



Yuxin Wen¹, Yuanlong Yue¹, Xin Zuo^{1,*} and Xiaoguang Li²

- ¹ Department of Automation, China University of Petroleum, Beijing 102249, China; 2021211246@student.cup.edu.cn (Y.W.); yueyuanlong@cup.edu.cn (Y.Y.)
- ² Offshore Oil Engineering Co., Ltd., Tianjin 300461, China; lixg12@cooec.com.cn
- * Correspondence: zuox@cup.edu.cn

Abstract: The subsea control system, a pivotal element of the subsea production system, plays an essential role in collecting production data and real-time operational monitoring, crucial for the consistent and stable output of offshore oil and gas fields. The increasing demand for secure offshore oil and gas extraction underscores the necessity for advanced reliability modeling and effective maintenance strategies for subsea control systems. Given the enhanced reliability of subsea equipment due to technological advancements, resulting in scarce failure data, traditional reliability modeling methods reliant on historical failure data are becoming inadequate. This paper proposes an innovative reliability modeling technique for subsea control systems that integrates a Wiener degradation model affected by random shocks and utilizes the Copula function to compute the joint reliability of components and their backups. This approach considers the unique challenges of the subsea environment and the complex interplay between components under variable loads, improving model accuracy. This study also examines the effects of imperfect maintenance (PM), optimized against reliability and economic considerations. Numerical simulations on a Subsea Control Module demonstrate the effectiveness of the developed models.

Keywords: subsea control system; preventive maintenance; reliability model; imperfect maintenance

1. Introduction

The reliability of subsea control systems is critical for the operational efficiency and safety of offshore oil subsea production systems [1–3]. These systems regulate vital components such as pipelines, Christmas trees (a kind of wellhead control device), and various instruments, ensuring operations proceed safely and smoothly [4–6]. According to the Offshore Reliability Data Handbook (OREDA) by Det Norske Veritas (DNV), which includes reliability and maintenance data for a wide range of equipment and conditions, the subsea control system is divided into surface and subsea segments, connected by an umbilical cable [7]. The structure of subsea control system is shown in Figure 1.

On the surface, the control segment includes the Uninterruptible Power Supply (UPS), Main Control Station (MCS), Electrical Power Unit (EPU), Hydraulic Power Unit (HPU), Chemical Injection Unit (CIU), and Modem. The MCS acts as the operational core, managing data, human–machine interactions, and overall control, including monitoring and emergency responses. The UPS ensures a steady power supply, with the Modem integrating control signals into the power stream. The EPU and HPU provide electrical and hydraulic power, respectively, while the CIU is responsible for chemical injections.

Beneath the surface, the Subsea Distribution Unit (SDU) directs signals to production equipment, and the Subsea Control Module (SCM), comprising the Subsea Electronic Module (SEM) and Directional Control Valve (DCV), handles signal processing for equipment operation. Sensors (PT/TT) capture temperature and pressure data, relaying them back for real-time monitoring and maintaining safety and reliability.



Citation: Wen, Y.; Yue, Y.; Zuo, X.; Li, X. Reliability-Based Preventive Maintenance Strategy for Subsea Control System. *Processes* **2024**, *12*, 761. https://doi.org/10.3390/pr12040761

Academic Editor: Hsin-Jang Shieh

Received: 5 March 2024 Revised: 26 March 2024 Accepted: 5 April 2024 Published: 9 April 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/).



Figure 1. Subsea control system structure.

Advancements in subsea production technology have significantly enhanced equipment reliability, leading to a scarcity of historical failure data [8]. This lack of data, alongside harsh operating conditions, complex maintenance needs, significant costs, and weather constraints, challenges maintenance management. Research indicates that conventional monitoring techniques based on static p-f indicators are increasingly outdated, resulting in unnecessary energy inefficiency and loss of power [9]. There is a pressing demand for updating traditional reliability models that rely on fault data to better suit complex operations and include proactive maintenance strategies to minimize failures and losses [10].

In developing more reliable models for subsea control systems, researchers have explored various methodologies. Liu et al. [11] examined the effects of extreme shocks on equipment performance degradation, establishing a model that links sudden failures with degradation thresholds. Liu et al. [12] introduced a Bayesian network approach, performing a reliability analysis of the electrical control system of subsea BOP. Ali et al. [13] explored the risks and reliability of subsea production systems, particularly focusing on the "Xmas tree" system, offering risk mitigation and maintenance recommendations. Si et al. [14] combined life distribution and degradation models, using the Wiener model for better parameterization. Narayanaswamy et al. [15] utilized Reliability, Availability, and Maintainability (RAM) analysis for the ROSUB 6000 ROV, leading to an improved design with an MTBF of 4.9 and 6.2 years for ROV-TMS docking and manipulator system operations.

These studies advance precision in reliability modeling but often overlook the effects of external environmental factors and the interplay between system components, creating a gap between theoretical models and actual operational scenarios.

In the domain of production equipment safety, emphasizing preventive maintenance based on the reliability of equipment performance is crucial. This area has seen significant contributions from researchers. Liu et al. [16] utilized uncertain fault tree analysis and established two optimization models for systems with epistemic uncertainty, enhancing subsea system maintenance and extending service life. Zhou et al. [17] introduced a multi-machine maintenance strategy focusing on equipment availability through Markov state analysis, applicable to multi-component systems. Zhen et al. [18] proposed an optimization method for determining optimal preventive maintenance intervals based on risk and cost criteria. Sun et al. [19] applied the improved beluga whale optimization algorithm for estimating Weibull distribution parameters of bogie components. Wu et al. [20] combined the Markov process with general generating function techniques to evaluate system availability and develop a model for the optimal preventive maintenance interval. Pereira et al. [21] presented a reliability model that included incomplete preventive maintenance and a variable improvement factor based on age reduction. Zhao et al. [22] analyzed the pros and cons of single-component versus multi-component maintenance strategies, focusing on maintenance timing. Yonit et al. [23,24] explored periodic system inspections and the impact of repair-extent-dependent maintenance costs, also considering

an R-out-of-N system for optimizing group maintenance strategies. Abbou [25] created a degradation model for systems with partially observable failure states, using dynamic programming to identify an optimal maintenance strategy. Liang et al. [26] optimized the predictive group maintenance policy for multi-system, multi-component networks (MSMCNs) through analytical and numerical methods. Julie et al. [27] suggested a clustering approach to maintenance that gained effectiveness in early development stages. Eleonora et al. [28] explored roller bearing diagnostics, enhancing selection and maintenance strategies. Borivoj et al. [29] demonstrated that laser alignment and vibro-diagnostics on industrial fans can quadruple bearing life, contrasting sharply with the early failures of misaligned systems. These studies collectively advance the field of preventive maintenance, highlighting its importance in enhancing production equipment safety and efficiency. Implementing modern maintenance and diagnostic methods optimizes industrial systems with minimal downtime.

The described methods often rely on extensive degradation data and historical maintenance records for optimizing the maintenance model. Additionally, these strategies are highly specific. However, applying these maintenance models to subsea control system equipment in practice presents certain challenges. This paper addresses the complexity and reliability challenges of subsea control systems in harsh seabed environments. It proposes a novel reliability modeling approach that accounts for equipment redundancy and operational shocks, alongside an optimized preventive maintenance (PM) model for the system's entire lifecycle, offering a valuable reference for maintenance strategies.

Equipment failures arise from both internal causes, like wear and corrosion, and external factors, such as stochastic shocks from natural events [30,31]. Electronic elements are particularly sensitive to both internal degradation factors like thermal stress and external stochastic shocks, which may not affect mechanical components in the same manner. Since the majority of components in subsea control systems are electronic, distinguishing their degradation process from that of mechanical components, this study employs the Wiener process to simulate the degradation of subsea control system components. Utilizing the Wiener process, it models the degradation of these components, acknowledging the distinct effects on operational versus redundant equipment. The Wiener process, known for capturing fluctuations in degradation [32], has been applied successfully in electronic equipment degradation modeling, including life prediction and estimating remaining service life.

The main innovations can be summarized as follows:

1. Combines the Wiener process with a stochastic shock model to addresses the impact of harsh marine conditions on equipment performance alongside natural degradation.

2. Incorporates Copula functions for reliability modeling within subsea control systems to realize inter-device coupling effects and operational load variability.

3. Introduces a preventive maintenance model based on reliability constraints throughout the equipment's lifecycle, with a focus on economic optimization. In summary, the methodology proposed in this paper realizes a major innovation in subsea control system reliability modeling and maintenance strategy development, and provides a new theoretical framework and practical way to improve the reliability and maintenance efficiency of deep-sea exploration equipment.

The remainder of this manuscript is structured as follows:

Section 2 introduces the performance degradation analysis methods and reliability analysis methods for considering random shocks in subsea control systems.

Section 3 expands on the mathematical redundant reliability model of subsea control systems, as well as considering a full life cycle preventive maintenance cost model.

Section 4 presents the results of the simulation modeling and compares these results with the data collected in previous studies.

Section 5 concludes the paper with a summary of the key findings and proposes directions for future research.

2. Methodology

2.1. Definition and Properties of Wiener Process

The Wiener process, also known as Brownian motion, is a type of continuous stochastic process characterized by continuous paths and Markov properties. Initially proposed by Norbert in the early 20th century, its mathematical definition and properties have wide applications in the theory of stochastic processes.

The Wiener process is a continuous-time stochastic process whose sample paths are continuous over any time interval and exhibit the Markov process's independent increments. The Wiener process satisfies the following properties:

- 1. X(0) = 0;
- 2. For any $0 < t_1 < t_2 < \cdots t_n$, random variables, and the increments $X(t_1) X(t_0), \ldots$, $X(t_2) X(t_1), X(t_n) X(t_{n-1})$ are mutually independent random variables;
- 3. For any $t \ge 0$, $\tau > 0$, $X(t + \tau) X(t) \sim N(\mu_1 \tau, \sigma_1^2 \tau)$, where μ_1 is the drift coefficient and σ_1 is the diffusion coefficient.

The evolution of the Wiener degradation state over time is described by the following equation:

$$X(t) = \mu_1 t + \sigma_1 W(t) \tag{1}$$

where W(t) is the standard Brownian motion.

The literature [33] suggests that the failure threshold L can be established based on equipment design manuals, industrial standards, and expert insights. The failure of degraded products is determined by the failure criterion, which, in practical projects, may be a fixed value or a random variable. This paper assumes a fixed value for the failure threshold of the equipment. Therefore, the lifetime of the degraded failed product is described by the following equation:

$$T = \inf\{t|X(t) \ge L, t \ge 0\}$$
(2)

From the above equation, it can be seen that the life of the device can be described by the inverse Gaussian distribution, and because the inverse Gaussian distribution is exactly used to describe the waiting time for the first time to reach a fixed level in Brownian motion, it can be seen that the degradation of the device X(t) is a Wiener process, then the distribution function of the life of the *T* and the probability density function are described by the following equations:

$$F(t) = \Phi\left(\frac{\mu_1 t - L}{\sigma_1}\right) + \exp\left(\frac{2\mu_1 L}{\sigma_1^2}\right) \Phi\left(\frac{-L - \mu_1 t}{\sigma_1 \sqrt{t}}\right)$$
(3)

$$f(t) = \frac{L}{\sqrt{2\pi\sigma_1^2 t^3}} \exp\left[-\frac{(L-\mu_1 t)^2}{2\sigma_1^2 t}\right]$$
(4)

Since W(t) is a standard Brownian motion with expectation 0, the growth of X(t) on long time scales is mainly determined by the drift term μt , and the average path of the degradation process is approximately a straight line with slope μ , the expectation and the variance of the lifetime *T* are described by the following equation:

$$E(t) = \frac{L}{\mu_1} \tag{5}$$

$$Var(T) = \frac{L\sigma_1^2}{\mu_1^3} \tag{6}$$

2.2. Parameter Estimation of the Wiener Process

Assume that there are *N* devices of the same type for performance degradation testing. For sample *i*, the amount of degradation at the initial moment $X_{i0} = 0$, and the amount

of performance degradation of the product is measured at the moments t_1, t_2, \dots, t_r to obtain its measured values, respectively. $\Delta X_{i,j} = X_{i,j} - X_{i,j-1}$ is the amount of performance degradation of the sample *i* between moments t_{j-1} and t_j . According to the nature of the Wiener process, $\Delta X_{i,j}$ is described by the following equation:

$$\Delta X_{i,j} \sim N(\mu_1 \Delta t_{i,j}, \sigma_1^2 \Delta t_{i,j}) \tag{7}$$

where $\Delta t_{i,j} = t_{i,j} - t_{i,j-1}$; $j = 1, 2, \cdots, r$.

From the performance degradation data, the likelihood function is described by the following equation:

$$L(\mu_{1},\sigma_{1}) = \prod_{i=1}^{N} \prod_{j=1}^{r} \frac{1}{\sqrt{2\sigma_{1}^{2} \pi \Delta t_{i,j}}} \exp\left(-\frac{\left(\Delta x_{i,j} - \mu_{1} \Delta t_{i,j}\right)^{2}}{2\sigma_{1}^{2} \Delta t_{i,j}}\right)$$
(8)

The log-likelihood function is described by the following equation:

$$l(\mu_{1},\sigma_{1}) = \prod_{i=1}^{N} \ln\left(\prod_{j=1}^{r} \frac{1}{\sqrt{2\sigma_{1}^{2}\pi\Delta t_{i,j}}} \exp\left(-\frac{\left(\Delta x_{i,j} - \mu_{1}\Delta t_{i,j}\right)^{2}}{2\sigma_{1}^{2}\Delta t_{i,j}}\right)\right)$$
(9)

Take the partial derivatives of the parameters μ_1 and σ_1 respectively and make them equal to zero.

$$\frac{\partial l(\mu_1, \sigma_1)}{\partial \mu_1} = \sum_{i=1}^N \sum_{j=1}^r \frac{\Delta x_{i,j} - \mu_1 \Delta t_{i,j}}{\sigma_1^2} = 0$$
(10)

$$\frac{\partial l(\mu_1, \sigma_1)}{\partial \sigma_1} = -\frac{rN}{\sigma_1} + \sum_{i=1}^N \sum_{j=1}^r \frac{\left(\Delta x_{i,j} - \mu_1 \Delta t_{i,j}\right)^2}{\sigma_1^3 \Delta t_{i,j}} = 0$$
(11)

From the above equation, the maximum likelihood estimation of the parameters can be described by the following equations:

$$\hat{\mu}_{1} = \frac{\sum_{i=1}^{N} x_{i,r}}{\sum_{i=1}^{N} t_{i,r}}$$
(12)

$$\hat{\sigma}_{1} = \sqrt{\frac{1}{rN} \sum_{i=1}^{N} \sum_{j=1}^{r} \frac{\left(\Delta x_{i,j} - \mu_{1} \Delta t_{i,j}\right)^{2}}{\Delta t_{i,j}}}$$
(13)

2.3. Reliability Modeling Method Based on Wiener Process

The degradation process of the subsea control system is assumed to be a drift Wiener process X(t), which can be described by the following equation:

$$X(t) = \mu_1 t + \sigma_1 W(t) \tag{14}$$

When subsea control system equipment performance degradation satisfies Wiener, its MTBF (mean time between failures) is described by the following equation:

$$MTBF = E(t) = \frac{L}{\mu_1} \tag{15}$$

The reliability of equipment in a subsea control system $R_1(t)$ that complies with the Wiener degradation process is described by the following equation:

$$R_1(t) = P\{X(t) < L\} = \Phi\left(\frac{L - \mu_1 t}{\sigma_1 \sqrt{t}}\right)$$
(16)

where *P* indicates the probability that the device does not reach the failure threshold and $\Phi(\cdot)$ indicates that the function conforms to a standard normal distribution.

2.4. Reliability Modeling Method Considering the Impact of Random Shocks

Traditional reliability models for subsea control systems often overlook the impact of sudden external random shocks, such as oceanic natural disasters and ship anchor impacts, leading to significant performance degradation. To address this gap, this study introduces a shock degradation model for assessing the influence of random shocks on subsea control systems. This model characterizes the degradation process as a diffusion process, triggered by random shocks occurring according to a chi-square Poisson process with a rate of λ . For a system subjected to random shocks over time *t*, denoted by N(t), the probability distribution for the system experiencing *n* shocks within time *t* is described by the following equation:

$$P\{N(t) = n\} = \frac{(\lambda t^2)}{n!} e^{-\lambda t}$$
(17)

We assume that the degradation increments from random shocks are independent, identically distributed standard normal variables without causing catastrophic losses, with the equipment's degradation rate remaining constant post-shock. The degradation process is depicted in Figure 2.





The instantaneous degradation caused by the *j* random shock is assumed to be Y_j , j = 1, 2, 3, ... The degradation of all random shocks S(t) is described by the following equation:

$$S(t) = \sum_{j=1}^{N(t)} Y_j$$
(18)

Therefore, the degradation over the lifecycle of the component equipment is described by the following equation:

$$D(t) = X(t) + S(t) = \mu_1 t + \sigma_1 W(t) + \sum_{j=1}^{N(t)} Y_j$$
(19)

The reliability considering random shocks $R_2(t)$ can be described by the following equation:

$$R_2(t) = P\left(\mu_1 t + \sigma_1 W(t) + \sum_{j=1}^{N(t)} Y_j + < L\right)$$
(20)

Assume that the instantaneous degradations caused by random shocks are independent of each other and follow a normal distribution at $Y_j \sim (\mu_2, \sigma_2^2)$. The overall degradation of the device is obtained as $D(t) \sim (\mu_1 + n\mu_2, \sigma_1^2 + n\sigma_2^2)$, so the reliability function of the device considering random shocks $R_2(t)$ is described by the following equation:

$$R_{2}(t) = P(D(t) < L) = \sum_{n=0}^{\infty} \Phi\left(\frac{L - \mu_{1}t - n\mu_{2}}{\sqrt{\sigma_{1}^{2}t + n\sigma_{2}^{2}}}\right) \frac{e^{-\lambda t}(\lambda t)^{n}}{n!}$$
(21)

3. Modeling

This paper outlines assumptions based on the operational characteristics and preventive maintenance (PM) features of subsea control systems to develop a comprehensive reliability model:

- The Wiener process, preferred over the Weibull and Gamma distributions for its accuracy in representing the natural degradation of electronic components, is adopted due to its compatibility with the redundancy and complex coupling in subsea control systems. It aligns more closely with observed reliability trends.
- 2. It is presumed that all components are new at commissioning and receive timely maintenance to avert potential operational failures, where "timely" implies immediate action upon detecting degradation or reaching a maintenance interval.
- 3. The model accounts for external shocks such as natural disasters or sudden operational changes, considering their discrete, sudden nature and independence. It aims to quantify their impact in terms of frequency and intensity.
- 4. Replacement is anticipated during the Nth PM cycle, triggered when reliability dips below a pre-determined threshold, which is informed by historical data analysis and equipment performance criteria, to ensure maintenance precedes significant deterioration.

These assumptions support the establishment of a reliability model that encapsulates both internal degradation and external shocks' effects on subsea control systems. Further content will explore these assumptions' application in case studies and their influence on model accuracy and predictive performance, underlining the model's applicability and effectiveness.

The model notations used in this paper are described in Table 1.

Notation	Description	Notation	Description
X(t)	Degradation process	W(t)	Standard Brownian motion
μ_1	Drift parameter of Wiener process	σ_1	Diffusion parameter of Wiener process
Ĺ	Failure threshold	T	Lifetime
$R_1(t)$	Reliability of equipment	$\Phi(\cdot)$	Standard normal distribution
N(t)	Number of shocks	λ	Incidence rate of random shock
Y_i	Degradation of each random shock	S(t)	Degradation of all random shock
D(t)	Degradation over lifecycle	$R_2(t)$	Reliability considering random shock
μ_2	Expectation of normal distribution	σ_2	Standard deviation of normal distribution
C	Copula function	θ	Critical parameter

Table 1. Model notations.

Notation	Description	Notation	Description
K	Number of parameters in the model	L_E	Maximum likelihood estimate
$R_3(t)$	Joint reliability	k	Number of imperfect maintenance rounds
D_k	Initial degradation after repair	γ_k	Residual degradation factor
L_k'	Relative failure threshold	ω	Degradation rate influence factor
T_i	Interval between PM	T_m	Time required for PM
R_t	Reliability threshold	Ν	Number of PM times
C_d	Average daily maintenance cost	C_c	Cost of repairing the fault
C_p	Cost of the preparatory work	C_m	Cost of PM
C_t	Loss of shutdown	Cr	Cost of equipment replacement

Table 1. Cont.

3.1. Redundant System Reliability Modeling

In the quest to enhance the reliability of subsea control systems, fault-tolerant technologies are widely employed. A prime example of these technologies is redundancy design. This approach typically involves the integration of additional spare components to bolster system reliability. The failure of certain components within the subsea control system can precipitate critical malfunctions in the production system and pose significant replacement challenges. Consequently, the implementation of a redundant configuration for these components is of paramount importance.

Traditional reliability analysis of parallel dual redundancy often assumes independence between the two units. However, in actual working conditions, the interdependence between the main equipment and its redundancy is uncertain, with notable disparities in the loads they bear. Treating the main equipment and its redundancy as identical components in conventional reliability models is overly simplistic and fails to capture the nuanced coupling relationships and load variations within the system. To overcome these limitations, this paper employs the Copula function to develop a joint reliability function for the components of the subsea control system. This approach offers a more precise representation of the system's internal dynamics and the differing load distributions, enhancing the accuracy of reliability assessments.

Copula is a function used to describe the relationship between the marginal distributions among multi-dimensional random variables and their joint distribution [34]. It serves as an effective method for modeling and quantifying dependencies among components within complex systems, particularly when these components have correlated inter-dependencies or failure modes. By employing a Copula, the precision in estimating the overall system's reliability is significantly enhanced, offering a more nuanced understanding of the intricate relationships within the system.

Suppose there are two random variables *X* and *Y*, and the marginal distribution functions of these variables are $F_X(x)$ and $F_Y(y)$. In this context, the Copula function *C* is used to construct the joint distribution function H(x,y); H(x,y) is described by the following equation:

$$H(x,y) = C(F_X(x), F_Y(y))$$
(22)

where θ is the critical parameter of the Copula function, which is used to quantify and adjust the dependencies between different components. The maximum likelihood method is chosen to obtain an estimate of $\hat{\theta}$, which can be described by the following equation:

$$\hat{\theta} = \operatorname{argmax} \sum_{n=1}^{\infty} \ln C(u_n, v_n; \theta)$$
(23)

where *u* and *v* represent the cumulative probabilities of the two marginal distributions, respectively, and *n* is the number of samples.

Common types of Copulas include the following:

1. Clayton Copula;

$$C_{\theta}^{Clayton}(u,v) = \max(u^{-\theta} + v^{-\theta} - 1, 0)^{-1/\theta} \quad \theta > 0$$
(24)

2. Gumbel Copula.

$$C_{\theta}^{\text{Gumbel}}(u,v) = \exp\left(-\left[\left(-\ln(u)\right)^{\theta} + \left(-\ln(v)\right)^{\theta}\right]^{1/\theta}\right) \quad \theta \ge 1$$
(25)

The Akaike Information Criterion (AIC) was used to select the appropriate Copula model. A lower value of *AIC* indicates that the model is more effective in fitting the data, and the *AIC* was calculated using the following equation:

$$AIC = 2K - 2\ln(L_E) \tag{26}$$

where *K* is the number of parameters in the model and L_E is the maximum likelihood estimate of the model.

The joint reliability function of the subsea control system equipment $R_3(t)$ is described by the following equation:

$$R_3(3) = C(R_1(t), R_2(t))$$
(27)

3.2. Imperfect PM Modeling

The equipment in actual operation will undergo irreversible changes and breaks, and it is impossible to achieve the state of the equipment when it was just put into production through maintenance, a situation known as imperfect maintenance. In addition, the increase in the number of repairs and the age of the equipment in service lead to a rise in the rate of degradation of the equipment and a certain degree of initial degradation of the equipment after each maintenance; its basic process is shown in Figure 3.



Figure 3. The degradation process of imperfect PM.

As can be seen from Figure 3, imperfect PM was carried out at t_1 , t_2 and t_3 . $T_1 = t_1$, $T_2 = t_2 - t_1$ and $T_3 = t_3 - t_2$ show the PM interval during the first three repair cycles, $t_1 > t_2 - t_1 > t_3 - t_2$.

According to the literature [35,36], PM is considered an imperfect activity, implying that the repaired equipment's degraded state falls between its initial degraded state and the PM threshold. This paper defines the residual degradation caused after k maintenance activities as D_k , and it is a random number conforming to a normal distribution with a range of values; k is the number of times imperfect maintenance has been carried out. D_k is described by the following equation:

$$D_k = \gamma_k L \tag{28}$$

where γ_k is the residual degradation amount coefficient after *k* rounds of maintenance, and $\gamma_k \sim N(1 - \exp(-ak), b)$.

We introduce the degradation rate influence factor ω , with a range of values $\omega \in (0, +\infty)$, define the expression for the drift rate of the device after imperfect maintenance as the following equation:

$$\mu_k = \mu_{k-1}(1 + \omega D_k) \tag{29}$$

Therefore, the degradation process after *k* repairs is described by the following equation:

$$X(t) = D_k + \mu_k t + \sigma_1 W(t)$$
(30)

At this point, the relative failure threshold L'_k of the constituent equipment is described by the following equation:

$$L'_k = L - D_k \tag{31}$$

Markou et al. [37] used reliability theory and empirical data to enhance hydraulic systems' efficiency, demonstrating improvements through simulation and statistical modeling. This indicates that reliability theory can help improve efficiency, so this paper constructs a preventive maintenance model based on reliability theory. The malfunction of critical equipment within subsea control systems can result in significant losses, necessitating the minimization of unplanned shutdowns. This paper introduces a preventive maintenance strategy aimed at minimizing the average daily maintenance cost rate while optimizing maintenance intervals within reliability constraints to prevent equipment failures. The proposed optimization model presumes equipment replacement after k times preventive maintenance and mandates maintenance when equipment reliability dips below a predefined threshold. The repair process, constrained by this reliability threshold, is illustrated in Figure 4.



Figure 4. Preventive maintenance under reliability threshold constraints.

$$T_i = \{t | R(t) = R_t\}$$
(32)

Supposing that the maintenance cost over the subsea control system's lifecycle comprises several components, N represents the number of PM times, C_c indicates the cost of repairing the faults, C_p represents the cost of the preparatory work for maintenance, including the cost of mobilizing the vessel and summoning the maintenance personnel, C_m means the cost of PM, C_t denotes the loss caused by the shutdown of oil and gas fields, and C_r is the cost of equipment replacement. Define the average daily maintenance cost of equipment over its entire lifecycle C_d as the following equation:

$$C_{d} = \frac{N[C_{c}(-\ln R_{t}) + C_{p} + C_{m} + C_{T}T_{m}] + C_{r}}{\sum_{i=1}^{N} T_{i} + NT_{m}}$$
(33)

We introduce the degradation rate influence factor to represent the influence of imperfect maintenance, taking the time when the equipment reaches the set reliability threshold as the PM interval and taking the set reliability threshold value and the number of PM times as the decision-making indexes. The basic process of the SCM reliability and PM model is shown in Figure 5.



Figure 5. Preventive maintenance strategy map of subsea control system.

This preventive maintenance map first analyzes the performance degradation characteristics of subsea control equipment, combines the Wiener process and the stochastic process to simulate its degradation process, and selects appropriate physical quantities to quantify the degree of degradation. Next, the monitoring data are processed using the great likelihood estimation method, so as to accurately obtain the parameters of the degradation process and derive the reliability function of the device accordingly. Next, the most suitable Copula model is selected by applying the AIC information criterion in order to calculate the joint reliability function of the equipment. Finally, the reliability model is reconstructed considering the effect of imperfect maintenance on equipment degradation, and based on this, a maintenance strategy model with the objective of minimizing the average daily maintenance cost over the whole lifecycle is developed. The optimal maintenance reliability threshold and maintenance frequency are determined through an iterative method of optimization.

4. SCM Maintenance Case Simulation

In this paper, the important functional equipment SCM in the subsea control system is selected as the simulation object, and in order to ensure the high reliability of the system in the actual project, the SCM is redundantly configured. The components are not independent of each other, and there are complex dependencies between them. The structure of the subsea control system is shown in Figure 6.



Figure 6. Subsea control system structure model.

As the core component of the subsea control system, the SCM is responsible for processing signals from the surface of the water to drive the solenoid valve (DCV) and actuator operations. This process places high demands on signal clarity and real-time processing to ensure accuracy and reliability in subsea operations. However, the performance of the SCM may degrade over time and with continued use of the equipment. This degradation process is mainly reflected in signal attenuation, which is a key factor affecting SCM performance. When the signal strength is reduced to a certain degree, the SCM may not be able to accurately parse the received signal, resulting in delayed or incorrect transmission of commands to the DCV, and the reliability of the entire subsea control system will be affected, increasing the risk of subsea operations.

In the study of subsea control systems, signal acquisition is conducted through various sensors installed within the system, capable of converting physical quantities (such as pressure and temperature) into electrical signals. The collected signals are pre-processed by signal-processing modules, involving filtering, amplification, and digitalization, primarily relying on Analog-to-Digital Converters (ADCs) and Digital Signal Processors (DSPs). To analyze signal attenuation or strength, signals are converted into decibels (*dB*). The calculation formula for the decibel value of a signal is the following equation:

$$dB = 10 \times \log_{10} \left(\frac{P}{P_0}\right) \tag{34}$$

where *P* is the power of the measured signal, and P_0 is the reference power.

4.1. SCM Performance Degradation Data Simulation

In this section, firstly, the degradation data of the SCM are simulated based on the mean time between failures according to the relationship between performance degradation and lifetime distribution. Based on the degradation data, the maximum likelihood estimation method is used for the estimation of the degradation model parameters.

According to the content introduced in the summary of Section 2.3, it can be learned that when the degradation law of the device can be described by the Wiener process, the MTBF is related to the parameters of the Wiener process as well as the failure threshold. According to the reliability parameters of SCM obtained from the actual data collected in an oil field project, the MTBF is about 8424 h, equivalent to 351 days. Based on the actual engineering requirements, the maximum signal attenuation threshold of the SCM is set to 29 dB, which is used as the threshold *L* for maintenance, and it can be calculated according to Equation (15) that the drift parameter $\mu_1 = 0.0826$. We set $\sigma_1 = 0.025$ to generate 100 sets of SCM natural performance degradation data, as shown in Figure 7. The colored lines in Figure 7 indicate different degradation processes.



Figure 7. Simulation of SCM degradation process.

The generated 100 sets of SCM degradation data were substituted into Equations (12) and (13) for calculation. The 95% confidence intervals were selected and the values of the parameters μ_1 and σ_1 are estimated in Table 2.

Table 2. Parameter estimation.

Parameter	Estimated Value	Lower Limit	Upper Limit
μ_1	0.0824	0.0809	0.0837
σ_1	0.1569	0.1558	0.1581

4.2. SCM Reliability Modeling Simulation

According to the literature [38], the amount of instantaneous degradation caused by random shocks obeys a normal distribution. Assuming $\mu_2 = 5.2$, $\sigma_2 = 0.1$, and $\lambda = 0.001$ based on the experience of the experts. Substituting the relevant parameters into Equations (16) and (21), the reliability function considering the effect of random shocks and the reliability function under natural degradation is obtained as shown in Figure 8.



Figure 8. Reliability of SCM.

To establish the joint reliability distribution function of the redundant system, Gumbel Copula and Clayton Copula functions are used to fit the reliability of the SCM, and the AIC values of the two types of Copula functions calculated according to Equation (26) are shown in Table 3.

Table 3. AIC values of different Copula functions.

Functional Model	AIC Value	
Gumbel Copula	-4970.2815	
Clayton Copula	-4191.3035	

In Table 3, the Gumbel Copula function has a lower AIC check value, so this function is chosen to model the redundancy system reliability.

The traditional redundancy reliability calculation method assumes that the SCMs are independent of each other; the redundancy reliability is described by the following equation:

$$R_{independent} = 1 - (1 - R_1)(1 - R_2) \tag{35}$$

The method proposed in this paper is based on Equation (27). The different reliability functions are shown in Figure 9.

As can be seen from the figure, if the operating and redundant modules are assumed to be independent of each other according to the traditional parallel system reliability calculation method, the joint reliability of the modules will be overestimated, which is not conducive to grasping the evolutionary pattern of their performance.





4.3. Imperfect PM strategies of SCM

Based on information from an oil field project, the basic parameters for the PM of the SCM are shown in Table 4 below.

Parameter	Value
<i>C_m</i>	USD 1 million
C_r	USD 12.5 million
C_c	USD 3 million
C_p	USD 0.5 million
$\dot{C_t}$	USD 3.5 million
T_m	1 day

The degradation process modeling method under the influence of imperfect maintenance proposed in this chapter is used to model the degradation process of the SCM after imperfect maintenance, and the values of model parameter settings are shown in Table 5.

 Table 5. SCM imperfect maintenance process parameters.

Parameter	Value
а	0.1
b	0.001
ω	0.1

Substituting the given reliability thresholds with the joint reliability into Equation (32) solves for the operating time within a single preventive maintenance cycle for different numbers of repairs and uses it as the preventive maintenance interval. The reliability function curves of the SCM after different numbers of repairs were obtained as shown in Figure 10.



Figure 10. Different reliability curves of SCM at various maintenance frequencies.

As can be seen from Figure 10, the decay rate of the reliability of the SCM after many repairs is constantly accelerating. If the preventive maintenance work continues to be carried out according to a fixed cycle, failures may occur before the arrival of maintenance personnel, causing unplanned downtime and significant economic losses, so it is necessary to re-plan the preventive maintenance cycle according to the change rule of reliability.

We substitute preventive maintenance intervals into Equation (33), set the preventive maintenance reliability threshold to vary between 0.8 and 0.9, and increase the number of preventive maintenance times in the whole lifecycle from 1 to 6 times. The iterative method is used to optimize the SCM with the goal of minimizing the maintenance cost rate in the whole lifecycle of the SCM is shown in Figure 11.



Figure 11. PM cost rate for the entire lifecycle of SCM.

It is observed that the maintenance cost rate initially decreases with an increase in PM frequency, but subsequently rises as the number of maintenance procedures increases. The minimum point in the graph indicates that, over the SCM's entire lifecycle, the daily

average maintenance cost is lowest—at USD 68,485 per day—when the reliability threshold is set at 0.82 and after two rounds of PM, and the preventive maintenance intervals are 213 days and 132 days.

Based on the reliability analysis conducted in this paper, utilizing traditional scheduled maintenance for the SCM would result in equipment failure before the commencement of the second maintenance activity. In contrast, the preventive maintenance (PM) approach proposed in this article, which timely adjusts the maintenance intervals based on the degradation pattern of equipment reliability, effectively prevents unplanned downtimes. This significantly enhances the economic efficiency and reliability of oil and gas production activities, showcasing the distinct advantages of this method over traditional approaches in avoiding production halts and improving operational reliability.

5. Conclusions

This paper develops a PM strategy for subsea control systems, focusing on reliability and cost-effectiveness, in light of internal natural degradation, random shocks, and imperfect maintenance of the system's components. The key findings are as follows:

Firstly, the use of the Wiener process and random shock models for modeling the performance degradation of subsea control system equipment allows for a more accurate depiction of reliability variations under different operating conditions. Additionally, this paper examines the overall reliability of redundant systems, acknowledging their use in practical applications to reduce system failure risk.

Secondly, the impact of imperfect maintenance on PM strategies is analyzed. With increasing instances of imperfect maintenance and equipment age, the degradation rate accelerates. Therefore, a degradation rate influence factor is introduced to characterize this change, optimizing the PM strategy to balance cost and reliability.

Lastly, the maintenance strategy proposed in this paper not only supports the sustainability goals of long-term stable operation of equipment but also aligns with the energybased maintenance requirements of reducing energy wastage and environmental impacts.

This article still has some limitations. Primarily, the research falls short in providing a comprehensive analysis of the impact of different decision criteria on maintenance strategies. Additionally, this investigation does not extend to the examination of group maintenance strategies for multi-component systems, which is critical to the holistic application of these strategies within intricate engineering settings.

Future research directions should focus on evaluating the influence of diverse decisionmaking criteria on maintenance strategies and exploring group maintenance approaches for multi-component systems, incorporating fault diagnosis techniques to enhance predictive maintenance capabilities and system reliability.

Author Contributions: Conceptualization, Y.W. and X.Z.; methodology, Y.W.; validation, Y.W., Y.Y. and X.Z.; formal analysis, Y.W.; investigation, Y.W.; resources, X.L.; data curation, Y.W.; writing—original draft preparation, Y.W.; writing—review and editing, Y.Y.; supervision, X.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

Conflicts of Interest: Xiaoguang Li was employed by the company Offshore Oil Engineering Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

References

 Yamamoto, M.; Almeida, C.F.; Angelico, B.A.; Colon, D.; Salles, M.B. Integrated subsea production system: An overview on energy distribution and remote control. In Proceedings of the 2014 IEEE Petroleum and Chemical Industry Conference—Brasil (PCIC Brasil), Rio de Janeiro, Brazil, 25–27 August 2014; pp. 173–181. [CrossRef]

- Hansen, R.L.; Rickey, W.P. Evolution of subsea production systems: A worldwide overview. J. Pet. Technol. 1995, 47, 675–680. [CrossRef]
- 3. Bednar, J.M. Zinc subsea production system: An overview. J. Pet. Technol. 1994, 46, 346–352. [CrossRef]
- 4. Sotoodeh, K. A review on subsea process and valve technology. Mar. Syst. Ocean. Technol. 2019, 14, 210–219. [CrossRef]
- Lyalla, I.; Arulliah, E.; Innes, D. A critical analysis of open protocol for subsea production controls system communication. In Proceedings of the OCEANS 2017, Aberdeen, UK, 19–22 June 2017; pp. 1–7. [CrossRef]
- Zhang, Y.; Tang, W.; Du, J. Development of subsea production system and its control system. In Proceedings of the 2017 4th International Conference on Information, Cybernetics and Computational Social Systems (ICCSS), Dalian, China, 24–26 July 2017; pp. 117–122. [CrossRef]
- SINTEF. OREDA Offshore and Onshore Reliability Data Handbook, Vol 1.—Topside Equipment and Vol. 2—Subsea Equipment, 6th ed.; OREDA Participants: Høvik, Norway, 2015.
- 8. Zuo, X.; Yu, X.; Yue, Y.; Yin, F.; Zhu, C. Reliability Study of Parameter Uncertainty Based on Time-Varying Failure Rates with an Application to Subsea Oil and Gas Production Emergency Shutdown Systems. *Processes* **2021**, *9*, 2214. [CrossRef]
- 9. Orošnjak, M.; Brkljač, N.; Šević, D.; Čavić, M.; Oros, D.; Penčić, M. From predictive to energy-based maintenance paradigm: Achieving cleaner production through functional-productiveness. *J. Clean. Prod.* **2023**, 408, 137177. [CrossRef]
- Gao, H.; Cui, L.; Qiu, Q. Reliability Modeling for Degradation-Shock Dependence Systems with Multiple Species of Shock. *Reliab. Eng. Syst. Saf.* 2019, 185, 133–143. [CrossRef]
- 11. Liu, W.; Wang, X.; Li, S. Study on the Effect of Failure Threshold Change Rate on Product Reliability Based on Performance Degradation. *J. Fail. Anal. Prev.* **2020**, *20*, 448–454. [CrossRef]
- 12. Liu, Z.; Liu, Y. A Bayesian network based method for reliability analysis of subsea blowout preventer control system. *J. Loss Prev. Process Ind.* **2019**, *59*, 44–53. [CrossRef]
- Ali, L.; Jin, S.; Bai, Y. Risk Assessment and reliability analysis of subsea production Systems. In Proceedings of the International Conference on Offshore Mechanics and Arctic Engineering, American Society of Mechanical Engineers, Virtual, 3–7 August 2020. V02AT02A076. [CrossRef]
- 14. Si, X.; Wang, W.; Hu, C.; Chen, M.; Zhou, D. A Wiener-process-based degradation model with a recursive filter algorithm for remaining useful life estimation. *Mech. Syst. Signal Process.* **2013**, *35*, 219–237. [CrossRef]
- 15. Narayanaswamy, V.; Raju, R.; Durairaj, M.; Ananthapadmanabhan, A.; Annamalai, S.; Ramadass, G.A.; Atmanand, M.A. Reliability-centered development of deep water ROV ROSUB 6000. *Mar. Technol. Soc. J.* **2013**, 47, 55–71. [CrossRef]
- 16. Liu, Y.; Ma, L.; Sun, L.; Zhang, X.; Yang, Y.; Zhao, Q.; Qu, Z. Risk-Based Maintenance Optimization for a Subsea Production System with Epistemic Uncertainty. *Symmetry* **2022**, *14*, 1672. [CrossRef]
- 17. Zhou, B.; Shi, Y.; Zhang, Y. Availability-centered Maintenance Policies for Degrading Manufacturing Systems Considering Product Quality. J. Northeast. Univ. (Nat. Sci.) 2021, 42, 814–820. [CrossRef]
- 18. Zhen, X.; Hua, Y.; Huang, Y. Optimization of preventive maintenance intervals integrating risk and cost for safety critical barriers on offshore petroleum installations. *Process Saf. Environ. Prot.* **2021**, *152*, 230–239. [CrossRef]
- 19. Sun, H.; He, D.; Zhong, J. Preventive maintenance optimization for key components of subway train bogie with consideration of failure risk. *Eng. Fail. Anal.* 2023, 154, 107634. [CrossRef]
- 20. Wu, C.; Pan, R.; Zhao, X.; Wang, X. Designing preventive maintenance for multi-state systems with performance sharing. *Reliab. Eng. Syst. Saf.* **2024**, 241, 109661. [CrossRef]
- 21. Pereira, F.H.; Melani, A.H.D.A.; Kashiwagi, F.N.; Rosa, T.G.D.; Santos, U.S.D.; Souza, G.F.M.D. Imperfect Preventive Maintenance Optimization with Variable Age Reduction Factor and Independent Intervention Level. *Appl. Sci.* 2023, *13*, 10210. [CrossRef]
- 22. Zhao, J.; Gao, C.; Tang, T. A Review of Sustainable Maintenance Strategies for Single Component and Multicomponent Equipment. *Sustainability* 2022, 14, 2992. [CrossRef]
- 23. Barron, Y.; Yechiali, U. Generalized control-limit preventive repair policies for deteriorating cold and warm standby Markovian system. *Lise. Trans.* **2017**, *49*, 1031–1049. [CrossRef]
- 24. Barron, Y. Group maintenance policies for an R-out-of-N system with phase-type distribution. *Ann. Oper. Res.* **2018**, *261*, 79–105. [CrossRef]
- 25. Abbou, A. Maintenance Optimization of Partially Observable Complex Systems. Ph.D. Thesis, University of Toronto, Toronto, ON, Canda, 2020.
- 26. Liang, Z.; Parlikad, A.K. Predictive group maintenance for multi-system multi-component networks. *Reliab. Eng. Syst. Saf.* 2020, 195, 106704. [CrossRef]
- 27. Agergaard, J.K.; Sigsgaard, K.V.; Mortensen, N.H.; Ge, J.; Hansen, K.B. Quantifying the impact of early-stage maintenance clustering. *J. Qual. Maint. Eng.* **2023**, *29*, 1–15. [CrossRef]
- Desnica, E.; Ašonja, A.; Radovanović, L.; Palinkaš, I.; Kiss, I. Selection, dimensioning and maintenance of roller bearings. *Lect.* Notes Netw. Syst. 2022, 592, 133–142. [CrossRef]
- 29. Novaković, B.; Desnica, E.; Radovanović, L.J.; Ivetić, R.; Đorđević, L.; Labović Vukić, D. Optimization of industrial fan system using methods laser alignment. *Appl. Eng. Lett.* **2021**, *6*, 62–68. [CrossRef]
- 30. Ye, Z.; Xie, M.; Tang, L. Degradation-Based Bum-in Planning under Competing Risks. Technometrics 2012, 54, 159–168. [CrossRef]
- 31. Huang, W.; Askin, R.G. Reliability Analysis of Electronic Devices with Multiple Competing Failure Modes Involving Performance Aging Degradation. *Qual. Reliab. Eng. Int.* **2003**, *19*, 241–254. [CrossRef]

- 32. Zhang, Z.; Si, X.; Hu, C.; Hu, C. Degradation data analysis and remaining useful life estimation: A review on Wiener-process-based methods. *Eur. J. Oper. Res.* 2018, 271, 775–796. [CrossRef]
- Wang, Z.; Hu, C.; Wang, W. An Additive Wiener Process-Based Prognostic Model for Hybrid Deteriorating Systems. *IEEE Trans. Reliab.* 2014, 63, 208–222. [CrossRef]
- Zhang, Y.; Sun, Y.; Li, L. Copula-Based Reliability Analysis for a Parallel System with a Cold Standby. *Commun. Stat. -Theory Methods* 2018, 47, 562–582. [CrossRef]
- 35. Pang, Z.; Pei, H.; Li, T.; Hu, C.; Si, X. Remaining Useful Lifetime Prognostic Approach for Stochastic Degradation Equipment Considering Imperfect Maintenance Activities. J. Mech. Eng. 2023, 59, 14–29. [CrossRef]
- 36. Changhua, H.; Hong, P.; Zhaoqiang, W.; Xiaosheng, S.; Zhengxin, Z. A new remaining useful life estimation method for equipment subjected to intervention of imperfect maintenance activities. *Chin. J. Aeronaut.* **2018**, *31*, 514–695. [CrossRef]
- Marko, O.; Mitar, J.; Velibor, K. Quality analysis of hydraulic systems in function of reliability theory. In Proceedings of the 27th DAAM International Symposium of Intelligent Manufacturing and Automation, Mostar, Bosnia and Herzegovina, 26–29 October 2016; Volume 27, pp. 569–577. [CrossRef]
- Rafiee, K.; Feng, Q.; Coit, D.W. Reliability modeling for dependent competing failure processes with changing degradation rate. *IIE Trans.* 2014, 46, 483–496. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.