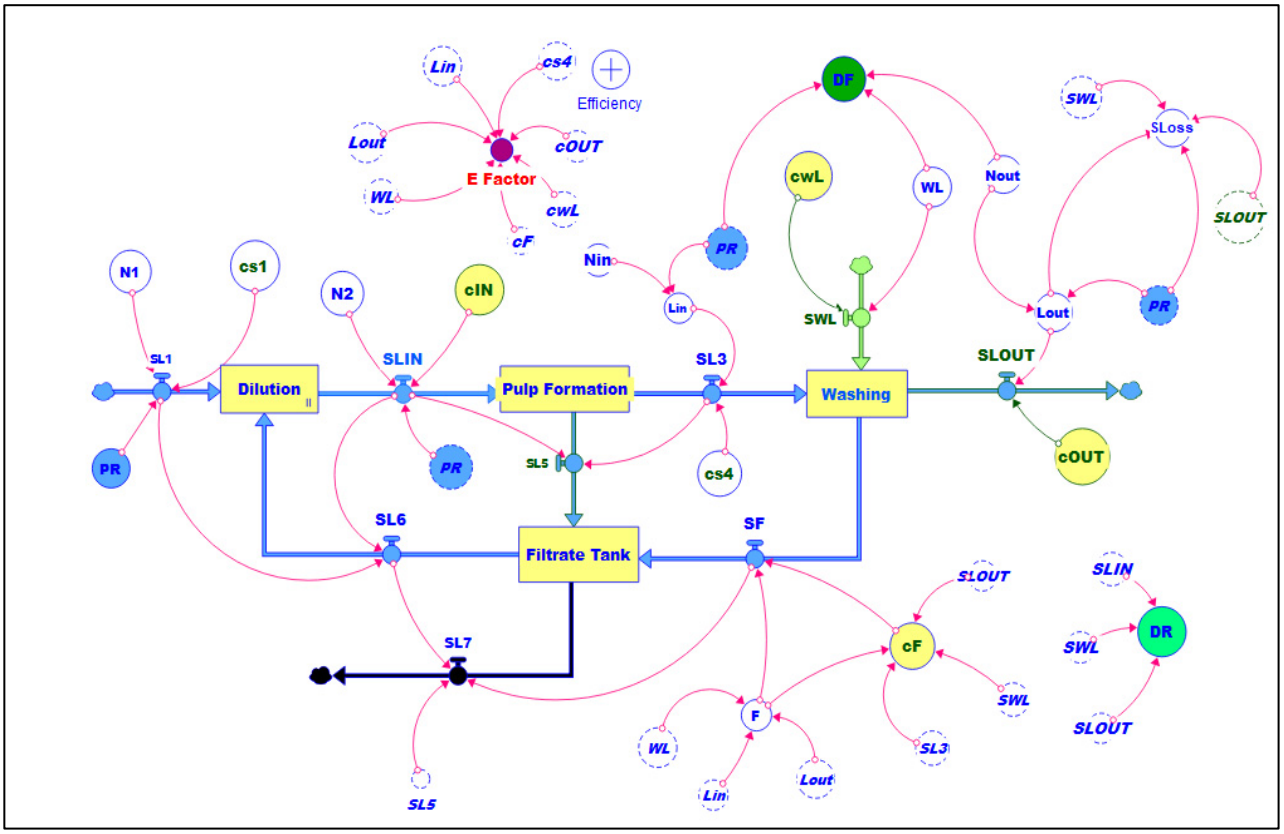


Figure 13. System dynamics model of the dissolved solid stream.

The value of the concentration of the soluble substance is c_F = 33,000 mg/kg. A meaningful measure of the washing efficiency is the displacement ratio. Using the above notations, the displacement ratio *DR* is defined as the actual washing performance related to the maximum possible washing performance at ideal displacement, expressed in concentrations

$$DR = \frac{c_{in} - c_{out}}{c_{in} - c_{WL}} \tag{12}$$

Using the notation from the system dynamics model in Figure 13, DF's dilution factor can be calculated as follows: *DF* = dilution factor (t odt^{-1}), and P_R = pulp flow rate (odt h^{-1}).

$$DF = \frac{WL - L_{OUT}}{P_R} \tag{13}$$

The displacement ratio depends on the amount of liquor used for washing, the feed and discharged consistency, and the dilution factor. Introducing random variations in the production rate and wash liquor can disrupt the dynamic system, enabling a closer examination of the relationship between key parameters within the model. Among these parameters, the system dynamics model determines the dilution factor, displacement, ratio, wash loss, efficiency factor, and the correlation between them. Illustrated in Figure 14 is the correlation between the dilution factor and displacement ratio, derived from analyzing how the system dynamics model responds to random disturbance. These findings have

been verified by field experiments documented in various literature sources. The dilution factor should be above 1 to guarantee effective washer performance, but for rotary vacuum drums, an optimal dilution factor typically falls within the range of 1 to 3.

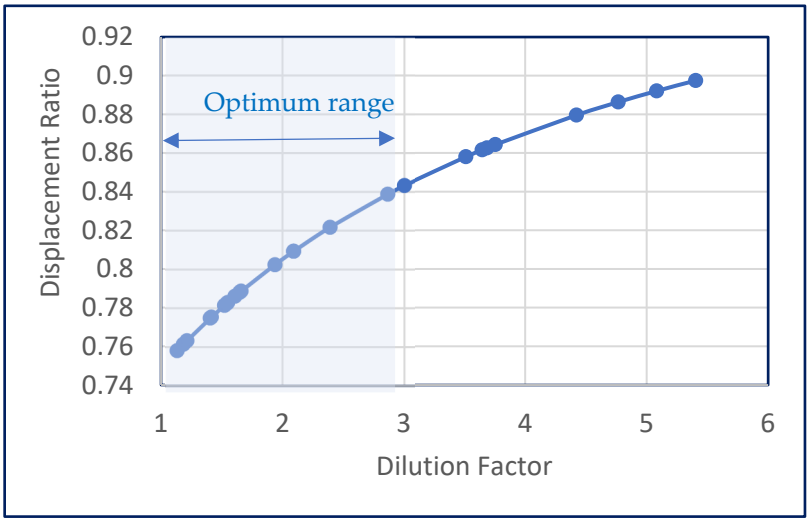


Figure 14. Correlation between dilution factor and the displacement ratio.

Wash loss is the dissolved impurities that remain in the discharged pulp slurry. The spent washing liquor containing the impurities is then collected, and the solids content can be recovered as desired. For optimum washing efficiency, cleaning with the least possible amount of wash loss and the least possible dilution of the recovered black liquor is desirable.

As calculated in the model, the wash loss per unit of pulp production (SL) is the difference between the amount of solid discharged per unit of pulp and the fraction of dissolved solids added to the pulp with the shower water:

$$SL = (c_{out} - c_{WL})\frac{L_{out}}{P_R} \tag{14}$$

where

c_{out}—solid concentration in discharged liquor mg solids/kg liquor;
L_{out}—flow rate of the discharged liquor kg/h;
c_{WL}—solid concentration in washer liquor mg solids/kg liquor;
P_R—flow rate of discharged pulp kg/h.

Figure 15 illustrates the relationship between wash loss and dilution factor computed from random disturbance. Washing loss and dilution factor are interrelated variables. For constant processing conditions and washing efficiency, the relationship of wash loss to dilution factor shows that as dilution factor increases, i.e., more wash water is used, the washing loss decreases, i.e., more dissolvable solids are recovered by washing. This figure also shows a diminishing return of dissolvable solids recovered as the dilution factor increases and more significant amounts of wash water are used. The concept of washing system efficiency, expressing the fraction of dissolved solids removed in the system, is calculated according to the Norden efficiency factor, E-factor [33]. With the notation used in the system dynamics model of the dissolved solids stream in Figure 13, the E-factor is calculated according to the formula:

$$E = \frac{log\left(\frac{L_{in}}{L_{out}}\frac{c_{s4}-c_F}{c_{out}-c_{WL}}\right)}{log\left(\frac{WL}{L_{out}}\right)} \tag{15}$$

where

c_{s4} is the concentration in pulp feed (mg kg^{-1});
c_{out} is the concentration in pulp discharge (mg kg^{-1});
c_{WL} is the concentration in wash liquor in (mg kg^{-1});
c_F is the concentration in filtrate in mg (kg^{-1});
L_{in} is the feed pulp liquor flow rate (t h^{-1});
L_{out} is the discharged pulp liquor (t h^{-1});
WL is the wash liquor flow rate (t h^{-1}).

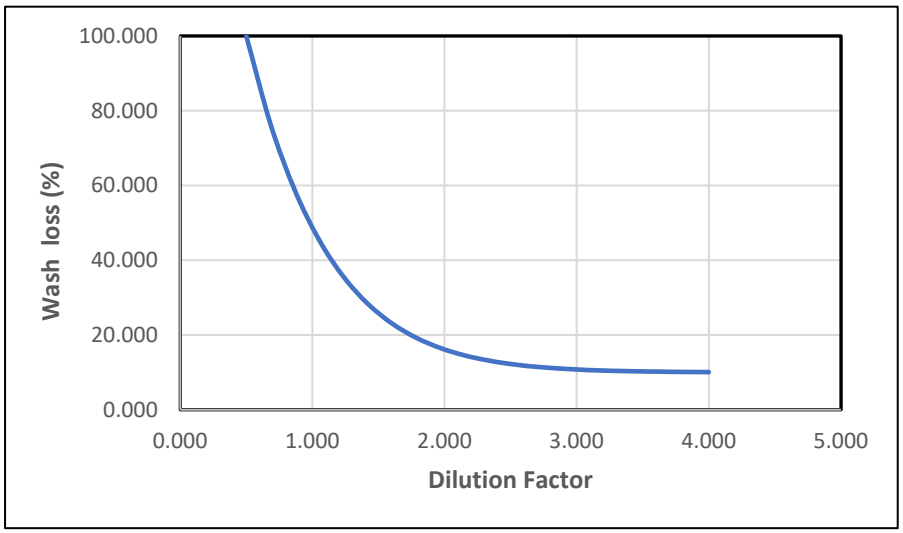


Figure 15. Correlation between dilution factor and wash loss.

Figure 16 illustrates the efficiency factor dependency on the dilution factor for a constant feed consistency Nin = 1.2%. For the optimal range of dilution factor 1 to 3, the corresponding efficiency factor range is between 11 and 22. The optimal wash liquor flow rate can be computed from the model with these values. The results are represented in Figure 17. With the selected feed pulp consistency, the optimal range of wash liquor lies between 400 and 470 t h^{-1}.

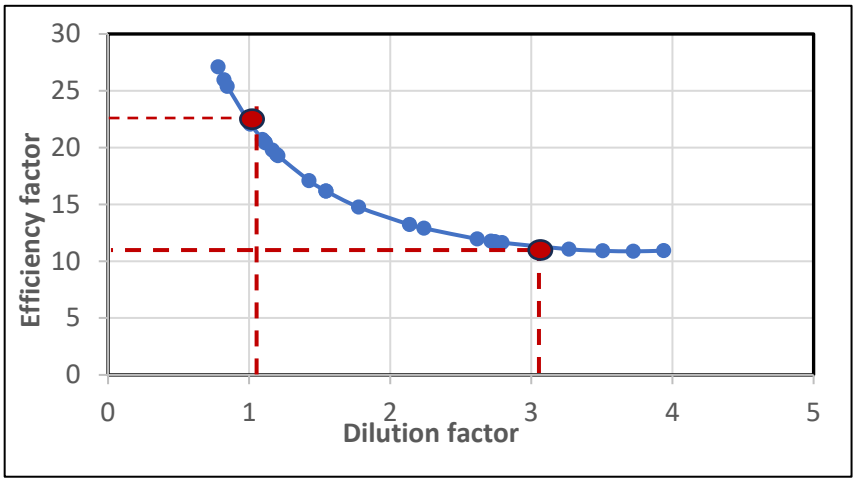


Figure 16. The effect of dilution factor on efficiency factor at a feed consistency Nin = 1.2%.

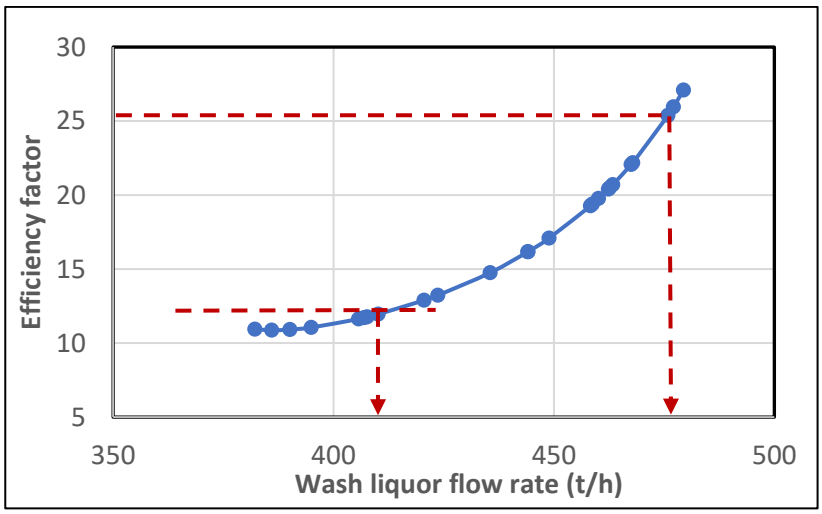


Figure 17. Selection of the wash liquor flow rate based on the efficiency factor optimum range.

5. Multistage Brown Stock Washing Line

Multistage washing is effective when a single washing stage may not suffice to achieve the necessary cleanliness. This method involves using multiple individual washers in series or employing specialized multistage washing equipment. The decision on the required stages is influenced by factors such as the desired washing efficiency, the nature of the pulp supply, the type of equipment used, and how the washing liquids are managed.

While theoretically, optimal solute removal would be achieved by washing pulp with fresh water in each stage of a multistage system, this approach results in a significant volume of highly diluted filtrate, making it impractical for industrial applications. Modern multistage wash plants have adopted the countercurrent principle to enhance efficiency. In this setup, the wash medium flows toward the pulp flow [33]. The countercurrent design ensures that the pulp comes into contact with the cleanest available wash liquor before exiting the final washing stage. The filtrate from the last stage is redirected back towards the initial stages against the pulp flow, serving as wash liquor for each subsequent stage until it reaches the first washing stage again. As a result, the filtrate from the initial stage has accumulated higher concentrations and is suitable for further processing in other areas of the facility, such as an evaporation plant. After the third-stage washing cycle, the purified pulp stock is gathered in the storage tank and awaits transfer to the bleaching plant. This innovative approach optimizes resources and minimizes wastage in industrial washing processes. The system dynamics methodology is employed in this paper to examine the effectiveness of a multistage washing line equipped with three washers, depicted in Figure 18. The model incorporates all three material streams, comprehensively analyzing the system's operations. The pulp stream line's input is represented by the vat's actual level, which is regulated by the drum speed of the washers. By employing this methodology, the system dynamics approach allows for a detailed examination of how different variables interact within the system, offering a holistic perspective on its functionality and potential areas for improvement. Cost savings in brown stock washing necessitate control of the dilution factor and efficiency.

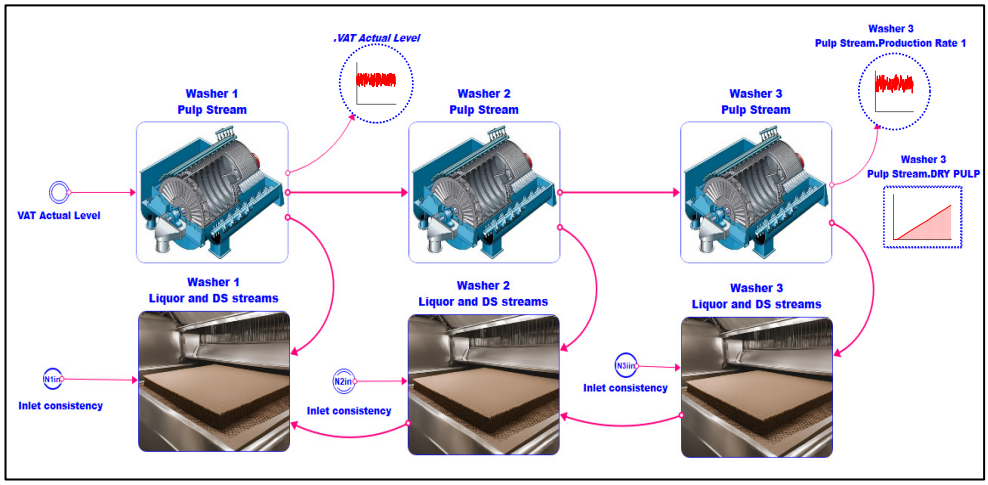


Figure 18. Multistage brownstock washing line.

Figure 19 presents the efficiency of the multistage washing line as a function of the inlet consistency. The efficiency factor is calculated using the Norden approach [34]. The overall mass balance for a dissolvable solid substance over all the mixing stages can be written in the following form:

$$L_o\, c_{L,0} - F_1 c_{F,1} - L_3 c_{L,3} + F_3\, c_{F,3} = 0 \tag{16}$$

where L_i is the flow rate of liquor associated with the pulp (kg s^{-1}) at the inlet in the first stage and the discharge in the last stage, respectively; F is the flow rate of filtrate (kg s^{-1}); $c_{L,\,i}$ = concentration of the soluble substance in liquor associated to the pulp (kg kg^{-1}); and $c_{F,\,i}$ is the concentration of the soluble substance in the filtrate (kg kg^{-1}).

$$E = \frac{log\left(\frac{L_{in}}{L_{out}}\frac{c_{in}-c_{out}}{c_{out}-c_{WL}}\right)}{log\frac{WL}{L_{out}}} \tag{17}$$

where WL is the wash liquor flow rate in t h^{-1}, c_{in} is the concentration in pulp feed, kg kg^{-1}; c_{out} is the concentration in pulp discharge kg kg^{-1}, and c_{wL} is the concentration in the wash liquor.

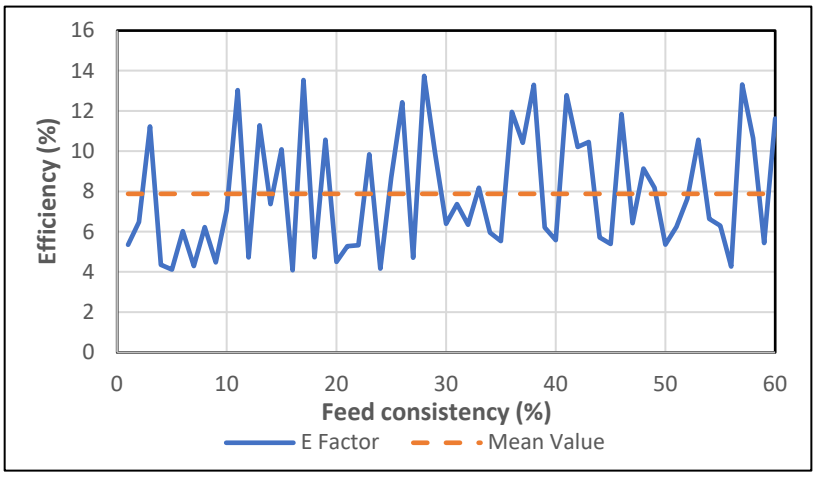


Figure 19. Efficiency of the brown stock washing line.

6. Discussion

Countercurrent washing is a widely recognized and efficient technique used in multi-stage kraft pulping to enhance chemical recovery and reduce environmental impact. This method, involving the flow of black and white liquor in opposite directions, allows for the effective extraction of chemicals and impurities from the pulp, thereby improving the overall efficiency of the pulping process. Mills can recover valuable pulping chemicals while minimizing water consumption and chemical waste by optimizing the flow dynamics in countercurrent systems. These advantages are crucial for the sustainability of kraft pulping operations in the paper industry. As illustrated in the brown stock washing line model (Figure 18), our findings show a significant improvement in wash liquor savings—an 8-ton per hour reduction in wash liquor consumption in a countercurrent configuration compared to a conventional configuration (Figure 20). This result is consistent with previous studies that have demonstrated the benefits of countercurrent washing. For example, Kamal et al. (2011) [34] found a similar reduction in wash liquor usage when switching to countercurrent configurations in kraft pulping systems, highlighting the method's potential to improve chemical recovery and reduce environmental impact. Additionally, Mori et al. (2019) [35] reported that countercurrent washing reduces water consumption and enhances chemical recovery efficiency, aligning with our observations in the current study. The improvements observed in our study further emphasize the importance of countercurrent washing in optimizing pulp mill operations. While previous investigations have focused on the theoretical benefits, our results demonstrate how countercurrent washing can save operational resources when scaled to industrial production rates. These findings support the growing body of literature advocating for countercurrent systems to improve the economic and environmental performance of kraft pulping operations. More to the point, as a prospective study, sensitivity analysis on key parameters like dilution factor and feed consistency to validate model robustness could be performed.

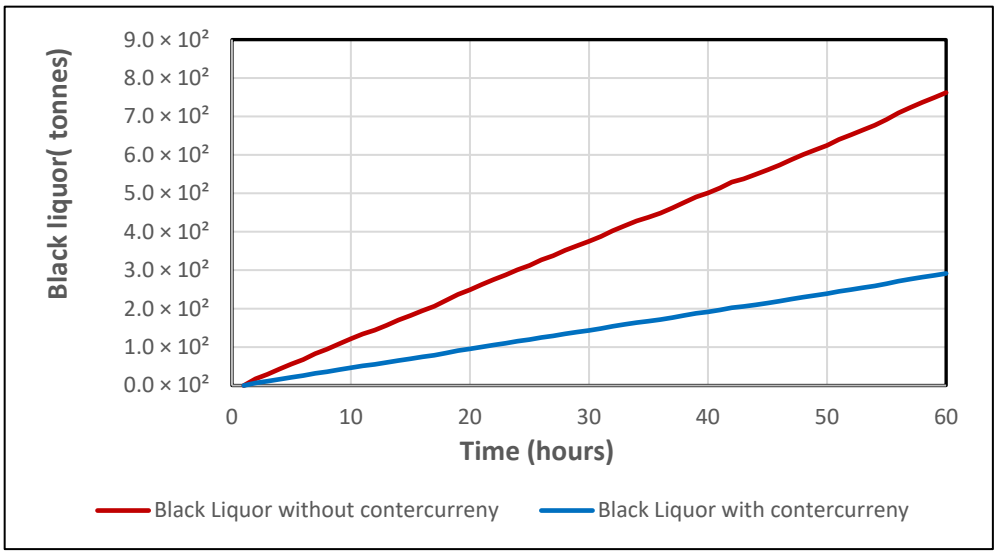


Figure 20. Water savings using the countercurrent technique.

In the brown stock washing process, ensuring the thorough removal of cooking chemicals during the washing process is critical. However, the efficiency of washing is typically evaluated based on two key parameters: wash loss and dilution factor. A lower dilution factor reduces energy consumption and contributes to cost savings in operations. On the other hand, a higher dilution factor results in a cleaner pulp output, which is essential for maintaining product quality standards. This relationship between the dilution factor and process efficiency is clearly illustrated in Figure 21, where it can be observed that as the pro-

duction rate increases, process efficiency also rises. However, an increased wash liquor flow rate leads to decreased process efficiency. Finding the right balance between these parameters is imperative to optimize washing efficiency and achieve desired outcomes.

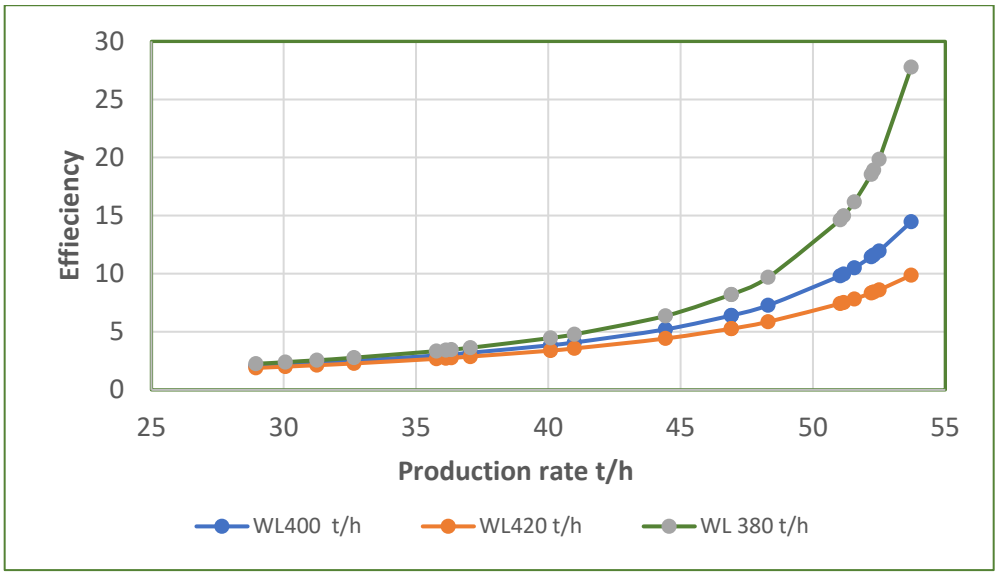


Figure 21. Process efficiency depends on the production and wash liquor flow rates.

In a countercurrent washing line, an important function that characterizes the efficiency of a rotary drum pressure washer is the relative washing capacity, which, according to the empiric expression (2), depends on four parameters: washer drum rotation rate, pulp feed consistency, temperature, air content in feedstock, and the adequate filtration pressure.

Maintaining the ideal inlet consistency in the pulp mat formation process is essential for achieving uniform fiber distribution. A consistent feed consistency of around 1.25% results in a fiber mat with even thickness and porosity throughout the drum's surface. However, it is essential to note that feed consistency affects fiber distribution and significantly influences power consumption and equipment design. When feed consistency is low, large volumes of dilution filtrate must be pumped to various points before reaching the washer inlet, increasing pumping energy requirements. The amount of filtrate that needs to be removed in the dewatering and mat-forming zones of the washer is also determined by feed consistency. Lower inlet consistencies necessitate the removal of larger quantities of filtrate [16]. Lower inlet consistencies require larger filtrate tanks, impacting overall washer capacity per Equation (2) [36]. An illustration depicting the correlation between inlet consistency and rotary pressure washer capacity can be seen in Figure 22. In addition to affecting fiber distribution and power consumption, maintaining optimal inlet consistency plays a significant role in the efficiency and operation of the pulp mat formation process. It is essential to ensure consistent quality output, minimize energy wastage, and optimize equipment performance in paper mills.

Washer drum speed is typically between 1 and 4 rpm. The faster the drum rotates at a fixed production rate, the thinner the pulp mat becomes, resulting in potential drum sealing and increased rewet potential. Ideally, the pulp mat should be about 25–50 mm thick to ensure a good lift from the air doctor, resulting in a clean face wire going back into the vat to pick up another mat. As inlet consistency increases, the drum speed typically must increase to maintain the fiber mat thickness that produces optimal washer performance [37].

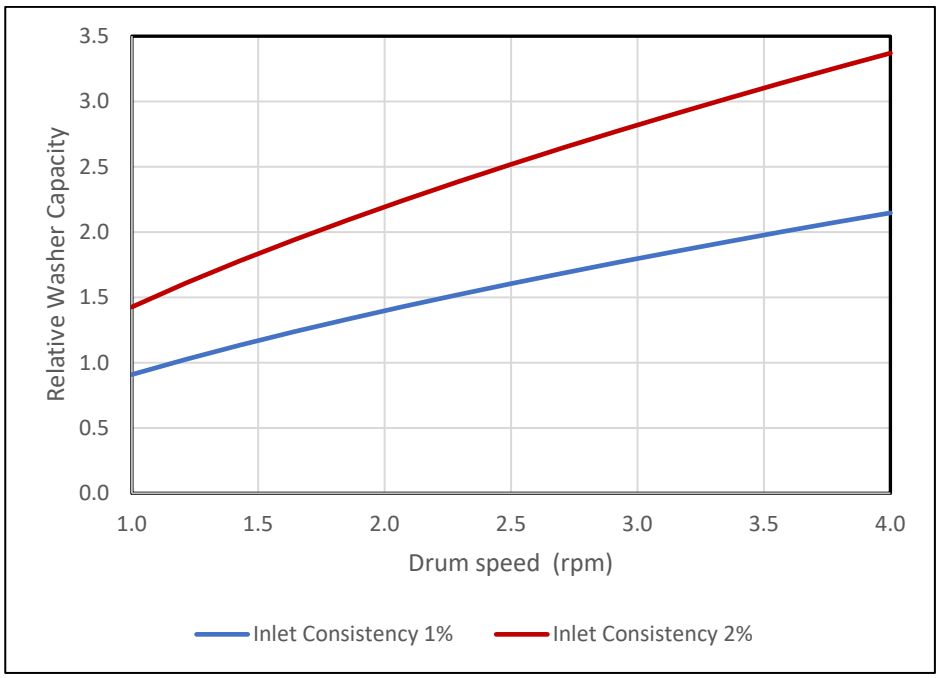


Figure 22. Effect of drum rotational speed on the relative washer capacity.

7. Conclusions

The optimization of the brown stock washing process is critical for improving pulp quality, enhancing production efficiency, and meeting environmental standards within the pulp and paper industry. This research employs system dynamics modeling to provide a structured and adaptable approach to streamlining the process. The investigations aimed to analyze and optimize the brown stock washing process to minimize wash loss and ensure optimal performance across individual and multistage washing lines. Key variables for modeling a single-stage washing line included pulp flow rate, washing liquor flow rate, and the amount of solids in washing liquids. These parameters formed the foundation for mass balance equations used in system dynamics simulations. VAT dilution flow and drum speed control were also incorporated into the model to enhance accuracy and performance. Dividing the process into four distinct operations—dilution, pulp formation, washing, and filtration—further improves modeling efficiency and enables a deeper understanding of system interactions. Implementing PI control techniques is pivotal in minimizing wash loss and maintaining stability while evaluating the system's response to disturbances across various parameters. A notable outcome of this research is reducing wash water usage, processing 200–300 tons of water per hour, significantly minimizing energy consumption and operational costs while maintaining high product quality. Additionally, this study emphasizes sustainability by aligning operations with environmental goals, improving water management practices, and extending equipment lifespan through reduced corrosion and buildup. This research provides a replicable methodology for process optimization across the pulp and paper industry by offering actionable, model-based insights and bridging the gap between theory and practice. Its contributions set new benchmarks for sustainable and efficient operations, underscoring the effectiveness of system dynamics modeling in refining the brown stock washing process and advancing resource conservation, energy efficiency, and environmental compliance.

Author Contributions: Conceptualization, all authors; methodology, B.K.; software, B.K.; validation, B.K.; formal analysis, all authors; investigation, all authors; resources, all authors; data curation, all authors; writing—original draft preparation, B.K.; writing—review and editing, all authors;

visualization, all authors; supervision, M.E.B. and H.E.Z. All authors have read and agreed to the published version of the manuscript.

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