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A Robust Process Identification Method under Deterministic Disturbance

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Abstract: This study introduces a novel process identification method aimed at overcoming the challenge of accurately estimating process models when faced with deterministic disturbances, a common limitation in conventional identification methods. The proposed method tackles the difficult modeling problems due to deterministic disturbances by representing the disturbances as a linear combination of Laguerre polynomials and applies an integral transform with frequency weighting to estimate the process model in a numerically robust and stable manner. By utilizing a least squares approach for parameter estimation, it sidesteps the complexities inherent in iterative optimization processes, thereby ensuring heightened accuracy and robustness from a numerical analysis perspective. Comprehensive simulation results across various process types demonstrate the superior capability of the proposed method in accurately estimating the model parameters, even in the presence of significant deterministic disturbances. Moreover, it shows promising results in providing a reasonably accurate disturbance model despite structural disparities between the actual disturbance and the model. By improving the precision of process models under deterministic disturbances, the proposed method paves the way for developing refined and reliable control strategies, aligning with the evolving demands of modern industries and laying solid groundwork for future research aimed at broadening application across diverse industrial practices.

Keywords: disturbance modeling; deterministic disturbance; process identification; integral transform; Laguerre polynomials

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1. Introduction

The landscape of modern industrial processes is becoming increasingly complex, and the objectives guiding these operations are diversifying rapidly. This evolving environment underscores the critical importance of implementing effective control strategies, which are essential not only for ensuring economic and environmental sustainability but also for accommodating inevitable fluctuations in operating schedules. Against this backdrop, model-based approaches, such as model predictive and adaptive controls, have emerged as promising solutions for achieving optimal control performance [1–8]. However, the efficacy of these approaches heavily relies on the accuracy of the employed process model. The accuracy of these models, therefore, becomes a cornerstone for the successful application of model-based control strategies, catalyzing the development and deployment of various process identification methods. Consequently, various process identification methodologies have been developed and implemented across diverse industrial domains [9–20].

Since the pioneering introduction of the relay feedback method for identifying the ultimate frequency response data of processes, the field has seen the emergence of numerous advanced techniques within the literature [21–24]. These methodologies have made notable strides in enhancing the precision of frequency response data estimation, particularly for the desired frequency regions [25–29]. Among the array of techniques developed, parametric identification methods such as subspace, prediction error, and instrumental variable methods stand out for their efficacy and widespread application [30–37]. Furthermore, the advent of parametric identification techniques utilizing various weight functions and integral transforms represents an advancement in the identification of continuous-time processes [38–42]. In parallel, the development of nonparametric identification methods has introduced a new dimension of flexibility and robustness [43–48]. These methods distinguish themselves by not necessitating prior knowledge of the process dynamics, thus offering a powerful tool for accurate modeling.

However, despite these advances, the array of previous process identification methods reveals limitations when confronted with the complexity of real-world processes. This is predominantly due to the inherent challenges associated with uncertainty, an inescapable aspect of determining the process model for industrial applications, which remains a critical hurdle to overcome. While certain published identification methods consider un-certainties such as measurement noise and disturbances during parameter estimation [49–53], the majority have been built upon the assumption of stochastic disturbances. This assumption tends to overlook the prevalence and impact of deterministic disturbances in practical processes, a reality that can significantly undermine the effectiveness of an estimated model. These deterministic disturbances can significantly degrade the performance of estimated process models.

Addressing this critical gap, we introduce a novel process identification method aimed at explicitly neutralizing the influence of deterministic disturbances during parameter estimation. By embracing a novel approach that models deterministic disturbances as linear combinations of Laguerre polynomials and employing an integral transform with frequency weighting for the estimation of parameters, our proposed method stands out for its exceptional accuracy. This is further complemented by numerical stability and robustness from strategic use of the least squares method for parameter estimation, which can eliminate the reliance on complex iterative search-based nonlinear optimization methods. The performance of the proposed method was confirmed through a simulation study, which attests to the remarkable ability of the method to accurately model processes, even processes heavily corrupted by deterministic disturbances. The proposed method heralds a significant leap forward from the previous identification methods, which encounter difficulties in guaranteeing model accuracy in the face of deterministic disturbance, showcasing the potential of our method to substantially improve the fidelity of process models.

2. Theoretical Development of the Proposed Method

This study adopts a continuous-time differential equation as the process model, perturbed by a deterministic disturbance, represented as the following:

$$\begin{aligned} y(t) + a_1\frac{dy_p(t)}{dt} + \cdots + a_{n-1}\frac{d^{n-1}y_p(t)}{dt^{n-1}} + a_n\frac{d^n y_p(t)}{dt^n} \\ = b_0u(t) + b_1\frac{du(t)}{dt} + \cdots + b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}} + b_m\frac{d^m u(t)}{dt^m} \end{aligned} \tag{1}$$

$$y(t) = y_p(t) + D(t) \tag{2}$$

where y_p, y, and u represent the disturbance-free process output, the measured process output disturbed by deterministic disturbances, and the process input, respectively. $a_i,\ i = 1, 2, \cdots, n$ and $b_i,\ i = 0, 1, \cdots, m$ are the model parameters that the proposed identification method should provide. This study assumes that the disturbance $D(t)$ can be expressed by the linear combination of basis functions as follows:

$$D(t) = d_0f_0(t) + d_1f_1(t) + \cdots + d_{p-1}f_{p-1}(t) + d_pf_p(t) \tag{3}$$

Here, $f_i(t), i = 0, 1, \cdots, p$ denotes the i-th basis function and $d_i,\ i = 0, 1, \cdots, p$ are the model parameters of the disturbance model that the proposed identification method should provide. The basis function for the proposed method can be a variety of formulae [54], but this study adopts the following Laguerre polynomials in Equation (4).

$$f_i(t) = L_i(t) = \sum_{k=0}^{i} \left(\frac{(-1)^k}{k!} \frac{i!}{k!(i-k)!} t^k \right),\ i = 0, 1, 2, \cdots, p \tag{4}$$

Substituting Equation (2) into Equation (1), we obtain the following:

$$\begin{aligned} y(t) + a_1 \tfrac{dy(t)}{dt} + \cdots + a_n \tfrac{d^n y(t)}{dt^n} + a_{n-1} \tfrac{d^{n-1} y(t)}{dt^{n-1}} \\ = b_0 u(t) + b_1 \tfrac{du(t)}{dt} + \cdots + b_{m-1} \tfrac{d^{m-1} u(t)}{dt^{m-1}} + b_m \tfrac{d^m u(t)}{dt^m} + \\ D(t) + a_1 \tfrac{dD(t)}{dt} + \cdots + a_{n-1} \tfrac{d^{n-1} D(t)}{dt^{n-1}} + a_n \tfrac{d^n D(t)}{dt^n} \end{aligned} \tag{5}$$

This equation can be further manipulated by substituting Equation (3) into Equation (5) and considering that $D(t)$ is a linear combination of polynomials, as shown in Equations (3) and (4), resulting in the following:

$$\begin{aligned} y(t) + a_1 \tfrac{dy(t)}{dt} + \cdots + a_n \tfrac{d^n y(t)}{dt^n} + a_{n-1} \tfrac{d^{n-1} y(t)}{dt^{n-1}} \\ = b_0 u(t) + b_1 \tfrac{du(t)}{dt} + \cdots + b_{m-1} \tfrac{d^{m-1} u(t)}{dt^{m-1}} + b_m \tfrac{d^m u(t)}{dt^m} + \\ c_0 f_0(t) + c_1 f_0(t) + \cdots + c_{p-1} f_{p-1}(t) + c_p f_p(t) \end{aligned} \tag{6}$$

Traditional methods have primarily focused on accurately identifying process models perturbed by stochastic disturbances, typically in the form of high-frequency noise with a mean of zero. However, disturbances in practical processes often exhibit characteristics such as fluctuating means and irregular low-frequency dynamics. Consequently, the accuracy of process models derived using earlier identification techniques is significantly degraded when deterministic disturbances are present. In this study, a novel process identification method is developed to overcome the limitations of existing approaches.

2.1. Integral Transform

This study employs an integral transform from $t = 0.$ and $t = t_{end}.$ to estimate model parameters with frequency weighting [55].

$$Y(n, m, \omega) = \int_0^{t_{end}} \frac{d^n w(\omega, \tau)}{dt^n} \frac{d^m y(\tau)}{dt^m} d\tau \tag{7}$$

$$U(n, m, \omega) = \int_0^{t_{end}} \frac{d^n w(\omega, \tau)}{dt^n} \frac{d^m u(\tau)}{dt^m} d\tau \tag{8}$$

$$F_i(0, 0, \omega) = \int_0^{t_{end}} w(\omega, \tau) f_i(\tau) d\tau,\ i = 1, 2, \cdots, p \tag{9}$$

Here, $w(\omega, t)$ represents a frequency weight function aimed at mitigating the effects of the initial and final values of the signal. By applying integration by parts to Equations (7) and (8), the following properties are derived:

$$Y(n-1, m, \omega) = -Y(n, m-1, \omega) + \left. \frac{d^{n-1} w(\omega, t)}{dt^{n-1}} \frac{d^{m-1} y(t)}{dt^{m-1}} \right|_{t=t_{end}} - \left. \frac{d^{n-1} w(\omega, t)}{dt^{n-1}} \frac{d^{m-1} y(t)}{dt^{m-1}} \right|_{t=0} \tag{10}$$

$$U(n-1, m, \omega) = -U(n, m-1, \omega) + \left. \frac{d^{n-1} w(\omega, t)}{dt^{n-1}} \frac{d^{m-1} u(t)}{dt^{m-1}} \right|_{t=t_{end}} - \left. \frac{d^{n-1} w(\omega, t)}{dt^{n-1}} \frac{d^{m-1} u(t)}{dt^{m-1}} \right|_{t=0} \tag{11}$$

Simplification of Equations (10) and (11) is possible under the condition that the weight function satisfies

$$\left. \frac{d^i w(\omega, t)}{dt^i} \right|_{t=0} = \left. \frac{d^i w(\omega, t)}{dt^i} \right|_{t=t_{end}} = w(\omega, 0) = w(\omega, t_{end}) = 0,\ i = 1, 2, \cdots, n \tag{12}$$

This results in

$$Y(n-1,m,\omega) = -Y(n,m-1,\omega) \tag{13}$$

$$U(n-1,m,\omega) = -U(n,m-1,\omega) \tag{14}$$

Repeating Equations (13) and (14) leads to the following expressions:

$$Y(0,k,\omega) = (-1)^k Y(k,0,\omega), k = 1,2,\cdots,n-1 \tag{15}$$

$$U(0,k,\omega) = (-1)^k U(k,0,\omega), k = 1,2,\cdots,n-1 \tag{16}$$

Consequently, a significant observation emerges: the integral transform of the n-th derivative of a signal (e.g., $d^n y(t)/dt^n$) can be determined from the integral transform of the 0-th derivative of the signal (e.g., $y(t)$), without the need to consider the signal's initial and final values.

This study adopts the weight function proposed by Sung in their work [54], defined as follows:

$$w(\omega,t) = \frac{t^q(t-t_{end})^q}{t_{end}^{2q}} \exp(-i\omega t) \tag{17}$$

Here,q represents the order of the weight function, which must be greater than the process order n.

2.2. Process Identification Using Least Squares Method

The process model provided in Equation (6) can be transformed into Equation (18) by applying the integral transform outlined in Equations (7)–(9) to Equation (6):

$$\begin{aligned} &Y(0,0,\omega) + a_1 Y(0,1,\omega) + a_2 Y(0,2,\omega) + \cdots + a_n Y(0,n,\omega) \\ &= b_0 U(0,0,\omega) + \cdots + b_m U(0,m,\omega) + c_0 F_0(0,0,\omega) + \cdots + c_p F_p(0,0,\omega) \end{aligned} \tag{18}$$

By utilizing Equations (15)–(16) and (18), we arrive at Equation (19):

$$\begin{aligned} &Y(0,0,\omega) = -a_1(-1)Y(1,0,\omega) - a_2(-1)^2 Y(2,0,\omega) - \cdots - a_n(-1)^n Y(n,0,\omega) + \\ &b_0 U(0,0,\omega) + b_1(-1)U(1,0,\omega) + \cdots + b_m(-1)^m U(m,0,\omega) + \\ &c_0 F_0(0,0,\omega) + \cdots + c_p F_p(0,0,\omega) \end{aligned} \tag{19}$$

where

$$Y(0,0,\omega) = \int_0^{t_{end}} w(\omega,\tau) y(\tau) d\tau \tag{20}$$

$$Y(k,0,\omega) = \int_0^{t_{end}} \frac{d^k w(\omega,\tau)}{dt^k} y(\tau) d\tau,\ k = 1,2,\cdots,n \tag{21}$$

$$U(0,0,\omega) = \int_0^{t_{end}} w(\omega,\tau) u(\tau) d\tau \tag{22}$$

$$U(k,0,\omega) = \int_0^{t_{end}} \frac{d^k w(\omega,\tau)}{dt^k} u(\tau) d\tau,\ k = 1,2,\cdots,m \tag{23}$$

$$F_i(0,0,\omega) = \int_0^{t_{end}} w(\omega,\tau) f_i(\tau) d\tau,\ i = 0,1,2\cdots,p \tag{24}$$

Given that the analytic derivatives of the weight function ($d^k w(\omega,\tau)/dt^k$) are easily derived, the values of Equations (20)–(24) can be computed through numerical integration with process input and output data, yielding n_ω equations of Equation (19) corresponding to multiple frequencies ($\omega = \omega_k,\ k = 1,2,\cdots,n_\omega$). Since these equations are valid for both the real and imaginary parts of the complex number, $2n_\omega$ equations are obtained. Consequently, the model parameters $a_i, i = 1,2,\cdots,n$ and $b_j, j = 0,1,\cdots,m$ can be estimated straightforwardly by applying a simple least squares method to the $2n_\omega$ equations.

2.3. Disturbance Modeling and Initial State Estimation

In this section, a novel identification method is proposed to estimate the initial values of state variables and the model parameters ($d_0, d_1, \cdots, d_p$) of the disturbance model based on the model parameters ($a_i, i = 1, 2, \cdots, n$ and $b_j, j = 0, 1, \cdots, m$) of the process model estimated in the previous section. Equation (5) can be represented as a state–space model:

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t),\ y_p(t) = Cx(t) \tag{25}$$

$$y(t) = y_p(t) + d_0 f_0(t) + d_1 f_1(t) + \cdots + d_p f_p(t) \tag{26}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & -1/a_n \\ 1 & 0 & 0 & \cdots & 0 & -a_1/a_n \\ 0 & 1 & 0 & \cdots & 0 & -a_2/a_n \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & 0 & -a_{n-2}/a_n \\ 0 & 0 & 0 & 0 & 1 & -a_{n-1}/a_n \end{bmatrix},\ B = \begin{bmatrix} b_0/a_n \\ b_1/a_n \\ b_2/a_n \\ \vdots \\ b_{m-1}/a_n \\ b_m/a_n \end{bmatrix},\ C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}^{\mathrm{T}},\ x(0) = \begin{bmatrix} x_{1,0} \\ x_{2,0} \\ x_{3,0} \\ \vdots \\ x_{n-1,0} \\ x_{n,0} \end{bmatrix} \tag{27}$$

where $x(0)$ denotes the initial value of state variables at time $t = 0$.

This study estimates the disturbance model parameters (d_0, d_1, $\cdots$, d_p) and the initial state values ($x_{0,0}$, $x_{1,0}$, $\cdots$, $x_{n,0}$) by solving an optimization problem where the cost function $V(x_{1,0}, x_{2,0}, \cdots, x_{n,0}, d_0, d_1, \cdots, d_p)$ is the sum of the squares of the modeling errors.

$$\begin{aligned} &\min\left[V(x_{1,0}, x_{2,0}, \cdots, x_{n,0}, d_0, d_1, \cdots, d_p)\right] \\ &\quad = 0.5\sum_{k=1}^{N}\left(y(k\Delta t) - y_p(k\Delta t) - d_0 f_0(k\Delta t) - d_1 f_1(k\Delta t) - \cdots - d_p f_p(k\Delta t)\right)^2 \end{aligned} \tag{28}$$

The optimal solution minimizing $V(x_{1,0}, x_{2,0}, \cdots, x_{n,0}, d_0, d_1, \cdots, d_p)$ can be analytically derived, since Equation (28) is a quadratic function with respect to the initial values and the model parameters, as shown in the following proof:

$$\frac{\partial V}{\partial \theta} = -\sum_{k=1}^{N}\left(y(k\Delta t) - y_p(k\Delta t) - d_0 f_0(k\Delta t) - \cdots - d_p f_p(k\Delta t)\right) Z(k\Delta t) \tag{29}$$

$$\frac{\partial^2 V}{\partial \theta^2} = Z(k\Delta t) Z^{\mathrm{T}}(k\Delta t) \tag{30}$$

Here, $Z(t)$ and θ are defined as follows:

$$Z(t) = \begin{bmatrix} \frac{\partial y_p(t)}{\partial x_{1,0}} & \frac{\partial y_p(t)}{\partial x_{2,0}} & \cdots & \frac{\partial y_p(t)}{\partial x_{n,0}} & f_0(t) & f_1(t) & \cdots & f_p(t) \end{bmatrix} \tag{31}$$

$$\theta = \begin{bmatrix} x_{1,0} & x_{2,0} & \cdots & x_{n,0} & d_0 & d_1 & \cdots & d_p \end{bmatrix}^{\mathrm{T}} \tag{32}$$

Additionally, the derivatives of the state variable ($x(t)$) with respect to $x_{i,0}$ at time 0 are constant, as indicated by Equation (34).

$$\frac{d}{dt}\left(\frac{\partial x(t)}{\partial x_{i,0}}\right) = A\left(\frac{\partial x(t)}{\partial x_{i,0}}\right),\ \frac{\partial y_p(t)}{\partial x_{i,0}} = C\left(\frac{\partial x(t)}{\partial x_{i,0}}\right),\ i = 1, 2, \cdots, n \tag{33}$$

$$\left[\frac{\partial x(t)}{\partial x_{i,0}}\right]_{t=0} = [\underbrace{0\ 0\ \cdots\ 0\ 1}_{i}\ 0\ 0\ \cdots\ 0]^{\mathrm{T}},\ i = 1, 2, \cdots, n \tag{34}$$

Therefore, the first-order derivatives of the state variable ($x(t)$) with respect to the initial values of the state variable ($x_{i,0}$) at time 0 are constant, which leads to the fact that the second- or higher-order derivatives are zero via Equation (33). As a result, the cost function can be formulated in a quadratic form.

The first derivative of the cost function can be represented as follows:

$$\frac{\partial V(\theta)}{\partial \theta} = \left[\frac{\partial V}{\partial \theta}\right]_{\theta=\theta_0} + \left[\frac{\partial^2 V}{\partial \theta^2}\right]_{\theta=\theta_0} (\theta - \theta_0),\ \theta_0 = [0\ 0\ \cdots\ 0]^{\mathrm{T}} \tag{35}$$

Since $\partial V(\theta)/\partial\theta = 0$ at the optimal solution, Equation (35) can be written as follows:

$$\theta_1 = \theta_0 - \left[\frac{\partial^2 V}{\partial \theta^2}\right]^{-1}_{\theta=\theta_0} \left[\frac{\partial V}{\partial \theta}\right]_{\theta=\theta_0} \tag{36}$$

While θ_1 represents the theoretical optimal solution, θ_2 is chosen as the practical optimal solution to mitigate the effects of numerical errors:

$$\theta_2 = \theta_1 - \left[\frac{\partial^2 V}{\partial \theta^2}\right]^{-1}_{\theta=\theta_1} \left[\frac{\partial V}{\partial \theta}\right]_{\theta=\theta_1} \tag{37}$$

In summary, the model parameters of the disturbance model and the initial values of the state variables can be estimated straightforwardly with Equations (36) and (37).

3. Simulation Study

The performance of the proposed identification method was validated through simulation studies, juxtaposed with that of a previous identification method.

3.1. Case 1: Low-Order Process with Measurement Noises

Consider the below low-order plus time delay process with a deterministic disturbance:

$$G(s) = \frac{y_p(s)}{u(s)} = \frac{1}{s^2 + 2s + 1},\ y_p(0) = 0.3,\ \left.\frac{dy_p(t)}{dt}\right|_{t=0} = 0.5 \tag{38}$$

$$y(s) = y_p(s) + D(s) \tag{39}$$

$$D(t) = 2(3t/80)\exp(-3t/80) \tag{40}$$

The process measurement is contaminated by uniformly distributed random noises between -0.05 and 0.05. The process is excited by the following PI controller with $k_c = 1.5$ and $\tau_i = 3.0$, as represented in Figure 1.

$$u(t) = k_c(y_s(t) - y(t)) + \frac{k_c}{\tau_i}\int_0^t (y_s(\tau) - y(\tau))d\tau \tag{41}$$

$$y_s(t) = 0 \text{ for } t < 20,\ y_s(t) = 2 \text{ for } 20 \le t < 50, \text{ and } y_s(t) = 1 \text{ for } t \ge 50 \tag{42}$$

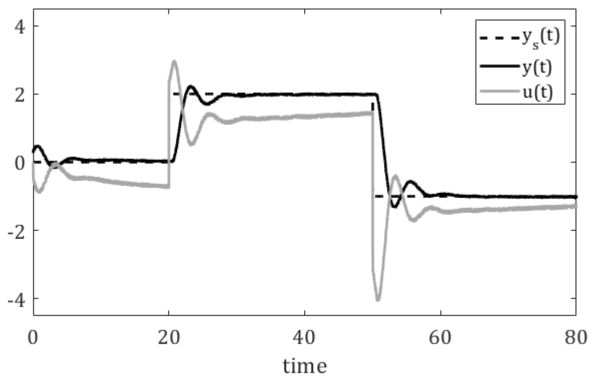

Figure 1. Low-order process excited by the PI controller.

The estimated model parameters from each method are enumerated in Table 1. While the previous method fails to address the effects of the deterministic disturbance, resulting in inaccurate models (Figure 2a), the proposed method effectively deals with the deterministic disturbance, yielding accurate frequency estimates even under significant disturbance.

Table 1. Estimated process model and disturbance model for Case 1.

		Previous Method			
a_1	6.1250	a_2	6.2385	a_3	3.9880
b_0	1.4235	b_1	1.9224	b_2	−0.7699
d_0	–	d_1	–	d_2	–
d_3	–	d_4	–	d_5	–
		Proposed Method			
a_1	2.1501	a_2	1.3265	a_3	0.1534
b_0	0.9926	b_1	−0.3280	b_2	0.0356
d_0	0.0902	d_1	–0.0591	d_2	–0.0039
d_3	–0.0002	d_4	0.0000	d_5	0.0000

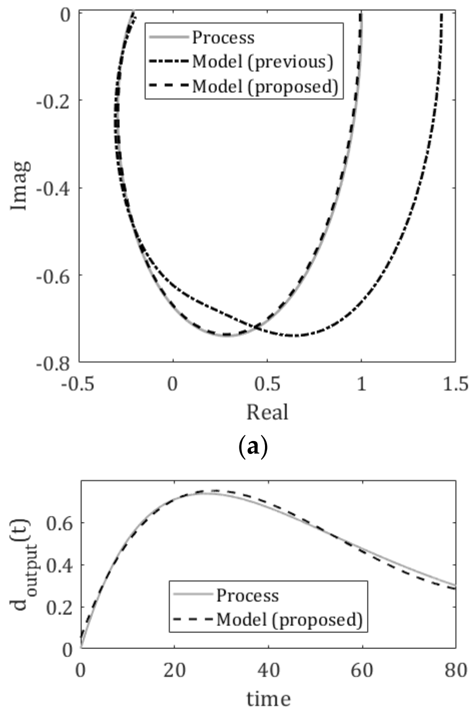

Figure 2. Identification results for Case 1; (**a**) frequency responses and (**b**) disturbance model.

The parameters of the disturbance model in Equation (3) and the initial values of the state variable x are estimated by the proposed method as follows:

$$x(0) = [6.7543\ 2.9181\ 0.2720]^T$$

$$d_0 = 0.0902,\ d_1 = -0.0591,\ d_3 = -0.0039,\ d_4 = -0.0002,\ d_5 = 0.000,\ d_6 = 0.000$$

As confirmed in Figure 2b, the proposed method can provide a fairly accurate disturbance model despite the structural difference between the actual deterministic disturbance and the disturbance model using Laguerre polynomials. Moreover, the proposed method can estimate the initial values of the state variable without any complicated iterative searching-based optimization.

3.2. Case 2: High-Order Process

Consider the fifth-order process with a deterministic disturbance:

$$G(s) = \frac{y_p(s)}{u(s)} = \frac{1}{s^5+5s^4+10s^3+10s^2+5s+1},$$
$$y_p(0) = \left.\frac{dy_p(t)}{dt}\right|_{t=0} = \left.\frac{d^2y_p(t)}{dt^2}\right|_{t=0} = \left.\frac{d^3y_p(t)}{dt^3}\right|_{t=0} = \left.\frac{d^4y_p(t)}{dt^4}\right|_{t=0} = 0.0 \quad (43)$$

$$D(t) = 2(3t/80)\exp(-3t/80) \quad (44)$$

Figure 3 displays the process input and output data excited by the PI controller with $k_c = 1.5$ and $\tau_i = 10$ with the setpoint (y_s) as $y_s(t) = 0$ for $t < 30$, $y_s(t) = 2$ for $30 \le t < 60$, and $y_s(t) = 1$ for $t \ge 60$. The parameters of the process model estimated by the proposed method and previous method, along with disturbance model parameters, are enumerated in Table 2. The performance comparison in Figure 4a underscores the superiority of the proposed method in providing accurate models, unlike the previous method. Additionally, Figure 4b confirms the proposed method's ability to estimate disturbance models and initial state variables ($x(0) = [-0.0777 \;\; -0.0885 \;\; -0.0877]^T$) with satisfactory precision.

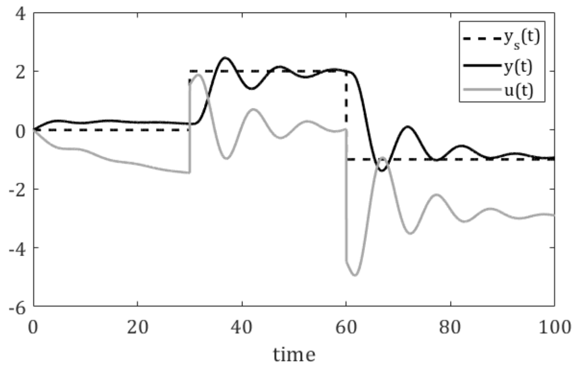

Figure 3. High-order process excited by the PI controller.

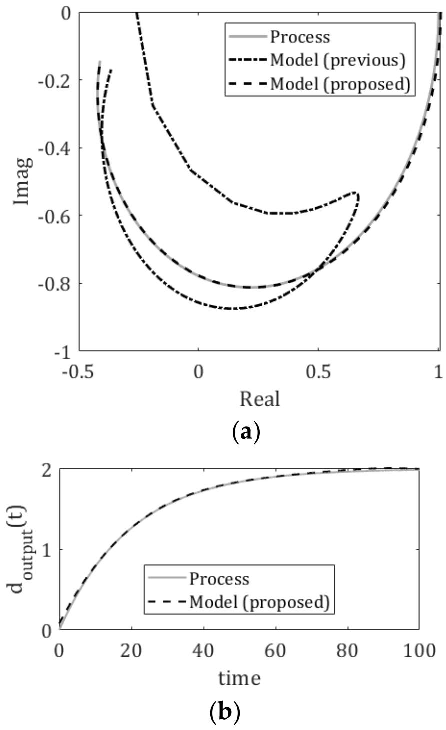

Figure 4. Identification results for Case 2; (**a**) frequency responses and (**b**) disturbance model.

Table 2. Estimated process model and disturbance model for Case 2.

Previous Method					
a_1	−38.945	a_2	−120.08	a_3	−259.76
b_0	−0.2592	b_1	−36.252	b_2	26.913
d_0	–	d_1	–	d_2	–
d_3	–	d_4	–	d_5	–
Proposed Method					
a_1	4.3151	a_2	6.5867	a_3	4.1972
b_0	1.0084	b_1	−0.7186	b_2	0.2726
d_0	0.0902	d_1	−0.0591	d_2	−0.0039
d_3	−0.0002	d_4	0.0000	d_5	0.0000

3.3. Case 3: Non-Minimum-Phase Process

Consider the second-order non-minimum-phase process:

$$G(s) = \frac{y_p(s)}{u(s)} = \frac{(1-0.3s)}{s^2+2s+1}\exp(-0.2s),\ y_p(0) = 0.3,\ \left.\frac{dy_p(t)}{dt}\right|_{t=0} = 0.5 \tag{45}$$

$$D(t) = 2(3t/80)\exp(-3t/80) \tag{46}$$

The process is excited by the PI controller, of which the proportional gain $k_c = 1.5$ and the integral time $\tau_i = 3.0$, with the set point designed as $y_s(t) = 0$ for $t < 20$, $y_s(t) = 2$ for $20 \le t < 50$, and $y_s(t) = 1$ for $t \ge 50$, as shown in Figure 5. The estimated model and disturbance parameters are represented in Table 3. As anticipated, the proposed method exhibits significantly better model performance compared to the previous method, as evidenced in Figure 6a. Moreover, Figure 6b underscores the proposed method's proficiency in estimating disturbance models and initial state variables ($x(0) = [7.0505\ 2.9391\ 0.3060]^T$).

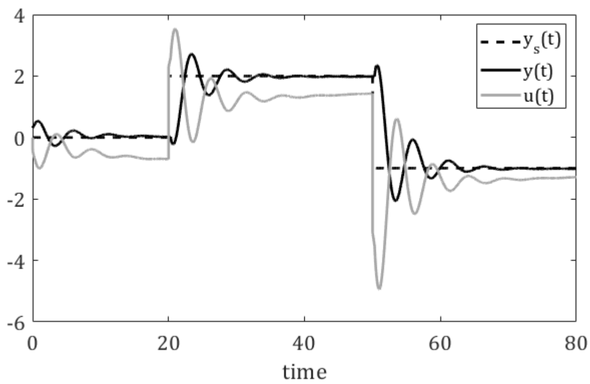

Figure 5. Non-minimum-phase process excited by the PI controller.

Table 3. Estimated process model and disturbance model for Case 3.

Previous Method					
a_1	6.0117	a_2	6.1022	a_3	4.0156
b_0	1.4276	b_1	1.3971	b_2	−1.3956
d_0	–	d_1	–	d_2	–
d_3	–	d_4	–	d_5	–
Proposed Method					
a_1	2.1493	a_2	1.2983	a_3	0.1545
b_0	1.0005	b_1	−0.6464	b_2	0.0757
d_0	0.0746	d_1	−0.0633	d_2	−0.0045
d_3	−0.0002	d_4	0.0000	d_5	0.0000

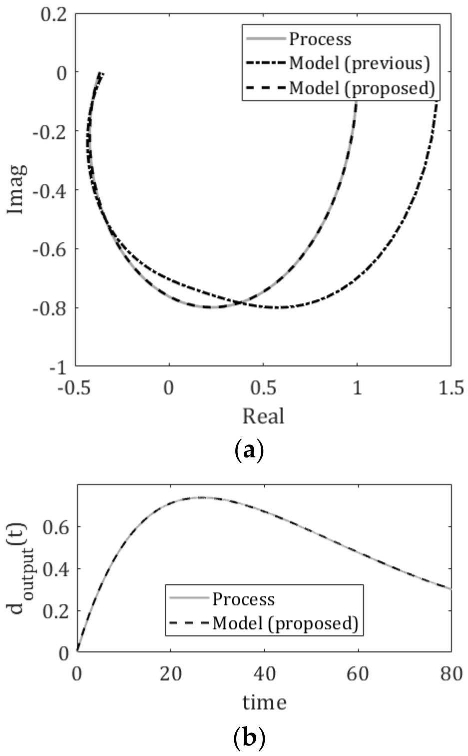

Figure 6. Identification results; (**a**) frequency responses and (**b**) disturbance model.

3.4. Case 4: Underdamped Process

Consider the underdamped process:

$$G(s) = \frac{y_p(s)}{u(s)} = \frac{\exp(-0.5s)}{s^2 + 1.4s + 1},\ y_p(0) = 0.3,\ \left.\frac{dy_p(t)}{dt}\right|_{t=0} = 0.5 \tag{47}$$

$$D(t) = 2(3t/80)\exp(-3t/80) \tag{48}$$

Figure 7 shows the process input and output data excited by the PI controller with $k_c = 2.0$ and $\tau_i = 3.0$. The set point (y_s) is designed as $y_s(t) = 0$ for $t < 5$, $y_s(t) = 1$ for $5 \leq t < 15$, and $y_s(t) = 0$ for $t \geq 15$. Table 4 lists the model parameters estimated by both methods. Frequency responses in Figure 8a demonstrate the proposed method's superior model performance. Furthermore, Figure 8b reaffirms the accuracy of the proposed method in estimating disturbance models and initial state variables ($x(0) = [5.0361\ 2.5354\ 0.3043]^T$).

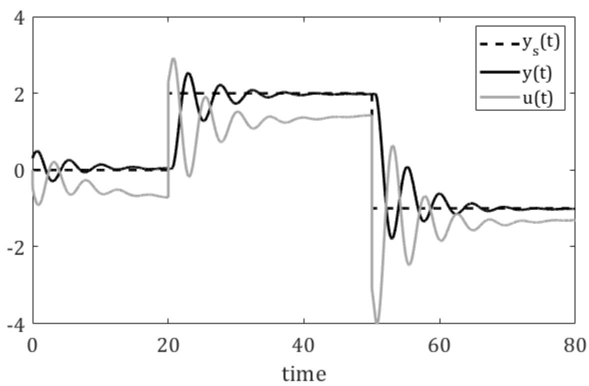

Figure 7. Underdamped process excited by the PI controller.

Table 4. Estimated process model and disturbance model for Case 4.

Previous Method					
a_1	7.0575	a_2	7.1168	a_3	5.9039
b_0	1.4314	b_1	3.5842	b_2	−1.4273
d_0	–	d_1	–	d_2	–
d_3	–	d_4	–	d_5	–
Proposed Method					
a_1	1.5794	a_2	1.2514	a_3	0.1829
b_0	1.0004	b_1	−0.3160	b_2	0.0366
d_0	0.0357	d_1	–0.0320	d_2	–0.0023
d_3	–0.0001	d_4	0.0000	d_5	0.0000

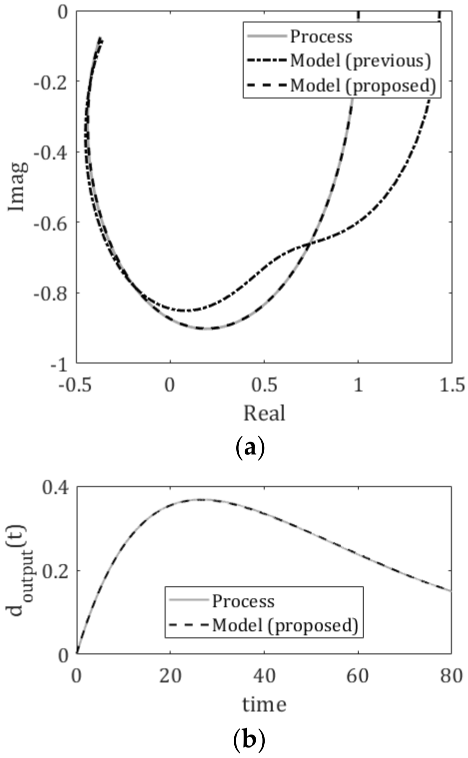

Figure 8. Identification results for Case 4; (**a**) frequency responses and (**b**) disturbance model.

4. Conclusions

This study presents a novel process identification method designed to effectively overcome the prominent challenges of accurately modeling industrial processes in the presence of deterministic disturbances. This challenge represents a significant limitation of previous identification methods, which often fall short in practical applications where disturbances do not follow predictable stochastic disturbance. Our method introduces a unique conceptualization of deterministic disturbances as a linear combination of Laguerre polynomials, coupled with integral transformation featuring frequency weighting for precise model parameter estimation, significantly enhancing overall model accuracy as well as robustness. A notable aspect of our approach is its reliance on the least squares method for parameter estimation. By circumventing the complexities associated with iterative nonlinear optimization, this method improves robustness and accuracy, particularly from a numerical analysis perspective.

Through extensive simulation studies encompassing a wide variety of process types, including lower-order, higher-order, non-minimum phase, and underdamped processes, we have unequivocally demonstrated exceptional performance of our method in faithfully modeling processes, even in the face of significant deterministic disturbances. Our simulations unequivocally illustrate that our method outperforms existing process identification techniques by accurately estimating both process and disturbance models under challenging conditions, marking a substantial advancement over previous approaches that often struggle with accuracy in the presence of deterministic disturbances.

Moreover, the capability of our approach to precisely identify initial state variables alongside the disturbance model is particularly noteworthy, offering comprehensive insight into process dynamics. Remarkably, the proposed method demonstrated the ability to estimate the behavior of disturbances, achieving near-accurate results even when the structure of the proposed disturbance model differs from the actual disturbance. This characteristic is especially advantageous for industrial processes where the exact nature of disturbances remains unknown, potentially enhancing the practical utility of our method significantly. For example, many chemical processes frequently encounter deterministic disturbances such as variations in feed composition and fluctuations in utility supplies. Under these real operational conditions, the proposed method can provide a fairly accurate process model. Then, it is possible to design a high-performance control system or control performance-monitoring system on the basis of the process model the proposed method provides, resulting in improving the product quality and yield as well as increasing production rate. By enhancing the accuracy of process models under deterministic disturbances, our method creates new possibilities for developing refined and dependable control strategies that align with the evolving needs of modern industries. This work sets a solid foundation for future research endeavors aimed at further refining its application across diverse industrial scenarios and seamlessly integrating it with advanced control systems. Furthermore, the proposed method can effectively identify unknown deterministic disturbances, suggesting potential applications beyond industry, including various social phenomena.

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