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Risk Assessment Method for Analyzing Borehole Instability Considering Formation Heterogeneity

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Abstract: In the study of borehole instability, the majority of input parameters often rely on the average values that are treated as fixed values. However, in practical engineering scenarios, these input parameters are often accompanied by a high degree of uncertainty. To address this limitation, this paper establishes a borehole stability model considering the uncertainty of input parameters, adopts the Monte Carlo method to calculate the borehole stability reliability at different drilling fluid densities, evaluates the sensitivity of borehole instability to a single parameter, and studies the safe drilling fluid density window at different borehole stability reliability values under multi-parameter uncertainties. The results show that the uncertainty of rock cohesion has a great influence on the fracture pressure of the vertical and horizontal wells. The minimum horizontal stress has the greatest influence on the fracture pressure of the vertical and horizontal wells, followed by pore pressure. In the analysis of borehole stability, the accuracy of cohesion and minimum horizontal stress parameters should be improved. In scenarios involving multiple parameter uncertainties, while the overall trend of the analysis results remains consistent with the conventional borehole stability outcomes, there is a noteworthy narrowing of the safe drilling fluid density window. This suggests that relying on conventional borehole stability analysis methods for designing the safe drilling fluid density window can considerably increase the risks of borehole instability. Uncertainty assessment is crucial to determine the uncertainties associated with the minimum required mud pressure, thereby ensuring more informed decision-making during drilling operations. To meet practical application demands, structure and boundary condition uncertainties should be implemented for a more comprehensive assessment of borehole stability.

Keywords: borehole stability; instability risk; heterogeneity; coefficient of variation; sensitivity analysis

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Citation: Gao, X.; Wang, M.; Shi, X.; Li, C.; Zhang, M. Risk Assessment Method for Analyzing Borehole Instability Considering Formation Heterogeneity. *Processes* **2024**, *12*, 70. https://doi.org/10.3390/pr12010070

Academic Editors: Ali Habibi, Jan Vinogradov and Zhengyuan Luo

Received: 26 November 2023
Revised: 11 December 2023
Accepted: 14 December 2023
Published: 28 December 2023

1. Introduction

Borehole instability is one of the key factors affecting borehole rules and comprehensive drilling benefits. Statistics show that borehole instability causes losses of USD 500~600 million in the worldwide oil industry every year [1–4]. It is one of the world's most concerned problems to drilling workers at home and abroad, and it is also an important matter that scientific and technological workers have been making unremitting efforts to tackle [5–10]. To minimize the risks of borehole instability and its attendant technical and economic repercussions, scholars, both domestically and internationally, have developed a diverse array of borehole stability analysis models. These models aim to elucidate the mechanisms underlying borehole instability across various lithologies and in the presence of different coupling fields [11–14]. However, in the construction of these models, the uncertainty associated with their parameters is often reduced to enhance their predictive capabilities and practical applicability. When implementing these models in real-world scenarios, it is advisable to account for parameter uncertainty to ensure a more precise and dependable analysis [15–17].

Geo-mechanical modeling is an effective solution to ensure borehole safety and reduce development costs, especially in the drilling design stage. A geo-mechanical model presents the theoretical or empirical relationship between the stress state and the rock mechanics parameters of the drilled strata section. The establishment of a geo-mechanical model needs to integrate data from various sources to accurately describe the geo-mechanical properties of the strata. Therefore, the more accurate the data obtained, the more reliable the model being built. However, in reality, not all geo-mechanical parameters are completely available and accurate [18,19]. This is due to the lack of geo-mechanical parameters of continuous strata, and the high cost and longtime of coring test can only provide partial information of the characteristic depth of the strata. At the same time, the geo-mechanical parameters of formation are mostly converted from seismic data, well-logging data, or core analysis data, and there are great uncertainties. In addition, there is also a large heterogeneity in the properties of the rock itself, which will lead to distortion of the predicted results [20–25]. Obviously, rock is a material with strong heterogeneity, and it is difficult to measure and determine the mechanical parameters that can represent the overall situation of the formation in an experiment. If the input parameters are limited in representing the formation information, it will lead to deviation in the borehole stability output results, and even lead to wrong conclusions and understandings, resulting in a large risk of borehole instability in the drilling process, which is difficult for conventional borehole stability analysis methods to overcome.

Ai [26] proposed a method for evaluating borehole stability with limited data. This method uses common reservoir data, conducts sensitivity analysis based on various reservoir parameters, and confirms the reliability of the correlation between logging data and geo-mechanistic data. Al-Ajmi and Al-Harthy [27] developed a probabilistic borehole stability model that captures uncertainty in the input variables by running Monte Carlo simulations to calculate drilling fluid pressure values as a probability distribution. The results show that the maximum horizontal stress, formation cohesion, and friction angle are the most critical factors affecting borehole stability. Fontoura et al. [28] used the statistical error analysis method, the first-order second-moment method, and the first-order reliability method to evaluate the influence of parameter uncertainty on borehole failure process. The results of these three methods are similar to those of the more commonly used Monte Carlo method. Wen et al. [29] pointed out that the probability of well stability does not change significantly under different azimuths. All parameters in their model are assumed to be normally distributed. Ma et al. [4] studied the influence of uncertainty of input parameters on the risk of borehole instability in an inclined well, and the results showed that, considering the influence of parameter uncertainty, collapse pressure increased, fracture pressure decreased, and safety density window narrowed, with the most significant factors affecting borehole stability being in situ stress, pore pressure, and rock strength in turn. Chen et al. [20] combined the first-order second-moment method under the reliability theory, established a risk assessment method for borehole instability based on the reliability theory, and investigated the influence of the uncertainty degree of in situ stress, rock mechanics, and other parameters on the analysis results of borehole stability. Zhao Liping et al. [22] established a borehole stability model of brittle bedding fractured mud shale based on the characteristics of the drilled formation, used the quantitative risk analysis method based on the limited collapse width to reasonably assess the borehole stability probability, and selected the drilling fluid density window. Their case analysis concluded that, in the case of limited caving, by using the quantitative risk analysis method, dynamic optimization of the drilling fluid density window in the stable and unstable sections of the borehole can improve ROP and reduce drilling cost. The estimated probability of borehole stability can reach 91~98%, and the calculated results are basically consistent with the actual situation. Considering the uncertainty of model input parameters, Udegbunam et al. [3] used Monte

Carlo simulation to study the impact of probabilistic input parameters on borehole stability, and the results showed that uncertainty modeling could obtain a more realistic operating window and improve underbalanced drilling design and operations. Hilgedick et al. [30] used the Monte Carlo random sampling method to calculate the numerical solution of the risk probability of borehole instability, and compared it to the traditional borehole stability coefficient. The results showed that the borehole instability risk evaluation method established based on the reliability theory could quantitatively evaluate borehole stability. The evaluation index of borehole instability risk probability could provide a basis for selecting traditional borehole stability coefficient and evaluating borehole stability under uncertain conditions [31–35]. Sheng et al. [36] analyzed the sensitivity of factors affecting the reliability of borehole stability to stress, rock strength, and other factors by using the method of borehole instability risk so as to help improve the accuracy of sensitive factors by taking certain measures. For insensitive factors, their ranges can be estimated according to the statistical data of boreholes or drilled wells. Finally, the uncertainty of collapse and fracture pressure prediction results is reduced, and the reliability of the borehole stability analysis is improved. Huang Yi et al. [37] established a quantitative characterization method for geo-mechanical parameter uncertainty, with the input parameters conforming to different probability distribution characteristics in the borehole stability model, and performed quantitative evaluation and analysis of formation borehole stability based on quantitative risk assessment methods.

The uncertainties surrounding in situ stresses, along with those related to rock strength and deformation properties, significantly widen the range of outcomes in the probability distribution for both minimum and maximum mud weights required to prevent fracturing and cracking [38–42]. Given this complexity, when embarking on the development of new fields, it is advisable to adopt a deterministic approach that relies on the most reliable estimates of input parameters to ascertain borehole stability [43–48]. In this scholarly contribution, we established a risk assessment model specifically tailored to evaluate borehole stability. This model is versatile and can assess reliability under scenarios involving either single-parameter uncertainty or more complex multi-parameter uncertainties, offering a comprehensive tool for decision-makers in the field. The research results are helpful to determine the most sensitive factors affecting the reliability of borehole stability analysis, and this study used certain methods to improve the accuracy of sensitive factors and, finally, to reduce the uncertainty of the prediction results of collapse pressure and fracture pressure so as to improve the reliability of borehole stability analysis.

2. Prediction Model of Borehole Instability

2.1. Stress Distribution around Borehole

For any trajectory hole, the in-situ stress should be transferred from the geodetic coordinate system to the borehole rectangular coordinate system, and the conversion equation is shown in Equation (1):

$$\begin{bmatrix} \sigma_x^b \\ \sigma_y^b \\ \sigma_z^b \\ \tau_{yz}^b \\ \tau_{zx}^b \\ \tau_{xy}^b \end{bmatrix} = \begin{bmatrix} \sin^2\gamma & \cos^2\gamma\cos^2\varphi & \cos^2\gamma\sin^2\varphi \\ 0 & \sin^2\varphi & \cos^2\varphi \\ \cos^2\gamma & \sin^2\gamma\cos^2\varphi & \sin^2\gamma\sin^2\varphi \\ 0 & -\sin\varphi\cos\varphi\sin\gamma & \sin\varphi\cos\varphi\sin\gamma \\ -\sin\gamma\cos\gamma & \sin\gamma\cos\gamma\cos^2\varphi & \sin\gamma\cos\gamma\sin^2\varphi \\ 0 & -\sin\varphi\cos\varphi\cos\gamma & \sin\varphi\cos\varphi\cos\gamma \end{bmatrix} \begin{bmatrix} \sigma_v \\ \sigma_H \\ \sigma_h \end{bmatrix} \tag{1}$$

where $\begin{bmatrix} \sigma_x & \sigma_y & \sigma_z & \tau_{yz} & \tau_{xz} & \tau_{xy} \end{bmatrix}$ is the stress component of the in situ stress transformed to the borehole Cartesian coordinate system, in MPa; $\begin{bmatrix} \sigma_v & \sigma_H & \sigma_h \end{bmatrix}$ represent, respectively, the vertical in situ stress, horizontal maximum in situ stress, and horizontal minimum in situ stress, in MPa; φ is the hole azimuth angle, that is, the angle between the projection of the hole on the horizontal plane and the direction of the horizontal maximum in situ stress along the clockwise direction, in °; and γ is the inclination angle, that is, the angle between the borehole axis and the plumb line, in °. The process of coordinate conversion and the diagram of borehole azimuth and inclination are shown in Figure 1. The in situ stresses should be transformed to the geodetic coordinate system firstly; then, the stress component of the in situ stresses in the geodetic coordinate system should be converted to the borehole coordinate system, and finally, they should be converted to the borehole polar coordinate system.

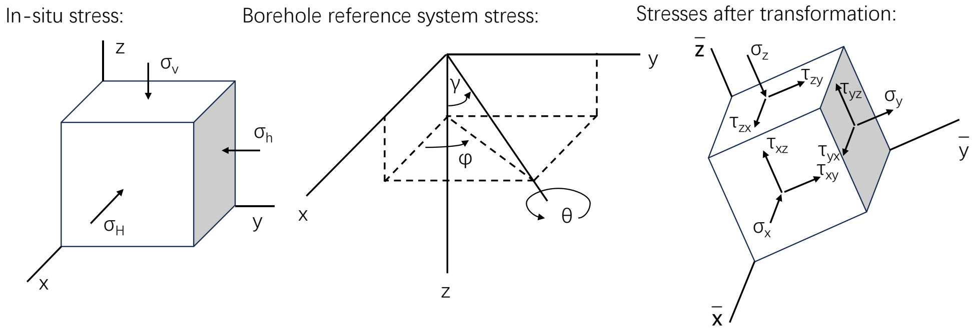

Figure 1. Transformation diagram of stress coordinates around a borehole.

After obtaining the stress component of the in situ stresses in the borehole rectangular coordinate system, the stress distribution of the superimposed borehole pressure around the borehole, the peripheral stress component caused by the temperature difference between the drilling fluid and formation, and considering the effective stress of Biot, the peripheral stress component in the borehole column coordinate system are obtained as shown in Equation (2).

$$\begin{cases} \sigma_r = \frac{(\sigma_x^b+\sigma_y^b)}{2}\left(1-\frac{r_w^2}{r^2}\right)+\frac{(\sigma_x^b-\sigma_y^b)}{2}\left(1-4\frac{r_w^2}{r^2}+3\frac{r_w^4}{r^4}\right)\cos 2\theta+\tau_{xy}^b\left(1-4\frac{r_w^2}{r^2}+3\frac{r_w^4}{r^4}\right)\sin 2\theta \\ \quad +P_w\frac{r_w^2}{r^2}-\alpha P_p+\frac{E\alpha_T}{3(1-\upsilon)}\frac{1}{2}\left(T_w-T_p\right)\left(1-\frac{r_w^2}{r^2}\right) \\ \sigma_\theta = \frac{(\sigma_x^b+\sigma_y^b)}{2}\left(1+\frac{r_w^2}{r^2}\right)-\frac{(\sigma_x^b-\sigma_y^b)}{2}\left(1+3\frac{r_w^4}{r^4}\right)\cos 2\theta+\tau_{xy}^b\left(1+3\frac{r_w^4}{r^4}\right)\sin 2\theta-P_w\frac{r_w^2}{r^2}-\alpha P_p \\ \quad +\frac{E\alpha_T}{3(1-\upsilon)}\frac{1}{2}\left(T_w-T_p\right)\left(1+\frac{r_w^2}{r^2}\right) \\ \sigma_z = \sigma_z^b-2\nu\left(\sigma_x^b-\sigma_y^b\right)\frac{r_w^2}{r^2}\cos 2\theta-4\nu\tau_{xy}^b\frac{r_w^2}{r^2}\sin 2\theta-\alpha P_p+\frac{E\alpha_T}{3(1-\upsilon)}\left(T_w-T_p\right) \\ \tau_{r\theta} = \left[\frac{(\sigma_x^b+\sigma_y^b)}{2}\sin 2\theta+\tau_{xy}^b\cos 2\theta\right]\left(1+2\frac{r_w^2}{r^2}-3\frac{r_w^4}{r^4}\right) \\ \tau_{rz} = \left[\tau_{yz}^b\sin\theta+\tau_{xz}^b\cos\theta\right]\left(1-\frac{r_w^2}{r^2}\right) \\ \tau_{\theta z} = \left[-\tau_{xz}^b\sin\theta+\tau_{yz}^b\cos\theta\right]\left(1+\frac{r_w^2}{r^2}\right) \end{cases} \quad (2)$$

2.2. Strength Criterion

In 1977, Lade proposed a strength model for non-cohesive soil, called the Lade criterion, which considers the effects of all principal stresses or stress invariants on yield and failure [10]. The yield surface is smooth and without edges, reflecting the influence of hydrostatic pressure on yield and the nonlinear relationship between the yield curve and hydrostatic pressure under a high-stress environment. The triaxial tensile strength and compressive strength can be distinguished, but this criterion does not consider the effect of cohesion and is not suitable for predicting the strength characteristics of rocks and cohesive soil. In order to make the Lade criterion more applicable to rock and soil materials with tensile strength and high cohesion, Ewy introduced the coefficient S, which reflects the effect of cohesion on rock yield and failure, into the Lade criterion in 1999 and established the modified Lade criterion:

$$I_1''^3/I_3'' = \eta + 27 \quad (3)$$

$$\begin{cases} I_1'' = (\sigma_1+S)+(\sigma_2+S)+(\sigma_3+S) \\ I_3'' = (\sigma_1+S)(\sigma_2+S)(\sigma_3+S) \end{cases} \quad (4)$$

The relationship between the material parameters (S and η) in the modified Lade criterion and the cohesion and internal friction angle is shown in the following equation:

$$S = \frac{c_o}{\tan\phi_o}, \eta = \frac{4\tan^2\phi_o(9 - 7\sin\phi_o)}{1 - \sin\phi_o} \tag{5}$$

The yield surface of the modified Lade criterion in the principal stress space is shown in Figure 2a, which is a curved cone, and the meridian changes linearly with an increase in hydrostatic pressure. Under a certain hydrostatic pressure, the yield curve of the π-plane is shown in Figure 2b. The yield curve of the criterion complies with Drucker's postulate, and is smooth and convex in the π-plane. The relationship is simple, which is convenient for numerical calculation and has high fitting accuracy for true triaxial strength experimental data.

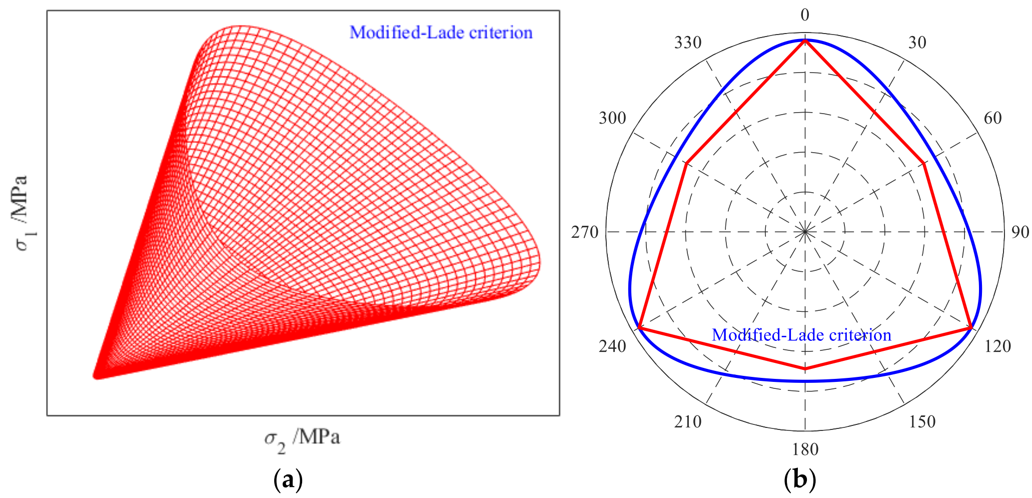

Figure 2. The failure envelops the characteristics of the modified Lade criterion in the principal stress space and π-plane. (**a**) Yield surface in the principal stress space. (**b**) Yield curve in the π-plane (Blue line).

2.3. Model Solution

After obtaining the distribution of borehole stresses, the borehole stresses should also be substituted into the rock strength criterion to judge the stability of the rock around the well. Since the rock strength criterion is expressed in the form of principal stress, the borehole stresses should be converted into the form of principal stress, and the equation for converting borehole stresses into principal stresses under polar coordinates is shown in Equation (6):

$$\begin{cases} \sigma_i = \sigma_r \\ \sigma_j = (\sigma_\theta + \sigma_z)/2 + \sqrt{(\sigma_\theta + \sigma_z)^2 + 4\tau_{\theta z}^2}/2 \\ \sigma_k = (\sigma_\theta + \sigma_z)/2 - \sqrt{(\sigma_\theta + \sigma_z)^2 + 4\tau_{\theta z}^2}/2 \end{cases} \tag{6}$$

The relative magnitude of the three principal stresses at any point around the well varies with the change in the liquid column pressure at the bottom of the well. In order to accurately distinguish the three major principal stresses around the well, Equation (7) is substituted with $\sigma_i, \sigma_j, \sigma_k$ obtained from Equation (6):

$$\begin{cases} \sigma_1 = \max(\sigma_i, \sigma_j, \sigma_k) \\ \sigma_3 = \min(\sigma_i, \sigma_j, \sigma_k) \\ \sigma_2 = \sigma_i + \sigma_j + \sigma_k - \sigma_1 - \sigma_3 \end{cases} \tag{7}$$

The lower limit of the safe drilling fluid density to maintain borehole stability can be obtained by bringing the principal stresses around the well into the modified Lade criterion.

With an increase in the liquid column pressure in the well, the principal stresses around the well will produce tensile stress. When the value of any principal stress around the well exceeds the tensile strength of the rock, the borehole will be broken and an induced crack will be produced in the borehole wall. Since the pressure stress around the well is defined as positive and the tension stress around the well is negative, the solution equation for the wall fracture pressure is shown in Equation (8):

$$\sigma_3 \leq -\sigma_t \tag{8}$$

where σ_3 is the minimum principal stress around the borehole, in MPa; and σ_t is the tensile strength of the rock around the borehole, in MPa.

$$\mathrm{UCS} = \frac{2c_o \cos\varphi_o}{1 - \sin\varphi_o} \tag{9}$$

where UCS is the uniaxial compressive strength of the rock, in MPa. Due to the characteristics of the compressive strength and non-tensile strength of the rock, the tensile strength is only 1/10 of the uniaxial compressive strength of the rock. According to the relationship between the uniaxial compressive strength of the rock, cohesion, and internal friction angle, as shown in Formula (9), the tensile strength of the rock can be obtained, and then the safe upper limit of the drilling fluid density for maintaining borehole stability can be obtained by solving Equation (8).

3. Quantitative Characterization of Stratigraphic Heterogeneity

Due to the old geological age, deep burial, multiple tectonic movements and accumulation periods, and late transformation, the deep strata in China have complex characteristics such as a high temperature, a high-steep structure, multiple faults, a huge thick salt paste layer and a mixed gravel layer, carbonate fractures, solution pores, developed caves, great difference between pore fluid pressure and in situ stress distribution, poor regularity, and strong uncertainty. The input parameters of the borehole stability analysis model are seriously uncertain, and it is difficult to adapt the prediction results to the formation under such uncertain parameter conditions, which makes the risk problems in the drilling process more prominent, and is mainly reflected as serious borehole instability in the complex intervals, frequent occurrence of blowout loss, serious inclination of the high-steep structure, and frequent occurrence of drilling tool accidents [20,48].

In the early stage of oil and gas field exploration and development, the size and distribution of geological parameters, such as in situ stress and formation pressure, are highly uncertain. Due to the lack of a large amount of geological statistical data, it is difficult to accurately calculate the distribution of the pressure profile. Therefore, the uncertainty of geological parameters can only be analyzed by using a subjective probability estimation method. The input parameters of conventional borehole instability analysis method are represented by a probability distribution function instead of a specific value so as to conduct the quantitative evaluation of borehole instability risk under the condition of parameter uncertainty, which can more accurately evaluate the reliability of the borehole stability analysis.

The probability theory is used to reveal the statistical regularity of random phenomena; it is a breakthrough in precise mathematics and can describe random information well. Among the many subjective probability analysis methods available for determining uncertainties, the more widely used are triangular distribution, normal distribution, binomial distribution, and Weber distribution. Through the collection of drilling data in the same block, data statistics, screening, and induction analysis, it is found that in most cases, the distribution of geological parameters obeys the normal distribution. When a certain geological parameter x follows a normal distribution, its probability density function is shown in Equation (10):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}(x-\Delta)^2\right) \tag{10}$$

The corresponding distribution function is

$$F(x) = \int_0^t \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2\sigma^2}(x-\Delta)^2\right)dx \tag{11}$$

According to the distribution function, the mean and variance of the normal distribution are calculated as follows:

$$\begin{cases} \mu = \sum x_i/N \\ \Delta = \sqrt{\sum(x_i-\Delta)^2/N} \end{cases} \tag{12}$$

After obtaining the parameter distribution characteristics of the influencing factors of borehole instability, the Monte Carlo random sampling method was used to calculate the risk probability of borehole instability. Under the condition of uncertain input parameters, the main control factors of borehole instability were studied, and the reliability of borehole stability was quantitatively evaluated. Finally, a borehole instability risk evaluation model was established. It provides the scientific decision-making basis for deep drilling design and risk control in China.

To eliminate the influence of the measurement scale and dimension, the variation coefficient of the output value of the predicted borehole safety density window is used to evaluate the sensitivity of borehole stability to each influencing factor. In the probability theory and statistics, the coefficient of variation, also known as the "dispersion coefficient", is a normalized measure of the degree of dispersion of the probability distribution, defined as the ratio of the standard deviation to the mean, as shown in Equation (13):

$$c_v = \frac{\sigma}{\mu} \tag{13}$$

where σ is the mean value of the data and μ is the standard deviation of the data.

According to the normal distribution probability density function, a group of normally distributed arrays is defined by the mean value and standard deviation. Therefore, the input parameters of the summarized borehole instability risk assessment model are shown in Table 1. According to the geological, drilling, and logging data of a block, the mean value of the cohesive force of the borehole surrounding rock at the depth of 1573.00 m in the block is 6.38 MPa. The mean internal friction angle is 30.42°, the mean horizontal maximum in situ stress is 30.54 MPa, the mean horizontal minimum in situ stress is 23.32 MPa, the mean vertical in situ stress is 35.5 MPa, the mean pore pressure is 17.15 MPa, and the mean Poisson's ratio is 0.25.

Table 1. Distribution characteristics of input parameters.

Inputting Parameters	Cohesion/MPa	Internal Friction Angle/°	Horizontal Maximum In Situ Stress/MPa	Horizontal Minimum In Situ Stress/MPa	Vertical In Situ Stress/MPa	Pore Pressure/MPa	Poisson's Ratio
Mean value	6.38	30.42	30.54	23.32	35.5	17.15	0.25
Standard deviation	2	2	2	2	2	2	2

In order to study the sensitivity of borehole instability to the input parameters, the standard deviation of the parameters is set to 2. To obtain stable distribution characteristics of the input parameters, it is necessary to have a large enough number of samples. When the sampling frequency is 10^4, the probability density and cumulative probability distribution curves of cohesion and internal friction angle are obtained, as shown in Figure 3. This indicates that smooth probability density and cumulative probability distribution curves can be obtained when the sampling frequency reaches 10^4, and the distribution characteristics of the input parameters are stable. Therefore, the sampling times in this analysis were all set to 10^4.

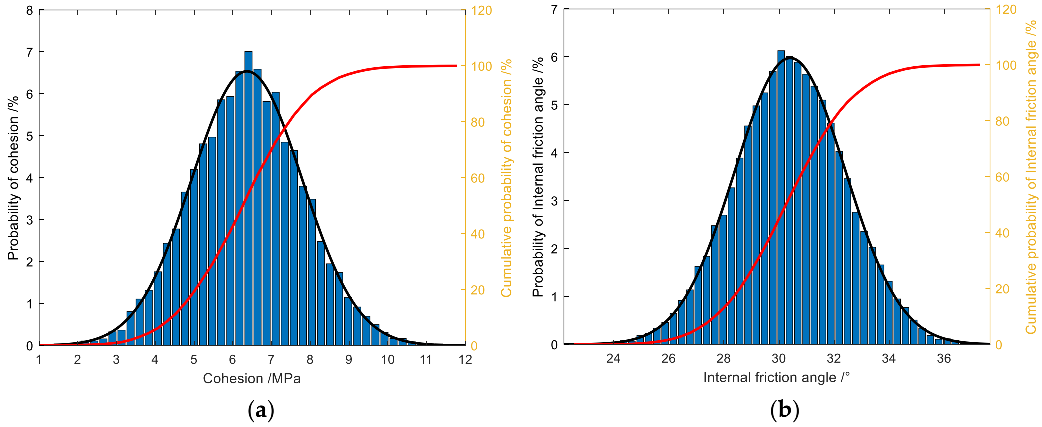

Figure 3. Distribution characteristics of input parameters: (**a**) cohesion and (**b**) internal friction angle.

4. Risk Analysis of Borehole Instability

4.1. Influence of Single Parameter on the Risk of Borehole Instability

By studying the reliability of borehole stability under the condition of single-parameter uncertainty, the sensitivity of borehole stability to each input parameter can be obtained. When a studied parameter is entered into the borehole instability risk prediction model as normally distributed data, the remaining parameters are fixed at the mean value, and the result can be obtained under the condition of each parameter uncertainty. The mean value, standard deviation, and variability of the lower-limit output results of the safe drilling fluid density in the vertical well and horizontal well drilled along the direction of the horizontal maximum in situ stress are shown in Table 2.

Table 2. Sensitivity analysis of effects of input parameters on borehole collapse.

Well Type	**Vertical Well**			**Horizontal Well**		
Influencing Factors	**Mean Value (g/cm^3)**	**Standard Deviation (g/cm^3)**	**Coefficient of Variation (%)**	**Mean Value (g/cm^3)**	**Standard Deviation (g/cm^3)**	**Coefficient of Variation (%)**
Cohesion	1.331	0.115	8.613	1.577	0.114	7.209
Internal friction angle	1.333	0.026	1.986	1.577	0.041	2.599
Horizontal maximum in situ stress	1.37	0.192	7.34	1.5	0.292	0.000
Horizontal minimum in situ stress	1.333	0.033	2.45	1.576	0.033	2.073
Vertical in situ stress	1.332	0	0	1.576	0.098	6.243
Pore pressure	1.333	0.067	5.023	1.576	0.067	4.227
Poisson's ratio	1.127	0	0	1.126	0.00004	0.004

The influence of the input parameters on the vertical borehole collapse pressure was analyzed. The variation coefficient of the lower limit of the safe drilling fluid density output according to the borehole stability and reliability evaluation model is shown in Figure 4.

Since the borehole is a vertical well and parallel to the vertical in situ stress, the borehole collapse pressure has nothing to do with the vertical in situ stress. The influence of other factors on borehole stability was analyzed. According to the coefficient of variation, the most influential factors on the borehole instability of the vertical well are in the order of cohesion > maximum horizontal in situ stress > pore pressure > minimum horizontal in situ stress > angle of internal friction > Poisson's ratio.

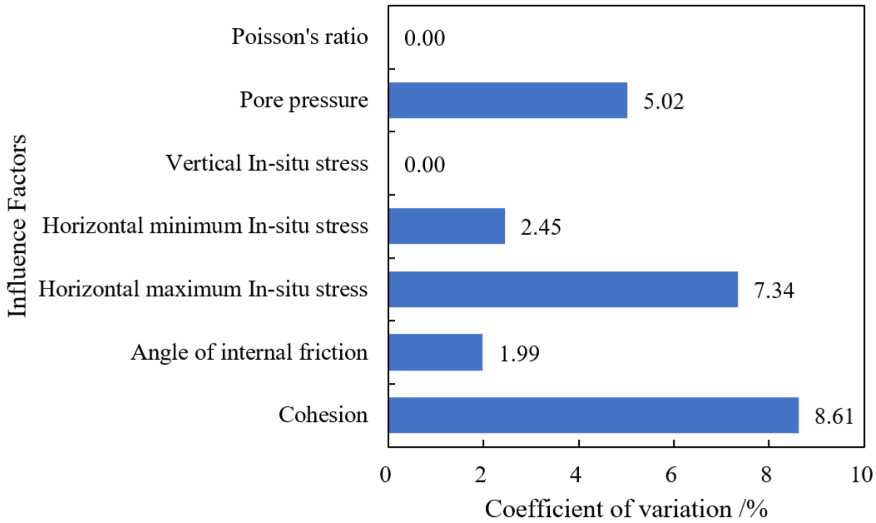

Figure 4. Sensitivity analysis of factors affecting vertical borehole collapse.

The influence of the input parameters on the horizontal borehole collapse pressure was analyzed. The variation coefficient of the lower limit of the safe drilling fluid density output according to the borehole stability and reliability evaluation model is shown in Figure 5. For the horizontal well drilled along the direction of the horizontal maximum in situ stress, the borehole is parallel to the horizontal maximum in situ stress, so the borehole collapse pressure has nothing to do with the horizontal maximum in situ stress. The influence of other factors on borehole stability was analyzed. According to the coefficient of variation, the most influential factors on the borehole instability of the horizontal well are in the order of cohesion > vertical in situ stress > pore pressure > internal friction angle > horizontal minimum in situ stress > Poisson's ratio.

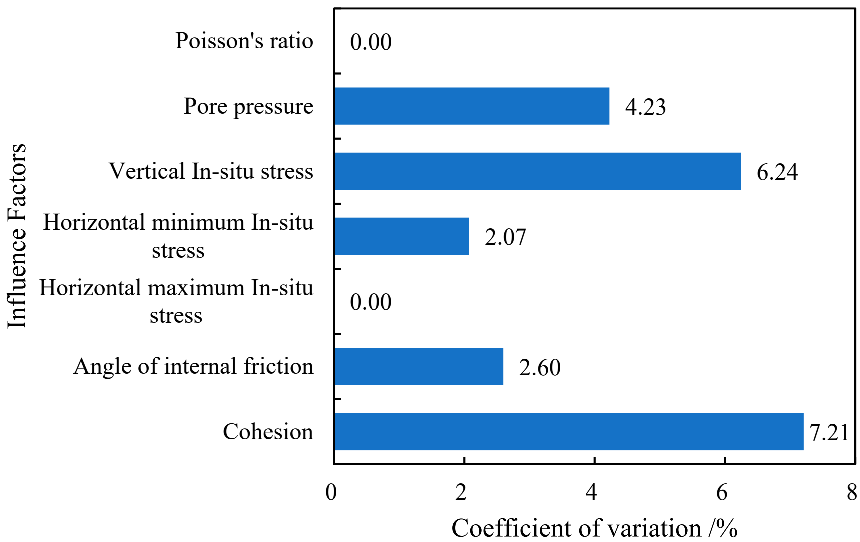

Figure 5. Sensitivity analysis of factors affecting borehole collapse for the horizontal well drilled in the direction of the maximum in situ stress.

When the liquid column pressure in the well is too low, it is not enough to support the borehole, resulting in the collapse and instability of the borehole; when the liquid column pressure in the well is too high, it presses onto the formation, resulting in complex conditions such as drilling fluid loss. The wall fracture pressure is also an important factor to be considered in drilling design. The risk of borehole fracture increases when the risk of borehole collapse decreases. The degree of influence of the influencing factors of borehole instability on borehole fracture instability was quantitatively analyzed, and the variation coefficient of borehole fracture pressure in the vertical and horizontal wells under uncertain parameter distribution was obtained, as shown in Table 3.

Table 3. Sensitivity analysis of effects of input parameters on borehole breakout.

Well Type	**Vertical Well**			**Horizontal Well**		
Influencing Factors	**Mean Value (g/cm^3)**	**Standard Deviation (g/cm^3)**	**Coefficient of Variation (%)**	**Mean Value (g/cm^3)**	**Standard Deviation (g/cm^3)**	**Coefficient of Variation (%)**
Cohesion	1.63	0.05	2.88	1.74	0.19	10.63
Internal friction angle	1.63	0.01	0.37	1.74	0.02	1.38
Horizontal maximum in situ stress	1.62	0.3	8.04	1.74	0	0
Horizontal minimum in situ stress	1.63	0.2	12.23	1.75	0.2	11.42
Vertical in situ stress	1.63	0	0	1.74	0.13	7.57
Pore pressure	1.63	0.13	7.74	1.74	0.09	5.4
Poisson's ratio	1.63	0	0	1.74	0	0

The influence of the input parameters on the fracture pressure of the vertical and horizontal wells was analyzed. The coefficient of variation of the upper limit of the safe drilling fluid density output according to the evaluation model of borehole stability and reliability is shown in Figures 6 and 7.

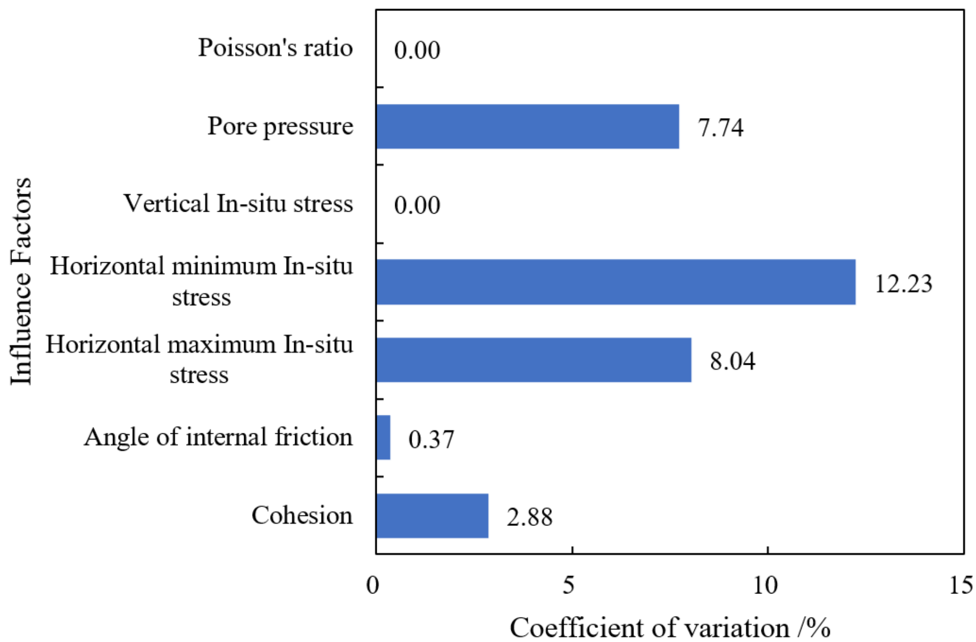

Figure 6. Sensitivity analysis of factors affecting vertical borehole breakout.

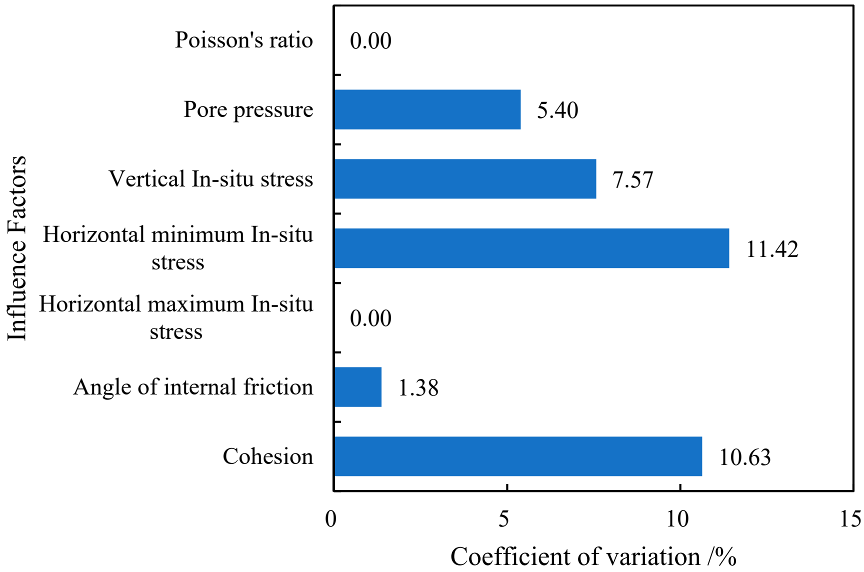

Figure 7. Sensitivity analysis of factors affecting borehole breakout for the horizontal well drilled in the direction of the maximum in situ stress.

For the vertical well, the vertical in situ stress and Poisson's ratio also have no influence on the wall fracture pressure, so their coefficient of variation is 0. The sensitivity of borehole fracture to other influencing factors was analyzed, and the parameters with a greater influence are ordered successively as follows: minimum horizontal in situ stress > maximum horizontal in situ stress > pore pressure > cohesion > internal friction angle.

For the horizontal well drilled along the direction of the horizontal maximum in situ stress, the Poisson's ratio has no influence on the wall fracture pressure, and its coefficient of variation is 0. The borehole is parallel to the direction of the horizontal maximum in situ stress, so the value of the horizontal maximum in situ stress has no influence on the wall fracture pressure. The influence of other factors on the fracture pressure of the borehole wall in descending order is horizontal minimum in situ stress, cohesion, vertical in situ stress, pore pressure, and, finally, internal friction angle.

Based on the above analysis, it can be seen that borehole collapse and instability of the vertical and horizontal wells are most sensitive to rock cohesion, that is, the uncertainty of rock cohesion has a great influence on borehole collapse. In the analysis of borehole collapse and instability, the testing and analysis of rock cohesion parameters should be strengthened to obtain more accurate cohesion values and distribution range. It is helpful to reduce the uncertainty of the analysis results of borehole collapse instability. For the fracture pressure, the horizontal minimum in situ stress has the greatest influence on both the vertical and horizontal wells, followed by pore pressure. The difference is that the horizontal well is more sensitive to changes in cohesion. The sensitivity analysis results may be different from the results reported in previous research studies [26,27,29,31]; this may be caused by different in situ stress mechanisms and distribution characteristics of the input parameters. In the analysis of borehole fracture pressure in a reservoir of interest, multiple methods should be adopted to measure the horizontal minimum in situ stress to obtain a higher measurement accuracy of the horizontal minimum in situ stress.

4.2. Influence of Multiple Parameters on the Risk of Borehole Instability

Borehole instability is the result of the coupling of multiple factors. Compared to studying the sensitivity of borehole instability to a single influencing factor, the obtained probability of borehole instability is more in line with the actual situation when considering the uncertainty of all influencing factors. The Monte Carlo method was used to study the instability risk of borehole wall under the condition of multi-parameter uncertainty, and the distribution characteristics of the input parameters were unchanged according to the data in Table 1.

Firstly, the influence of sampling times on borehole instability risk was studied, and the results were obtained when the sampling times were 10,000, 50,000 and 10,000; the borehole collapse and instability risk of the vertical well is shown in Figure 8. As can be seen from Figure 8, when the sampling times exceed 10,000, the borehole stability reliability curves basically coincide. To save calculation time, the sampling times in subsequent calculations were set to be 10,000 times.

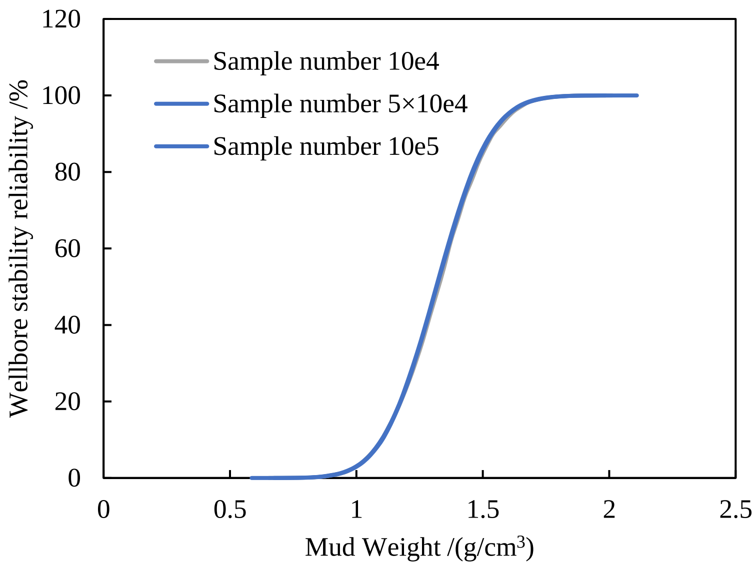

Figure 8. Influence of sampling times on borehole stability of vertical well.

The rule of borehole instability in the vertical well under multi-parameter uncertainty is shown in Figure 9. The blue bars in the chart show the probability density to prevent borehole collapse and instability in the vertical well under different drilling fluid densities. The red bars in the chart show the probability density to prevent vertical borehole breakout and instability under different drilling fluid densities. The blue and red curves show the cumulative probability to prevent borehole collapse and fracture instability (also known as borehole stability reliability) under different drilling fluid densities. It can be seen from the figure that the probability density to prevent collapse and the probability density to prevent fracture overlap with each other. The larger the overlap area, the lower the probability of borehole stability. At the intersection of the blue curve and the red curve, the drilling fluid density is 1.61 g/cm^3, and the probability that the borehole wall will not collapse or break is 95.895%. The stability and reliability of the borehole wall are the highest at this drilling fluid density, but the safety density window is extremely narrow. Considering construction measures such as the start and stop of drilling fluid pumps and the lifting and dropping of drilling tools, it is difficult to maintain the equivalent drilling fluid density at 1.61 g/cm^3. In practice, the equivalent drilling fluid density has a certain window, and the larger the window of the equivalent drilling fluid density that the borehole wall can withstand, the better the stability of the borehole wall. Similarly, by controlling the equivalent drilling fluid density window, the reliability of borehole stability can be controlled within a certain range, and the higher the reliability of borehole stability, the better. When the borehole stability reliability is 90% (that is, the cumulative probability of no borehole collapse or fracture and instability is 90%), the equivalent drilling fluid density window is 1.55~1.67 g/cm^3, indicating that the drilling fluid density variation range is only 0.16 g/cm^3. When the borehole stability reliability is reduced to 80%, the equivalent drilling fluid density window changes to 1.46~1.79 g/cm^3, and the safe drilling fluid density range is 0.33 g/cm^3. When the borehole stability reliability is 70%, the equivalent drilling fluid density window is 1.43~1.85 g/cm^3, and the safe drilling fluid density range is 0.42 g/cm^3.

When the stability reliability of the borehole wall is further reduced to 60%, the safe drilling fluid density ranges from 1.36 g/cm^3 to 1.97 g/cm^3, and the safe drilling fluid density range is 0.61 g/cm^3. With an increase in drilling fluid density window, the risk of borehole instability gradually increases.

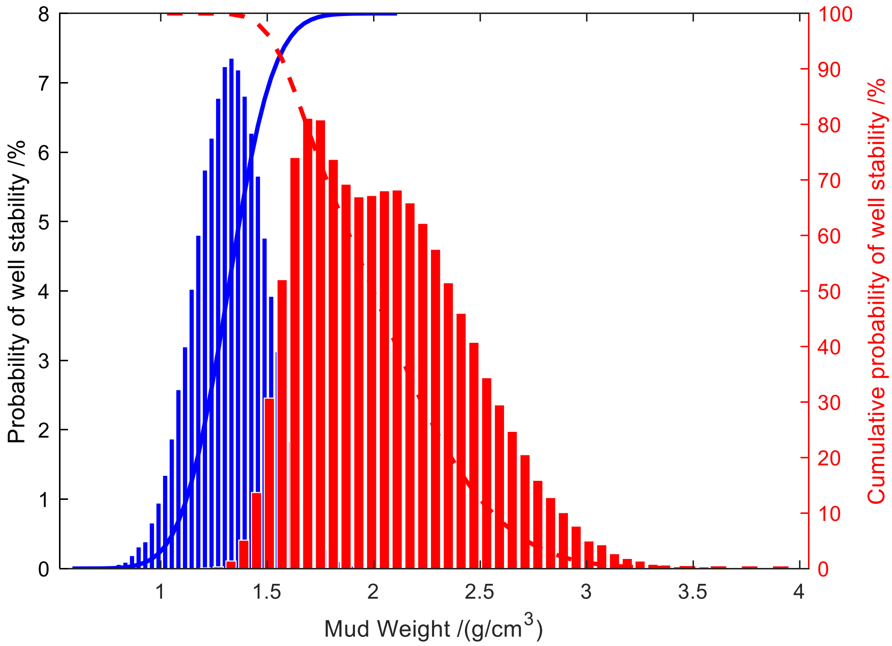

Figure 9. Influence of multi-parameter uncertainty on borehole instability risk in vertical well.

Under the condition of multi-parameter uncertainty, the instability of the horizontal borehole wall drilled along the direction of the maximum horizontal in situ stress is shown in Figure 10. The analysis shows that at the intersection of the cumulative probability curve of no borehole collapse and instability, the drilling fluid density is 1.76 g/cm^3, and the reliability of borehole stability is 87.43%, which indicates that the horizontal well in this block has a higher risk of borehole instability and a narrower window of the safe drilling fluid density than the vertical well. When the borehole stability reliability is 80%, the equivalent drilling fluid density window is 1.73~1.82 g/cm^3, and the safe drilling fluid density variation range is only 0.09 g/cm^3, which is obviously difficult to achieve for the current borehole pressure control ability. When the borehole stability reliability is reduced to 70%, the equivalent drilling fluid density window is 1.67~1.94 g/cm^3, and the safe drilling fluid density variation range is 0.27 g/cm^3. When the wall stability reliability is further reduced to 60%, the equivalent drilling fluid density window is 1.61~2.12 g/cm^3, and the safe drilling fluid density variation range is 0.51 g/cm^3. At this time, the pressure control of the fluid column in the borehole is easier, but the risk of borehole instability is as high as 40%.

Further analysis shows that without considering the influence of parameter uncertainty, the lower limit of the safe drilling fluid density in the vertical well in this block is 1.33 g/cm^3, the upper limit is 2.13 g/cm^3, and the range of the safe drilling fluid density is 0.8 g/cm^3. For the horizontal well, the lower limit of the safe drilling fluid density is 1.58 g/cm^3, the upper limit is 2.24 g/cm^3, and the range of the safe drilling fluid density is 0.66 g/cm^3. It can be seen that compared to the results of the conventional borehole stability analysis, although the overall change trend is consistent after considering the influence of parameter uncertainty, the obtained safe drilling fluid density window is further narrowed. If the safe drilling fluid density window is designed according to the conventional borehole stability analysis method, the reliability of borehole stability will be reduced and the risk of instability will be significantly increased. Conventional borehole stability analysis methods cannot evaluate the impact of uncertainty of input parameters and construction parameters. However, the evaluation model of borehole instability risk established in this paper can quantitatively evaluate the impact of parameter uncertainty on borehole instability risk, which can provide more accurate and effective decision-making basis for drilling technicians and construction personnel.

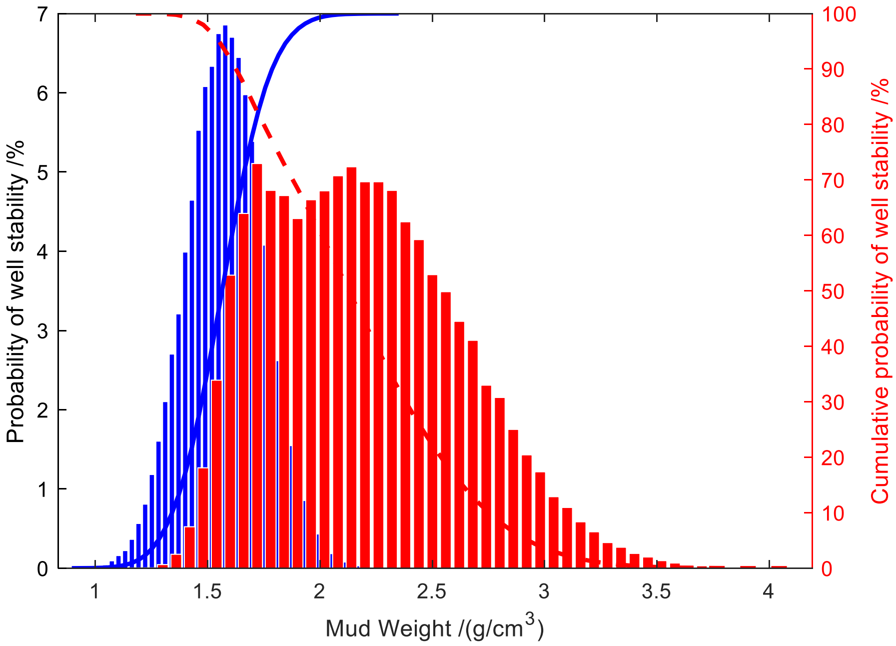

Figure 10. Influence of multi-parameter uncertainty on borehole instability risk of horizontal well.

5. Conclusions

By using the Monte Carlo method, a risk assessment model of borehole instability was established, and the influence degree of uncertainty of a single influencing factor, as well as the influence of uncertainty due to the coupling of multiple influencing factors, on borehole stability reliability was analyzed. The research results show that first, borehole collapse and instability of the vertical and horizontal wells are most sensitive to rock cohesion. In the analysis of borehole collapse and instability, the testing and analysis of rock cohesion parameters should be strengthened to obtain more accurate cohesion values and distribution range, which is helpful to reduce the uncertainty of the analysis results of borehole collapse and instability. Second, the horizontal minimum in situ stress has the greatest influence on the fracture pressure of the vertical and horizontal wells, followed by pore pressure. In the analysis of borehole fracture and instability, multiple methods should be used to measure the horizontal minimum in situ stress to provide a higher measurement accuracy of the horizontal minimum in situ stress. Third, the evaluation of borehole instability risk shows that the borehole instability risk of the horizontal well in this block is higher than that of the vertical well, which is consistent with the overall change trend of the conventional borehole stability analysis results. However, the safe drilling fluid density window is further narrowed, indicating that the design of the safe drilling fluid density window according to the conventional borehole stability analysis method significantly increases the risk of borehole instability. The evaluation model of borehole instability risk established in this paper can analyze the influence of geo-mechanical parameter uncertainty on borehole stability, obtain the function curve of borehole stability probability and drilling fluid density in the drilling process, and optimize the drilling fluid density.

While this study primarily focused on input parameter uncertainty, borehole stability may also be influenced by other uncertainty sources, such as model structure and boundary conditions. Future research could collectively consider these uncertainties for a more comprehensive assessment of borehole stability.

Author Contributions: Conceptualization, M.Z. and X.G.; Methodology, X.G. and M.Z.; Formal Analysis, M.W. and M.Z.; Resources, X.G. and C.L.; Data Curation, M.W. and C.L.; Writing—Original Draft Preparation, X.G. and M.Z.; Writing—Review and Editing, X.G., C.L. and M.Z.; Visualization, X.G. and M.Z.; Supervision, M.Z. and X.G.; Project Administration, X.S. All authors have read and agreed to the published version of the manuscript.

Funding: This study was supported by the team construction project under the Young Innovative Talents Introduction and Cultivation Program of Shandong Province "Research and Innovation Team of Complex Oil and Gas Well Drilling Engineering" (Grant No. 2019035).

Institutional Review Board Statement: All research activities were conducted in accordance with the ethical guidelines and principles outlined by the Committee on Publication Ethics.

Informed Consent Statement: All individuals involved in this study have provided their consent for the publication of the study findings. Any personal or identifying information that could potentially compromise privacy has been carefully removed or anonymized.

Data Availability Statement: The data and materials used in this study are available upon request. Please contact Mingming Zhang (zhangmm60862.sripe@sinopec.com) to inquire about the availability of data and materials, including any restrictions that may apply due to privacy or confidentiality concerns.

Conflicts of Interest: Mingming Zhang was employed by Sinopec Research Institute of Petroleum Engineering Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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