

## Article

# Heat Transfer Modeling of Geothermal Wells in Fractured Aquifers Considering the Joule–Thompson Effect

Mengmeng Li <sup>1</sup>, Gang Bi <sup>2,\*</sup>, Yu Shi <sup>2</sup> and Yang Wang <sup>1</sup><sup>1</sup> College of New Energy, Xi'an Shiyou University, Xi'an 710065, China; 190605@xsyu.edu.cn (M.L.); wyang2235@126.com (Y.W.)<sup>2</sup> College of Petroleum Engineering, Xi'an Shiyou University, Xi'an 710065, China; 181206@xsyu.edu.cn

\* Correspondence: big@xsyu.edu.cn; Tel.: +86-150-9175-9039

**Abstract:** Geothermal energy, as a clean, low-carbon, widely distributed, renewable and environmentally friendly energy source, plays an important role in the transition from traditional energy sources dominated by coal and oil to clean energy. Ground source heat pump technology is a key technological tool for developing geothermal energy for widespread use. Coaxial-cased heat exchangers are the core component of the ground source heat pump system, and their heat transfer performance directly affects the heat transfer efficiency and service life of the ground source heat pump system. According to the actual working conditions of coaxial-cased heat exchangers in fractured aquifers, the coupled pressure–temperature model of the heat transfer outside the borehole was created by considering the influence of the Joule–Thompson effect. For heat transfer inside the wellbore, a multi-layer long concentric cylinder wall model was developed to obtain the fluid temperature distribution within the wellbore. Results show that the heat transfer efficiency increases with the increase of thermal conductivity, water production and effective permeability of fractures. The positive and negative values of the Joule–Thompson coefficient reflect the trend of fluid temperature changing with pressure. The larger the absolute value is, the greater the temperature change is. The increase in the initial temperature of the injected water will lead to a decrease in the theoretical heat transfer. With the increase of the water injection rate and horizontal wellbore length, the heat recovery power will also increase significantly, but the optimal value needs to be considered comprehensively. The findings of the study can not only lay a theoretical foundation for the performance evaluation and optimal design of coaxial-cased heat exchangers but also have great significance in promoting the efficient development of geothermal energy.

**Keywords:** geothermal well; heat transfer model; coaxial-cased heat exchanger; Joule–Thompson effect; fractured aquifer



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## 1. Introduction

With the changes in the world's energy landscape, the supply–demand conflict in the fossil energy market has continued to intensify, energy prices have been at a high level of volatility, and the development of clean and renewable energy sources to better ensure the security of energy supply has become a mainstream trend [1–3]. Geothermal energy is a renewable, carbon-free and environmentally reliable source of energy. For many countries, geothermal energy reduces their dependence on imported energy sources while reducing atmospheric pollutants such as carbon particles and greenhouse gases. According to statistics, of the approximately 90 countries in the world with available geothermal energy, only 24 countries are developing geothermal energy, and the existing utilization rate is less

than 15 percent [4]. The current exploration and development level of geothermal resources is not yet high, and there is a great potential for development. In the exploitation process of geothermal energy, in order to avoid problems such as the decline of geothermal water level and the shortening of thermal storage life due to the difficulty of recharge [5], the project often adopts the water retention and heat transfer technology, sending the circulating water to the concentric tube heat exchanger, and realizes the mode of heating the outer layer of circulating water through the deep rock thermal storage to realize taking heat without water for medium and deep layer of geothermal energy [6]. Concentric tube heat exchanger mainly realizes heat transfer through heat conduction, convection, radiation, etc. It is a kind of heat exchanger that adopts inner and outer tubes arranged concentrically, with sufficient space between two tubes so that the fluid can form a flow between the inner and outer tubes. It mainly utilizes the heat energy in the formation and the work materials to exchange heat without extracting or recharging groundwater in large quantities, which reduces the potential impact on the groundwater level and water quality and has little impact on the groundwater resources. In order to better develop geothermal energy as well as to alleviate the current resource critical situation, it is necessary and realistic to study the heat transfer mechanism of concentric tube heat exchangers.

In the study of the heat transfer model of concentric tube heat exchangers, many scholars have carried out in this field. The results of the study proved the rationality of the concept of coaxial-cased heat exchangers as well as the great potential of geothermal energy utilization and achieved a lot of meaningful results [7–9]. The coaxial-cased heat exchanger is an important component in the process of extracting medium and deep geothermal heat, and the construction of a heat transfer model is very important for the design and performance analysis of the coaxial-cased heat exchanger. Thus, how to construct the heat transfer model has received a lot of attention [10–12]. The heat transfer process in the coaxial-cased heat exchanger is mainly divided into two parts. The first part is the heat transfer process between the fluid inside the borehole and the wall of the wellbore. The second part is the heat transfer in the formation outside the borehole. Considering the complexity of the geological conditions and the depth of the heat exchanger, numerical analyses, semi-analytical and analytical methods are used to study the heat transfer in the two parts.

Roland N. Horne [13] was the first to assume that there is only heat conduction in thermal storage and constructed a one-dimensional steady-state heat transfer model, which allowed us to calculate the outlet temperature of the working fluid in the heat extraction cycle. Kohl et al. [14] revealed that the interfacial thermal resistance between the cementitious material and the casing has a detrimental effect on heat transfer by constructing a 2D unsteady state heat transfer mathematical model. Zanchini et al. [15] proposed a novel 2D non-stationary heat transfer model by COMSOL Multiphysics 3.4 software and found that the heat transfer characteristics of the cementitious material have a great influence on its heat transfer characteristics. Beier et al. [16] proposed a transient heat transfer model based on Laplace transform method. This model can simulate both the surrounding soil temperature and the transient state change process of the pipes inside the heat exchanger. Holmberg et al. [17] evaluated the thermal power of concentric tube heat exchangers and showed that the effect of the flow direction of the circulating mass on the heat extraction power decreases when the well depth decreases. Saadi et al. [18] simulated and investigated the thermal performance of different nanofluids as geothermal borehole heat exchanger media and found that there exists a specific ratio of inner and outer tube diameters that minimizes the pressure loss of the media in the heat exchanger. Morchio et al. [19] developed a heat transfer model based on a hybrid approach of thermal resistance concept and finite difference to study the heat transfer performance of deep coaxial heat exchangers in dense urban geothermal applications. Luo et al. [20] proposed

a hierarchical segmented finite line source model to simulate the fluid and formation temperature distribution in a coaxial-cased heat exchanger. This model considers the fact that the fluid flow in the borehole is characterized by dynamic boundary conditions and in-homogeneous temperature boundaries on the well wall, which allows the calculation of the excess temperature transferred by the heat source when passing through multiple formations depending on the location of the heat source and the calculation point. This approach provides a simple and effective way of dealing with layering.

Regarding the study of heat transfer models for fractured reservoirs, Barrenblatt [21] proposed a novel mathematical model to characterize naturally fractured reservoirs, which provided strong theoretical support for the subsequent study of heat transfer models for fractured reservoirs. Since then, the research focus has gradually shifted to the heat transfer characteristics in porous media, and the calculation methods of the wellbore temperature field have been explored in depth. Subsequently, Ramey [22] proposed a landmark methodology aimed at accurately predicting the spatial distribution of fluid temperature inside the wellbore. The method assumes that the fluid temperature is affected by the depth of the well and the transient heat transfer mechanism of the surrounding formation and derives an equation for the temperature distribution in a production well by introducing the concept of the integrated heat transfer coefficient. The presentation of this classical method has stimulated the interest of many scholars, who have carried out their own research for the calculation of temperature distributions based on the method and theory. Given the complexity of heat transfer mechanisms in porous media, traditional analytical models are difficult to provide satisfactory solutions when dealing with practical challenges. In order to more accurately model and analyze heat transfer phenomena in porous media, most scholars have turned to numerical analysis methods for their studies.

Stefensen R.J. [23] supposed that the presence of the Joule–Thomson effect may have a significant impact on the accuracy of temperature profiles and temperature logging data after well shut-in. In fact, the temperature profile after shut-in is not only directly affected by changes in the downhole flow profile but also indirectly affected by the Joule–Thomson effect. Miller et al. [24] proposed a classical reservoir temperature model, i.e., the initial transient temperature model, which considers the dynamic effects of the inflow or outflow of fluids in the wellbore on the reservoir temperature change and lays a solid foundation for subsequent reservoir thermodynamic studies. The Ramey method was subsequently generalized by Alves [25] to make the model more comprehensive and applicable to real gas situations. This generalization allows the model to handle multiphase flows, arbitrary real gases, and a variety of dip cases, enhancing its applicability and accuracy in formation temperature prediction. Alive's work provides a more flexible and comprehensive tool for reservoir temperature modeling, facilitating research and applications in the field of reservoir temperature. Hagoort. J. [26] pointed out that there is a significant deviation between the temperatures calculated by the Ramey model between the early transient state and the actual observed temperatures. They modified the Ramey wellbore heat transfer model to improve its accuracy and reliability.

Muradov, K.M. [27] emphasized that temperature information should be considered as an important complement to pressure data, which plays a vital role in the analysis of formation thermodynamic characteristics. In addition, their results further indicate that the temperature data show higher reliability and more accurately reflect the characteristics of the subsurface region compared to the pressure data in multiple completion operations. Nozomu Yoshida [28] developed a flow and thermal model of a transversely fractured horizontal well system under single-phase gas flow conditions. They concluded that wellbore temperature is sensitive to both fracture geometry and conductivity but exhibits different characteristics over time. Thus, real-time post-fracture temperature data

measurements provide a basis for evaluating the creation of fracture parameters. H.L.C. Galvão [29] developed a mathematical model of heat transfer in vertical and inclined well electro-stripper systems based on black oil fluid models for single-phase and multiphase flows. John Killough [30] proposed three computational methods for predicting the outlet temperature of a concentric two-pipe heat exchanger, and the results show that the constant fluid characteristic model could not accurately estimate the pressure loss of the system. They proposed a novel, fully analytical solution to economically analyze to calculate the optimum insulation thickness and mass flow rate, minimizing the cost of power generation. Joachim F. Oppelt [31] compared the transient temperature distribution obtained from the finite element analysis software ANSYS-FLUENT 6.3 with a CFD model. The results show that the new model has significant advantages over the CFD model in terms of computational time and power. Magnus Wangen [32] divided a coaxial-cased heat exchanger into multiple segments and used a semi-analytical approach to simulate and analyze the temperature distribution and heat transfer patterns in a multi-segmented coaxial heat exchanger. Segmented models can be used instead of numerical methods to simulate complex borehole heat transfer processes and make calculations easier.

The analysis shows that there is much research on the heat transfer model and heat transfer mechanism of concentric double-tube heat transfer, but there are few studies on the heat transfer characteristics of geothermal wells in fractured aquifers. In this paper, the heat transfer characteristics of fractured aquifers are analyzed by proposing the coupled pressure–temperature heat transfer model considering the Joule–Thompson effect. At the same time, by developing a multi-layer long concentric cylinder wall model in the borehole, the heat transfer characteristics and heat exchange efficiency of the geothermal well are analyzed. The analysis of heat transfer characteristics inside and outside the wellbore can improve the theoretical knowledge of concentric tube heat exchangers in geothermal wells in fractured aquifers and expand the practical application range of concentric tube heat exchangers.

## 2. Modeling of Heat Transfer Outside the Wellbore

The geothermal heat exchanger is one of the most critical components of the ground source heat pump system, which realizes the collection and utilization of underground heat energy by connecting the evaporator underground and the indoor condenser. Geothermal heat exchangers can be divided into horizontal buried pipes and vertical buried pipes according to the different forms of arrangement. Horizontal buried tube geothermal heat exchangers are susceptible to climate change for heat transfer performance and take up large areas of upland resources. Therefore, vertically buried pipes for coaxial-cased heat exchangers are more widely used. In order to study the heat transfer performance of coaxial-cased heat exchangers in fractured aquifers, a heat transfer model outside the borehole is developed based on the theory of heat transfer and groundwater seepage, as seen in Figure 1.

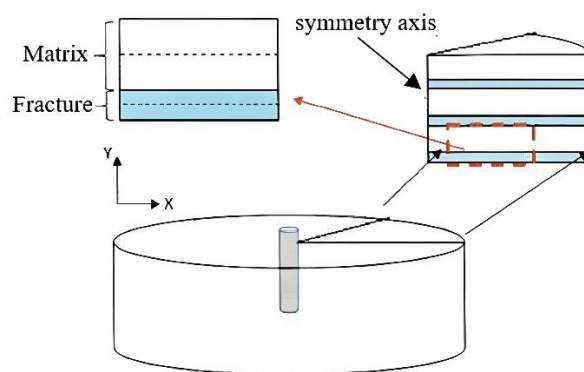


**Figure 1.** Physical model of heat transfer outside the wellbore.

### 2.1. Physical Model

- (1) There is a geothermal well in the center of the infinite aquifer producing at a constant flow rate. The thermal physical properties and the initial temperature of the formation are homogeneous. The surface temperature remains constant and equals to the value of the initial formation temperature.
- (2) The formation is a homogeneous fractured artesian aquifer, and the dual pore medium formed by matrix and fractures is characterized by the Kazemi plate model.
- (3) The formation around the wellbore is homogeneous, and the fluid physical properties and the rock's thermodynamic properties remain unchanged during the process of heat exchange between the fluid in the wellbore and the formation.
- (4) The temperature change in the continuity equation is negligible. Only the flow in the horizontal direction is considered, and the gravity effect is neglected.
- (5) The change of fluid temperature due to fluid expansion caused by a pressure drop is considered; i.e., the model includes the influence of the Joule–Thompson effect.

Since the temperature at the top of the formation is constant, the fluid flow in the aquifer can be approximated as an isenthalpic flow process. Changes in fluid pressure will cause compression or expansion of the fluid volume, resulting in the Joule–Thompson effect, affecting the change of temperature field. Therefore, the pressure field model is coupled with the temperature field model to develop the heat transfer model outside the borehole with the consideration of the Joule–Thomson effect.

### 2.2. Pressure Field Modeling

The fractured porous media are characterized by the Kazemi flat plate model [33], and in the dual-medium model, the flow in the matrix can be neglected and considered as  $K_M = 0$  since  $K_F > K_M$  and  $\varphi_M > \varphi_F$ . The fluid mass change due to fluid compressibility in fractures is negligible compared to the fluid mass change due to seepage along the pores. The differential equation for seepage in fractured aquifers is:

$$\frac{K_F}{\mu} \operatorname{div}(\nabla p_F) + \frac{\alpha}{\mu} (p_M - p_F) = 0 \quad (1)$$

$$\phi_M C_M \frac{\partial p_M}{\partial t} + \frac{\alpha}{\mu} (p_M - p_F) = 0 \quad (2)$$

Eliminating  $p_M$  by combining Equations (1) and (2), the pressure distribution equation describing the seepage process in fractures can be obtained:

$$C_0 \frac{\partial p_F}{\partial t} - \operatorname{div} \left[ \frac{K_F}{\mu} \nabla p_F + \eta C_0 \frac{\partial}{\partial t} \nabla p_F \right] = 0 \quad (3)$$

where the coefficient  $\eta$  is a characteristic quantity that characterizes fractured rocks; as  $\eta$  tends to zero, the unit volume of the matrix shrinks, and the development of rock fractures increases. When  $\eta$  tends to infinity, then the above equation is transformed into a pressure distribution model describing the fluid elastic unsteady seepage in a single porous medium. The aquifer is a completely penetrated infinite formation. Therefore, the well diameter is small and negligible. The seepage flow is axisymmetric, then the seepage control equation [34] can be expressed as:

$$\frac{\partial p_F}{\partial t} - \eta \frac{\partial}{\partial t} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p_F}{\partial r} \right) \right] = \eta \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p_F}{\partial r} \right) \quad (4)$$

Initial conditions:

$$p_F(r, 0)_0 = p_i \quad (5)$$

Inner boundary conditions:

$$-\frac{Q\mu}{2\pi K_F h} = \left( r \frac{\partial p_F}{\partial r} \right)_{r \rightarrow +\infty} + \frac{\eta}{\zeta} \frac{\partial}{\partial t} \left( r \frac{\partial p}{\partial r} \right)_{r=0} \quad (6)$$

where,

$$\zeta = \frac{K_F}{\mu(C_{\phi M} + \phi_0 C_\rho)} \quad (7)$$

When  $t = 0$ ,  $\left( r \frac{\partial p_F}{\partial r} \right)_{r \rightarrow +\infty} = 0$ , the final form of the boundary conditions can be expressed as:

$$\left( r \frac{\partial p_F}{\partial r} \right)_{r \rightarrow 0} = -\frac{Q\mu}{2\pi K_F h} \left( 1 - e^{\frac{\zeta t}{\eta}} \right) \quad (8)$$

Introducing Laplace transform, the Equations (4)–(8) can be simplified as

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{dU}{dr} \right) - \frac{\lambda}{\zeta + \lambda\eta} U = 0 \quad (9)$$

$$\left( r \frac{dU}{dr} \right)_{r=0} = -\frac{\zeta}{\lambda(\zeta + \lambda\eta)} \quad (10)$$

Substituting the boundary conditions gives:

$$U_{(r,\lambda)} = \frac{\zeta}{\lambda(\zeta + \lambda\eta)} K_0 \left( \sqrt{\frac{\lambda}{\zeta + \lambda\eta}} r \right) \quad (11)$$

where,

$U_{(r,\lambda)}$ —MacDonald cylindrical functions. According to certain transformation rules,

$$U_{(r,t)} = \frac{\zeta}{2\pi i} \int_{r-t^\infty}^{r+t^\infty} \frac{e^{\zeta}}{\lambda(\zeta + \lambda\eta)} K_0 \left( \sqrt{\frac{\lambda}{\zeta + \lambda\eta}} r \right) d\lambda \quad (12)$$

Integrating Equation (12), the equation for the integral in the fracture can be written as:

$$p_F(r,t) = p_i + \frac{Q\mu}{2\pi K_F h} \int_0^\infty \frac{I_0(U,r)}{U} \left[ 1 - e^{\left( \frac{U^2}{1-U^2\eta} \right)} \right] dU \quad (13)$$

### 2.3. Temperature Field Modeling

Similar to the process of deriving the dual-porosity media pressure equation, the fracture energy equation can also be derived from the average fracture thickness. The expression for the unsteady state energy equation within the fracture can be written as:

$$\begin{aligned} \overline{\rho c_{PF}} \frac{\partial T_F}{\partial t} + c_{pf} \rho_f \vec{u}_r \cdot \nabla_r T_F &= \nabla_r \cdot (\bar{\lambda}_F \nabla_r T_F) - \frac{2}{h_F} q_z \\ + \phi_F \frac{\partial P_F}{\partial t} + \phi_F \rho_f c_{pf} \mu_{JT} \left( \frac{\partial P_F}{\partial t} + \vec{v}_r \cdot \nabla_r P_F \right) \end{aligned} \quad (14)$$

Assuming that the thermodynamic equilibrium between the matrix and the fracture is achieved, while neglecting the heat transfer item  $q_z$ . The equivalent energy equation for the homogeneous aquifer can be expressed as:

$$\overline{\rho c_{PR}} \frac{\partial T_F}{\partial t} + \chi_F c_{pf} \rho_f \vec{u}_r \cdot \nabla_r T_F = \nabla_r \cdot (\chi_F \bar{\lambda}_{TF} \nabla_r T_F) + \phi_R \frac{\partial P_F}{\partial t} + \rho_f c_{pf} \mu_{JT} \left( \phi_R \frac{\partial P_F}{\partial t} + \chi_F \vec{u}_r \cdot \nabla_r P_F \right) \quad (15)$$

where,

$$\overline{\rho c_{PR}} = \chi_F \overline{\rho c_{PF}} + (1 - \chi_F) \overline{\rho c_{PM}} = \rho_f c_{pf} (\phi_F \chi_F + \phi_M \chi_M) + \rho_s c_{ps} (1 - \phi_F \chi_F - \phi_M \chi_M) \quad (16)$$

#### 2.4. Coupled Solution of Pressure and Temperature Field Model

In order to facilitate the calculation, dimensionless variables [34] are introduced as seen in Table 1. The basic seepage differential equation for the coupled pressure and temperature can be obtained as follows:

$$\frac{\partial T_D}{\partial t_D} + \omega(r_D, t_D) \frac{\partial T_D}{\partial r_D} = f(r_D, t_D) \quad (17)$$

where,

$$\omega(r_D, t_D) = \frac{\partial P_D}{\partial t_D} \quad (18)$$

$$f(r_D, t_D) = -P_e \left( \zeta \frac{\partial P_D}{\partial t_D} + \left( \frac{\partial P_D}{\partial r_D} \right)^2 \right) \quad (19)$$

**Table 1.** Definition of dimensionless parameters.

| Parameters                                                                           | Formula                                                                                                                                   |
|--------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| Dimensionless elastic storage capacity ratio                                         | $\omega_H = \frac{\phi_F \chi_F}{\phi_F \chi_F + \phi_M \chi_M}$                                                                          |
| Dimensionless heat storage capacity ratio                                            | $\varepsilon_T = \frac{\phi_2 \rho_f c_{pf}}{(\phi_1 + \phi_2) \rho_f c_{pf} + \rho_s c_{ps} (1 - \phi_1 - \phi_2)}$                      |
| Dimensionless crossflow coefficient                                                  | $\alpha_H = \omega_H \frac{k_{fb}}{\mu c_f \phi_2}$                                                                                       |
| Dimensionless thermal diffusion coefficient                                          | $\alpha_T = \frac{\lambda_f \phi_2 + \lambda_s (\chi_F - \phi_2)}{(\phi_1 + \phi_2) \rho_f c_{pf} + \rho_s c_{ps} (1 - \phi_1 - \phi_2)}$ |
| Dimensionless pressure                                                               | $P_D = \frac{2\pi k_{fb} H_w}{q_w B_0 \mu} (P_i - P_F(r, t))$                                                                             |
| Dimensionless temperature                                                            | $T_D = (T_i - T_F(r, t)) \frac{c_f}{-\mu_{JT}} \frac{\varepsilon_T \alpha_H}{\omega_H \alpha_T}$                                          |
| Dimensionless Peclet number                                                          | $P_e = \frac{q_w B_0}{2\pi H_w \phi_2} \frac{\varepsilon_T}{\alpha_T}$                                                                    |
| Dimensionless radius                                                                 | $r_D = \frac{r}{r_w}$                                                                                                                     |
| Dimensionless time                                                                   | $t_D = t \frac{P_e \alpha_T}{r_w^2}$                                                                                                      |
| Ratio of dimensionless Joule–Thompson coefficient to adiabatic expansion coefficient | $\zeta = \frac{\varepsilon_T}{\omega_H} \frac{\eta}{\mu_{JT}}$                                                                            |

Equation (17) is also known as the convective diffusion equation and can be solved by the characteristic line method. In order to determine the value of  $\zeta$  during the solution process, it is necessary to ensure that the integration curve satisfies the following characteristic equation:

$$\Gamma(\xi, s) = \left( \frac{dt_D}{ds}, \frac{dr_D}{ds} \right) = (1, \omega) \quad (20)$$

where,

$\xi$ —Characteristic parameters of different integral curves, dimensionless;

$s$ —Value at any position of the integration curve direction, dimensionless.

The solution of the integral curve can be expressed as:

$$T_D(\xi, s) = \int_0^s f(\xi, s^*) ds^* \quad (21)$$

Thus, the dimensionless temperature can be written as:

$$T_D(\xi, s) = -P_e \zeta P_D(\xi, s) - P_e (1 - \zeta) \int_0^s \left( \frac{\partial P_D}{\partial r_D} \right)^2 (r_D(\xi, s^*), s^*) ds^* \quad (22)$$

In the process of formation temperature analysis, the small influence of late-time heat transfer effects is negligible, and usually the temperature change in the later period is

mainly determined by thermal convection. Therefore, the approximate solution expression for the dimensionless temperature can be written as [34]:

$$T_D(r_D = 1, t_D) = -P_e \zeta P_D(r_D = 1, t_D) - P_e \frac{1 - \zeta}{2} \ln(1 + 2t_D) \quad (23)$$

### 3. Influencing Factor Analysis for Heat Transfer Outside the Wellbore

In order to analyze the heat transfer mechanism of the coaxial-cased heat exchanger outside the wellbore, the formation heat transfer characteristic curves are plotted according to the proposed heat transfer model, and the effects of different main control factors on the heat transfer performance are analyzed.

#### 3.1. Matrix Thermal Conductivity

As physical qualities measure the matrix thermal conductivity, the greater the value of matrix thermal conductivity, the stronger the heat transfer capacity. Figure 2 reflects the variation of formation temperature when the values of thermal conductivity are 1, 3, 5, and 9. It can be seen from the figure that the heat exchange between the matrix and the fluid in the wellbore leads to a gradual decrease in temperature until it stabilizes. This is due to the fact that the temperature varies significantly during the initial stage due to the boundary conditions and initial conditions, but the effect of these factors diminishes with time, and the temperature stabilizes. As the thermal conductivity of the matrix increases, the temperature decreases significantly, reflecting the fact that heat transfer from the matrix to the wellbore is more rapid at high thermal conductivity, improving the heat transfer efficiency and the heat exchange with the fluid in the wellbore.

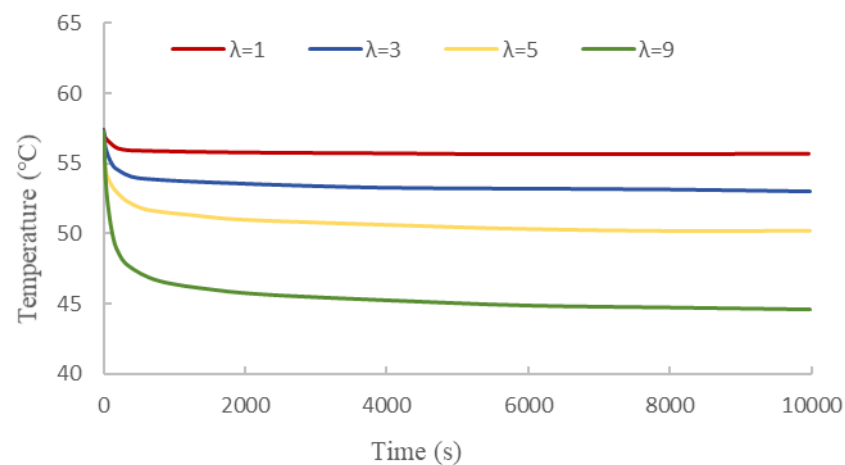


**Figure 2.** Heat transfer characteristic curve with different matrix thermal conductivity.

#### 3.2. Joule–Thompson Coefficient

The Joule–Thompson coefficient,  $\mu_{JT}$ , is a key parameter describing the change of formation temperature caused by fluid expansion when the pressure drops. It has a major effect on the temperature gradient, the dynamic relationship of heat flow and the thermal coupling effect, and the magnitude of its value reflects the tendency of temperature change with the pressure under the condition of fluid expansion at constant external pressure. When  $\mu_{JT}$  is negative, the pressure decreases, leading to a temperature increase due to the constant enthalpy of the throttling process, and the increase in internal energy induces heat exchange between the fluid and the formation. The absolute value of  $\mu_{JT}$  is directly related to the temperature change. The larger the absolute value of the Joule–Thomson coefficient, the greater the magnitude of the temperature change. Figure 3 shows the formation of heat transfer characteristic curves with different Joule–Thompson coefficients. It can be

seen that when  $\mu_{JT}$  is negative, and the absolute value is large, the temperature increases rapidly, showing high conduction efficiency. In contrast, when the absolute value is small, the temperature rises slowly, mainly due to the obstruction of heat conduction. When  $\mu_{JT}$  is positive, and the absolute value is small, the temperature decreases slowly. This is mainly due to the enhanced inter-molecular interaction and increased energy exchange efficiency. The curve stabilizes over time to a constant temperature, showing the balance of heat input and output within the system.

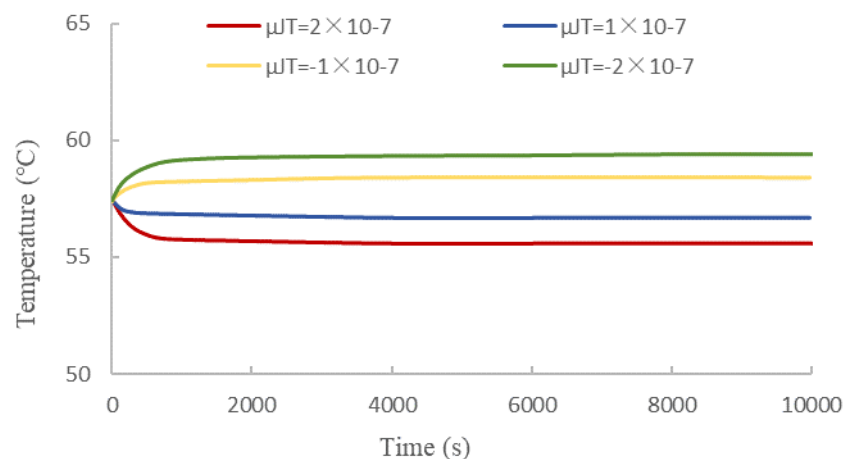


**Figure 3.** Heat transfer characteristic curves with different Joule–Thompson coefficients.

### 3.3. Water Production Rate

Figure 4 shows the formation of heat transfer characteristic curves under different water production conditions. In the early stages of production, the increase in water production leads to a significant temperature difference between the formation and the fluid, and the rapid exchange of heat leads to a rapid drop in temperature. However, as time progresses, the temperature difference decreases, and the heat transfer rate slows down. Eventually, when the heat exchange between the system and the environment reaches equilibrium, the temperature no longer changes significantly. A comparison of the curves in the figure shows that when the water production rate is high, the inflow of large amounts of fluid rapidly reduces the average formation temperature, impelling it to approach the ambient temperature more quickly. When the water production rate is low, the temperature decreases relatively slowly due to less heat dissipation. The fluids within the formation may reach a new thermal equilibrium with the external environment over time, and the temperature profile will gradually rebound to a new stable level.

### 3.4. Effective Permeability of Fracture

Effective fracture permeability determines the fluid flow capacity in formation. Changes in permeability will directly affect the efficiency of heat conduction and convective velocity transfer within the system. Figure 5 shows the heat transfer characteristic curve with different fracture effective permeability. From the figure, it can be seen that when the fracture permeability is high, the fracture provides a more open channel for fluid convection, which accelerates the heat dissipation rate from the formation. Due to the more efficient heat transfer, the final temperature of the system will decrease greatly. In contrast, lower fracture permeability limits the ability of heat exchange between the interior matrix and the external environment, and heat cannot be effectively dissipated from the fractures, leading to the accumulation of internal heat, which in turn slows down the temperature drop.

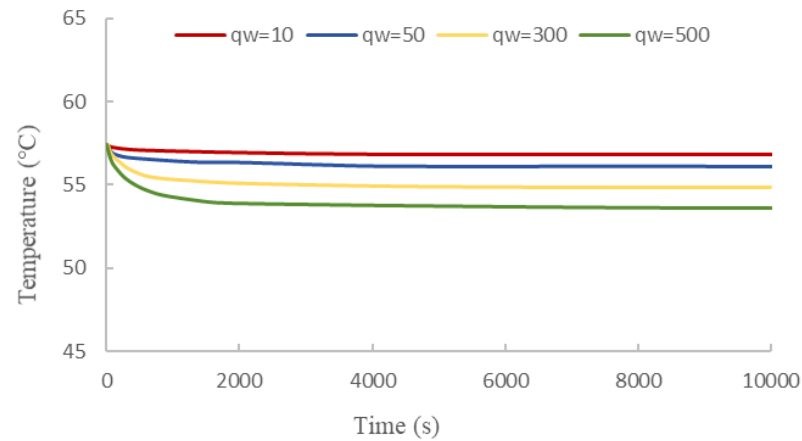


**Figure 4.** Heat transfer characteristic curves with different water production rates.

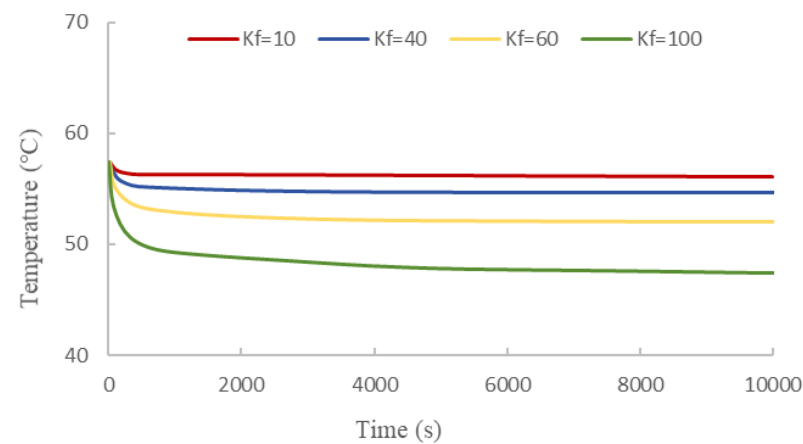


**Figure 5.** Heat transfer characteristic curves with different fracture effective permeabilities.

## 4. Modeling of Heat Transfer Inside the Wellbore

### 4.1. Physical Models and Assumptions

The process of heat transfer in geothermal wells is a complex system in which geothermal energy is transferred into the geothermal wells mainly through the thermal conductivity of the rocks. The thermal conductivity of the rock determines the efficiency of heat transfer. Inside the well, the geothermal fluid brings heat from the high-temperature region underground to the surface by convection, and fluid convection is the main mechanism of heat exchange in geothermal wells. In the process of heat exchange in geothermal wells, the casing wall only plays a role in heat transfer and does not change in temperature because of the adequate recharge of heat from the formation. The casing wall temperature remains constant at a certain stratigraphic depth, and casing temperature variations at different depths are consistent with the ground temperature gradient. Based on the consideration of the heat transfer mechanism between the formation and the wellbore, the following assumptions are made for the in-borehole heat transfer model (shown in Figure 6):

- (1) The temperature of the formation around the wellbore is linear, with equal temperatures at each point in any cross-section in the wellbore;
- (2) The initial temperature of the formation is uniform, and the heat transfer between the fluid in the wellbore and the surrounding formation is a one-dimensional steady-state heat transfer process;
- (3) The casing and cement ring are concentric tubes, and the fluid is injected into the well from the casing at the surface and then returned to the surface from the center tubing,

ignoring the frictional heat generation of the fluid in the wellbore in the process of flow;

- (4) Heat transfer between the wellbore and the formation is transverse, ignoring longitudinal heat transfer processes along the wellbore direction.

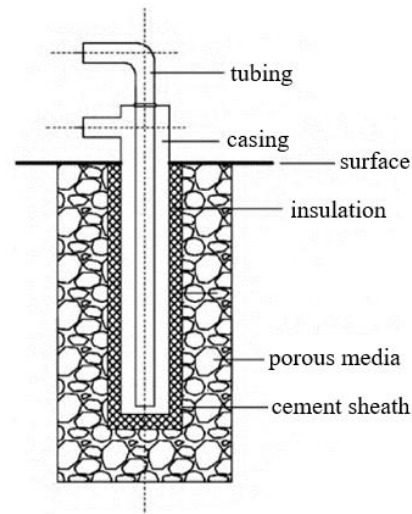


**Figure 6.** Schematic diagram of heat transfer model inside the borehole.

#### 4.2. Mathematical Model

Based on the above assumptions, the heat transfer within the borehole of a geothermal well is regarded as a multilayer long concentric cylinder wall model, and its heat transfer process can be regarded as a radial one-dimensional steady-state heat conduction problem in the absence of an internal heat source and a constant wall temperature. Assuming that the temperatures at the inner and outer side of the wellbore are,  $t_\infty$ ,  $t_f$ , their differential equations [22] for thermal conductivity are:

$$\frac{d}{dr} \left( r \frac{dt}{dr} \right) = 0 \quad (24)$$

Boundary condition:

$$-\lambda 2\pi r_1 l \frac{dt}{dr} \Big|_{r=r_1} = h_1 2\pi r_1 l (t_{w1} - t_f) \quad (25)$$

$$-\lambda 2\pi r_2 l \frac{dt}{dr} \Big|_{r=r_2} = h_2 2\pi r_2 l (t_\infty - t_{w2}) \quad (26)$$

where  $\lambda$  is the thermal conductivity, W/(m·K);  $t_{w1}$ ,  $t_{w2}$  are the temperatures on both sides of the wellbore wall.

For the thermal conductivity of an n-layer cylindrical wall, the flux is calculated as:

$$\Phi = \frac{t_{w1} - t_{w(n+1)}}{\sum_{i=1}^n \frac{1}{2\pi\lambda_i l} \ln \frac{d_{i+1}}{d_i}} W \quad (27)$$

- (1) The thermal resistance to convective heat transfer between the circulating fluid and the inner wall of the casing is:

$$R_{tc} = \frac{1}{2\pi r_{ci} h_f} \quad (28)$$

where,

$R_{tc}$ —Convective heat transfer thermal resistance between the heat transfer medium and the inner wall of the casing,  $\text{m}\cdot\text{K}/\text{W}$ .

$r_{ci}$ —Inner radius of casing,  $\text{m}$ .

$h_f$ —Convective heat transfer coefficient between the heat transfer medium and the inner wall of the casing,  $\text{W}/\text{m}^2\cdot\text{K}$ .

(2) Casing thermal conductivity thermal resistance

$$R_{cas} = \frac{1}{2\pi\lambda_{cas}} \ln \frac{r_{co}}{r_{ci}} \quad (29)$$

where,

$R_{cas}$ —Thermal resistance of casing wall,  $\text{m}\cdot\text{K}/\text{W}$ ;

$\lambda_{cas}$ —Casing thermal conductivity,  $\text{m}\cdot\text{K}$ ;

$r_{co}$ —Outer radius of casing,  $\text{m}$ ;

(3) Cement ring thermal conductivity thermal resistance

$$R_{cem} = \frac{1}{2\pi\lambda_{cem}} \ln \frac{r_h}{r_\infty} \quad (30)$$

where,

$R_{cem}$ —Cement ring thermal conductivity thermal resistance,  $\text{m}\cdot\text{K}/\text{W}$ ;

$\lambda_{cem}$ —Thermal conductivity of cement ring,  $\text{m}\cdot\text{K}$ ;

$r_h$ —Cement ring radius,  $\text{m}$ ;

(4) Thermal resistance from the borehole wall to the formation

$$R_e = \frac{1}{2\pi\lambda_e} I\left(\frac{r_b}{2\sqrt{\alpha T}}\right) \quad (31)$$

$$I(u) = \frac{1}{2} \int_u^\infty \frac{e^{-s}}{s} ds \quad (32)$$

where,

$R_e$ —formation thermal resistance,  $\text{m}\cdot\text{K}/\text{W}$ ;

$\alpha$ —Thermal diffusivity of formation,  $\text{m}^2/\text{s}$ ;

$\lambda_e$ —Thermal conductivity of formation,  $\text{m}\cdot\text{K}$ ;

$I$ —Exponential integral formula;

$r_b$ —Radius of the borehole,  $\text{m}$ ;

$\tau$ —running time,  $\text{s}$ ;

(5) Calculation formula for heat transfer in drilled holes

$$Q = \sum_i^n \left( \frac{1}{R_i} L_i \Delta t_i \right) \quad (33)$$

where,

$\Delta t_i$ —Difference between the average temperature of each formation and the average temperature in the tube of each formation;

$R_i$ —Total thermal resistance of each formation,  $\text{m}\cdot\text{K}/\text{W}$ ;

$L_i$ —Length of each formation,  $\text{m}$ ;

$Q$ —Heat transfer from a single borehole,  $\text{W}$ ;

$i$ —Number for each formation.

(6) Calculation of heat exchange per unit for geothermal wells

$$q_h = \frac{t_\infty - t_f}{R_{tc} + R_{cas} + R_{cem} + R_e \times F_h} \quad (34)$$

$$F_h = \frac{T_{h1}}{T_{h2}} \quad (35)$$

where,

$t_\infty$ —formation temperature;

$t_f$ —Temperature of heat transfer medium;

$T_{h1}$ —Unit heating operating hours;

$T_{h2}$ —Hours in a heating season.

#### 4.3. Field Analysis and Model Validation

A geothermal well with a coaxial heat exchanger in the field was selected to analyze the heat transfer property and ensure the accuracy and reliability of the model. The design depth of the geothermal well is 2400 m, and the vertical depth of the well is 2388 m. The design data of the wellbore structure are shown in Table 2. The wellbore temperature data and the thermal physical parameters of each formation are listed in Table 3.

**Table 2.** The design data of wellbore structure.

| Structure      | Well Depth/m | Casing Depth/m | Cement Back Top    |
|----------------|--------------|----------------|--------------------|
| catheter       | 120          | 120            | back to the ground |
| first opening  | 500          | 500            | back to the ground |
| Second opening | 2388         | 2400           | back to the ground |

**Table 3.** Data of wellbore temperature and thermal physical parameters of each formation.

| Well Depths /m | Temperature /°C | Density kg/m <sup>3</sup> | Thermal Conductivity W/(m·k) | Thermal Diffusivity m <sup>2</sup> /h | Isobaric Specific Heat Capacity J/(kg·k) |
|----------------|-----------------|---------------------------|------------------------------|---------------------------------------|------------------------------------------|
| 0–406          | 27.4            | 1940.2                    | 1.752                        | 0.00324                               | 1085                                     |
| 406–1120       | 42.5            | 2036.5                    | 1.845                        | 0.00257                               | 965                                      |
| 1120–1756      | 53.4            | 2094.3                    | 1.769                        | 0.00286                               | 1123                                     |
| 1756–2400      | 65.2            | 2167.1                    | 1.634                        | 0.00298                               | 1036                                     |

It is assumed that the initial temperature of the injected water is 10 °C. The water flow rate is 0.3 m/s. The specific heat capacity of the water is  $4.2 \times 10^3$  J/(kg·K). The casing diameter is 121.36 mm and the outer diameter of the casing is 133.27 mm. The thickness of the casing wall is 11.91 mm. The thermal conductivity of the casing material is  $\lambda_{cas} = 0.104$  W/(mm·k). The thickness of the cement ring is 37.42 mm and the thermal conductivity of the cement material is  $\lambda_{cem} = 0.6 \times 10^{-3}$  W/(mm·k). The temperature along the horizontal direction is assumed to be constant.

Substituting the above data into Equations (28)–(30), we can obtain:

$$R_{tc} = \frac{1}{2\pi r_{ci} h_f} = \frac{1}{2\pi \times 0.12136 \times 1836} = 0.0007 \quad (36)$$

$$R_{cas} = \frac{1}{2\pi \lambda_{cas}} \ln \frac{r_{co}}{r_{ci}} = \frac{1}{2\pi \times 0.104} \ln \frac{133.27}{121.36} = 0.143 \quad (37)$$

$$R_{cem} = \frac{1}{2\pi \lambda_{cem}} \ln \frac{r_h}{r_{co}} = \frac{1}{2\pi \times 0.6 \times 10^{-3}} \ln \frac{170.69}{133.27} = 65.643 \quad (38)$$

Substituting  $\alpha = 0.6 \times 10^{-6}$  m<sup>2</sup>/s into the Equation (31), it can be written as:

$$R_e = \frac{1}{2\pi \lambda_e} I \left( \frac{1}{2\sqrt{\alpha T}} \right) = 0.201 \quad (39)$$

The unit thermal resistance of a geothermal well can be calculated as

$$R = 0.0007 + 0.143 + 65.643 + 0.201 = 65.988 \quad (40)$$

It is supposed that in the well section of 0–406 m, the water mass in the 406 m long string is  $1.428 \times 10^4$  kg, and the flow duration time is 1800 s. The formation temperature is  $27.4^\circ\text{C}$ , and the temperature at the exit of the well section can be calculated as

$$q = \frac{\Delta t}{R} = \frac{27.4 - 10}{66} \text{ W/mm} = 0.264 \text{ W/mm} \quad (41)$$

$$\Phi = lq = 0.264 \times 406 \times 10^3 \text{ W} = 1.072 \times 10^5 \text{ W} \quad (42)$$

$$Q = \Phi \cdot t = 1.072 \times 10^5 \times 1800 = 1.93 \times 10^8 \text{ J} \quad (43)$$

$$Q = cm\Delta t = 4.2 \times 10^3 \times 1.428 \times 10^4 \cdot \Delta t = 1.93 \times 10^8 \quad (44)$$

It can be obtained that  $\Delta t = 3.22^\circ\text{C}$  from Equation (44); thus, the exit temperature in a well depth of 406 m is  $13.22^\circ\text{C}$ .

By continuing to calculate using Equations (36)–(44) for loop iteration, the fluid temperature in the casing at any depth can be obtained. The fluid temperature in the wellbore at a depth of 2400 m is  $40.36^\circ\text{C}$  by calculation. If the heat loss of the fluid in the casing during the production process is ignored, the temperature of the wellhead produced water can be approximately considered to  $40.36^\circ\text{C}$ .

The model solutions are validated by field data of the HGP-A well in Hawaii [35–37]. Although there is a power failure during the test, it does not influence the trend of the temperature change. Thus, the data are reasonable. Figure 7 shows the results of experimental data and the solution by this model. As shown, there is a good agreement between the proposed model and the field test, indicating that the calculation and analysis method of the model is correct.

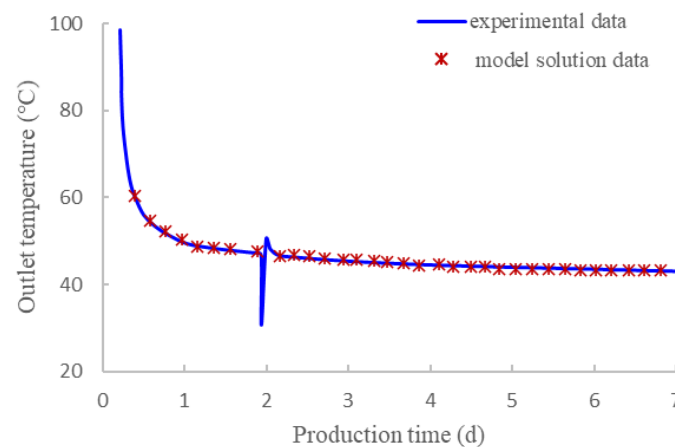


Figure 7. Comparison between model solution and experimental data.

## 5. Analysis of Factors Affecting Heat Transfer Inside the Wellbore

In the process of heat transfer inside the wellbore, different well types, water injection volumes, water injection temperatures, and the wellbore length all affect the efficiency of heat extraction from a geothermal well. In order to analyze the influence of these parameters on the geothermal heat recovery properties and realize the sustainable development of

the geothermal heat recovery, the factors influencing the heat transfer in the vertical and horizontal wells were analyzed according to the heat transfer model inside the wellbore.

#### 5.1. Effect of Injection Temperature on Heat Transfer

In order to meet the heating requirements, the determination of the water injection temperature needs to consider the water injection rate and the lifting temperature difference comprehensively. The lifting power increases with the increase of lifting temperature difference. In addition, excessively high temperatures should be avoided in order to improve the utilization of geothermal water wells. Therefore, the water injection temperature is taken in the range of 5–25 °C. The heat transfer parameters for vertical and horizontal wells are shown in Tables 4 and 5.

**Table 4.** Heat transfer efficiency data of vertical well with different injection temperatures.

| Initial Injection Temperature/°C | Well Depths/m | Injection Rate/m <sup>3</sup> | Final Outlet Temperature/°C | Amount of Temperature Change/ $\Delta t$ | Theoretical Heat Exchange/kW |
|----------------------------------|---------------|-------------------------------|-----------------------------|------------------------------------------|------------------------------|
| 5                                | 2400          | 20                            | 24.83                       | 19.83                                    | 432.39                       |
| 10                               | 2400          | 20                            | 28.04                       | 18.04                                    | 393.36                       |
| 15                               | 2400          | 20                            | 31.76                       | 16.75                                    | 365.23                       |
| 20                               | 2400          | 20                            | 35.18                       | 15.18                                    | 331.00                       |
| 25                               | 2400          | 20                            | 38.73                       | 13.73                                    | 299.38                       |

**Table 5.** Heat transfer efficiency data of horizontal well with different injection temperatures.

| Initial Injection Temperature/°C | Well Depths/m | Horizontal Wellbore Length/m | Injection Rate/m <sup>3</sup> | Final Outlet Temperature/°C | Amount of Temperature Change/ $\Delta t$ | Theoretical Heat Exchange/kW |
|----------------------------------|---------------|------------------------------|-------------------------------|-----------------------------|------------------------------------------|------------------------------|
| 5                                | 2400          | 400                          | 20                            | 40.87                       | 35.87                                    | 782.14                       |
| 10                               | 2400          | 400                          | 20                            | 43.45                       | 33.45                                    | 732.56                       |
| 15                               | 2400          | 400                          | 20                            | 45.46                       | 30.46                                    | 664.18                       |
| 20                               | 2400          | 400                          | 20                            | 48.12                       | 28.12                                    | 613.15                       |
| 25                               | 2400          | 400                          | 20                            | 50.04                       | 25.04                                    | 545.99                       |

Comparing the data in the tables, it can be seen that the higher the water injection temperature, the smaller the temperature difference between the work material and the geothermal reservoir. The wellbore outlet water temperature decreases with less theoretical heat exchange. The reason is that the reduction of injection temperature will lead to a low-temperature difference between the fluid and the formation, reducing the heat transfer efficiency. When the initial injection temperature of both straight and horizontal wells is 5 °C, and the water injection volume is 20 m<sup>3</sup>, the temperature change of the horizontal well is 35.87 °C and the theoretical heat exchange is 782.14 kW, while the temperature change of the straight well is 19.83 °C and the theoretical heat exchange is 432.39 kW. Thus, the outlet water temperature of the horizontal well is significantly higher than that of the straight well, indicating that the horizontal wellbore length increases the contact area and heat exchange time between the injected work material and the surrounding formation and improves the heat exchange efficiency.

#### 5.2. Influence of Water Injection Rate on Heat Transfer

In the case of heat extraction without water, the injection rate has a significant effect on the heat extraction performance. Therefore, the water injection rate was analyzed as an important factor, and the heat transfer parameters for vertical and horizontal wells are shown in Tables 6 and 7.

**Table 6.** Heat transfer efficiency data of vertical wells with different injection rates.

| Initial Injection Temperature/°C | Well Depths/m | Horizontal Wellbore Length/m | Injection Rate/m <sup>3</sup> | Velocity m/s | Final Outlet Temperature/°C | Amount of Temperature Change/ $\Delta t$ | Theoretical Heat Exchange/kW |
|----------------------------------|---------------|------------------------------|-------------------------------|--------------|-----------------------------|------------------------------------------|------------------------------|
| 10                               | 2400          | 0                            | 10                            | 0.5          | 28.04                       | 18.04                                    | 393.36                       |
| 10                               | 2400          | 0                            | 50                            | 2.7          | 71.20                       | 61.20                                    | 1334.47                      |
| 10                               | 2400          | 0                            | 90                            | 4.9          | 86.90                       | 76.90                                    | 1676.80                      |
| 10                               | 2400          | 0                            | 130                           | 7.0          | 80.13                       | 70.13                                    | 1529.18                      |

**Table 7.** Heat transfer efficiency data of horizontal wells with different injection rate.

| Initial Injection Temperature/°C | Well Depths/m | Horizontal Wellbore Length/m | Injection Rate/m <sup>3</sup> | Velocity m/s | Final Outlet Temperature/°C | Amount of Temperature Change/ $\Delta t$ | Theoretical Heat Exchange/kW |
|----------------------------------|---------------|------------------------------|-------------------------------|--------------|-----------------------------|------------------------------------------|------------------------------|
| 10                               | 2400          | 400                          | 10                            | 0.5          | 40.87                       | 30.87                                    | 673.12                       |
| 10                               | 2400          | 400                          | 50                            | 2.7          | 56.38                       | 46.38                                    | 1011.31                      |
| 10                               | 2400          | 400                          | 90                            | 4.9          | 51.97                       | 41.97                                    | 915.15                       |
| 10                               | 2400          | 400                          | 130                           | 7.0          | 48.29                       | 38.29                                    | 834.91                       |

Analyzing the data in the table, it can be seen that when the injection volume is low, increasing the injection volume can significantly improve the heat recovery power. However, as the injection volume continues to increase, when the injection volume reaches a certain value, the heat recovery power enhancement decreases, and the heat recovery power enhancement has been very limited. In the practical application, the injection and extraction flow rate should be estimated first based on parameters such as drilling diameter and pump power to control the cost. In addition, the larger the injection volume, the smaller the average geothermal output temperature reduction between different heating seasons, which is due to the increase of injection volume so that the fluid in the wellbore will quickly take away the heat of the formation near the wellbore, while the thermal conductivity of the rock is small, and the heat far away from the wellbore cannot be replenished in time. Therefore, the output temperature reduction in different heating seasons decreases.

### 5.3. Effect of Horizontal Wellbore Length on Heat Transfer

Since the selection of horizontal wellbore length is limited by the construction difficulty and cost, the horizontal wellbore length of 200–600 m was selected to study the effect on the extracted water temperature of the injection well. Table 8 shows the calculated heat transfer efficiency data of heat transfer parameters for different horizontal wellbore lengths.

**Table 8.** Calculated heat transfer efficiency data for different horizontal wellbore lengths.

| Horizontal Segment Length/m | 200    | 400    | 600    |
|-----------------------------|--------|--------|--------|
| Outlet temperature °C       | 41.17  | 43.45  | 47.6   |
| Average heating power/kW    | 679.66 | 729.37 | 819.86 |

It can be seen from the table that as the horizontal wellbore length increases, the exit temperature and heat recovery power of the horizontal well increases. This means that by increasing the length of the horizontal wellbore, the geothermal energy has sufficient time to exchange heat with the work material, thus increasing the outlet temperature and heat exchange efficiency. In practice, the heat exchange capacity can be increased by increasing the length of the horizontal wellbore to meet greater heating demand. However, an increase in the horizontal wellbore length will also lead to an increase in drilling and maintenance costs. Hence, it is necessary to consider comprehensive factors to determine the optimal length of the horizontal wellbore.

## 6. Conclusions

The heat transfer performance for the coaxial-cased heat exchanger for ground source heat pump systems in fractured aquifers has been investigated on the basis of heat transfer and groundwater seepage theory. Aiming at the geological characteristics of fractured formation and the engineering practice of geothermal heat exchangers, the heat transfer model has been presented to analyze the heat transfer characteristics. The following conclusions are drawn:

- (1) In the process of developing the heat transfer model outside the wellbore, Kazemi's flat plate model was used to characterize the fractured formation. A coupled heat transfer model of temperature and pressure field was proposed by considering the influence of the Joule–Thompson effect. The heat transfer model inside the wellbore is a one-dimensional steady-state multi-layer long concentric cylinder wall model. The model provides a theoretical basis for the analysis of heat transfer performance of the coaxial-cased heat exchanger.
- (2) When the matrix thermal conductivity increases, the heat transfer capacity will be significantly enhanced. The value of the Joule–Thompson coefficient reflects the temperature change with pressure. When it is negative, the decrease in pressure leads to an increase in temperature. In addition, the larger the absolute value of the Joule–Thompson coefficient, the greater the magnitude of the temperature change.
- (3) More heat will be carried away with the increase in water production. When the fracture's effective permeability is larger, the capacity of heat transfer and convection increases. The higher the injection temperature, the smaller the temperature difference between the working fluid and the formation, the lower the outlet water temperature and the theoretical heat exchange.
- (4) The outlet water temperature of the horizontal well is significantly higher than that of the vertical well, indicating that the horizontal wellbore length increases the contact area and heat exchange time between the injected working fluid and the surrounding formation and improves the heat exchange efficiency.
- (5) The water injection rate can significantly improve the heat recovery power. However, the heat recovery power improvement decreases when the injection rate reaches a certain value. As the length of the horizontal wellbore increases, the exit temperature and heat recovery power increase, but it also leads to an increase in the drilling and maintenance cost. Thus, it is necessary to determine the optimal wellbore length by considering various factors.

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