

Article

Rock Brittleness Evaluation Index Based on Ultimate Elastic Strain Energy

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Abstract: Brittleness is an essential parameter to determine the deformation and failure behavior of rocks, and it is useful to quantify the brittleness of rocks in numerous engineering practices. A novel energy-based brittleness evaluation index is proposed in this study, which redefines the dissipated proportion of ultimate elastic strain energy relative to post-peak failure energy and residual elastic strain energy. A series of conventional triaxial compression (CTC) tests were performed on shale rock to verify the reliability and accuracy of the brittleness index. The results show that the proposed index can precisely reflect the deformation and failure characteristic of rocks under different confining pressures. Based on the testing data from six types of rocks in previous studies, the universality of the novel index was verified. According to comparison with existing brittleness indices, the new brittleness index can more precisely characterize the brittleness of rock.

Keywords: rock brittleness; energy evolution; triaxial compression; stress–strain curve; failure behavior



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1. Introduction

Brittleness is a crucial parameter describing the behavior of rock and soil material failure [1–3], and is closely influenced by mineral composition [4], loading condition [5], micro-fracture [6], physical properties [7,8] and mechanical characteristics [9–11]. In underground engineering field, the brittleness index provides a convenient and effective method to assess the mechanical properties and failure mode of surrounding rock. In shale gas production, the brittleness of shale is key index to evaluate the mechanical properties of shale gas reservoirs. Generally, low brittleness of shale means the shale gas reservoirs have typically low porosity and low permeability; development of this type of shale gas is difficult, and economic benefits are minimal [12,13]. In underground excavation and tunnel engineering, the brittleness of surrounding rock has been widely utilized to judge the stability of rock masses and the tendency of rock burst [14–16]. Therefore, precisely quantifying the brittleness of rock plays a significant role in deep rock mass engineering construction projects and disaster prevention measures.

Recently, Ref. [17] suggested that the deformation and failure behavior of rock is a destabilizing phenomenon accompanied by the storage, dissipation, transformation, and release of energy. Thus, a series of brittleness indices have been proposed to evaluate rock brittleness based on energy evolution in recent years. Ref [18] believed that the more elastic strain energy is stored in the rock during pre-peak stage, the more brittle the rock is. Ref. [19] stated that the more brittle rock is, the more the elastic strain energy and less additional input energy is required for failure. Ref [20] suggested that brittle rock requires more elastic strain energy than additional input energy during the failure process. Ref. [21] suggested that brittle rock could generate little dissipated energy in the pre-peak stage, and the dissipated energy has a negative impact on brittle rock failure. Ref. [22] stated that

the more elastic strain energy is stored in the pre-peak stage and used for failure in the post-peak stage, the more brittle the rock is. Ref. [23] concluded that brittle rock could store a large amount of elastic strain energy in the pre-peak stage and requires little additional input energy when failure occurs. Ref. [24] studied the influence of residual elastic strain energy on rock brittleness and defined two similar kinds of rock brittleness indices as the proportion of stored elastic strain energy required for failure. Ref. [25] stated that rock brittleness can be represented by the dissipation rate of the failure energy. Ref. [26] believed that the additional input energy could be transferred into total elastic strain energy. The aforementioned rock brittleness indices are listed in the Table 1.

Table 1. Summary of existing rock brittleness indices based on the energy evolution characteristic.

Brittleness Index	Remarks	Reference	Insufficient Issues
$BI_1 = \frac{W_u^e}{W_u^e + W_u^d}$	W_u^e is the ultimate elastic strain energy;	[19]	BI_1 only takes the pre-peak energy evolution of rock into consideration, whereas the post-peak stage also has a considerable influence on the rock brittleness.
$BI_2 = \frac{W_p^f}{W_c^e}$	W_u^d is the peak dissipated energy;	[20]	Both of BI_2 and BI_3 simplify the relationship of the pre-peak elastic modulus and post-peak deformation modulus, resulting a low accuracy of assessment.
$BI_3 = \frac{W_c^e}{W_p^f}$	W_p^f is the post-peak failure energy;	[21]	
$BI_4 = \frac{W_p^f + W_u^d}{W_u^e + W_u^d}$	W_r^e is the residual elastic strain energy;	[22]	The formulae of BI_4 and BI_5 are paradoxical.
$BI_5 = \frac{W_u^a}{W_c^e + W_r^e}$	W_u^a is the additional input energy;	[22]	
$BI_6 = \frac{1}{2} \left(\frac{W_c^e}{W_p^f} + \frac{W_c^e}{W_u^e + W_u^d} \right)$	W_c^e is the consumed elastic strain energy;	[23]	BI_6 defines the fracture energy as the area under the post-peak curve, subtracting the residual elastic strain energy. However, it also includes the peak elastic strain energy.
$BI_7 = \frac{W_u^a}{W_c^e} + \frac{W_u^e + W_u^d}{W_u^e}$	W^{ei} is the peak elastic strain energy of the absolute brittle rock;	[24]	BI_7 , BI_8 , BI_9 and BI_{10} join different factors with multiplication, which lacks a theoretical basis.
$BI_8 = \frac{W_u^e}{W_u^e + W_u^d} \times \frac{W_c^e}{W_p^f} \left(1 - \frac{W_c^e}{W_u^e} \right)$		[25]	
$BI_9 = \frac{W_u^e}{W_u^e + W_u^d} \times \frac{W_c^e}{W_p^f}$		[25]	
$BI_{10} = \frac{W_u^e}{W_u^e + W_u^d} \times \frac{W_c^e}{W_p^f} \left(1 - \frac{W_u^d}{W_u^e + W_p^f} \right)$		[26]	The calculation method of BI_{11} is not verified and thus not credible.
$BI_{11} = \frac{W_u^e}{W^{ei} + W_u^e} \times \frac{W_p^f}{W_u^a}$		[27]	

In this study, the energy evolution characteristics of rocks was analyzed during the whole deformation and failure process. A new energy-based brittleness index was redefined as the dissipated proportion of ultimate elastic strain energy relative to post-peak failure energy and residual elastic strain energy. A series of mechanical compression tests were conducted on shale samples to verify the reliability and accuracy of the new index. The results prove that the new index can present more kinds of rock and is more precise for characterization of the brittleness of rock than the other existing brittleness indices.

2. Establishment of the Brittleness Index

Based on previous studies [28,29], the deformation and failure characteristic and energy evolution of brittle rocks in the complete stress–strain curve are presented in Figure 1.

During rock deformation by an external force, the deformation of rock elements can be considered a closed system, assuming that the physical process has no heat exchange with the outside world, and the input energy during the pre-peak loading stage can be divided into two parts: elastic strain energy and dissipated energy [30]. The elastic strain energy is stored in the rock during loading, which is gradually released during unloading and is the major energy cause of rock deformation and failure. Then, the energy is dissipated due to plastic strain and microdamage of the rock, which is irrecoverable with the unloading of the stress [31]. When the elastic strain energy reaches the rock energy storage limit,

the rock enters the failure stage. One part of the concentrated elastic strain energy in the rock leads to rock failure, another part dissipates due to microdamage of the rock, and the residual elastic strain energy exhibits two different states depending on loading conditions. Regarding the uniaxial compression condition, the residual elastic strain energy is mainly be converted into kinetic energy when the rock fragments break. Nevertheless, for the triaxial compression condition, the residual elastic strain energy is stored in the rock, leading the rock to reach its residual strength phase.

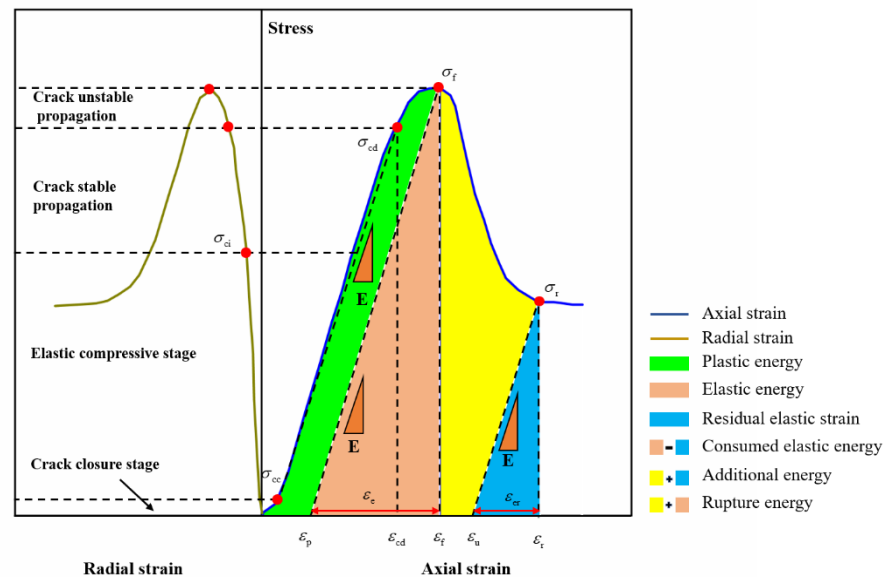


Figure 1. Deformation and failure characteristics and energy evolution of brittle rocks in a typical stress–strain curve (σ_{cc} is crack closure stress, σ_{ci} is crack initiation stress, σ_{cd} is crack damage stress, σ_f is failure stress, σ_r is residual stress, ε_{cd} is strain in crack damage state, ε_f is failure strain, ε_r is residual strain, ε_e is elastic strain, ε_{er} is elastic strain at residual state, and E is Young's modulus).

Based on the analysis of rock energy evolution characteristics and the test results of rocks with different brittleness degrees from previous studies, we found that the rock energy evolution characteristics in the post-peak failure stage can characterize the brittleness index of rocks, and it is believed that the more brittle the rocks are, the less stored pre-peak elastic strain energy converts into post-peak failure energy due to the microdamage of rocks and the less post-peak residual elastic strain energy stored in the rocks, resulting in reduced residual strength of the rocks. In addition, regarding rocks with higher brittleness, the post-peak failure energy is mainly converted from the elastic strain energy stored in the pre-peak stage; for rocks with lower brittleness, the post-peak failure energy is not only transformed from the elastic strain energy in the pre-peak stage but also provided by the additional input strain energy.

On this basis, the evaluation criterion of the rock brittleness index based on ultimate elastic strain energy is established as follows:

$$BI_{ES} = \frac{W_u^e - W_p^f - W_r^e}{W_u^e} \quad (1)$$

where W_u^e is the ultimate elastic strain energy, W_p^f is the post-peak failure energy, and W_r^e is the residual elastic strain energy. The higher the value of BI_{ES} , the more brittle the rock. When BI_{ES} reaches its maximum value of 1, the rock shows ideal brittleness characteristics. The lower the value of BI_{ES} , the more ultimate elastic strain energy is required for failure of the rock, and the plasticity of the rock presents more obviously. If the value of BI_{ES} is negative, the ultimate elastic strain energy is not sufficient to cause rock

failure, and additional input failure energy is required in the post-peak stage, which means that this kind of rock exhibits more pronounced plasticity than general rocks. Therefore, it is believed that the newly established rock brittleness index, BI_{ES} , can reflect a wider range of rock brittleness degrees than the previously proposed rock brittleness indices. The following series of laboratory tests were carried out to verify the reliability and universality of the brittleness index, BI_{ES} , based on the ultimate elastic strain energy.

3. Materials and Methodology

According to the literature [32,33], conventional triaxial compression (CTC) tests are crucial to evaluate the brittleness of rocks by the corresponding brittleness index, and the brittleness of the same kind of rock decreases with increasing confining pressure. Hence, the evaluation accuracy of a rock brittleness index can be judged by the variation trend of rock brittleness index values under different confining pressures.

3.1. Specimen Preparation

The specimens used in this research were shale rock; an intact shale block was mined from Shaanxi Province in China for test preparation. According to a suggestion by the International Society for Rock Mechanics (ISRM), the cylindrical granite specimens were cored with a diameter of 50 mm and a height of 100 mm from an identical block to minimize scattering of the test results. In particular, the shale strata were perpendicular to the axis of the cylindrical specimen. Additionally, sample ends were carefully ground and polished by a grinding machine until the deviation ranges of the flatness and roughness were less than 0.5 mm and 0.05 mm, respectively. The procedures of specimen preparation and stratigraphy formation of the shale rock are shown in Figure 2. Table 2 lists the basic physical and mechanical properties of the shale rock.

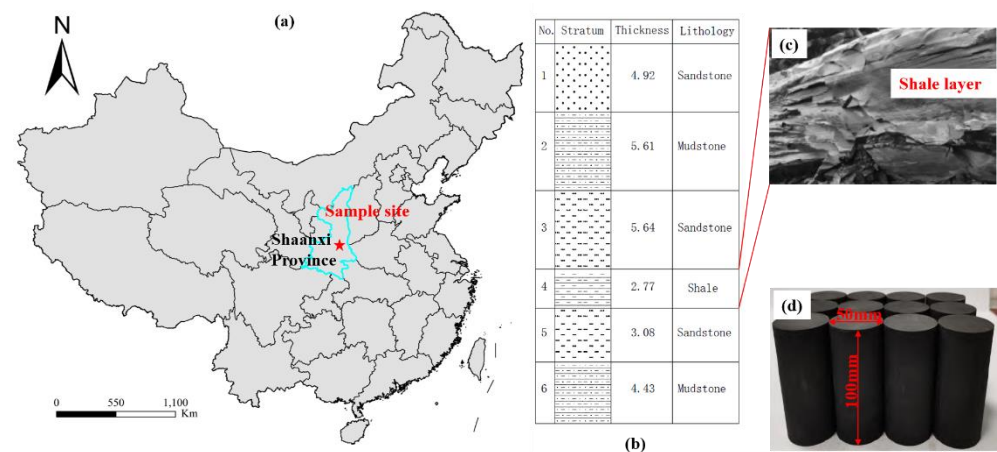


Figure 2. Specimen preparation: (a) sampling location; (b) stratigraphy formation; (c) shale layer in field; (d) prepared specimens.

Table 2. Physical and mechanical properties of the shale samples.

Density kg/m ³	P-Wave Velocity /m·s ⁻¹	Elastic Modulus /GPa	Uniaxial Peak Strength /MPa
2503	4156.62	18.94	172.43

3.2. Method and Instructions

Figure 3 shows the complex experimental process and equipment. The specific procedures are as follows:

- In the CTC tests, 7 levels of confining pressure (i.e., 0, 10, 20, 30, 50, and 70 MPa) were predetermined to investigate the brittleness evolution of the shale.

- Mass and volume measurements were employed using a balance and a conventional caliper, respectively. Bulk density of the shale samples was calculated from the ratio of the sample mass to the sample volume.
- An ultrasonic wave test was performed on the prepared samples. If the P-wave velocities of shale samples were abnormal, those samples were excluded to avoid the influence of microcracks on the test results.
- A total of 35 cylindrical shale specimens were divided into 7 groups based on the predetermined confining pressure.
- Before the mechanical tests, all the specimens were heated to 45 °C for 48 h in a dry oven and cooled to room temperature naturally to eliminate the influence of moisture.
- The laboratory experiments were conducted using an MTS-4000 rigid servo-controlled triaxial compression test system with a loading capacity of 2200 KN and a maximum confining pressure of 120 MPa. Axial deformation was measured by an MTS system in real time. Radial deformation was monitored using a radial linear variable differential transducer, which was wrapped tightly around the specimen.

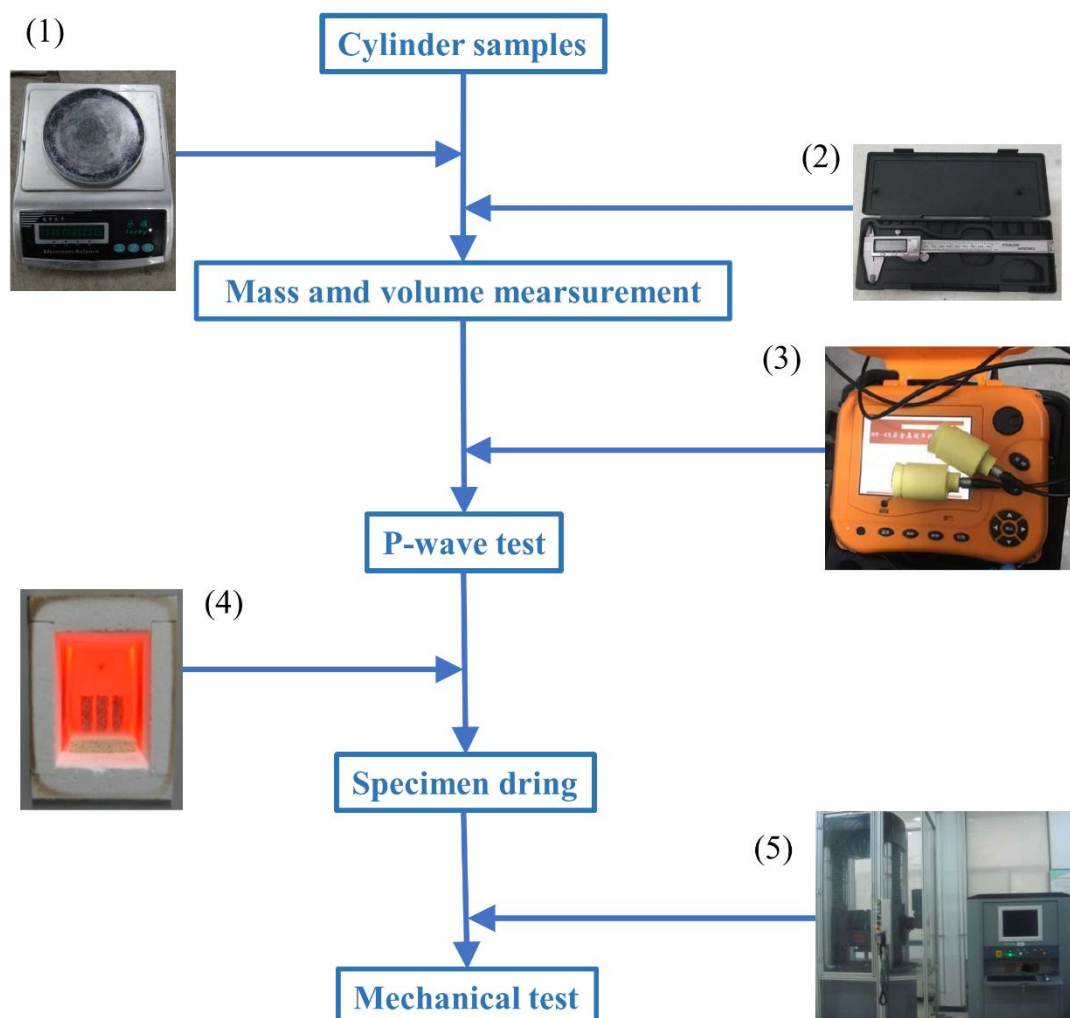


Figure 3. Experimental scheme and instructions: (1) mass measurement; (2) volume determination; (3) ultrasonic wave device; (4) heating oven; (5) mechanical test system.

3.3. Experimental Procedure

For comparison, a conventional uniaxial compression (CUC) test was first performed on the shale samples before CTC testing. A consistent loading mode of axial stress control was used for all CUC and CTC tests. To obtain reliable complete stress–strain curves,

the loading rate was set to a low value of 0.05 MPa/s. The CTC tests for shale samples were conducted for six different scenarios with confining pressures of 5, 10, 20, 30, 50, and 70 MPa. The experimental loading paths of the CTC tests are shown in Figure 4.

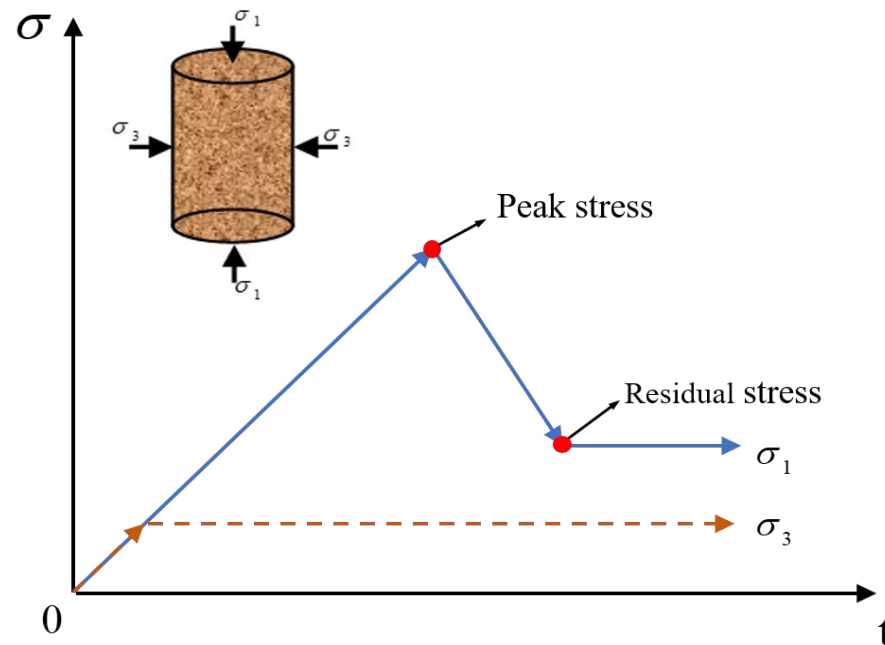


Figure 4. Loading path of CTC tests.

The specific operational procedures of the CTC laboratory experiments were as follows:

Step 1: at the beginning of the test, the shale sample was placed at the center of the test system platen, and a 1 KN axial load was applied to fix the shale sample. This prevented movement of the shale sample during the application of confining load, thus avoiding the impact of eccentric axial load on the test data.

Step 2: the axial pressure and confining pressure were simultaneously loaded to the test, with the confining pressure value set at a loading rate of 0.05 MPa/s; then, the confining pressure was kept constant during testing.

Step 3: the axial pressure was loaded to the surface of the shale sample at a constant axial stress control loading rate of 0.05 MPa/s until failure occurred.

To ensure the accuracy of the research data, each test scenario was repeated five times (i.e., each test scenario with five test cases under the same testing conditions).

4. Results

4.1. Stress–Strain Curves

Figure 5 shows the complete stress–strain curves of the shale specimens under different confining pressures. The statistical analysis results of the tested shale rock are listed in Tables 3 and 4.

The complete stress–strain response of the rock under different confining pressures can be divided into four stages: (1) natural crack closure stage: the natural microcracks of rocks gradually closed, and the rocks are compacted; (2) linear elastic deformation stage: the stress–strain curve is almost straight; (3) microcrack stable expansion stage: new microcracks stably develop in rocks; (4) microcrack unstable expansion stage: new microcracks connect with each other in rock, and macrocracks develop gradually until failure occurs. Additionally, when the confining pressure is 0, 5, or 10 MPa, the natural crack closure stage and the linear elastic deformation stage are obvious, whereas the microcrack stable expansion stage and the microcrack unstable expansion stage are almost nonexistent. This indicates that the rocks show high brittleness, with a high brittleness index. Furthermore, the natural crack closure stage almost disappears, whereas the microcrack

stable expansion stage presents obviously for rocks under confining pressures of 20 MPa and 30 MPa, which indicates that the brittleness of the rock decreases. Additionally, regarding rocks under 50 MPa and 70 MPa confining pressure, the microcrack unstable expansion stage slowly becomes evident, which means that the brittleness of the rock further declines. Overall, the brittleness of rocks gradually decreases with increased confining pressure, which is consistent with the results of previous studies [10,34]. The peak stress and residual strength of the rock increase with increasing confining pressure, indicating that more ultimate elastic strain energy is required for rock failure and more elastic strain energy is stored after rocks fail, with an increase in the confining pressure.

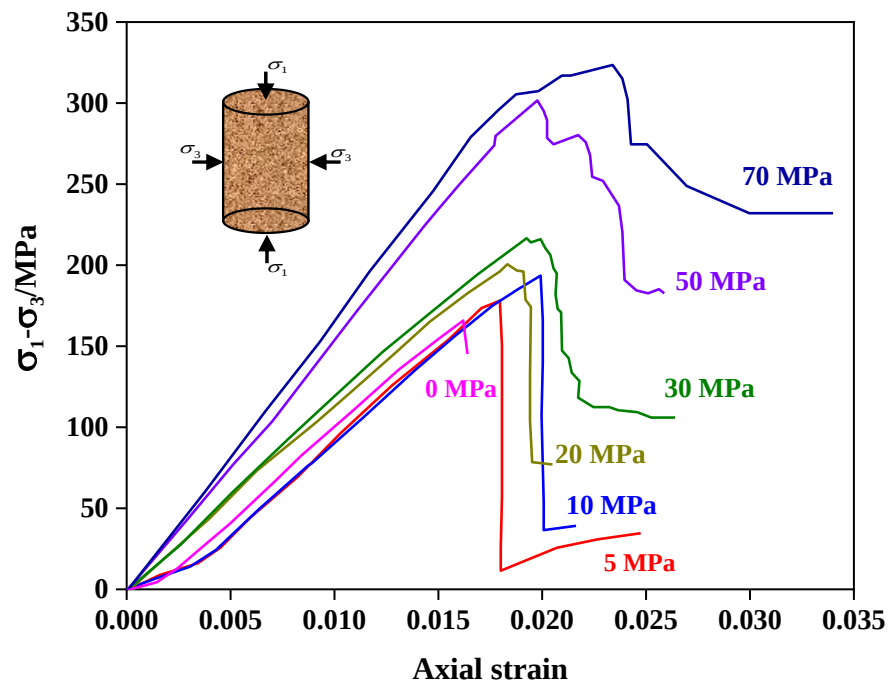


Figure 5. Typical stress–strain curve of shale samples in CTC tests.

Table 3. Statistical analysis results of peak stress in CTC tests.

Confining Pressure /MPa	Peak Stress /MPa	Mean Value /MPa	Absolute Deviation	Relative Deviation (%)	Standard Deviation /(m/s)	Coefficient of Variation (%)
0	165.86	167.70	1.8367	1.0952	0.6586	0.3927
	167.59		0.1067	0.0636		
	169.64		1.9433	1.1588		
	168.85		1.1533	0.6877		
	169.23		1.5333	0.9143		
5	181.78	179.99	1.7860	0.9923	0.5714	0.3175
	178.59		1.4040	0.7800		
	180.32		0.3260	0.1811		
	180.79		0.7960	0.4422		
	178.49		1.5040	0.8356		
10	194.79	194.57	0.2240	0.1151	0.3377	0.1736
	193.58		0.9860	0.5068		
	193.78		0.7860	0.4040		
	195.42		0.8540	0.4389		
	195.26		0.6940	0.3567		

Table 3. Cont.

Confining Pressure /MPa	Peak Stress /MPa	Mean Value /MPa	Absolute Deviation	Relative Deviation (%)	Standard Deviation /(m/s)	Coefficient of Variation (%)
20	201.59	200.50	1.0880	0.5426	0.4359	0.2174
	200.37		0.1320	0.0658		
	199.78		0.7220	0.3601		
	199.16		1.3420	0.6693		
	201.61		1.1080	0.5526		
30	217.64	217.03	0.6140	0.2829	0.2019	0.0930
	216.79		0.2360	0.1087		
	216.48		0.5460	0.2516		
	216.74		0.2860	0.1318		
	217.48		0.4540	0.2092		
50	302.78	302.44	0.3420	0.1131	0.2818	0.0932
	303.06		0.6220	0.2057		
	301.49		0.9480	0.3135		
	301.89		0.5480	0.1812		
	302.97		0.5320	0.1759		
70	324.95	324.80	0.1520	0.0468	0.2891	0.0890
	324.09		0.7080	0.2180		
	324.08		0.7180	0.2211		
	325.78		0.9820	0.3023		
	325.09		0.2920	0.0899		

Table 4. Statistical analysis results of strain corresponding with peak stress in CTC tests.

Confining Pressure /MPa	Strain /%	Mean Value /%	Absolute Deviation	Relative Deviation (%)	Standard Deviation /(m/s)	Coefficient of Variation (%)
0	1.62	1.63	0.0100	0.6135	0.0164	1.0043
	1.69		0.0600	3.6810		
	1.58		0.0500	3.0675		
	1.61		0.0200	1.2270		
	1.62		0.0100	0.6135		
5	1.71	1.71	0.0020	0.1171	0.0166	0.9713
	1.76		0.0520	3.0445		
	1.69		0.0180	1.0539		
	1.65		0.0580	3.3958		
	1.73		0.0220	1.2881		
10	1.99	1.98	0.0020	0.1004	0.0161	0.8082
	1.95		0.0420	2.1084		
	2.05		0.0580	2.9116		
	2.01		0.0180	0.9036		
	1.96		0.0320	1.6064		
20	1.83	1.83	0.0020	0.1092	0.0151	0.8228
	1.89		0.0580	3.1659		
	1.81		0.0220	1.2009		
	1.84		0.0080	0.4367		
	1.79		0.0420	2.2926		
30	1.99	1.99	0.0060	0.3006	0.0169	0.8455
	1.95		0.0460	2.3046		
	1.97		0.0260	1.3026		
	2.06		0.0640	3.2064		
	2.01		0.0140	0.7014		

Table 4. Cont.

Confining Pressure /MPa	Strain /%	Mean Value /%	Absolute Deviation	Relative Deviation (%)	Standard Deviation /(m/s)	Coefficient of Variation (%)
50	1.98	1.98	0.0020	0.1009	0.0195	0.9825
	1.97		0.0120	0.6054		
	2.04		0.0580	2.9263		
	2.01		0.0280	1.4127		
	1.91		0.0720	3.6327		
70	2.34	2.34	0.0000	0.0000	0.085	0.3626
	2.34		0.0000	0.0000		
	2.37		0.0300	1.2821		
	2.31		0.0300	1.2821		
	2.34		0.0000	0.0000		

4.2. Reliability Verification of BI_{ES}

According to the complete stress–strain curves of the rock, the rock brittleness index BI_{ES} based on the ultimate elastic strain energy can be calculated as follows:

$$W_u^e = \frac{\sigma_c^2}{2E} \quad (2)$$

$$W_r^e = \frac{\sigma_r^2}{2E} \quad (3)$$

When the ultimate axial strain, ε_f , corresponding to the peak stress is less than the minimum axial strain, ε_u , in the ideal post-peak unloading stage, the post-peak failure energy, W_p^f , can be expressed as:

$$W_p^f = \int_{\varepsilon_p}^{\varepsilon_r} \sigma d\varepsilon - W_r^e \quad (4)$$

When the ultimate axial strain, ε_f , corresponding to the peak stress is higher than the minimum axial strain, ε_u , in the ideal post-peak unloading stage, the post-peak failure energy, W_p^f , can be calculated as follows:

$$W_p^f = \int_{\varepsilon_f}^{\varepsilon_r} \sigma d\varepsilon - W_r^e + \frac{1}{2}(\varepsilon_f - \varepsilon_u)^2 \times E \quad (5)$$

where E is the modulus of elasticity of the rock; and ε_f , ε_u , and ε_r are the ultimate axial strain corresponding to the peak stress, the minimum axial strain in the post-peak ideal unloading stage, and the initial axial strain in the residual strength stage, respectively (Figure 1). The post-peak ideal unloading line is defined as a straight line parallel to the line of the pre-peak elastic deformation loading stage from the starting point of residual strength stage.

Table 5 lists the calculated brittleness index, BI_{ES} , under different confining pressures. To intuitively exhibit the influence of confining pressure, Figure 6 demonstrates the variations in the brittleness index, BI_{ES} , with confining pressure.

The brittleness index, BI_{ES} , gradually decreases with an increase in confining pressure. Furthermore, based on the definition of the brittleness index, BI_{ES} (the lower the value of BI_{ES} , the less brittle the rock is), the variable trend indicates that the brittleness index, BI_{ES} , can sufficiently reflect the brittleness characteristics of rocks. In addition, results were in agreement with the characteristics of the stress–strain curves obtained from shale samples under different confining pressures; the rock shows higher brittleness, and the BI_{ES} value is higher under the confining pressures of 0 MPa, 5 MPa, and 10 MPa. The brittleness of the rock is lower, and the value of BI_{ES} is lower when the confining pressure is 20 MPa

or 30 MPa, and both values further decrease when the confining pressure is 50 MPa or 70 MPa.

Table 5. Calculated brittleness index, BI_{ES} , of the shale samples in the CTC tests.

Confining Pressure /MPa	Peak Strength /MPa	Residual Strength /MPa	W_u^e /(mJ/mm ³)	W_p^f /(mJ/mm ³)	W_r^e /(mJ/mm ³)	BI_{ES}
0	167.66	10.54	0.904	0.010	0.004	0.99
5	179.99	34.82	1.357	0.032	0.045	0.94
10	194.77	39.67	1.615	0.119	0.011	0.92
20	200.92	76.94	2.549	0.234	0.247	0.81
30	217.02	106.06	1.999	0.921	0.029	0.53
50	302.49	185.67	2.716	1.175	0.343	0.44
70	324.82	250.12	2.813	1.721	0.884	0.07

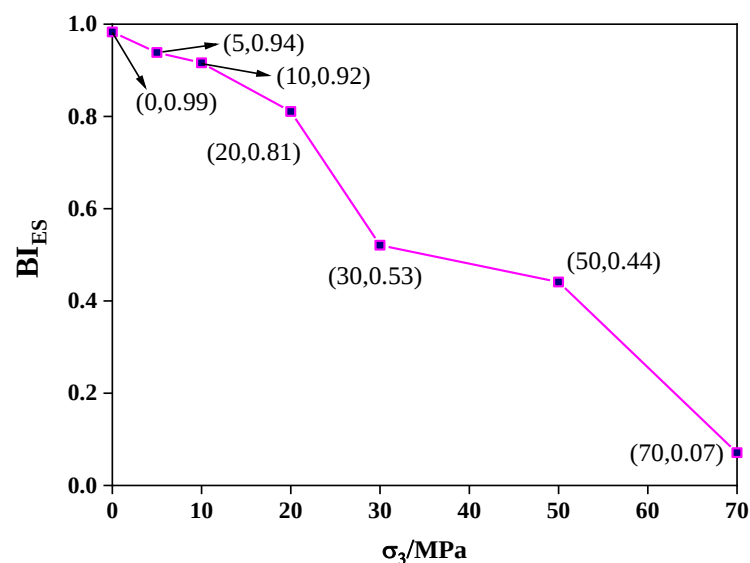


Figure 6. Relationship between BI_{ES} of the shale samples and the corresponding confining pressures.

4.3. Universality Verification of BI_{ES}

To verify the universality of the proposed brittleness index, BI_{ES} , the CTC test data of six types of rocks (i.e., sandstone [35], shale 1 [23], shale 2 [23], glutenite [26], granite [36], and gneiss [36]) are cited. The corresponding stress–strain curves are exhibited in Figure 7.

Table 6 lists the results of six types of rocks calculated by the proposed brittleness index, BI_{ES} , on the basis of experimental data. The variation of values of BI_{ES} are further demonstrated in Figure 8.

The calculated value of the brittleness index, BI_{ES} , for six types of rock gradually decreases with increasing confining pressure, which is consistent with the deformation and failure characteristics of brittle rocks, indicating that BI_{ES} can reflect a wide range of brittle rocks. In addition, negative values of BI_{ES} , as shown in Figure 8, show that such rocks exhibit more ductile characteristics under the corresponding confining pressures, resulting in the ultimate elastic strain energy not being sufficient to damage the rock and requiring additional input strain energy in the post-peak stage. In summary, the brittleness index, BI_{ES} , can reflect various types of rocks and characterize a wide range of rock brittleness.

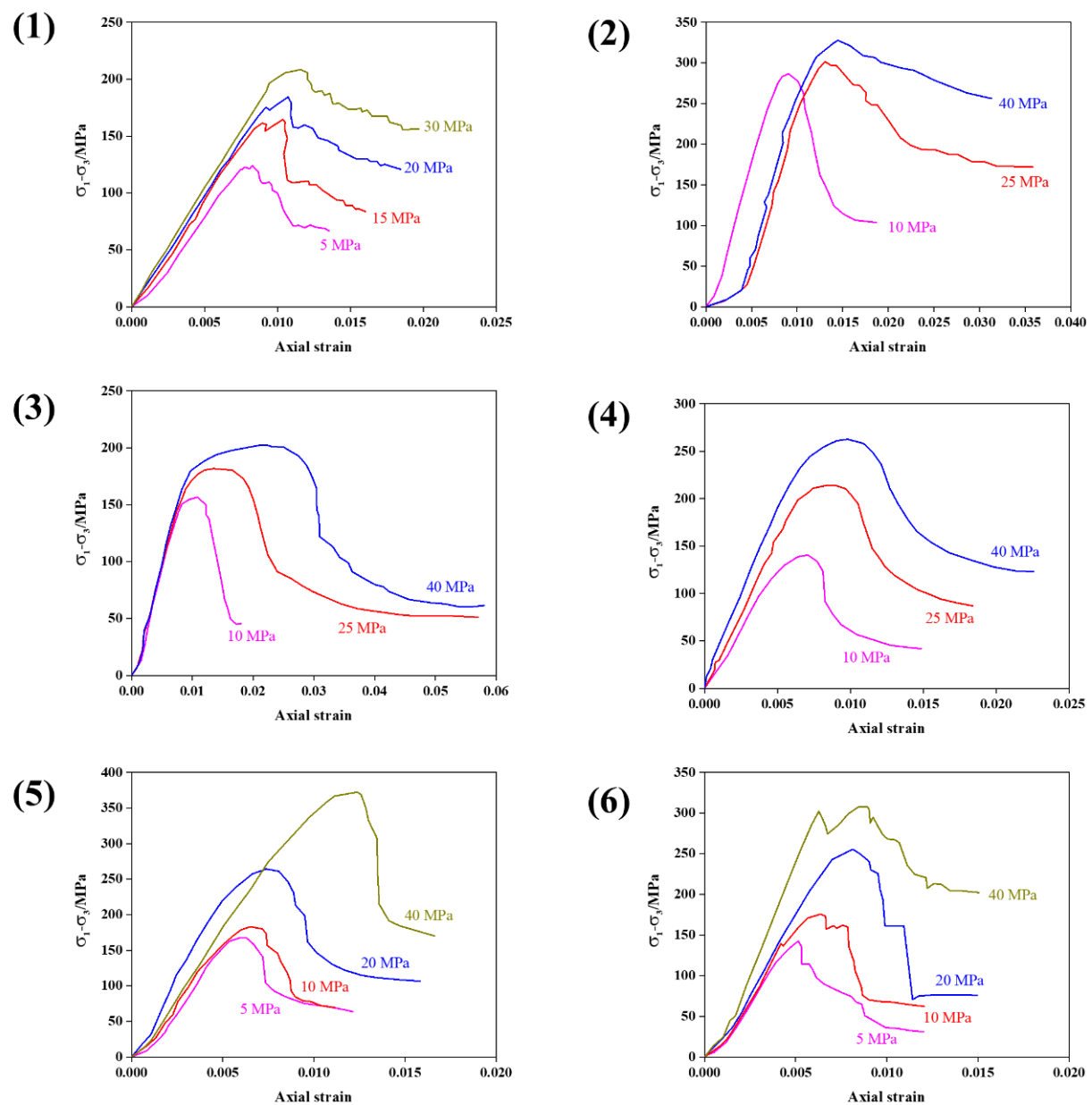


Figure 7. Stress–strain curves of six types of rocks: (1) Sandstone; (2) Shale 1; (3) Shale 2; (4) Glutenite; (5) Granite; (6) Gneiss.

Table 6. Results of six types of rocks calculated based on BI_{ES} .

Rock	Confining Pressure /MPa	Peak Strength /MPa	Residual Strength /MPa	BI_{ES}
Sandstone	5	136.52	64.89	0.53
	15	164.59	82.46	−0.18
	20	173.30	106.84	−0.35
	30	183.45	128.69	−1.2
Shale 1	10	301.56	106.79	−0.47
	25	332.91	195.41	−1.55
	40	369.54	264.39	−3.68
Shale 2	10	169.36	51.28	−0.19
	15	210.54	64.31	−2.59
	20	245.82	72.49	−2.78

Table 6. Cont.

Rock	Confining Pressure /MPa	Peak Strength /MPa	Residual Strength /MPa	BI_{ES}
Glutenite	10	142.34	47.30	−0.34
	15	215.38	88.94	−0.81
	20	266.40	125.93	−1.0824
Granite	5	172.64	95.34	0.57
	10	184.02	86.51	0.52
	20	267.43	119.06	0.41
Gneiss	40	375.09	186.75	0.36
	5	143.51	49.59	−0.11
	10	256.84	66.84	−0.20
	20	207.91	76.94	−0.31
	40	3.6.86	201.97	−0.45

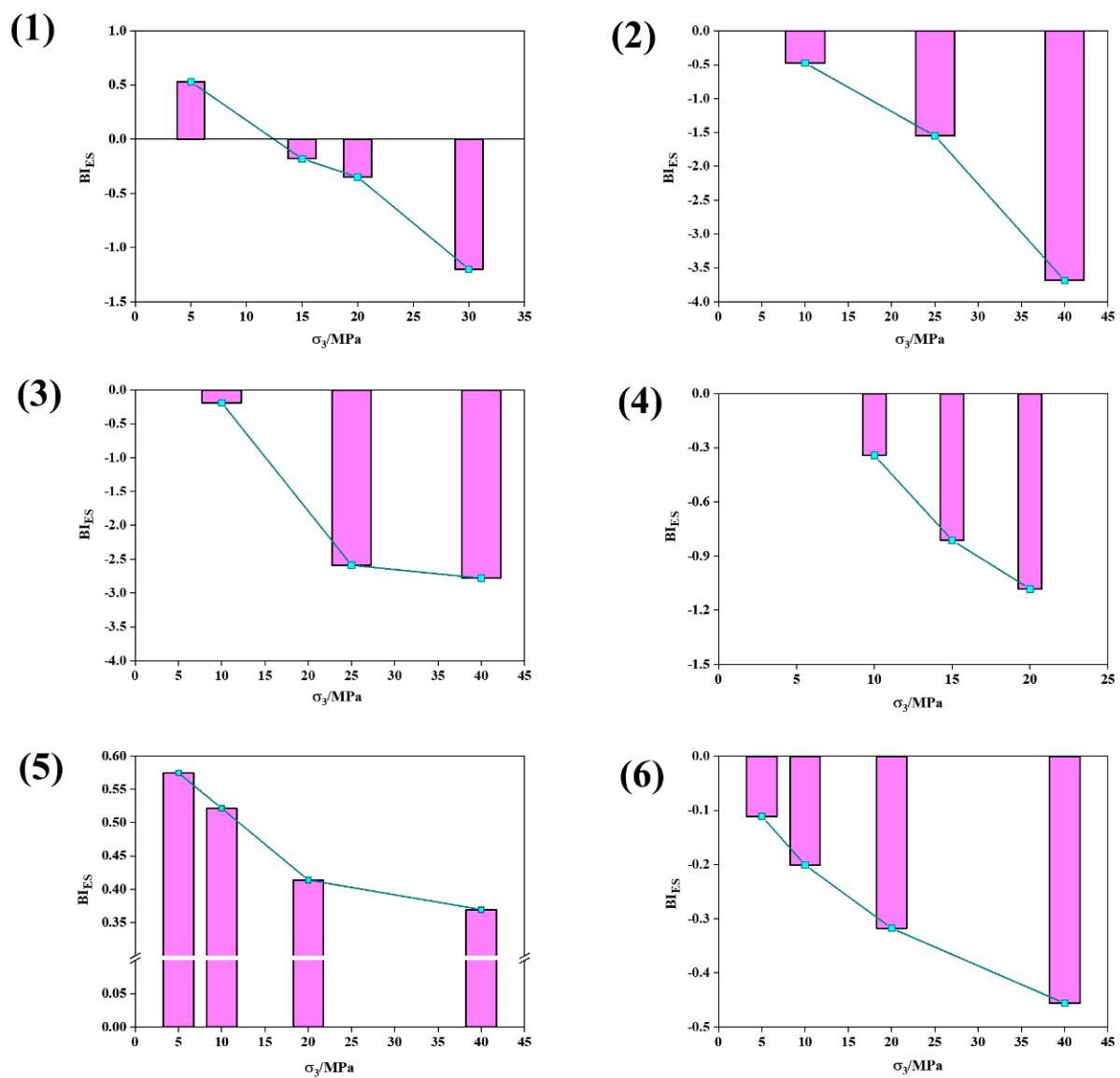


Figure 8. Calculation results of BI_{ES} of six types of rocks: (1) Sandstone; (2) Shale 1; (3) Shale 2; (4) Glutenite; (5) Granite; (6) Gneiss.

5. Discussion

A novel brittleness index, BI_{ES} , based on ultimate elastic strain energy is proposed, taking the relationship between the energy evolution characteristics and the brittleness of rocks into account. To verify the superiority of BI_{ES} , the new brittleness index was compared with 11 existing brittleness indices, which are summarized in Section 1. Table 7 lists the calculated results of 11 existing energy-based brittleness indices based on the experimental data of the shale samples in CTC tests. The variation trend of BI_{ES} and the 11 calculated brittleness indices are further exhibited in Figure 9 to compare their brittleness evaluation accuracies.

Table 7. Calculated results of various brittleness indices based on shale CTC test data.

		Brittleness Indices											
		BI_1	BI_2	BI_3	BI_4	BI_5	BI_6	BI_7	BI_8	BI_9	BI_{10}	BI_{11}	BI_{ES}
σ_3	0	0.97	1.02	0.01	1.01	0.01	0.97	1.03	0.96	0.99	0.96	0.27	0.98
	5	0.86	1.03	0.02	1.02	0.02	0.97	1.19	0.79	0.84	0.79	0.26	0.94
	10	0.77	1.01	0.01	1.01	0.01	0.94	1.28	0.70	0.77	0.70	0.55	0.90
	20	0.90	1.02	0.02	1.02	0.02	0.90	1.12	0.74	0.89	0.74	0.42	0.77
	30	0.80	1.03	0.02	1.02	0.03	0.86	1.28	0.59	0.77	0.59	0.23	0.51
	50	0.87	1.69	0.69	1.56	0.56	0.59	1.83	0.30	0.52	0.30	0.02	0.42
	70	0.61	1.77	0.76	1.38	0.38	0.60	2.38	0.21	0.35	0.21	0.00	0.14

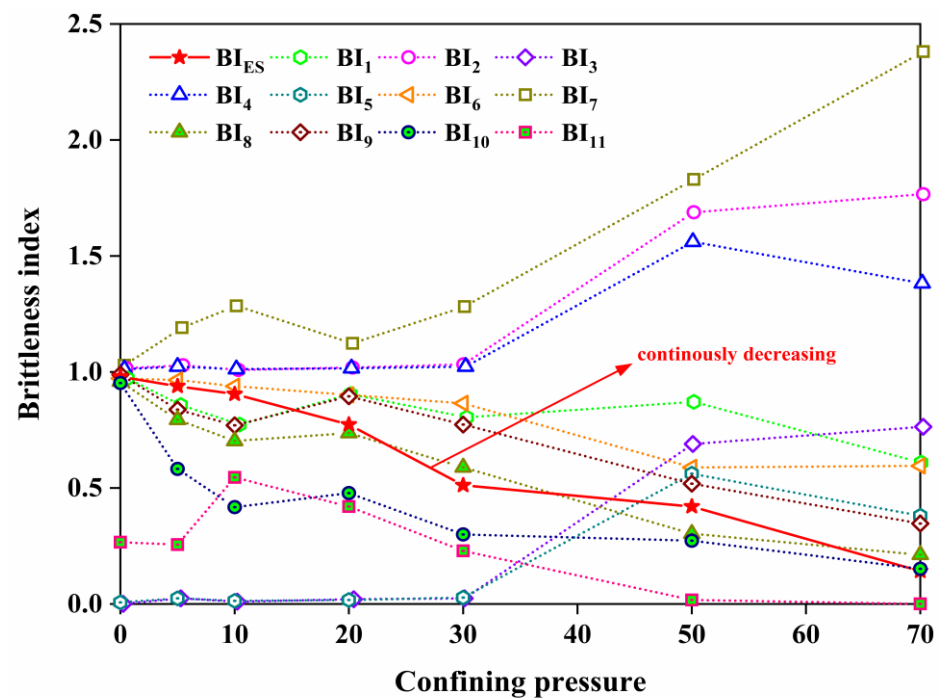


Figure 9. Comparison of BI_{ES} with existing energy-based brittleness indices based on the shale samples in the CTC tests.

The brittleness indices, except the proposed BI_{ES} , exhibit fluctuations with an increase in the confining pressure, which failed to reflect the brittleness characteristics of shale samples. The fluctuating change trends mean that those brittleness indices cannot evaluate rock brittleness accurately. The proposed brittleness index decreases continuously with increasing confining pressure, which is consistent with the deformation and failure characteristics of rocks. The decreasing tendency of the BI_{ES} value as with increasing confining pressure indicates that the new brittleness index can accurately assess the brittleness evolution of rocks. Consequently, it can be concluded that the novel brittleness index characterizes the brittleness of rocks more accurately than existing indices.

All the brittleness indices compared in Figure 9 can reflect rock brittleness accurately in one of the typical aspects, but there is not a unified judgment standard for their accuracies. Therefore, the evaluation accuracy of rock brittleness can only be judged by the variation trend, with rock brittleness continuously decreasing with increasing confining pressures, as shown in Figure 9.

6. Conclusions

In this study, a new brittleness evolution index is proposed on the basis of energy evolution analysis during the whole process of rock deformation and failure. The following conclusions can be drawn from the results of this work:

1. Based on the ultimate elastic strain energy, a novel rock brittleness index is proposed to characterize brittleness evolution during the whole process of deformation and failure of rocks.
2. A series of shale CTC tests were conducted to verify the reliability of the new brittleness index, and the testing results indicate that the new index can precisely reflect rock brittleness. Furthermore, the CTC testing data of six types of rocks were cited to verify the universality of the new brittleness index, and the results show that the new brittleness index can reflect various kinds of rock brittleness.
3. The new brittleness index was compared with 11 existing brittleness indices based on the shale CTC testing data; the calculation results of existing brittleness indices exhibit fluctuations with increasing confining pressure, whereas the values of the new index continuously decrease, indicating that the new brittleness index can characterize rock brittleness more precisely than the existing brittleness indices.

In this study, we conducted a series of rock CTC tests to reveal the energy storage law. However, the energy storage law exists in rock under one-dimensional, two-dimensional, and three-dimensional loading conditions. Therefore, further research is required, including a series of cycling, loading, and unloading triaxial compression tests to analyze the three-dimensional compression energy storage characteristics. Moreover, an accurate calculation method of peak elastic strain energy and residual elastic strain energy can be obtained.

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