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Trading Portfolio Strategy Optimization via Mean-Variance Model Considering Multiple Energy Derivatives

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Abstract: Energy retailers that sell energy at fixed prices are at risk of bankruptcy due to instantaneous fluctuations in wholesale electricity prices. Energy derivatives, e.g., electricity options, can be purchased by energy retailers then sold to customers as one potential risk-mitigation tool. A class of energy retailers that trade energy derivatives, including the electricity option, the carbon option and the green certificate, is considered in this paper. In terms of energy retailers, a strategy that can maximize the value of the purchased energy derivatives over a period of time and minimize the risk due to the stochastic price fluctuations is developed. Firstly, the dynamic prices of the electricity option as well as the carbon option are described by stochastic differential equations, and the dynamic prices of the green certificate are described by ordinary differential equations. Historical price data are used to obtain the parameters of both stochastic and ordinary differential equations by maximum likelihood estimation. Next, an investment portfolio is established as a mean-variance portfolio selection problem where the retailer maintains the satisfactory asset value and minimizes the risk simultaneously. Then, the problem is transformed into a stochastic optimal control problem which can be solved analytically by using the linear-quadratic method. Finally, the numerical simulations illustrate the feasibility of the proposed method.

Keywords: energy retailer; linear-quadratic control; mean-variance portfolio selection; stochastic differential equation

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1. Introduction

In the energy market, the energy retailer is obliged to serve users' demands by purchasing and selling energy, e.g., electricity [1]. In the past few years, more than 40 energy retailers, e.g., Otima Energie (Germany) [2], Together Energy Retail (the UK) [3], GNC Holdings Inc (the US) [4], declared their bankruptcy, which has caused great concern around the world. The bankruptcy of energy retailers has a detrimental effect on the operation of worldwide enterprises [5]. Thus, it is necessary to analyze the reasons for bankruptcy and put forward effective measures.

The wholesale price volatility of electricity within a certain period has been regarded as the main cause of the bankruptcy of energy retailers [6]. In the opening electricity market, real-time changes in power supply and demand results in instantaneous random fluctuations of wholesale electricity price. Such fluctuations increase the cost of purchased electricity for retailers, who sell energy at fixed prices [2,3]. Thus, they tend to pass on the increased cost to customers, which exposes them to significant risk of losses of earnings. For example, Otima Energie, a German retail company, declared bankruptcy in 2021 due to an extreme rise in wholesale electricity price which raised the cost of purchase [2]. Although the emergence of energy storage devices enables the mitigation of risk caused by wholesale electricity price instantaneous fluctuations, it is uneconomical to store electricity on a large scale.

The market for power lacks alternatives such as financial means (e.g., derivatives including futures and options) to mitigate the detrimental effect aroused by the stochastic electricity price in the wholesale market. To fill this need, the electricity option was developed and applied. The option is an effective tool for managing energy trading risk, and it refers to the right that the holder can purchase the underlying assets on or before a certain date with the price specified in the option contract [7]. The electricity option refers to the option with electricity as its underlying asset. With the introduction of the electricity option, energy retailers can take the initiative in locking electricity price for future trading, therefore reducing risk caused by sharp fluctuations in wholesale electricity market effectively [8,9]. Currently, there are a number of studies focusing on the electricity option. The seasonality of implied volatility of electricity option is studied in [10]. Trading modes of the electricity option in the Nasdaq market are studied in [11]. In the future, most of the available electricity will be traded in the bilateral contract market [12], supplemented by this option. It is shown in [13] that the electricity option can act as an efficient means for mitigating risks in situations where spot price fluctuation occurs.

On the other hand, the current power structure remains dominated by traditional fossil energy, which results in carbon emissions that has caused the world to face extraordinary climate change and environmental pollution [14]. In order to mitigate this problem, many scholars have studied the development of energy from the perspective of planning. The correct energy planning can reduce carbon emissions effectively. The integrated hybrid methodology is proposed in [15,16] for the analysis of the energy strategy alternatives and priorities to adjust its energy planning in Turkey and Pakistan. Renewable energy communities are defined in European Clean Energy Package to promote the overall success of the low-carbon energy transition [17]. The potential and future role of clean energy communities (CECs) are discussed by analyzing various form of CECs including Australia in [18] towards a low-carbon energy transition. In addition to national energy planning, small and medium cities also participate in reducing carbon emissions [19]. In order to ensure a sustainable development of the power industry and promote the realization of zero emissions targets in countries, e.g., China, Japan, UK [20], the application of energy derivatives which can facilitate carbon reduction in the power industry needs to be advocated vigorously [21]. Evidence suggests that trading energy derivatives such as the carbon emission allowance and the green certificate can reduce carbon emissions effectively [22–24].

Carbon emission allowance, an effective tool for carbon emissions reduction and low-carbon development promotion [22], refers to the total amount of carbon dioxide that an energy-using enterprise is permitted to emit each year. If the actual amount of carbon emitted by the enterprise is higher than the amount of the allowance, the exceeding carbon emissions will be punished. Then, the required excess carbon emission allowance needs to be purchased from those who have additional ones, and vice versa [25]. An appropriate carbon emission allowance price, which can be formed by combining financial products and carbon emission allowance into carbon derivatives [26], has assistance in facilitating the development of global carbon emissions reduction [27]. Currently, carbon derivatives, including carbon forward, future, option, swap, etc., have been traded in many countries, e.g., USA [28], Japan [29], Australia [30], etc. Among these derivatives, the carbon option refers to the option that trades carbon emission allowance and delivers them in the future, which is similar to the electricity option, which can lock in the future trading price and mitigate risk effectively. It is, therefore, essential that the energy retailers trade carbon options when the electricity option is traded, which can facilitate low-carbon development in the power industry.

Carbon trading and the green certificate trading are considered to create a mutually reinforcing and promoting relationship, and their joint implementation can have a positive effect on the reduction in carbon emissions in the power industry [31]. Meanwhile, the utilization of renewable energy is considered as an advisable way to reduce carbon emissions [32–34]. The green certificate is regarded as another tool for carbon reduction in the power industry. The green certificate refers to the proof that a certain amount of renewable energy is used by an enterprise. In many countries, e.g., USA [35], Australia [36], UK [37], etc., the renewable portfolio standard implemented by their governments stipulated compulsorily that a certain quota of renewable energy should be used by an enterprise each year. In terms of the enterprise, failing to meet quota stipulations ones requires the purchase of green certificate in order to promote renewable energy and reduce penalties [38]. At present, the transaction of a green certificate is mainly carried out on a trading platform [39], and the re-trading of green certificates is not allowed [40]. This transaction mode leads to a lack of liquidity in the green certificate market. In the near future, reasonable and gradually increasing quota stipulations will be implemented for the enterprise with the global promotion of carbon emissions reduction, which might result in the increasing demand for a green certificate [41]. The increased demand for the green certificate encourages a third party, the energy retailer, to participate in the certificate transaction. At present, Sweden [42] has implemented the practice of the retailer, instead of end users, to purchase green certificate whose price fluctuates in real time. The practice results in the market inefficiencies and retailer's earnings risk in the case of price fluctuation. It is, therefore, particularly worthwhile to carry out a study for the energy retailer on how to properly trade the green certificate considering price fluctuations, which is a promoter of the reduction in carbon emissions and risk mitigation.

With the increasing and rather enormous pressure on carbon emissions reduction for the power industry [43], in this paper, three closely related energy derivatives, including the electricity option, the carbon option and the green certificate, are considered to be traded simultaneously by a class of energy retailers that is likely to emerge soon. An investment portfolio among these three derivatives is designed for such energy retailer with a certain initial fund, in order to maximize the asset value and minimize the risk simultaneously.

Similar to the time-varying electricity price, the price of electricity option is also affected by the future power supply and demand and fluctuates randomly in real-time [44,45]. There are some similarities in the time varying price trend for carbon emission allowance and its options [36]. In terms of the green certificate, the change in its supply and demand would also result in the instantaneous fluctuation in its price [46,47]. The government's certain restrictions imposed on the price of the green certificate [48] makes a divergence of the price trend for the green certificate and two other derivatives (the electricity option and the carbon option). A trend of steady growth rather than random fluctuation is shown in the green certificate price. Considering the difference in trends that are exhibited in the price of each derivative, the inappropriate investment portfolio might lead the energy retailer to buy high and sell low, which results in the risk of earnings losses.

Motivated by this, this paper aims to design the investment portfolio that maximizes the asset value and minimizes the risk simultaneously for an energy retailer to purchase each derivative rationally. In this article, stochastic differential equations (SDEs) [44] are considered to describe the dynamic price of the electricity option as well as the carbon option, and ordinary differential equations (ODEs) are applied to model the dynamic price of the green certificate. Then, how to allocate the initial funds to purchase each derivative is formulated as a mean-variance portfolio selection problem, which is, indeed, a stochastic optimal control problem. For portfolio selection problems, from the control perspective, the initial funds for purchasing each energy derivative are regarded as the system controller. Linear-quadratic (LQ) [49] control methods are used to solve such control problem with analytical solutions. Finally, to verify the feasibility and correctness of the proposed portfolio trading strategy, a series of case studies are considered.

This paper makes four contributions to the field. The main contributions of this paper are as follows:

1. The appropriate portfolio trading strategies with given initial funds are obtained via the LQ control method, which is one of the most popular methods in the field of financial

mathematics. This is the very first time that the portfolio selection problem has been able to be solved, considering the energy option and the green certificate with LQ control method, and it is notable that such portfolio can maximize the asset value of the purchased energy derivatives and mitigate the risk simultaneously. Analytical solutions, which are easier to be implemented compared with the numerical ones mentioned in [50–52], are obtained via the LQ control method.
2. SDEs [53], which are driven by Brownian motion, are used to model the dynamic price of electricity option and carbon option through the up-to-date real data in European Energy Exchange AG (EEX) [54]. Using SDEs to describe the dynamic price of derivatives has the advantage of considering the randomness of prices as well as the correlation with time series, and accurately describing the probability distribution of randomness. Notably, these advantages are not available in the methods used in [55–57]. Furthermore, the architecture of SDE also makes the problem of mean-variance portfolio selection mathematically complex and challenging.
3. Facing the risk of energy price volatility and the pressure of carbon emissions reduction in the power industry, this paper represents the first effort that multiple types of energy derivatives, including the electricity option, the carbon option and the green certificate, are simultaneously considered to be traded by a class of energy retailers. Focusing on the cross fields of energy and economy, we emphasize that it is forward-looking to study the optimal trading strategy for such a scenario that is likely to exist in the future.
4. In this paper, when the prices of electricity option and carbon option are predicted, the real data of EEX [54] is used. Based on the electricity option prices data in EEX which plays a key role in European financial derivatives market, we predict the price of electricity option. Based on the European Union Allowance (EUA) option data which is currently one of the main international carbon options, we predict the price of carbon option. In addition to the price prediction of energy derivatives, we demonstrate the correctness of the proposed portfolio trading strategy, and study the relationship between the asset value of derivatives and the risk aversion of an energy retailer. Thereby, our simulation is meaningful, and provides useful information for countries that have not carried out or have little research on option trading. Meanwhile, guidance is provided for the energy retailer aiming to obtain high asset value while mitigating risk.

The paper is constructed as follows. Section 2 establishes price models of each energy derivative, by which the function of the problem is formulated. The portfolio selection problem is solved analytically via LQ method in Section 3. The case studies are designed in Section 4, which demonstrate the feasibility of the proposed portfolio trading strategy. The conclusions and the prospects of this paper are discussed in Section 5.

2. System Modeling and Problem Formulation

There are four key methodological steps in this section. Firstly, assumptions are provided to ensure the credibility of the research. Secondly, SDEs and ODEs are applied to describe the price models of energy derivatives, including the electricity option, the carbon option and the green certificate. Thirdly, the total asset value of energy derivatives is obtained mathematically. Fourthly, how to allocate initial funds to optimally purchase each derivative considering the price volatility is formulated as a mean-variance portfolio selection problem in the field of control theory.

2.1. Assumptions

Three energy derivatives including the electricity option, the carbon option, and the green certificate are assumed to be traded by a class of energy retailers with certain initial funds. To ensure the credibility of the research, the following assumptions are established:

1. This study focuses on the decision of one energy retailer, and game behavior among several energy retailers is not considered.

2. In the case of sufficient market liquidity, an energy retailer will be able to sell out all the energy derivatives which they purchased previously.
3. The energy retailer can only make one portfolio decision within the considered valid period.
4. The energy retailer is required to purchase at least two classes of derivatives.

2.2. Price Modeling of the Electricity Option

To better understand how the price model of electricity option can be constructed, a brief review of SDEs is first presented; see, e.g., [44]. The general form of SDEs is written as

$$dk(t) = f(k(t), t)dt + g(k(t), t)dW(t), \tag{1}$$

where $k(t)$ is the state of the object which is described by (1) at time t, $f(k(t), t)$ is the drift term used to describe the motion state of the object, $W(t)$ is the standard one-dimensional Brownian motion, and $g(k(t), t)$ is the volatility term. At present, SDEs are widely used in many fields, e.g., economic, biological, physical, medical fields, etc. Especially in the economic field, SDEs are extensively considered as a price model describing the price of option, stocks, and futures [44,58].

Next, the price model of electricity option is introduced. Affected by the changes in fuel price [59], the changes in supply and demand, and different power-production structures, the price of electricity option fluctuates randomly in real-time, which is similar to the price trend of stocks. Based on the extensive application of SDEs in the economic field, and considering real-time random fluctuation of electricity option price, SDEs are applied to describe the dynamic price of the electricity option. In addition, different types of electricity option, which are divided by exercise price, expiry date, etc., are considered in this paper. Let us assume there exist m electricity options to be traded. Then, the price of different types of electricity option can be expressed as

$$dF_i(t) = F_i(t)[\mu_i dt + \sigma_i dW(t)],\ i = 1, 2, \ldots, m, \tag{2}$$

where $F_i(t)$ (EUR/MWh) is the price of the i-th type of electricity option at time t, μ_i and σ_i are system parameters which can be obtained by maximum likelihood estimation (MSE) in Appendix A.

2.3. Price Modeling of the Carbon Option

The existing research has indicated that the price of carbon emission allowance satisfies with the geometric Brownian motion [38]. Remarkably, the price trend of option is analogous to that of the underlying asset [60]. Similar to the modeling of electricity option, SDEs are used to describe the generalized dynamic carbon option price, and different types of carbon option are also considered in this paper. Let us assume there exist n carbon option to be traded. Then, the prices of different types of carbon option can be expressed as:

$$dH_j(t) = H_j(t)\left[\alpha_j dt + \beta_j dW(t)\right],\ j = 1, 2, \ldots, n, \tag{3}$$

where $H_j(t)$ (EUR/ton) is the price of the j-th type of carbon option at time t, α_j and β_j are system parameters which can be obtained by MSE, similar to the approach introduced in Appendix A.

2.4. Price Modeling of the Green Certificate

Affected by the governments' restrictions in certificate price and the changes in certificate supply and demand, the price of green certificate shows an upward trend without random fluctuations in the future. This trend is similar to that of bond prices in the financial field. Due to this feature, ODEs, which have been used to describe the price of bonds, and bank accounts [61], are used to describe the dynamic price of green certificate in this paper:

$$dF_0(t) = \theta F_0(t)dt, \tag{4}$$

where $F_0(t)$ (EUR/MWh) is the price of green certificate at time t, θ represents the steady increase rate of green certificate price. θ is the system parameter and can be obtained by MSE, similar to the approach introduced in Appendix A.

2.5. Asset Value Modeling of Trading Portfolio Strategy

The asset value model of energy derivatives consists of the value of the electricity option, the carbon option and the green certificate at time t. Let $v_0(t)$ (MWh), $v_{1i}(t)$ (MWh), $v_{2j}(t)$ (ton) denote the amount of the green certificate, the electricity option and the carbon option purchased by energy retailers at time t, respectively. The total asset value obtained by derivatives price models can be expressed as:

$$x(t) = v_0(t)F_0(t) + \sum_{i=1}^{m} v_{1i}(t)F_i(t) + \sum_{j=1}^{n} v_{2j}(t)H_j(t), \tag{5}$$

where $x(t)$ (EUR) represents the value of all the energy derivatives at time t.

The incremental value of derivatives at time t can be written as:

$$dx(t) = v_0(t)dF_0(t) + \sum_{i=1}^{m} v_{1i}(t)dF_i(t) + \sum_{j=1}^{n} v_{2j}(t)dH_j(t). \tag{6}$$

Substituting Equations (2)–(4) into Equation (6), we obtain

$$\begin{aligned} dx(t) = \theta x(t)dt + \textstyle\sum_{i=1}^{m}(\mu_i - \theta)v_{1i}(t)F_i(t)dt + \sum_{j=1}^{n}\left(\alpha_j - \theta\right)v_{2j}(t)H_j(t)dt \\ +\left(\textstyle\sum_{i=1}^{m} v_{1i}(t)F_i(t)\sigma_i + \sum_{j=1}^{n} v_{2j}(t)H_j(t)\beta_j\right)dW(t), \end{aligned} \tag{7}$$

where $v_{1i}(t)F_i(t)$ and $v_{2j}(t)H_j(t)$ stand for the initial funds of purchasing electricity option and carbon option, respectively.

Mathematically, the value system (7) can be rewritten as

$$\begin{cases} dx(t) = [Ax(t) + Bu(t)]dt + Du(t)dW(t), \\ x(0) = x_0, \end{cases} \tag{8}$$

where x_0 (EUR) refers to the total initial funds owned by the energy retailer, that is, the total cost of all the energy derivatives purchased by the retailer. In (8), $u(t) = \left[v_{11}(t)F_1(t), \ldots, v_{1i}(t)F_i(t), v_{21}(t)H_1(t), \ldots, v_{2j}(t)H_j(t)\right]'$ is the control input. The coefficients A, B, and D in (8) are presented as $A = \theta$, $B = [\mu_1 - \theta,\ \mu_2 - \theta,\ \ldots,\ \mu_m - \theta,\ \alpha_1 - \theta,\ \alpha_2 - \theta, \ldots,\ \alpha_n - \theta]$, and $D = [\sigma_1,\ \sigma_2,\ \ldots,\ \sigma_m,\ \beta_1,\ \beta_2,\ \ldots, \beta_n]$. The technique of using such state space equation to describe the system dynamics has been adopted in many works; see, e.g., [34,62].

2.6. The Formulation of Objective Function

For the energy retailer, the asset value of energy derivatives is expected to be stable for a period of time. However, the random volatility of some energy derivatives price, e.g., the electricity option and the carbon option, lead to the instability of derivatives value, which might result in earnings risks of the energy retailer. Thus, the goals of risk minimization and value maximization shall be considered simultaneously when formulating energy portfolio trading strategies. Thus, the objective function of this paper can be written as:

$$min:\ J(T) = -E[x(T)] + \delta Var[x(T)], \tag{9}$$

where $E[\cdot]$ denotes the mathematical expectation, $E[x(T)]$ represents the expectation of asset value of energy derivatives, and $Var[\cdot]$ means the variance and $Var[x(T)]$ represents the "risk" of the energy retailer, which can be expressed as:

$$Var[x(T)] = E[x(T) - Ex(T)]^2 = Ex(T)^2 - [Ex(T)]^2. \tag{10}$$

In (9), δ is risk aversion parameter which represents the retailer's risk aversion and is selected based on the risk preferences of the retailer. A small value for δ is selected by a risk-taker retailer, whereas a large value of δ is selected by a risk-averse retailer. The parameter δ plays a key role in formulating the portfolio trading strategy for an energy retailer. Determination of this parameter is beyond the scope of this paper.

Next, the problem of energy retailer's portfolio trading strategy can be expressed as:

$$min: \ J(T), \ s.t. \ (8), \tag{11}$$

where $s.t.$ stands for subject to. In the field of control theory, this problem is called the mean-variance portfolio selection problem [61], which solution is presented in the next section.

3. Solution to the Optimal Control Problem

In this section, there are three key steps to solve Problem (11). Firstly, the mean-variance portfolio selection problem is transferred into an optimal control problem via the introduced auxiliary problem. Secondly, the solution of the general LQ control problem is presented. Thirdly, through the LQ control method, the analytical solution of the optimal control problem is obtained.

3.1. Introducing the Auxiliary Problem

It is notable that Problem (11) is not a standard stochastic optimal control problem, and its solution is hard to be obtained directly by general methods, e.g., dynamic programming, LQ control method, etc. Next, Problem (11) is transferred into a tractable auxiliary problem that can be solved via the LQ method after transformation. The auxiliary problem is set as follows:

$$min: \ J(T) = E\left\{\delta x(T)^2 - \lambda x(T)\right\}, \ s.t.(9), \tag{12}$$

where $\lambda = 1 + 2\delta E[x(T)]$. Here, λ is used to technically transfer Problem (11) into auxiliary Problem (12), and has no practical significance. Remarkably, Problem (11) and (12) have the same optimal solution, which can be proved by the properties of concave function. The specific proof procedures can be found in [61]. Here, we omit the details.

3.2. Solution to the General LQ Control Problem

Here, a general stochastic LQ problem is solved analytically. Consider the general LQ control problem as follows:

$$\begin{array}{l} min: \ J = E\int_0^T [x(t)'Qx(t) + u(t)'Ru(t)]dt + x(T)'Hx(T), \\ s.t. \ dx(t) = [Ax(t) + Bu(t) + C]dt + Du(t)dW(t), \end{array} \tag{13}$$

where $x(t)$ is the state variable change with time t and $u(t)$ is a control input.

The solution to Problem (13) is (see, e.g., [49])

$$u(t) = -\left(R + D'P(t)D\right)^{-1}B'(P(t)x(t) + g(t)), \tag{14}$$

where $P(t)$ is the solution to the following Riccati equation:

$$\begin{cases} \dot{P}(t) = -P(t)A - A'P(t) - Q + P(t)B(R + D'P(t)D)^{-1}B'P(t), \\ P(T) = H, \end{cases} \tag{15}$$

and $g(t)$ is the solution to the following differential equation:

$$\begin{cases} \dot{g}(t) = -A'g(t) + P(t)B(R + D'P(t)D)^{-1}B'g(t) - P(t)C, \\ g(T) = 0. \end{cases} \tag{16}$$

The specific proof procedures are presented in [61].

3.3. Solution to the Auxiliary Problem

Now, we return to solving Problem (12). Let us denote $\gamma = \frac{\lambda}{2\delta}$, $y(t) = x(t) - \gamma$, where γ is used to technically transfer (12) into (17) and has no practical significance. $y(t)$ is the system state of (17). Then, (12) is transformed into (17) as

$$\begin{aligned} &min : J(T) = E\{y(T)'\delta y(T)\}, \\ &s.t.\ dy(t) = [Ay(t) + Bu(t) + C]dt + Du(t)dW(t), \end{aligned} \tag{17}$$

where $C = \gamma r$.

It should be noted that Problem (17) is indeed a special form of the general LQ control problem which can be solved analytically. According to the solving procedures introduced in Section 3.2, the solution for Problem (17) can be obtained as follows:

$$u(t) = -(D'D)^{-1}B'\left(y(t) + \frac{g(t)}{P(t)}\right), \tag{18}$$

where $P(t)$ is the solution to the following Riccati equation:

$$\begin{cases} \dot{P}(t) = -P(t)A - A'P(t) + P(t)B(D'P(t)D)^{-1}B'P(t), \\ P(T) = \delta, \end{cases} \tag{19}$$

and $g(t)$ is the solution to

$$\begin{cases} \dot{g}(t) = -A'g(t) + P(t)B(D'P(t)D)^{-1}B'g(t) - P(t)C, \\ g(T) = 0. \end{cases} \tag{20}$$

By solving (19), the expression of $P(t)$ can be obtained as:

$$P(t) = \delta e^{-\int_t^T (B(D'D)^{-1}B' - 2A)ds}. \tag{21}$$

Next, we defined $h(t) = \frac{g(t)}{P(t)}$, and $\dot{h}(t) = rh(t) - \gamma r$. Combing the expressions of (20) and (21), the expression of $h(t)$ can be obtained as:

$$h(t) = \gamma\left(1 - e^{-\theta(T-t)}\right). \tag{22}$$

Substituting (22) into (18), the solution to Problem (17), i.e., Problem (11), can be obtained as:

$$u(t) = (D'D)^{-1}B'\left(\gamma e^{-\theta(T-t)} - x(t)\right), \tag{23}$$

which practically refers to the desired optimal portfolio trading strategy.

4. Numerical Simulations

In this section, three subsections are demonstrated to show the correctness and feasibility of the proposed portfolio trading strategy. Firstly, the source of data is introduced for parameter estimation of price models for the electricity option, the carbon option and the green certificate. We take the electricity option as an example to simulate the price, which verifies that our work is based on the correct system model. Secondly, the case study is designed to demonstrate the correctness of the portfolio trading strategy proposed in this paper. Thirdly, the relationship between the asset value of derivatives and the risk aversion of an energy retailer is studied.

4.1. Data Source and Parameter Acquisition

In this paper, the historical data in EEX [54] are used to obtain the system parameter of electricity option price model. Then, the comparison of the simulation results of the obtained electricity option's price model with historical price data is plotted. Finally, the electricity option price predictions for different days are presented, and the feasibility of using SDEs to describe electricity option price is demonstrated.

According to Appendix A, the drift term μ_i and volatility term σ_i of (2) can be obtained by the historical the electricity option price. In order to reduce the calculation difficulty, the price of one electricity option is modeled in this paper, that is, $i = 1$ and the parameters obtained are μ_1 and σ_1. Next, the source of the electricity option price data used in this paper is introduced.

The price of "French Power Options" contracts in EEX [54] from 7 February to 13 June in 2022 is used to obtain the value of μ_1 and σ_1. All the data of the electricity option price are recorded daily and are priced by EUR. Next, with the electricity option price data in [54], the parameters obtained by MSE for (2) are μ_1=1.611 and σ_1=1.580.

Based on the obtained values of μ_1 and σ_1, in Figure 1, we plot one typical simulated electricity option price curve for SDE together with the real electricity option price curve within same period (20 days). In Figure 1, the line in blue color with notation "Model" corresponds to the simulated electricity option's price obtained from (2) with determined parameters in one simulation. The real electricity option price data provided in [54] are drawn with red color and notation "Real". Within 100 times of simulations, almost all of the electricity option price curves of the price model (2) locate in the gray region shown in Figure 1. It can be seen from Figure 1 that the real price curve is mostly located in the gray area. In this sense, the validity of the used price modeling method has been demonstrated.

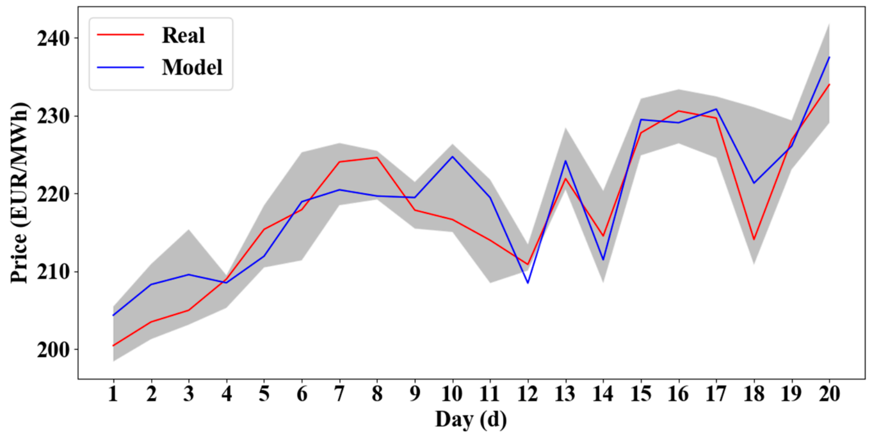

Figure 1. Comparison of the simulation results of electricity option's price model with historical price data.

Then, the price of electricity option with real-time fluctuation can be predicted accurately by (2) without difficulty. However, how long the price can be best predicted by model (2) is undiscovered. Next, the price data of the next 20 days (from 7 March 2022 to 8 April 2022), 40 days (from 7 March 2022 to 6 May 2022) and 60 days (from 7 March 2022 to 13 June 2022) are predicted by model (2) with the parameters obtained above in Figures 2–4, respectively, where the real electricity option price data is also included. Here, the periods of Figures 2–4 are 20 days, 40 days and 60 days, respectively.

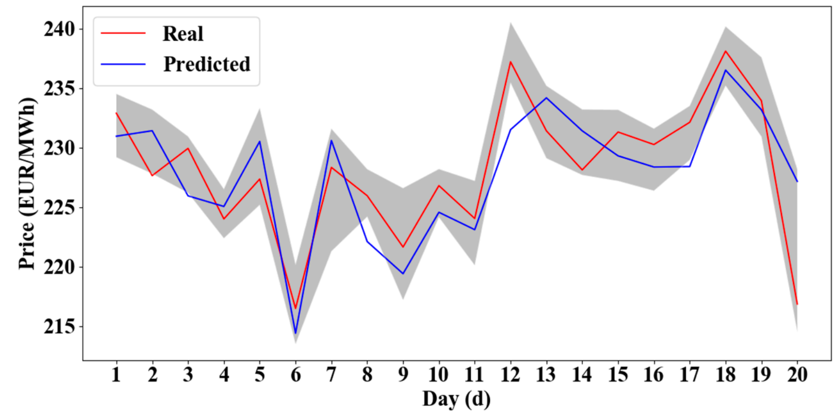

Figure 2. Prediction of the electricity option price for the next 20 days (from 7 March 2022 to 8 April 2022).

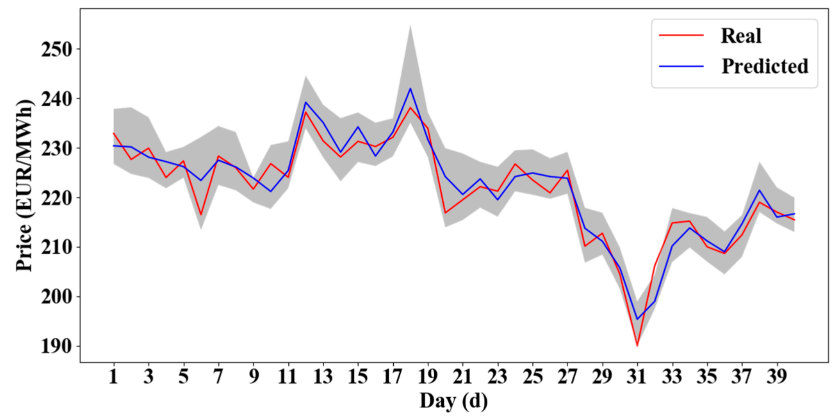

Figure 3. Prediction of the electricity option price for the next 40 days (from 7 March 2022 to 6 May 2022).

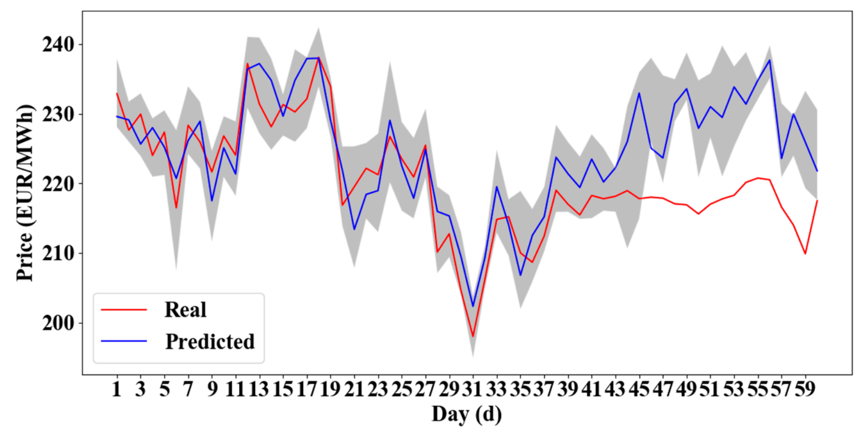

Figure 4. Prediction of the electricity option price for the next 60 days (from 7 March 2022 to 13 June 2022).

It can be seen from Figures 2–4 that the results of using model (2) to predict the price data for the next 20 days (Figure 2) and 40 days (Figure 3) is acceptable. However, the price predict deviation appears in Figure 4 after 40 days is unconvincing. This phenomenon is caused by the characteristics of differential equations which can only describe the dynamic processes in a relatively short-term period. In this sense, price data for 20 days are used to predict the price data from model (2) for the next 40 days, after which prices need to be re-predicted by reacquiring a series of parameter of (2). The corresponding parameters, re-acquirement and price re-prediction, are shown in Figure 5, where price data from 20 April 2022 to 16 May 2022 are used for the data re-prediction from 17 May 2022 to 11 July 2022. The results of parameters' re-acquirement are $\mu_1 = -0.296$ and $\beta_1 = 0.2889$. As can be seen from Figure 5, the price data re-prediction for the next 40 days is gratifying. This demonstrates the feasibility of the above prediction conclusions.

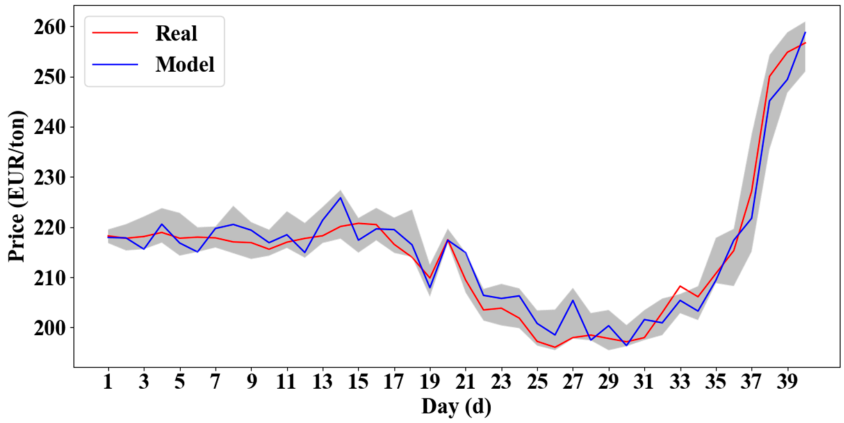

Figure 5. Re-prediction of the electricity option price for the next 40 days (from 17 May 2022 to 11 July 2022).

Similar to the approach to obtaining the parameters of electricity option price model, here, the historical data used to obtain the parameters of carbon option price model are briefly introduced. The EUA option prices [54] that expire in December 2020 from 7 February 2021 to 13 June 2021 are selected as the price data. All data for the carbon option price are recorded daily and are priced by EUR.

We adopt the approach similar to obtaining the parameters of the electricity option price model; the parameters of carbon option price model are obtained as α_1 = 0.132834, β_1 = 0.136222. Similar to the electricity option, the parameters obtained are suitable for the price prediction for the carbon option for 40 days.

For the green certificate, its price model is a special form of (2) and (3), and its parameter can be obtained by the approach used to obtain parameters of the electricity option and carbon option price model with historical price data. However, there are currently no appropriate price data to obtain the parameters of the green certificate price model. Nevertheless, the required parameter is assumed as θ = 0.03 without affecting the main contribution of this paper. The green certificate is priced by EUR, and the initial price of the green certificate is assumed as $F_0(0)$ = 13.6EUR.

4.2. Comparisons under Different Portfolios

The simulation duration is set to be 60 days. The initial funds used to purchase energy derivatives is set as 120,000 EUR, i.e., x_0 = 120,000 EUR. The energy retailer purchases energy derivatives on the first day and sells out all the derivatives previously purchased on the 60th day, which is the scenario set in this paper. The case study with the risk aversion parameter, δ = 0.6, is conducted by the parameters obtained above.

In Table 1, from which the optimality of the proposed portfolio trading strategy can be proved, the value and risk under different portfolios with the risk aversion parameter δ = 0.6 is shown. Among such portfolios, (i) is the optimal portfolio trading strategy that purchases the electricity option, the carbon option and the green certificate with EUR 49,733.2, EUR 35,748.3, and EUR 34,518.5, respectively. Such optimal portfolio trading strategy brings in maximum asset value of EUR 146,493.5 and minimum variance, i.e., risk, of EUR 12,905.1. Compared with other portfolio trading strategies (iii)–(v), the optimal portfolio trading strategy can increase the asset value of energy retailers by at least 27% and reduce the risk of energy retailers by at least 16.42%. Special portfolio trading strategies (v, vi) which conform to the normal distribution of the optimal portfolio trading strategy and have only a small deviation from the optimal one are designed.

Table 1. Value and variance under different portfolio trading strategies.

Property		Number	Initial Funds (EUR)			Objective	
			Electricity Options	Carbon Options	Green Certificates	Value (EUR)	Variance
Optimal portfolio trading strategy		(i)	49,733.2	35,748.3	34,518.5	146,493.5	12,905.1
Non-optimal portfolio trading strategies	Other portfolio trading strategies	(ii)	10,000	10,000	100,000	108,376.5	11,541.7
		(iii)	40,000	40,000	40,000	114,954.7	15,440.8
		(iv)	50,000	50,000	20,000	109,560.3	19,048.5
	Special portfolio trading strategies	(v)	47,596.5	34,310.9	38,092.6	135,890.5	14,475.1
		(vi)	51,869.9	37,185.7	30,944.4	138,633.9	15,900.9

Compared with (i), most of the initial funds in (ii) are used to purchase the green certificate, which brings in variance decrease of 10.56% but value decrease of 26.02%. Compared to other derivatives, the price of green certificate is less volatile, such that investing enormous initial funds in purchasing green certificate may decrease the degree of risk but decrease value.

Compared with (i), the initial funds of purchasing three derivatives are distributed averagely in (iii), which brings in value decrease of 21.53% and variance growth of 19.65%. As can be seen from (iii), the portfolio trading strategy can neither increase value nor decrease risk. In this sense, when formulating the energy portfolio trading strategy, the energy retailer needs to allocate the initial funds of purchasing derivatives reasonably according to the price characteristics of each energy derivative, rather than allocating the initial funds to purchase each derivative averagely.

Compared with (i), the initial funds of purchasing the electricity option and the carbon option are increased in (iv), which brings in value decrease of 25.21% and variance increase of 47.61%. Since the price of electricity option and carbon option fluctuates randomly, these two options cannot reduce risk for energy retailers effectively. When the price of option fluctuates to a very low price, it may bring in a decrease in terminal value.

The initial funds of purchasing three derivatives in (v, vi) conform to the normal distribution of the optimal portfolio trading strategy (i). It can be seen from the variance and value of (v, vi) that although the portfolio has only a little deviation, the value and variance will be less acceptable than that of (i), which demonstrates the optimality of our proposed portfolio trading strategy.

It can be demonstrated by the above comparison cases that the proposed optimal energy portfolio trading strategy is an important tool to increase the asset value and decrease the degree of risk.

4.3. The Relationship between Risk Aversion Parameter and Asset Value

Risk aversion parameter has a significant impact on the portfolio trading strategy. Here, the relationship between risk aversion parameter δ and the asset value is shown in Figure 6. Referring to [63], δ in this simulation is set from 0 to 50.

As can be seen from Figure 6, when the retailer is risk-neutral ($\delta = 0$), it has the highest asset value relatively. As δ increases (i.e., when the retailer tends to be more risk averse), the asset value is decreased, so that when $\delta = 50$ the retailer has the lowest asset value, which is similar to the conclusion of [63]. The retailer prefers to use the green certificate as an instrument for hedging against risk such that the instability of asset value caused by instantaneous volatility in option derivatives price is mitigated. However, a large amount of green certificate trading makes it difficult for the energy retailer to obtain high asset value due to the certificate's price trend of steady-state growth. Thus, as δ increases, retailer is increasingly risk-averse and more green certificates are chosen to be traded, such that the asset value is decreased.

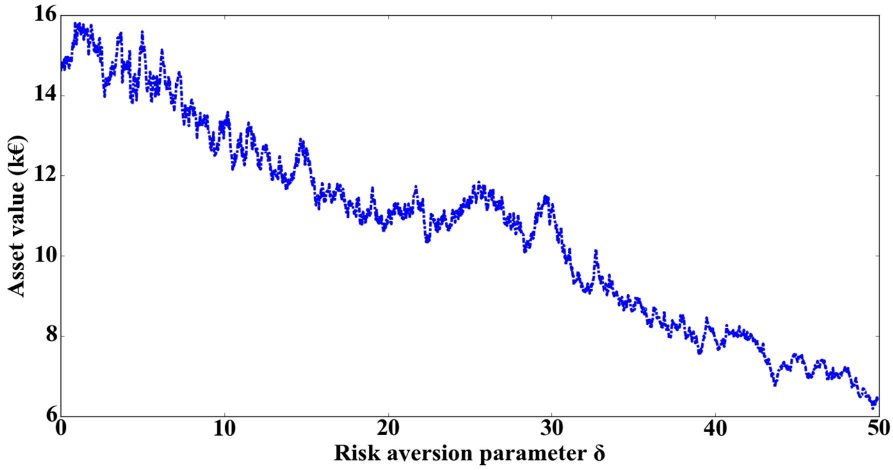

Figure 6. The relationship between the risk aversion parameter and asset value.

According to the simulation results in Figure 6, we can find that the risk aversion parameter has an important impact on the asset value of energy retailers.

5. Conclusions

Faced with the bankruptcy of many energy retailers and the increasing and rather enormous pressure of carbon emissions reduction for the power industry, in this paper, energy derivatives including the electricity option, the carbon option and the green certificate are considered to be traded simultaneously by a class of energy retailers that may emerge soon to reduce the risk of bankruptcy and promote carbon emissions reduction for the power industry. We described the price of the electricity option, the carbon option and the green certificate with SDEs and ODEs. Moreover, we constructed the problem of how retailers obtain satisfactory asset value and bear the lowest risk as the mean-variance portfolio selection problem. This problem is solved using the LQ control method. Finally, the simulation results show that the proposed strategy can increase the asset value of energy retailers by at least 27% and reduce the risk of energy retailers by at least 16.42%, compared with other portfolio trading strategies, which verifies that the feasibility of the proposed strategy.

6. Discussion

In addition to the research results mentioned above, we have some opinions which are worth discussing.

Firstly, in the finance field, SDE is widely used to describe the prices of commodities, e.g., stock, option, etc., at present. In the power field, SDE is used in the transient stability of power system. However, in the field of power finance, SDE is rarely used. Refer to [53,59], SDEs are used to predict the price of electricity option and carbon option with different days as the period. It can be seen from the simulation results in Section 4.1 that it is feasible to predict the price of this option for a period of 40 days.

Secondly, we claim that both the green certificate mechanism and the white certificate mechanism have an influence on the research results of this paper. For the green certificate mechanism, green certificate trading not only promotes the reduction in carbon emissions in the power industry, but also reduces the risk borne by energy retailers by hedging the risk caused by the price of other two derivatives, given the price of the green certificate showing an upward trend without random fluctuations. Since some geographical contexts have removed green certificate mechanism, for energy retailers, tools used to hedge trading risk are lacked, which may cause the decrease in the retailers' asset value and is unfavorable for risk-averse retailers. At the same time, when the price of electricity option and carbon option fluctuates to a favorable value for retailers, high asset value is likely to be obtained by retailers, which is beneficial to risk-taker retailers. The white certificate mechanism refers to the mechanism that limits the energy consumption of energy enterprises. The number, traded by energy retailers, of three energy derivatives is affected by the white certificate mechanism. The demand for the electricity option is reduced due to the implementation of the white certificate mechanism. This reduction leads to the decrease in demand for the carbon option and green certificates, which result in the decrease in asset value obtained by energy retailers.

Thirdly, we prospect the application scenarios of this paper and discuss the future research direction. The proposed portfolio has a broader application scenario. In addition to the energy derivatives mentioned in this paper, the scenario can be extended to the portfolio among other kinds of energy products whose price description is similar to that of the electricity option, the carbon option and the green certificate. The price of other energy products can be described by SDEs and ODEs, then the optimal trading portfolio strategy can be found by using the method proposed in this paper. In the future, we shall conduct the research from the determination of risk aversion parameter and the consideration of game behaviors among several energy retailers. The determination of risk aversion parameter in this paper is relatively simple to a certain extent, and the determination of this parameter by risk appetite, risk tolerance and investment time limitation of an energy retailer will be considered. In addition, non-cooperative game behaviors among several energy retailers will be considered. The problem of heavy computation and difficulty caused by game behaviors among several energy retailers can be solved by machine learning methods [64].

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Appendix A

According to [53,59], the approach to obtaining the values of parameters μ_i and σ_i in (2) is illustrated as follows.

For the i-th electricity option, by taking a small-time step Δt, SDE (2) can be discretized as:

$$\Delta F_i(t) = \mu_i F_i(t)\Delta t + \sigma_i F_i(t)(W(t+\Delta t) - W(t)). \tag{A1}$$

In (A1), $\Delta F_i(t)$ represents the increment of $F_i(t)$ at time t, and term $W(t+\Delta t) - W(t)$ conforms to normal distribution $\mathcal{N}(0, \Delta t)$ which is determined by the nature of Brownian motion. Based on the properties of normal distribution, transition probability $(F_i(t+\Delta t)|F_i(t), \mu_i, \sigma_i)$ can be obtained.

Next, the parameters of SDE, μ_i and σ_i can be estimated. The price of the i-th electricity option from time 0 to time N is denoted as $\{F_i(0), F_i(1), F_i(2), \ldots, F_i(N)\}$. Then, the log-likelihood function of transition probability is obtained in (A2).

$$ln(L(\mu_i, \sigma_i)) = \sum_{t=1}^{N} \ln((F_i(t)|F_i(t-1), \mu_i, \sigma_i)). \tag{A2}$$

Next, according to the MSE method, μ_i and σ_i, the parameters of (2), are found to maximize $\ln(L(\mu_i, \sigma_i))$. By solving two partial differential derivatives, $\frac{\partial \ln(L(\mu_i,\sigma_i))}{\partial \mu_i} = 0$ and $\frac{\partial \ln(L(\mu_i,\sigma_i))}{\partial \sigma_i} = 0$, the expressions of μ_i and σ_i can be obtained as:

$$\begin{cases} \mu_i = \frac{\sum_{t=1}^{N}(F_i(t)-F_i(t-1))F_i(t-1)}{\sum_{t=1}^{N}F_i(t-1)\Delta t}, \\ \sigma_i = \sqrt{\frac{\sum_{t=1}^{N}(F_i(t)-F_i(t-1)-\mu_i F_i(t-1)\Delta t)^2}{\sum_{t=1}^{N}(F_i(t-1))^2\Delta t}}. \end{cases} \tag{A3}$$

Similarly, the values of α_j and β_j can be obtained by the method mentioned above.

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