

Article

Novel Efficiency Calculation Model Based on Fine Particle Tracking Behavior

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Abstract: The underflow entrainment of fine particles occurs during the hydrocyclone separation process, resulting in a “fishhook” effect on grade efficiency. Traditional efficiency models fail to address this phenomenon. This study examines the tracking behavior of fine particles, using variations in centrifugal settling velocity to characterize separation performance. The effect of this behavior on particle separation is quantified through a tracking coefficient for small particles and an entrainment coefficient for large particles, together forming a novel efficiency calculation model. The experimental research shows that the new model is applicable for the efficiency calculation of particles with different shapes, and can calculate grade efficiency curves with fishhook segments. By comparing with the existing research results, the accuracy and universality of the new model have been demonstrated. This model facilitates the accurate computation of grade efficiency curves, thereby significantly enhancing the precision of efficiency calculations, which provides guidance for the design and selection of hydrocyclones.

Keywords: hydrocyclone; efficiency model; grade efficiency; fishhook effect; tracking characteristics



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1. Introduction

It is essential to calculate separation efficiency for the successful design and selection of hydrocyclones, which involves the structure and operational conditions of various models. Traditional models like the equilibrium orbit model [1], residence time theory [2], discharge blockage theory [3], and two-phase turbulence theory [4] can be used to analyze hydrocyclone efficiency. However, they do not address the underflow entrainment of fine particles, resulting in significant inaccuracies in efficiency calculations [5].

The underflow entrainment of fine particles occurs when a substantial number of fine particles, which should exit through the overflow, instead exit through the underflow [6,7]. This results in a “fishhook” effect in grade efficiency curves [8] and significant error in fine particle separation efficiency calculations [9]. This phenomenon is primarily attributed to the tracking behavior of small particles [10,11]; these particles, which ideally are separated, follow the wake of larger particles and exit through the underflow pipe.

The present study investigates how the tracking behavior of fine particles affects their velocity. The separation performance of small particles may be improved while controlling the behavior of large particles. A new model for calculating hydrocyclone separation efficiency is established and its accuracy is confirmed through experimental validation.

2. Materials and Methods

The experimental system is shown in Figure 1. Slurry tank 2 was filled with mixed liquid, which was evenly stirred by agitator 1 driven by motor 6. The mixed liquid was pressured by centrifugal pump 5, which was then introduced to hydrocyclone 11 along suction pipe 3. The separated liquid flowed into overflow tank 8 from the overflow pipe

and the sand-containing slurry was introduced to underflow tank 7 from the underflow pipe, which then converged again at slurry tank 2 to realize the circulating operation. When solid powder was added into the slurry tank to be blended with water, it was fully dispersed in the water by agitator stirring at high speed for more than five minutes.

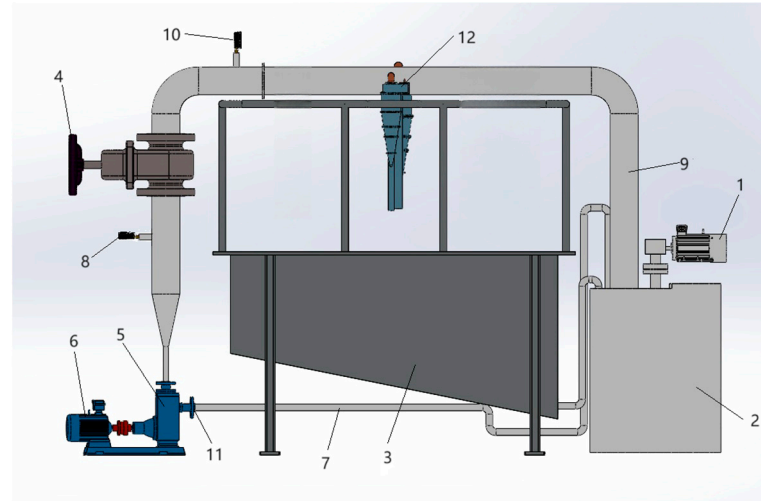


Figure 1. Loop of experimental system. 1—agitator; 2—slurry tank; 3—reflux tank; 4—main valve; 5—centrifugal pump; 6—motor; 7—circulating pipe; 8—electromagnetic flowmeter; 9—suction pipe; 10—pressure gauge; 11—bottom flow valve; 12—hydrocyclone.

Experiments were carried out under standard conditions of normal temperature and pressure. Water served as the continuous phase with a 3 wt.% solid mass concentration in the mixed liquid. Data were collected at two inlet velocities, 2.6 and 4.2 m/s, while the inlet pressure was 0.032 MPa and 0.083 MPa. The flow rate was measured by an LDG electromagnetic flowmeter with an accuracy of $1 \times 10^{-3} \text{ m}^3/\text{h}$. The inlet pressure was measured using a YN60BF pressure gauge with an accuracy of 0.01 MPa. The mass was weighed using an electronic scale with a minimum scale of 0.1 g. The mass flow of particles was analyzed by a sampling method, with samples from underflow and overflow ports taken at timed intervals to measure and calculate the flow rate. Then, the samples were dried and solid particle mass was determined to calculate the mass flow rate.

Two types of particles, quartz and glass frit, were used in the experiment. Though these particles share similar size distributions and densities, they differ significantly in shape. The particle size distribution and sphericity were measured using an Eyetechn laser analyzer (AmbiValue B.V., Dussen, The Netherlands). The particle density was measured using a pycnometer. The physical properties of the particles are described in Table 1 and the particle size distribution is illustrated in Figure 2.

Table 1. Physical properties of experimental powders.

Particles	D_{50}	Density	Sphericity
Quartz	35.76 μm	2.7 g/cm ³	0.74
Glass frit	40.52 μm	2.56 g/cm ³	0.98

The particle grade efficiency was the separation efficiency of particles with different sizes, defined as $Q_m(d_i) = Q_{mo}(d_i) + Q_{mu}(d_i)$, where d_i is a certain particle size level, such that $Q_m(d_i)$ is the mass flow rate for a particle with a size of d_i from the inlet and $Q_{mu}(d_i)$ from the underflow. The particle grade efficiency was expressed as Equation (1):

$$\eta(d_i) = \frac{Q_{mu}(d_i)}{Q_m(d_i)} = \frac{Q_{mu} \cdot f_u(d_i)}{Q_m \cdot f(d_i)} \quad (1)$$

where Q_m is the solid particle mass flow of the inlet and $f(d_i)$ and $f_u(d_i)$ are the mass percentage of particles with grade d_i in the inflow and underflow, respectively.

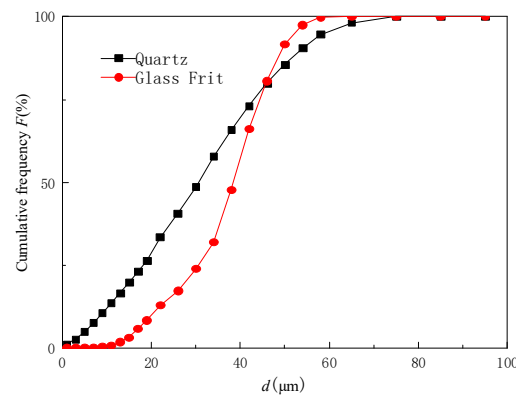


Figure 2. Powder particle size distributions.

An FX100 hydrocyclone was used as the hydrocyclone element in these experiments. Its geometry and size characteristics are shown in Figure 3.

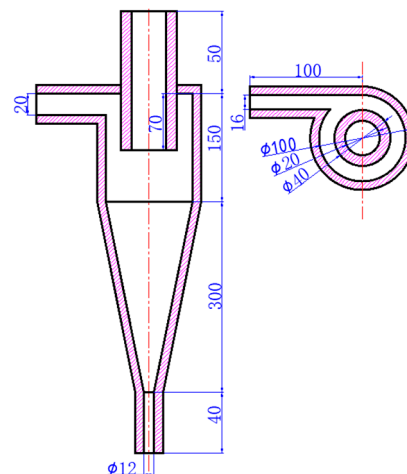


Figure 3. Hydrocyclone geometry (the unit in the figure is mm).

3. Experimental Results

The equilibrium orbit theory posits that a particle remains in equilibrium along a specific orbit due to the balance of centrifugal and drag forces acting on it. Particles of different sizes occupy distinct equilibrium orbits. Thus, particles outside the orbital plane are separated, while those within the plane may escape with the overflow [12]. The critical particle size at this boundary is denoted as d_{50c} . According to Bram's [13] research on the separation efficiency of hydrocyclones, the separation area falls outside the locus of zero vertical velocities (LZVV), which is approximately conical. The value of d_{50} could be calculated as follows:

$$d_{c50} = \frac{24.66 D_{in}^{0.87} D_z^{1.13}}{(1 - D_{in}/D)^{0.8}} \left[\frac{(1 - D_o/D)\mu}{(\rho_p - \rho)Qh_f} \right]^{0.5} \quad (2)$$

where D_{in} is the equivalent diameter of the inlet, D is the diameter of the hydrocyclone, D_z is the average LZVV diameter (approximately $0.43D$) [14], D_o is the diameter of the overflow pipe, Q is the flow rate, and h_f is the free vortex height, which is equal to the distance from the lower end of the cyclone overflow pipe to the end of the cyclone; μ denotes the fluid viscosity, ρ_p is the particle density, and ρ is the liquid density.

Substituting the calculated d_{c50} into the appropriate formula reveals the grade efficiency $\eta(d)$. According to Zhao [15], the grade efficiency can be represented by two parameters: d_{c50} and the separation limit coefficient c .

$$\eta(d) = \frac{\exp\left(c \frac{d}{d_{c50}}\right)}{\exp\left(c \frac{d}{d_{c50}}\right) + \exp(c)} \quad (3)$$

$$c = \frac{-\ln u_s(d)}{\sqrt{DD_t}} \quad (4)$$

where D_t is the turbulent diffusion coefficient; u_s is the centrifugal settling velocity of particles.

According to Schubert [16], D_t is related to pressure drop Δp :

$$D_t = 3.0784 \times 10^{-3} \times D_o \times \left(\frac{\Delta P}{\rho}\right)^{0.5} \quad (5)$$

where Δp is [17], which can be calculated as follows:

$$\Delta p = \frac{13.69c_v v_{in}^{1.75}}{2g} \left(\frac{D_{in}}{D}\right)^2 \left[\left(\frac{1.5D}{D_o - 0.85D_u}\right)^{1.58} - 1 \right] \quad (6)$$

Figure 4 provides a comparison of the calculated values from the equilibrium orbit model with experimental data from quartz separation efficiency trials. It shows significant deviations in the particle grade efficiency curves calculated by the model (M_B), particularly an inability to replicate the fishhook shape in smaller-size particles. Thus, the model fails to accurately predict efficiency during the underflow entrainment of fine particles. The model must be revised based on the particles' tracking characteristics.

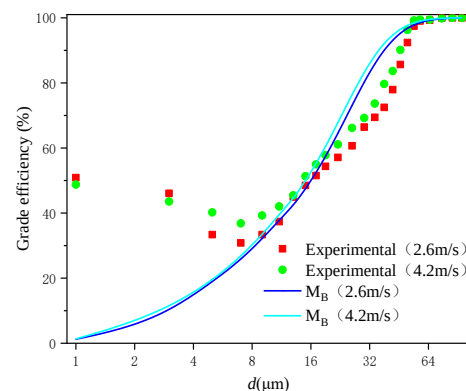


Figure 4. Comparison of quartz grade efficiencies.

4. Model Analysis and Correction Methods

As discussed above, substantial deviations in model calculations can be attributed to the underflow entrainment phenomenon of fine particles, which is driven by their tracking behavior. According to Kashiwaya [18], the separation performance of particles is positively correlated with their settling velocity. Therefore, an observed increase in the separation efficiency of fine particles can be equated to an increase in their centrifugal settling velocity, facilitated by their entrainment by larger particles. Conversely, some large particles experience a decrease in centrifugal settling velocity due to the drag effect exerted by smaller particles, resulting in a decrease in their separation efficiency.

This study aims to correct the d_{c50} value and separation limit coefficient c , both of which are influenced by changes in the centrifugal settling velocity of particles. The relationship between the centrifugal settling velocity $u_s(d)$ of particles and their tracking behavior is analyzed first. According to Dueck [19], in a gravitational settling field, the

settling velocity of fine particles increases as they track the wake of larger particles. In the same field, the settling velocity of some larger particles decreases due to this drag interaction; the velocity of some large particles decreases as they carry smaller particles. The extent of this change in velocity is related to factors such as particle concentration, size, and density [20].

The principles of centrifugal settling in cyclone separation mirror those of gravity settling, except that gravity acceleration g is replaced by centrifugal acceleration a_c . Based on these considerations, changes in particle separation performance driven by the degree of variation in centrifugal settling velocity are investigated in this study. The following equation incorporates particle tracking behavior into the expression of their centrifugal settling velocity:

$$\frac{u_s(d)}{u_h(d)} = 1 + d^{-3}g(c_v)\beta_t - (d^{0.35} - 1)g(c_v)f_e \quad (7)$$

where f_e is the entrainment coefficient, β_t is the tracking coefficient, c_v is the particle volume concentration, and $g(c_v)$ is a function of solid content.

$$g(c_v) = 2.25c_v^{2/3} \exp(-5c_v) \quad (8)$$

The component $u_h(d)$ modifies the settling velocity by adjusting the Stokes velocity of individual particles to account for the effects of concentration:

$$u_h = (1 - c_v)^{4.5} \frac{a_c d^2 (\rho_p - \rho)}{18\mu} \quad (9)$$

The second term on the right-hand side of Equation (7) reflects the increase in velocity of particles that follow in the wakes of larger particles, referred to as the “tracking term”. The third term indicates the decrease in velocity caused by the drag exerted by particles on one another, the “entrainment term”. The effectiveness of this model hinges on the specific formulations of the entrainment and tracking coefficients.

4.1. Tracking Coefficient

The tracking term indicates an increase in velocity. The tracking effect and underflow entrainment of fine particles are more pronounced when its value is higher. Research by Eswaraiyah [21] demonstrates that the effect of fine particle underflow entrainment is related to the size, shape, and concentration of particles as well as the operating conditions and structure of the hydrocyclone.

The term d^{-3} represents the influence of particle size on the tracking effect; smaller particles exhibit a stronger tracking effect. The function $g(c_v)$, representing the particle concentration, increases monotonically with a parabolic trend within a 1% to 15% range (Equation (8)). This indicates that higher particle concentrations enhance the tracking effect. The effects of particle shape, operating conditions, and hydrocyclone structure on the tracking effect are encapsulated in the coefficient β_t . A larger β_t value indicates stronger tracking effects and more severe fine particle underflow entrainment.

Majumder [22] compared the operating conditions and parameters of hydrocyclone structure parameters between an experiment and numerical model to find a certain relationship between β_t and the actual operating parameters. Specifically, the influence of operating conditions and hydrocyclone structure on β_t can be attributed to the split ratio R_u . Wan [23] found that sphericity Φ_p cannot be used to linearly characterize the influence of particle shape on the underflow entrainment of fine particles. Accordingly, a piecewise function is utilized here to represent the influence of particle shape on β_t :

$$\beta_t = \begin{cases} f_1(R_u)(\phi_p \leq 0.95) \\ f_2(R_u)(\phi_p > 0.95) \end{cases} \quad (10)$$

where R_u can be expressed as [24]:

$$R_u = \frac{0.818c_v^{0.232}}{Q^{0.148}} \left(\frac{D_u}{D_o} \right)^{2.512} \quad (11)$$

According to Equation (11), flow rates Q and D_o are negatively correlated with R_u , while c_v and D_u are positively correlated with R_u . Thus, R_u is proportional to the value of β_t in Equation (9).

4.2. Entrainment Coefficient

The entrainment term illustrates the decrease in velocity for large particles. The undertow entrainment of fine particles intensifies as this term increases. The tracking term and entrainment term are interactive, affecting each other's dynamics.

According to Equation (7), particle size and concentration are positively correlated with the entrainment effect. The influence of particle shape, operating conditions, and hydrocyclone structure on the entrainment effect are reflected in the entrainment coefficient f_e .

Spelay [25] suggests that the processes of entrainment and tracking are essentially the same mechanisms acting on different particles. Consequently, the relationship between f_e and the operating conditions can be described by β_t . For f_e , it is necessary to consider the potential drag effect caused by small particles tracking the wake of a larger particle in the flow field as per the entrainment coefficient in the integral form:

$$f_e = \int_0^d f(x) f_3(\beta_t x) dx \quad (12)$$

In this integral, $f_3(\beta_t x)$ represents the function relationship between the undetermined β_t and f_e , while $f(x)$ is the expression of the particle size distribution function $f(d)$ within the integral. Given that $f(d)$ may be discontinuous, it can be expressed in a summative form:

$$f_e = \sum_{i=1}^n f(d_i) f_3(\beta_t d_i) (d_n = d) \quad (13)$$

4.3. Cut Size d_{50c}

The tracking behavior influences not only the separation limit coefficient c but also d_{50c} . Tracking behavior, which involves changes in the centrifugal settling velocity of particles, typically leads to an increase in d_{50c} . This shift can still be quantified by the coefficient f_e . According to Qin [26], the change induced by tracking behavior can be equated to an increase in the average diameter of the LZVV in the equilibrium orbit model. Therefore, Equation (2) can be modified as follows:

$$d_{50c} = \frac{24.66 D_{in}^{0.87} f(f_e) D_z^{1.13}}{(1 - D_{in}/D)^{0.8}} \left[\frac{(1 - D_o/D) \mu}{(\rho_p - \rho) Q h_f} \right]^{0.5} \quad (14)$$

where f_e denotes all particles affected by the tracking behavior:

$$f_e = \int_0^{d_{max}} f(x) f_3(\beta_t x) dx \quad (15)$$

4.4. Model Formulation

According to Yang [27], regression fitting techniques can be employed to determine the specific formulation of Equation (10). The grade efficiencies corresponding to each particle size of quartz, obtained from the experimental results, are substituted into the model. Each data set is then fitted independently to ascertain the values of each parameter

within the mode. The accuracy of the fit can be quantified by the average deviation ε_g , defined as:

$$\varepsilon_g = \frac{\sum (\eta_{\text{cal}}(d_i) - \eta_{\text{exp}}(d_i))^2}{n} \quad (16)$$

where $\eta_{\text{cal}}(d_i)$ and $\eta_{\text{exp}}(d_i)$ represent the calculated and experimental values of the efficiency of a certain particle size, respectively; n is the number of fitted groups.

The model parameters in Equation (10) are compared with the actual operating parameters from the experiment. The value of β_t is tied to the small particle size range ($d < 10 \mu\text{m}$), so the value of grade efficiency in this range is taken for fitting. The relationship between β_t and R_u is shown in Figure 5, where the fitted equation is:

$$\beta_t = 484.25R_u^2 - 27.62R_u + 154.8 (\phi_p \leq 0.95) \quad (17)$$

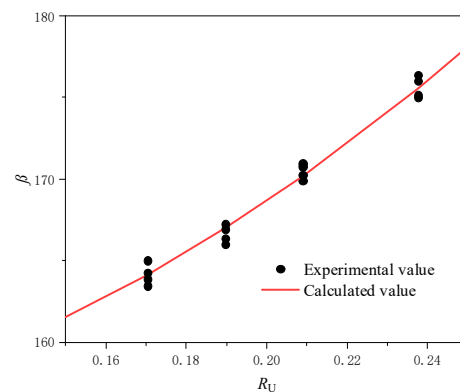


Figure 5. Relationship between model parameters and operating variants.

The average fitting deviation of this model is $\varepsilon_g = 2.42$, indicating that it is accurate.

The relationship between f_e and β_t is analyzed by comparison between the calculation results (Equation (13)) and experimental data. The value of f_e depends on the large particle size range ($d > 30 \mu\text{m}$), so the efficiency value of this particle size range is taken for fitting. The relationship between β_t and f_e is shown in Figure 6, where the fitted equation is:

$$f_e = 6.2 \int_0^d f(x) \beta_t x^{0.32} dx \quad (18)$$

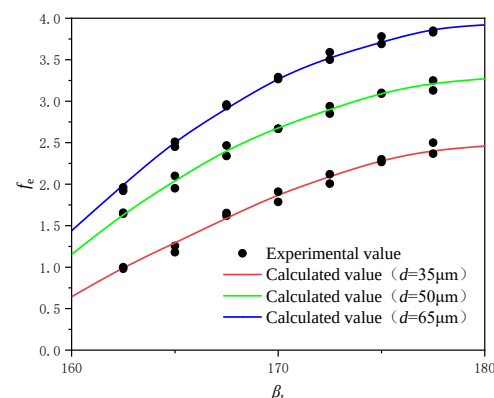


Figure 6. Fitting of model parameters.

This model is also accurate, as per its average fitting deviation of $\varepsilon_g = 2.53$.

For Equation (15), the quartz cut size obtained from the experimental $d_{50\text{ce}}$ was compared with the cut size calculated by the model $d_{50\text{cm}}$, then the form of $f(f_e)$ was determined

by fitting each set of data separately. The relationship between d_{50ce}/d_{50cm} and f_e is shown in Figure 7, expressed as follows:

$$d_{50c} = \frac{8.779D_{in}^{0.87}f_e^{0.46}D_z^{1.13}}{(1 - D_{in}/D)^{0.8}} \left[\frac{(1 - D_o/D)\mu}{(\rho_p - \rho)Qh_f} \right]^{0.5} \quad (19)$$

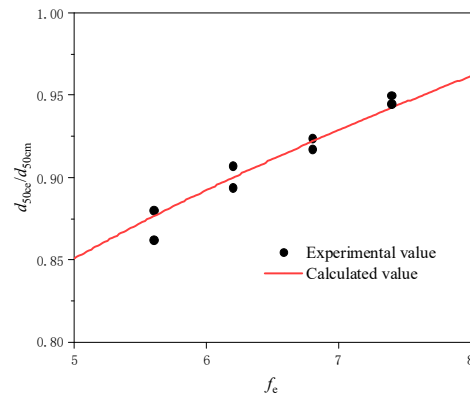


Figure 7. Fitting of model parameters for d_{50c} .

The average fitting deviation of the model is $\varepsilon_g = 2.08$, indicating high accuracy.

These results indicate that the proposed efficiency separation model can be formulated effectively by comprehensively considering the tracking effect of fine particles and the entrainment effect of large particles:

$$c = \frac{\ln \left\{ \left\langle 1 + 2.25c_v^{2/3}e^{-5c_v} [\beta_t d^{-3} - f_e(d^{0.35} - 1)] \right\rangle (1 - c_v)^{4.5} \frac{a_c d^2 (\rho_p - \rho)}{18\mu} \right\}}{-\sqrt{0.0030784DD_o} \left(\frac{\Delta P}{\rho} \right)^{0.5}} \quad (20)$$

$$d_{50c} = \frac{20.83f_e^{0.46}D_z^{1.13}D_{in}^{0.87}}{(1 - D_{in}/D)^{0.8}} \left[\frac{(1 - D_o/D)\mu}{(\rho_p - \rho)Qh_f} \right]^{0.5} \quad (21)$$

Within this model, the calculated separation efficiency is influenced not only by the particle size, medium properties, and concentration but also significantly by the particle size distribution. For small particles, the acceleration mechanism induced by the tracking effect is predominant, leading to much higher centrifugal speeds than those predicted by Stokes' law. In contrast, for large particles, the deceleration effect due to drag is predominant while changes in velocity due to entrainment are minimal. The actual centrifugal settling velocity for particles is slightly lower than that calculated by Stokes' law because suspensions have a higher density and viscosity than pure water.

5. Validation of Proposed Model

5.1. Comparison and Verification with Experimental Results

As depicted in Figure 8, the values calculated using the proposed model were compared with experimental values for quartz. The model generated grade efficiency curves with a fishhook shape, exhibiting only slight deviation from the experimental curves.

To determine the accuracy of the proposed model for other spherical particles, its calculated values were compared with the experimental values for glass frit. Highly spherical glass frit particles behave much differently than non-spherical particles, necessitating recalculation of β_t :

$$\beta_t = 496.25R_u^2 - 21.2R_u + 168.6(\Phi_p > 0.95) \quad (22)$$

This model was highly accurate, as evidenced by its average fitting deviation of $\varepsilon_g = 2.31$.

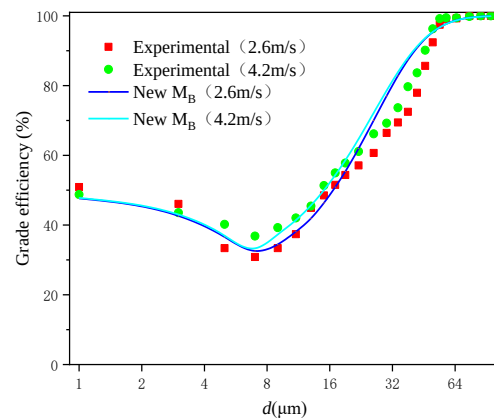


Figure 8. Comparison of quartz grade efficiencies between new model and experimental values.

Furthermore, according to Cao [28], replacing large particles has minimal impact on the underflow entrainment of fine particles; the influence of particle shape on f_e can be ignored. As shown in Figure 9, the proposed model accurately calculated the fishhook-shaped particle efficiency curves, with only minor deviations from the experimental results.

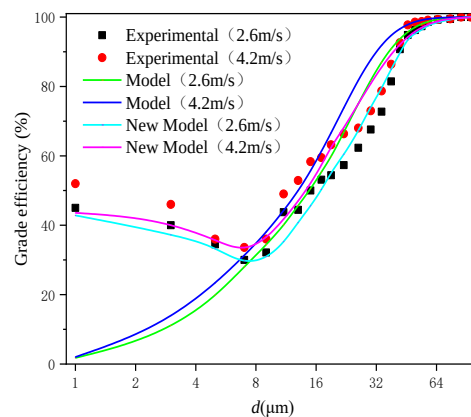


Figure 9. Comparison of glass frit grade efficiencies.

5.2. Verification of Proposed Model

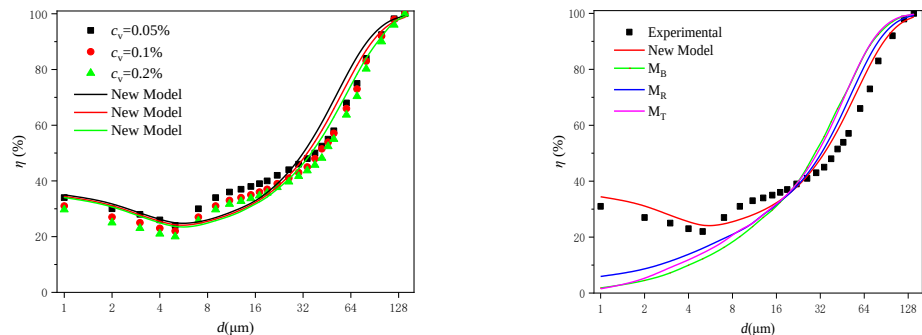
The broader applicability of the proposed model was evaluated by comparing its results with findings from existing research. Abdollahzadeh [29] conducted experimental research on the effect of grade efficiency concentration in small hydrocyclones, and Jiang [30] investigated the grade efficiencies of hydrocyclones with different overflow pipe diameters. The structures of the hydrocyclones they used are described in Table 2.

Table 2. Structural dimensions of experimental hydrocyclone models.

Structure Parameters	Abdollahzadeh (mm)	Jiang (mm)
D	15	75
D_{in}	4.2	18
D_o	5.1	25
H_1	5	120
H_2	79	97
D_u	3	15

Abdollahzadeh [29] used flake aluminum powder as a source of solid particles for their experiments, setting the inlet velocity to 5.05 m/s. Particle volume concentrations

were set to 0.05%, 0.1%, and 0.2%. The results of the proposed model were compared to Abdollahzadeh's results, as shown in Figure 10. Figure 10a provides a comparison across various concentrations, where grade efficiency slightly decreased as the concentration increased. The proposed model yielded results consistent with the experimental results.

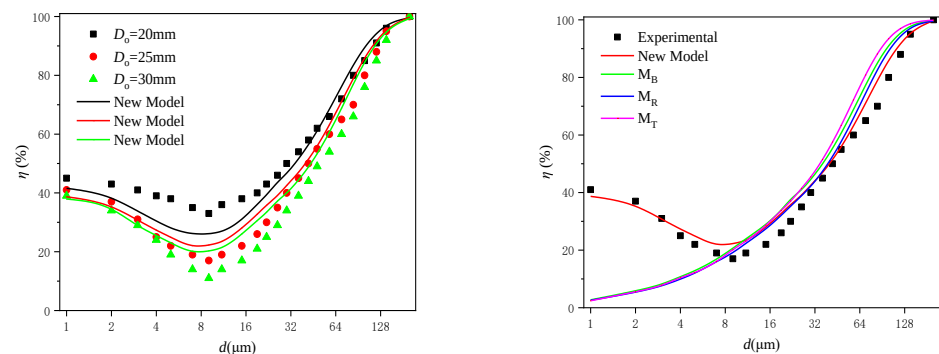


(a) Comparison across various concentrations. (b) Comparison of the calculation results for different models.

Figure 10. Comparison between proposed and Abdollahzadeh's models.

Figure 10b shows a comparison of the calculation results for different models at a consistent concentration of 0.1%, where M_B , M_R , and M_T represent the equilibrium orbit model, residence time model, and two-phase turbulence model, respectively. The proposed model not only accurately reflected the influence of concentration on grade efficiency, but also yielded calculations closer to the experimental results than the other three models.

Jiang [30] investigated the effect of different overflow pipe diameters on hydrocyclone performance through experiments conducted on a 5 mm-diameter hydrocyclone with non-spherical silica powder as the source of solid particles. The results of the proposed model were compared with Jiang's results, as shown in Figure 11. Figure 11a provides a comparison across various hydrocyclone structures, where the grade efficiency decreased as the diameter of the overflow pipe increased; again, the proposed model showed the same trends as the experimental data.



(a) Comparison across various hydrocyclone structures. (b) Comparison among different models.

Figure 11. Comparison between proposed model and Jiang's model.

Figure 11b shows a comparison among different models with a constant overflow pipe diameter of 25 mm. The proposed model accurately reflected the influence of different structures on grade efficiency while delivering results closer to the experimental data than the other three models.

6. Conclusions

A novel efficiency calculation method for hydrocyclones was established in this study through both theoretical analysis and mathematical fitting. The proposed model was designed to account for the underflow entrainment of fine particles. Its computational accuracy was evaluated through experimental validation and comparison against previous research findings. The main conclusions can be summarized as follows:

- (1) The proposed model leverages changes in centrifugal settling velocity to describe how underflow entrainment affects the separation performance of particles. Specifically, large particles experience a decrease in velocity due to the entrainment effect while small particles increase in velocity due to the tracking effect. These effects on particle velocity can be quantified using tracking and entrainment coefficients. The relationship between these coefficients and factors such as the structure of the hydrocyclone, operating conditions, and particle properties was successfully expressed through theoretical analysis and mathematical fitting.
- (2) Experimental results showed that, compared to the original model, the proposed model's efficiency calculations aligned more closely with experimental values and could accurately compute grade efficiency curves, including their fishhook segments. The model was validated based on experimental data from previous studies, demonstrating that despite variations in hydrocyclone structures, particle types, and particle concentrations, the proposed model captured the influence of various parameters and delivered consistently accurate calculation results.

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Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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