Article

A New Fractal Permeability Model Considering Tortuosity of Rock Fractures

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Abstract: During methane extraction, the permeability of a coal seam is the vital factor affecting recovery. Although the permeability of a coal seam and its relationship with porosity have been studied in a few works, the calculation process of coal seam permeability is usually too simplistic or neglects the influence of microscopic fracture structures. Statistical research shows that the permeability of coal seams with the same porosity and different fracture structures is quite different. For the purpose of quantitatively investigating the contribution of fractures and pore structure in coal seams, a fractal permeability model considering the microstructure of coal seam fracture is established in this paper. The correctness of the model is verified by comparing with the previous research results. Then, the influence of the microscopic fracture structure on the equivalent permeability is analyzed. The simulation results show that the permeability of fractured coal is directly proportional to the fractal dimension of the fracture, the maximum fracture length and the azimuth. It is inversely proportional to the tortuous fractal dimension and the dip angle of the fracture surface. This conclusion provides the foundation for revealing the microstructure mechanisms of macroscopic seepage characteristics of coal seams, and implementing effective strategies to enhance gas recovery rates under different geological structures.

Keywords: pore media; coal permeability; fracture network; fractal; fractal dimension

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1. Introduction

As the main channel for gas migration in a reservoir, fracture length is unevenly distributed, with varying degrees of tortuosity and a complex structural distribution. The connection state of the coal pore-throat network will directly affect oil and gas recovery [1–3]. Moreover, the coal fracture network is also complex [4–6]; therefore, accurately exposing the connectivity of the reservoir structure and quantitatively investigating the interaction between the reservoir microstructure and the macroscopic behavior at the micro scale is the key to improving gas recovery [7–10]. In recent years, an increasing number of studies have demonstrated that fractures in unconventional reservoirs have distinct fractal characteristics and that fractal theory is better able to quantitatively characterize the distribution of fracture structure networks.

The application of fractal geometry in porous media has been studied more extensively by scholars. On the basis of the concepts of the geometric model of volume and surface fractal dimension, Deinert et al. [11,12] established a relationship between fluid saturation and capillary pressure. Later, on the basis of the fractal geometric theory, Xu et al. [13–15] explored the heat transfer properties and percolation behavior of fractal tree networks.

In addition, Ishibashi et al. [16] explored the interaction law between fracture opening and seepage characteristics under different gas pressures and concluded that fracture permeability obeys a power-law distribution. On the basis of the fractal dimension of the pore space, Costa et al. [17] explored the relationship between the constant of the Kozeny-Carman equation and the porosity of the depth and derived the contribution of fractal permeability. Guarracino et al. [18] used the Sierpinski fractal model to predict hydraulic fracturing conductivity.

However, previous studies usually regard the fracture system as a simplified symmetric network or an empirical constant in the permeability model. Accurate and quantitative descriptions of seepage characteristics of different fracture network structures remain to be studied [19–22]. In this work, we established a permeability model, including fracture microstructures, such as fractal dimension of fracture dimension, fracture detour, fracture inclination, and fracture orientation, which does not contain empirical constants. In addition, the simulation results of the model are compared with the field-measured data, which has good consistency and verifies the correctness of the model. The internal relationship between the fracture macro permeability and micro-network structure is further analyzed.

2. Fractal Characterization of Coal Fracture Network

Statistics show that the length of coal fracture meets the fractal scaling law. Fractal theory defines the fractal power-law relation for fractals as [23–25]:

$$N(\geq l) \propto l^{-D_f} \quad (1)$$

where l is the single fracture length, $l_{\min} \leq l \leq l_{\max}$, N is the fracture number, D_f is the fracture fractal dimension, for 2D space, $1 < D_f < 2$. Therefore, Equation (1) can be simplified as:

$$N(\geq l) = kl^{-D_f} \quad (2)$$

Differentiating Equation (1), the number of fractures between l and $l + dl$ in length can be expressed as:

$$dN(l) = -kD_f l^{-D_f-1} dl \quad (3)$$

Using Equation (3), the probability density function of the fracture is expressed as:

$$f(l) = \frac{dN(l)}{N_t dl} = -\frac{k}{N_t} D_f l^{-D_f-1} \quad (4)$$

where N_t is the total number of the fractures. Normalizing the probability density function yields:

$$-\int_{l_{\min}}^{l_{\max}} \frac{k}{N_t} D_f l^{-D_f-1} dl = 1 \quad (5)$$

Thus:

$$\frac{k}{N_t} (l_{\min}^{-D_f} - l_{\max}^{-D_f}) = \frac{k l_{\min}^{-D_f}}{N_t} (1 - (\frac{l_{\min}}{l_{\max}})^{D_f}) = 1 \quad (6)$$

When the fracture network satisfies the fractal scaling law, fracture opening a and the length of fracture l satisfy the following relations [26,27]:

$$a = \beta l \quad (7)$$

where β is the reservoir matrix mechanical coefficient. The fractal dimension of the fracture length is:

$$D_f = d_E - \frac{\ln \phi}{\ln \frac{l_{\min}}{l_{\max}}} \quad (8)$$

where d_E represents the dimension of the calculation domain, for the two-dimensional domain, $d_E = 2$. If $l_{\min} \ll l_{\max}$, Equation (6) can be simplified as:

$$k = N_t l_{\min}^{D_f} \quad (9)$$

For natural fracture networks, the minimum length and the maximum length of the fractures usually satisfy $l_{\min}/l_{\max} \leq 10^{-2}$. In addition, on the basis of the porous media distribution law [28,29], the crack length and pore diameter are analogous. The total number of cracks for the reservoir is:

$$N_t(l) = (l_{\max}/l_{\min})^{D_f} \quad (10)$$

Combining Equations (9) and (10), we obtain:

$$k = l_{\max}^{D_f} \quad (11)$$

Substituting Equation (11) into Equation (3), it can be determine that the fractal power law expression of fracture length in fractal fracture network is:

$$dN(l) = -D_f l_{\max}^{D_f} l^{-D_f-1} dl \quad (12)$$

3. Fractal Permeability Model of Coal Fracture Network

In this section, we couple the fractal characteristic intrinsic constitutive equation derived above with the Hagen–Poiseuille equation and Darcy's law to derive a comprehensive seepage model for the analysis of reservoir fracture structures. The cubic law of single fracture seepage is the basis of seepage theory of a fracture network because of its simplicity and accuracy. The flow rate for a single horizontal fracture is [30,31]:

$$q(l) = \frac{a^3 l}{12\mu} \frac{\Delta p}{L_0} \quad (13)$$

where μ is gas hydrodynamic viscosity, Δp is difference of pressure for a fracture, L_0 is length for a characterization unit. When considering the spatial orientation of fractures (as illustrated in Figure 1), the flow rate for single crack yields [32–34]:

$$q(l) = \frac{\Delta P (1 - \cos^2 \alpha \sin^2 \theta)}{128 L_t(l)} \frac{\alpha^3 l}{\mu} = \frac{\Delta P (1 - \cos^2 \alpha \sin^2 \theta)}{128 L_t(l)} \frac{(\beta l)^3 l}{\mu} \quad (14)$$

where α is the fracture azimuth angle and θ is the dip angle.

Figure 1. Sketch map of the fracture azimuth.

Consequently, the total flow for all fractures in the analysis area (presented in Figure 2) can be obtained as:

$$Q = - \int_{l_{\min}}^{l_{\max}} q(l) dN(l) \quad (15)$$

Figure 2. Characterization unit of reservoir fracture structure.

The crack size satisfies [35,36]:

$$L_t(l) = l^{1-D_T} L_0^{D_T} \quad (16)$$

where D_T is the throat tortuosity fractal dimension, $L_t(l)$ is the length of the fracture in the fluid-flow direction.

Combining Equations (12), (15) and (17), we obtain:

$$\begin{aligned} Q &= - \int_{l_{\min}}^{l_{\max}} q(l) dN(l) = \frac{a^3}{128} \frac{\Delta P}{\mu} \frac{l_{\max}^{D_T} (1 - \cos^2 \alpha \sin^2 \theta)}{L_0^{D_T}} \frac{D_f}{D_T - D_f} \\ &= \frac{\beta^3}{128} \frac{\Delta P}{\mu} \frac{l_{\max}^{3+D_T} (1 - \cos^2 \alpha \sin^2 \theta)}{L_0^{D_T}} \frac{D_f}{3 + D_T - D_f} \end{aligned} \quad (17)$$

On the basis of Darcy's law:

$$k = \frac{\mu L_0 Q}{\Delta P A} \quad (18)$$

where k is the permeability of fractured coal. Simultaneous to Equations (17) and (18), the fractal permeability of fracture network is:

$$k = \frac{\beta^3}{128} \frac{L_0^{1-D_T} (1 - \cos^2 \alpha \sin^2 \theta)}{A} \frac{D_f}{3 + D_T - D_f} l_{\max}^{3+D_T} \quad (19)$$

Figure 3 illustrates the process of developing the reservoir fractal permeability model. The coal microstructure is simplified as a combination of fractures. We then construct a permeability model on the basis of the basic theory of fractal geometry, which enables quantitative investigation of maximum pore size, throat tortuosity, and pore scale. In the 2D calculation domain: $1 < D_f < 2$ and $1 < D_T < 2$.

Figure 3. Establishment of fractal permeability model.

4. Verification of the Permeability Model

The fractal dimension of the fracture length for 22 different natural fracture networks was calculated by Jafari and Babadagli [35] on the basis of the box-counting method, and then a fracture network of $100\text{ m} \times 100\text{ m}$ was constructed. The maximum length of the fracture was 2 m, and the fracture dip angle was 0. To verify the fractal fracture network model, the fractal model was compared with the above simulation results, as shown in Figure 4a. In addition, the simulation results of the fractal model in this paper were also compared with the seepage experimental results of rocks with different porosities [36] in order to further verify the rationality of the model, as shown in Figure 4b.

Figure 4. Cont.

Figure 4. Verification of the fractal permeability model. (a) Compared with the results of [35]; (b) Compared with the results of [36].

It is revealed in Figure 4 that the fractal permeability model proposed in this paper is in good agreement with the field permeability evolution results, verifying the accuracy of the fractal model.

5. Results and Discussion

The seepage network permeability is influenced by the microstructure parameters of fractures [37–40]. To discuss the impact of different structural parameters on reservoir permeability, we analyzed the influence mechanisms for microstructural parameters, including the fracture tortuosity fractal dimension, maximum fracture length, the azimuth angle of fracture, and the dip angle of fracture.

Figure 5 shows the curve of the permeability verified with D_T . According to Figure 5, the permeability gradually decreases with increasing D_T . This corresponds to the physical truth. Under the same inlet and outlet pressure difference, with increasing D_T , the bending degree of the crack increases, so the fluid resistance through the fracture increases and the permeability decreases.

The interaction between permeability and maximum fracture length is illustrated in Figure 6, i.e., k increases rapidly with increasing l_{max} . The main reason for this is that the total number of fractures decreases with the increasing length of a single fracture, and the small fractures tend to converge on several fractures, which reduces the flow resistance of the fluid.

As illustrated in Figure 7, when the azimuth angle of fractures α is constant, the permeability decreases with the increase in dip angle θ . When the dip angle is 0, the direction of the fluid flow is located on the fracture surface, and the resistance of the fluid flow is smallest, which is consistent with the physical truth. Furthermore, if the dip angle is constant, the permeability increases with the azimuth angle of fractures. When the fracture azimuth is $\alpha = \pi/2$, the direction of fracture coincides with the direction of the fluid flow, and the fluid resistance is smallest.

Figure 5. Permeability variation with the increasing tortuosity fractal dimension.

Figure 6. Permeability variation with the increase of the maximum fracture length.

Figure 7. Permeability variation with the increasing fracture azimuth.

6. Conclusions

To reveal the quantitative relationship between the macroscopic equivalent permeability and the micro-fracture structure of fractured coal, an equivalent permeability model, considering the microstructure of the fracture, is established on the basis of the fractal theory. The accuracy of the fractal permeability model is verified by comparing it with the previous simulation results.

Furthermore, by simulating the fluid flow in the fracture, the interactions of the microstructural parameters and permeability are studied, such as fracture fractal dimension, fracture tortuosity fractal dimension, maximum fracture length, dip angle of fracture, and azimuth angle of fracture. In this study, the calculation does not include the external force constraint and the process of coal deformation induced by adsorption and desorption in the gas flow. The evolution process of permeability and its effect on gas recovery under the combined effect of in situ stress and sorption–desorption-induced matrix deformation are interesting topics, and these will be our next work.

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