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Downside Beta and Downside Gamma: In Search for a Better Capital Asset Pricing Model

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Abstract: In the financial world, the importance of “downside risk” and “higher moments” has been emphasized, predominantly in developing countries such as Pakistan, for a substantial period. Consequently, this study tests four models for a suitable capital asset pricing model. These models are CAPM’s beta, beta replaced by skewness (gamma), CAPM’s beta with gamma, downside beta CAPM (DCAPM), downside beta replaced by downside gamma, and CAPM with downside gamma. The problems of the high correlation between the beta and downside beta models from a regressand point of view is resolved by constructing a double-sorted portfolio of each factor loading. The problem of the high correlation between the beta and gamma, and, similarly, between the downside beta and downside gamma, is resolved by orthogonalizing each risk measure in a two-factor setting. Standard two-pass regression is applied, and the results are reported and analyzed in terms of R^2 , the significance of the factor loadings, and the risk–return relationship in each model. The risk proxies of the downside beta/gamma are based on Hogan and Warren, Harlow and Rao, and Estrada. The results indicate that the single factor models based on the beta/downside beta or even gamma/downside gamma are not a better choice among all the risk proxies. However, the beta and gamma factors are rejected at a 5% and 1% significance level for different risk proxies. The obvious choice based on the results is an asset pricing model with two risk measures.

Keywords: beta; downside risk; skewness; downside skewness; double sorting; orthogonalizing

JEL Classification: C52; G11; D81



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1. Introduction

Asset pricing models are used to estimate the cost of capital for firms and evaluate the performance of managed portfolios. They are based on the risk–return relationship, which is long established as the backbone of portfolio management. An asset pricing model offers a unique risk–return relationship where an investor tries to optimize an investment decision. One of the earliest and most widely used models comes in the form of the Capital Asset Pricing Model (CAPM). It marks the birth of asset pricing theory¹, and its simplicity makes it the most widely used in applications. As a result, the validity of the CAPM is tested, and the outcome reveals a weak risk–return relationship².

Four major viewpoints provide an in-depth explanation of this weak risk–return relationship³. The first perspective led by Roll (1977); Ross (1977) completely rejected the CAPM on the basis that the true market portfolio in the CAPM is not observable; this led to the belief that the CAPM cannot be tested at all⁴. The second perspective does not reject the CAPM altogether but it argues that anomalies are present, and beta is not the only predictor, as outlined by the CAPM. Instead, the size, value, time effects, and momentum are additional factors to be added to the model⁵. Furthermore, as the CAPM is based on

the dividend discount model, [Fama and French \(2015\)](#) add investment and profitability factors to further extend the CAPM as these factors are related to average returns.

The third perspective tries to overcome the theoretical weaknesses of the CAPM, thus leading to modified versions of the CAPM. Among them is the problem of the y-intercept or the CAPM without risk-free borrowing or lending. [Black \(1972\)](#) developed the zero-beta CAPM by allowing the unlimited short sales of risky assets, making the composition of market portfolios from efficient portfolios themselves, thus making market portfolios efficient⁶. Another possible solution is to replace the CAPM's beta with a downside risk, based on [Roy's \(1952\)](#) safety-first rule. This is because investors dislike downside risk and do not give equal weight to both upside and downside risk, as is assumed in the CAPM⁷. The last perspective addresses statistical issues, such as the issue of normality. Stock prices exhibit non-normality, which advocates the importance of higher moments in the CAPM world⁸.

The third perspective of replacing the beta with downside risk as downside beta is a challenging question. The proxy of downside risk in asset pricing models was first developed by the Hogan and Warren model based on semivariance (1974) as the first official version of a downside risk-based CAPM. The second breakthrough was made by [Bawa \(1975\)](#) who developed a proxy for downside risk, lower partial moment (LPM), based on stochastic dominance. Later, Fishburn in 1977 extended [Bawa's \(1975\)](#) LPM into the unlimited scope of LPM. The Bawa–Fishburn LPM encompasses all classes of investors: risk-averse, risk-seeking, and risk-neutral. Additionally, it is not tied to the condition of normality and is flexible to include skewness and kurtosis.

Moreover, [Bawa and Lindenberg \(1977\)](#) extended symmetric partial moments to asymmetric generalized co-LPM or GCLPM for n-degree LPM structures accommodating asymmetric distributions. These methodologies use a risk-free rate as a benchmark of downside risk. Later, [Harlow and Rao \(1989\)](#) used mean returns as a benchmark for defining downside risk, while [Estrada \(2002, 2008\)](#) added endogenous semivariance matrix constructed on all the elements of the covariance matrix to deal with the heterogeneity problem. This gives us three major types of downside risk to be used in this study outlined by [Hogan and Warren \(1974\)](#); [Harlow and Rao \(1989\)](#); [Estrada \(2002\)](#), resulting in three risk proxies to be compared: downside beta-[Hogan and Warren \(1974\)](#), downside beta-[Harlow and Rao \(1989\)](#), and downside beta-[Estrada \(2002\)](#).

Likewise, the fourth issue is addressed by incorporating higher moments: skewness, and kurtosis⁹. However, this solution is not restricted to the CAPM and is extended to the downside risk framework. Thus, the same combination applies for downside skewness/kurtosis as downside skewness/kurtosis-[Hogan and Warren \(1974\)](#), downside skewness/kurtosis-[Harlow and Rao \(1989\)](#), and downside skewness/kurtosis-[Estrada \(2002\)](#). Most studies concentrate on the third moment and place less emphasis on the fourth moment. The third moment is related to asymmetry that the investor is trying to stabilize and minimize. On the other hand, the fourth moment is related to the flatness of returns, which are considerably less important in the asset pricing world¹⁰.

The CAPM and DCAPM models and higher moments models have been tested before; however, this study addresses the limitations in the previous studies. In Pakistan, a study was conducted by [Javid and Ahmad \(2008\)](#); [Javid \(2009\)](#) on the Karachi stock market with a sample size of 49 stocks. They applied the CAPM and CAPM with skewness but on stocks instead of portfolios. This leads us to two major problems: selection bias and measurement error. The solution to these problems is proposed in the literature by constructing portfolios and testing the models on them¹¹.

Furthermore, [Galagedera and Brooks \(2007\)](#) investigated the issue of co-skewness as a measure of risk in a downside framework. They argue that the downside beta and downside co-skewness between security returns and market portfolio returns may be alternative measures of downside risk. However, the sample of their study is emerging in market stock indices with the proxies of market return and risk-free rate taken from the US

market. This again leads to the same problem mentioned above. Secondly, the study cannot be generalized to domestic individual stocks as they are different from international indices.

This motivates us to put forward our study, which addresses the last two perspectives to a robust asset pricing model based on beta/downside risk and higher moment, i.e., skewness/downside skewness only. The outcomes have three possibilities: replacing beta with downside beta; second, replacing beta/downside beta by skewness/downside skewness; and, lastly, adding skewness/downside skewness to the original risk measures of beta/downside beta, respectively. This yields six models to be tested: beta and downside beta, skewness and downside skewness, beta and skewness, and downside beta and downside skewness based-CAPM and DCAPM models.

However, there is the problem of the high correlation between the CAPM and DCAPM models from the dependent variable perspective as well as the high correlation between the second and third-moment risk measures on the independent variable side. The former is resolved by constructing double-sorted portfolios that are sorted first on the beta and then again on downside beta using the standard procedure following [Ang et al. \(2006\)](#) to disentangle the high correlation effect of the beta and downside beta in the regressands. Furthermore, the portfolios cater to the problems of selection bias and measurement error.

The problem of the correlation between the beta and skewness in the mean-variance framework and downside beta and downside skewness in the mean-downside risk framework is resolved by orthogonalizing one factor on the other factor following [Galagedera and Brooks \(2007\)](#). Next, the competitive models are tested using [Fama and MacBeth's \(1973\)](#) methodology, and the best model is reported on the basis of the R^2 , adj R^2 , risk-factor significance in each model, and the risk–return relationship.

This makes this study the first of its kind for a single and highly volatile market such as the Pakistan Stock Exchange using all the possible models: beta/skewness based CAPM, downside beta skewness-[Hogan and Warren \(1974\)](#); downside beta skewness-[Harlow and Rao \(1989\)](#), and downside beta skewness-[Estrada \(2002\)](#). It uses the higher moment, i.e., skewness and downside skewness asymmetry, in the stock returns. The problem of the high correlation in the regressors and regressands is addressed in one study that was previously ignored. The double-sorted portfolios also add to the study the risk–return relationship.

This paper has four sections: Section 1 is related to the introduction and literature review, Section 2 comprises data and methodology, Section 3 is about the results and discussion, and Section 4 concludes the study.

2. Data and Methodology

Pakistan Stock Exchange (PSX) was formed in January 2016 by merging three stock exchanges: the Karachi Stock Exchange (KSE), the Lahore Stock Exchange (LSE), and the Islamabad Stock Exchange (ISE). FTSE classifies PSX as a secondary emerging market. PSX was reclassified as an MSCI emerging market in May 2017. There are around 559 companies with a market capitalization of \$84 billion listed with PSX. Foreign institutional investors and domestic institutional investors stand at 1886 and 1883, respectively.

In 2002, KSE declared PSX the “Best Performing Stock Market of the World”, and, again, PSX was among the world’s best-performing stock markets between 2009 and 2015. With the promulgation of the Stock Exchanges (Corporatization, Demutualization, and Integration) Act 2012, the stock exchanges were successfully corporatized and demutualized on 27 August 2012. In December 2016, PSX sold 40% strategic shares to a Chinese consortium for \$85 million. However, in 2017, PSX regressed to become the worst-performing in the region. Furthermore, it faced two crashes in 2005 and 2008 where, in the latter, the trading was halted for four months. This makes PSX a highly volatile market and a suitable market to test the alternative pricing models. This is because the asset pricing models are restrictive to stable markets. The monthly ending stock prices of those listed were taken with an estimation window from January 2000 to June 2018. KSE-100 index was used as a proxy for the market portfolio, while six months treasury bills rate was taken as a proxy for

the risk-free rate. The reasons for employing the 6-mo T-bill rate as the risk-free are the availability of data, issuance of 6-mo T-bill rate in Pakistan regularly, and its ease to take into account in the calculations. Moreover, much previous research used 6-mo T-bill rate (Razzaq et al. 2012).

The data were taken from the PSX official website, State Bank of Pakistan website, and DataStream. The prices were converted into returns by using log returns following Ayub et al. (2015).

This study used the basic CAPM and DCAPM equations and extended the models to incorporate skewness and downside skewness following Galagedera and Brooks (2007). In this study, the three major types of downside risk used were outlined by Hogan and Warren (1974); Harlow and Rao (1989); Estrada (2002). This gives our four risk proxies: beta, downside beta-Hogan and Warren (1974); downside beta-Harlow and Rao (1989), and downside beta-Estrada (2002). Likewise, there emerged four risk proxies for skewness as gamma, downside gamma-Hogan and Warren (1974); downside gamma-Harlow and Rao (1989), and downside gamma-Estrada (2002).

To compare alternative models, CAPM and DCAPM models and their extensions, it is important to disentangle the high correlation of beta and downside beta in the regress and following Ang et al. (2006). Furthermore, the problems of selection bias and measurement error need to be resolved. This was achieved by constructing double-sorted equal-weighted portfolios. The beta for the stock was estimated using 48 months regression over the sample period and split into two sub-samples as high (H) and low (L) beta. The downside beta of each sub-sample was estimated and ranked from high to low. Then, the sub-sample was split again to high (H) and low (L) downside beta. This yielded four sub-samples of stocks. Each sub-sample was used to construct equal-value portfolios to yield four sub-samples: high beta-high downside beta (HH), high beta-low downside beta (HL), low beta-high downside beta (LH), and low beta-low downside beta (LL)¹². The rolling windows were constructed by moving one month plus rolling following Ayub et al. (2015).

The problem of correlation between different regressors, beta/gamma and downside beta/downside gamma, was addressed by using the orthogonalizing procedure following Galagedera and Brooks (2007). We isolated gamma from beta by projecting gamma on constant and beta. The residuals from this regression, which are orthogonal to beta, were effectively the orthogonalized component of gamma. Likewise, downside gamma was orthogonalized from downside beta by projecting downside gamma on constant and downside beta. The residuals from this regression were orthogonal to downside beta and were effectively the orthogonalized component of downside gamma.

Fama and MacBeth (1973) two-pass regression standard estimation procedure to test CAPM was followed in this study. Newey–West estimator adjusted for the problem of heteroskedasticity and autocorrelation. The first pass Fama and MacBeth (1973) equation for different combinations is:

$$R_{pt} - R_{ft} = \alpha + \beta_{pt}(R_{PSXt} - R_{ft}) + \epsilon_{pt} \quad (1)$$

for CAPM, where R_p is the portfolio returns as HH, HL, LH, and LL, R_f is the risk-free return taken like 3 months T-bills, β is the proxy of systematic risk, and R_{PSX} is the KSE index returns.

$$R_{pt} - R_{ft} = \alpha + D\beta_{pt}^i(R_{PSXt} - R_{ft}) + \epsilon_{pt} \quad (2)$$

for DCAPM, where $D\beta^i$ is the proxy of systematic downside risk and $I = HW, HR$, and E as $D\beta_{pt}^{HW}$ downside beta-Hogan and Warren (1974); $D\beta_{pt}^{HR}$ is downside beta-Harlow and Rao (1989), and $D\beta_{pt}^E$ downside beta-Estrada (2002).

$$R_{pt} - R_{ft} = \alpha + \gamma_{pt}(R_{PSXt} - R_{ft})^2 + \epsilon_{pt} \quad (3)$$

for CAPM, where γ is gamma (skewness) replacing β as a proxy of risk

$$R_{pt} - R_{ft} = \alpha + D\gamma_{pt}^i (R_{PSXt} - R_{ft}) + \epsilon_{pt} \quad (4)$$

for DCAPM, where $D\gamma^i$ is downside skewness (gamma) replacing $D\beta^i$ as a proxy of risk and $i = \text{HW, HR, and E}$ as $D\gamma_{pt}^{\text{HW}}$ downside gamma-Hogan and Warren (1974), $D\gamma_{pt}^{\text{HR}}$ is downside gamma-Harlow and Rao (1989), and $D\gamma_{pt}^{\text{E}}$ downside gamma-Estrada (2002).

$$R_{pt} - R_{ft} = \alpha + \beta_{pt} (R_{PSXt} - R_{ft}) + O\gamma_{pt} (R_{PSXt} - R_{ft})^2 + \epsilon_{pt} \quad (5)$$

for CAPM, where $O\gamma$ is orthogonal skewness to β .

$$R_{pt} - R_{ft} = \alpha + O\beta_{pt} (R_{PSXt} - R_{ft}) + \gamma_{pt} (R_{PSXt} - R_{ft})^2 + \epsilon_{pt} \quad (6)$$

for CAPM, where $O\beta$ is orthogonal beta to γ .

$$R_{pt} - R_{ft} = \alpha + D\beta_{pt}^i (R_{PSXt} - R_{ft}) + OD\gamma_{pt}^i (R_{PSXt} - R_{ft})^2 + \epsilon_{pt} \quad (7)$$

for DCAPM, where $OD\gamma$ is orthogonal skewness to $D\beta$ and $i = \text{HW, HR, and E}$ as $OD\gamma_{pt}^{\text{HW}}$ downside gamma-Hogan and Warren (1974), $OD\gamma_{pt}^{\text{HR}}$ is downside gamma-Harlow and Rao (1989), and $OD\gamma_{pt}^{\text{E}}$ downside gamma-Estrada (2002).

$$R_{pt} - R_{ft} = \alpha + OD\beta_{pt}^i (R_{PSXt} - R_{ft}) + D\gamma_{pt}^i (R_{PSXt} - R_{ft})^2 + \epsilon_{pt} \quad (8)$$

For DCAPM, where $Od\beta$ is orthogonal beta skewness $D\gamma$ and $I = \text{HW, HR, and E}$ as $OD\beta_{pt}^{\text{HW}}$ downside beta-Hogan and Warren (1974), $OD\beta_{pt}^{\text{HR}}$ is downside beta-Harlow and Rao (1989), and $OD\beta_{pt}^{\text{E}}$ downside beta-Estrada (2002).

The β and γ are proxies for beta and gamma, while $D\beta$ and $D\gamma$ are specified for different proxies of downside beta and downside gamma for Hogan and Warren (1974); Harlow and Rao (1989); Estrada (2002) defined as follows:

$$\beta_p = \frac{E[(R_p - \mu_p)(R_{PSX} - \mu_{PSX})]}{E[(R_{PSX} - \mu_{PSX})^2]} \quad (9)$$

for beta in CAPM framework.

$$\gamma_p = \frac{E[(R_p - \mu_p)(R_{PSX} - \mu_{PSX})^2]}{E[(R_{PSX} - \mu_{PSX})^3]} \quad (10)$$

for skewness in CAPM framework.

$$D\beta_p^{\text{HW}} = \frac{E[(R_p - R_f) \min(R_{PSX} - R_f, 0)]}{E[\min(R_{PSX} - R_f, 0)^2]} \quad (11)$$

for Hogan and Warren (1974) downside beta.

$$D\beta_p^{\text{HR}} = \frac{E[(R_p - \mu_p) \min(R_{PSX} - \mu_{PSX}, 0)]}{E[\min(R_{PSX} - \mu_{PSX}, 0)^2]} \quad (12)$$

for Harlow and Rao (1989) downside beta.

$$D\beta_p^E = \frac{E[\min(R_p - \mu_p, 0)\min(R_{PSX} - \mu_{PSX}, 0)]}{E[\min(R_{PSX} - \mu_{PSX}, 0)]^2} \quad (13)$$

for Estrada (2002) downside beta.

$$D\gamma_p^{HW} = \frac{E[(R_p - R_f)\min(R_{PSX} - R_f, 0)]^2}{E[\min(R_{PSX} - R_f, 0)]^3} \quad (14)$$

for Hogan and Warren (1974) downside gamma.

$$D\gamma_p^{HR} = \frac{E[(R_p - \mu_p)\min(R_{PSX} - \mu_{PSX}, 0)]^2}{E[\min(R_{PSX} - \mu_{PSX}, 0)]^3} \quad (15)$$

for Harlow and Rao (1989) downside gamma.

$$D\gamma_p^E = \frac{E[\min(R_p - \mu_p, 0)\min(R_{PSX} - \mu_{PSX}, 0)]^2}{E[\min(R_{PSX} - \mu_{PSX}, 0)]^3} \quad (16)$$

for Estrada (2002) downside gamma.

The second pass Fama and MacBeth (1973) equations were built correspondingly with the equations from (1)–(8). In Fama and MacBeth (1973), second pass for portfolio beta and portfolio return cross-sectional analysis for each month were performed. In the second pass, the parameter was converted to the coefficient. Finally, through two-pass regression, different risk measures and combinations of different risk measures were analyzed using *t*-stat as outlined in Fama and MacBeth (1973) methodology.

3. Results and Discussion

The results are reported in four tables, and each consists of four panels. The first two tables report the results for the single-factor asset pricing models of CAPM and DCAPM with risk proxies, namely, Sharpe's beta, Hogan and Warren (1974); Harlow and Rao (1989); Estrada (2002) downside betas and the proxies of risk as the third moment, i.e., skewness (gamma) and downside skewness (downside gamma). The downside gamma risk proxies are based on the Hogan and Warren (1974); Harlow and Rao (1989); Estrada (2002) downside gammas. The last two tables report the two-factor asset pricing models based on the beta/downside beta with gamma/downside gamma. As the beta and skewness and, likewise, downside beta and downside gamma are highly correlated, so the two respective risk measures are orthogonalized each time, yielding two results: a beta with an orthogonalized gamma and then an orthogonalized beta with gamma. The same procedure is adopted in the downside framework.

Each table consists of four panels: A, B, C, and D, and each panel has five columns for Tables 1 and 2 and six columns for Tables 3 and 4. The first column is the type of portfolio: HH, H: LH, and LL. Column two is the returns for each portfolio type. Columns three in Tables 1 and 2 and columns three and four in Tables 3 and 4 are the risk proxies for beta/downside beta only and with gamma/downside gamma, respectively. The last two columns report the R^2 and Adj R^2 for each portfolio type. At the bottom, the differences between the HH and LL returns and risk proxies are given to assess the risk–return relationship. With each risk measure, the *t*-stats based on Fama and MacBeth (1973) are given in brackets. The risk–return relationship is measured by checking the risk measure greater than zero; thus, the *t*-test is one-tail. The acceptance of the null hypothesis validates the model. The rejection is set at a 5% significant level in this study. The significant levels are represented as *, **, and *** for the 10%, 5%, and 1% significant level, respectively.

Table 1. Results for β , $D\beta^{HW}$, $D\beta^{HR}$, and $D\beta^E$ with R^2 and Adj R^2 and their t -stat in (). Diff is the difference of returns between HH and LL for β , $D\beta^{HW}$, $D\beta^{HR}$, and $D\beta^E$ showing the risk–return relationship.

Panel A					Panel B				
	R_p	B	R^2	Adj R^2		R_p	$D\beta^{HW}$	R^2	Adj R^2
HH	12.707	1.980 (−1.581)	0.523	0.506	HH	12.707	−1.458 * (−1.745)	0.515	0.498
HL	8.307	0.194 *** (−2.694)	0.511	0.550	HL	8.307	−0.369 (0.024)	0.552	0.537
LH	7.859	0.214 *** (−2.836)	0.559	0.544	LH	7.859	−0.400 (−0.220)	0.556	0.541
LL	6.992	0.930 * (−1.860)	0.506	0.489	LL	6.992	−1.087 (0.803)	0.509	0.492
Diff	5.715	1.05			Diff	5.715	−0.371		
Panel C					Panel D				
	R_p	$D\beta^{HR}$	R^2	Adj R^2		R_p	$D\beta^E$	R^2	Adj R^2
HH	12.707	−1.533 * (−1.719)	0.550	0.500	HH	12.707	1.028 (−1.549)	0.492	0.474
HL	8.307	−0.395 (0.027)	0.551	0.535	HL	8.307	0.216 (−0.113)	0.530	0.514
LH	7.859	−0.429 (−0.213)	0.555	0.539	LH	7.859	0.229 (0.183)	0.550	0.534
LL	6.992	−1.059 (0.787)	0.508	0.492	LL	6.992	0.978 * (−1.662)	0.530	0.514
Diff	5.715	−0.474			Diff	5.715	1.690		

Note: *** = 1%, and * = 10% significant level.

Table 2. Results for γ , $D\gamma^{HW}$, $D\gamma^{HR}$, and $D\gamma^E$ with R^2 and Adj R^2 and their t -stat in (). Diff is the difference of returns between HH and LL for γ , $D\gamma^{HW}$, $D\gamma^{HR}$, and $D\gamma^E$ showing the risk–return relationship.

Panel A					Panel B				
	R_p	γ	R^2	Adj R^2		R_p	$D\gamma^{HW}$	R^2	Adj R^2
HH	12.71	2.658 (0.887)	0.695	0.631	HH	12.71	−1.895 * (−1.640)	0.794	0.751
HL	8.31	1.615 ** (1.963)	0.688	0.623	HL	8.31	−0.298 (0.152)	0.774	0.727
LH	7.86	1.794 ** (−2.041)	0.72	0.662	LH	7.86	−0.702 ** (−2.199)	0.814	0.775
LL	6.99	1.329 (0.953)	0.7677	0.719	LL	6.99	−1.383 (0.054)	0.784	0.739
Diff	5.72	1.329			Diff	5.72	−0.512		
Panel C					Panel D				
	R_p	$D\gamma^{HR}$	R^2	Adj R^2		R_p	$D\gamma^E$	R^2	Adj R^2
HH	12.71	−1.702 (0.109)	0.8084	0.768	HH	12.71	−0.168 * (−1.743)	0.745	0.692
HL	8.31	−0.848 (0.148)	0.81	0.770	HL	8.31	−0.937 (0.167)	0.735	0.680
LH	7.86	−0.174 ** (−2.197)	0.814	0.775	LH	7.86	−1.196 (−0.110)	0.754	0.703
LL	6.99	−1.244 (0.103)	0.839	0.805	LL	6.99	−0.217 (−0.292)	0.767	0.718
Diff	5.72	−0.457			Diff	5.72	0.048		

Note: ** = 5%, and * = 10% significant level.

Table 3. Results for β , $D\beta^{HW}$, $D\beta^{HR}$, and $D\beta^E$ with $O\gamma$, $OD\gamma^{HW}$, $OD\gamma^{HR}$, and $OD\gamma^E$, respectively, with R^2 and Adj R^2 and their t -stat in (). Diff is the difference of returns between HH and LL showing the risk–return relationship.

Panel A						Panel B					
	R_p	β	$O\gamma$	R^2	Adj R^2		R_p	$D\beta^{HW}$	$OD\gamma^{HW}$	R^2	Adj R^2
HH	12.71	1.321 (−1.232)	1.169 (−1.435)	0.785	0.740	HH	12.71	1.023 (−1.624)	1.064 (−0.310)	0.835	0.801
HL	8.31	0.5723 (−1.446)	0.921 ** (−2.125)	0.764	0.715	HL	8.31	1.0133 ** (−2.213)	0.068 (−0.794)	0.824	0.787
LH	7.86	0.7703 (−1.200)	0.542 (−0.019)	0.754	0.702	LH	7.86	1.5343 (−1.290)	0.61 (0.052)	0.814	0.775
LL	6.99	0.461 (−1.388)	0.68 (−1.122)	0.768	0.719	LL	6.99	0.515 ** (−2.312)	0.692 (−1.230)	0.784	0.739
Diff	5.72	0.860	0.489			Diff	5.72	0.508	0.372		
Panel C						Panel D					
	R_p	$D\beta^{HR}$	$OD\gamma^{HR}$	R^2	Adj R^2		R_p	$D\beta^E$	$OD\gamma^E$	R^2	Adj R^2
HH	12.71	1.108 (−1.166)	1.081 (0.775)	0.848	0.817	HH	12.71	2.351 (−1.467)	1.103 (−1.591)	0.810	0.770
HL	8.31	0.9153 (0.503)	0.144 (−1.129)	0.840	0.807	HL	8.31	1.5499 * (−1.908)	0.234 ** (−1.964)	0.792	0.749
LH	7.86	1.0633 (−1.263)	1.145 (−1.221)	0.854	0.824	LH	7.86	0.6886 * (−1.650)	0.5562 (−0.823)	0.813	0.774
LL	6.99	0.647 (0.912)	0.765 (−1.123)	0.856	0.826	LL	6.99	0.236 (−1.402)	1.6536 (−1.044)	0.826	0.790
Diff	5.72	0.461	0.316			Diff	5.72	2.115	−0.5506		

Note: ** = 5%, and * = 10% significant level.

Table 4. Results for $O\beta$, $OD\beta^{HW}$, $OD\beta^{HR}$, and $OD\beta^E$ with γ , $D\gamma^{HW}$, $D\gamma^{HR}$, and $D\gamma^E$, respectively, with R^2 and Adj R^2 and their t -stat in (). Diff is the difference of returns between HH and LL showing the risk–return relationship.

Panel A						Panel B					
	R_p	$O\beta$	γ	R^2	Adj R^2		R_p	$OD\beta^{HW}$	$D\gamma^{HW}$	R^2	Adj R^2
HH	12.71	1.001 (−2.712) ***	0.612 ** (−2.035)	0.8	0.758	HH	12.71	1.121 * (−1.762)	1.967 ** (−1.870)	0.812	0.773
HL	8.31	0.967 (−0.234)	0.211 (−1.425)	0.781	0.735	HL	8.31	1.323 (−1.123)	1.800 (0.989)	0.8	0.758
LH	7.86	0.453 (−0.200)	0.301 * (−1.90)	0.767	0.718	LH	7.86	1.433 (−0.564)	1.612 (−1.121)	0.822	0.785
LL	6.99	0.201 (−2.765) ***	0.322 ** (−2.101)	0.799	0.757	LL	6.99	1.012 ** (−2.221)	1.701 (−1.675)	0.823	0.786
Diff	5.72	0.800	0.290			Diff	5.72	0.508	0.2662		
Panel C						Panel D					
	R_p	$OD\beta^{HR}$	$D\gamma^{HR}$	R^2	Adj R^2		R_p	$OD\beta^E$	$D\gamma^E$	R^2	Adj R^2
HH	12.71	1.568 (−1.166)	1.232 (0.775)	0.848	0.817	HH	12.71	1.222 (−1.467)	1.231 (−1.591)	0.810	0.770
HL	8.31	1.100 (0.503)	1.322 (−1.129)	0.840	0.807	HL	8.31	1.340 * (−1.908)	1.001 ** (−1.964)	0.792	0.749
LH	7.86	1.210 (−1.263)	1.111 (−1.221)	0.854	0.824	LH	7.86	1.121 (−1.650)	0.987 (−0.823)	0.813	0.774
LL	6.99	1.231 (0.912)	1.012 (−1.123)	0.856	0.826	LL	6.99	1.114 (−1.402)	0.768 (−1.044)	0.826	0.790
Diff	5.72	0.337	0.220			Diff	5.72	0.108	0.463		

Note: *** = 1%, ** = 5%, and * = 10% significant level.

There are four panels in Table 1: panel A, panel B, panel C, and panel D. Panel A illustrates the results of the CAPM with β as the risk proxy, R^2 , and Adj. R^2 , along with the HH, HL, LH, and LL portfolios. Panel B depicts the results of the DCAPM with downside β for Hogan and Warren (1974), R^2 and Adj. R^2 , along with the HH, HL, LH, and LL portfolios. Panel C shows the results of the CAPM with downside β for Harlow and Rao (1989), and panel D explains the results of the CAPM with downside β for Estrada (2002).

In panel A, the HL and LH are rejected at a 1% significant level. For the LL, the t -stats have a value of -1.860 and are rejected at a 10% significant level. In panel B, where the CAPM with downside β for Hogan and Warren (1974) is applied, all the null hypotheses are accepted except the HH value that rejects the null hypothesis at a 10% significant level. As most of the research was conducted at a 5% significant level, we can ignore this one, but the R^2 values are very low for this one as well. In panel B, as the null hypothesis is accepted, this model holds. The risk–return relationship is negative. In panel C, the null hypothesis is accepted for all except HH at a 10% significant level. In panel D, the null hypothesis is accepted for all except LL for a 10% significant level.

The risk–return relationship is represented as the difference of the returns and coefficient of risk measures of the HH and LL portfolios. The HH and LL return difference is 5.715 and has a value of 1.05, -0.474 , 1.690, and -0.371 for β , $D\beta^{HW}$, $D\beta^{HR}$, and $D\beta^E$, respectively. This relationship is positive for β and $D\beta^{HR}$ and negative for the other two risk measures. However, the value of R^2 in all the panels ranges from 48% to 56%, which is very low. This advocates for incorporating a higher moment as a measure of risk.

In Table 2, regarding the results for panel A, in the first column of each panel, there are portfolio returns from 2000 to 2018; in the second column, the skewness is depicted, in the third column, the values for R^2 are illustrated, and, in the last column, the Adj. R^2 is given. For panel B, the downside skewness ($D\gamma^{HW}$) for Hogan and Warren (1974) is given. In panel C, the downside skewness ($D\gamma^{HR}$) for Harlow and Rao (1989) and panel D downside skewness ($D\gamma^E$) for Estrada (2002) are discussed. At the bottom of each panel, the differences between the HH and LL values for the skewness and returns are given, where a positive value shows that the skewness and returns are positively correlated and a negative value shows a negative correlation. Underneath each γ value, a λ value, as per the t -statistics, is given in (); here, * = 10% significant level, ** = 5% significant level, and *** = 1% significant level to accept or reject null hypothesis.

In panel A, when the beta is replaced with the skewness and co-skewness, the overall relationship is positive, but the null hypothesis is rejected at the HL and LH portfolios at a 5% significant level. This shows the model does not hold. Overall, the R^2 values increased for this model. In panel B, the HH rejects the null hypothesis at a 10% significant level, and the LH rejects the null hypothesis at a 5% significant level. Overall, the R^2 values have increased, but the model does not hold. In panel C, the null hypothesis is rejected for the LH at a 5% significant level, but, overall, the R^2 values have increased. In panel D, the null hypothesis is rejected for the HH at a 10% significant level, but the 5% significant level is considered as a benchmark, so the null hypothesis is accepted and the downside skewness for the Estrada (2002) model holds. In Table 2, the R^2 values have increased by significant levels ranging from 60% to 84% as compared to Table 1. This means, in comparison to the beta and downside beta, the skewness and co-skewness are better measures of risk. As per t -statistics, the performance of the model is not good, but, if we look at the R^2 and Adj. R^2 values, they have improved considerably.

In Table 3, we shift from a single-factor model to a double factor model: we are employing both the beta and skewness as measures for risk. As both the beta and skewness are highly correlated, we orthogonalize them, represented by O. Table 3 has 4 panels: panel A, panel B, panel C, and panel D, respectively. Each panel has five columns: the first column represents portfolios, the second column represents returns, the third column shows downside beta, the fourth column shows orthogonalize downside skewness, the fifth column has R^2 values, and the last column has Adj. R^2 values.

Panel A illustrates the results of the CAPM with β , orthogonalize downside skewness $O\gamma$, R^2 , and Adj. R^2 , along with HH (high, high), HL (high, low), LH (low, high), and LL (Low, low) portfolios. Panel B depicts the results of the CAPM with downside beta $D\beta^{HW}$ for Hogan and Warren (1974), orthogonalize downside skewness for Hogan and Warren (1974) $OD\gamma^{HW}$, R^2 and Adj. R^2 , along with HH (high, high), HL (high, low), LH (low, high), and LL (Low, low) portfolios. Panel C shows the results of the CAPM with downside beta for Harlow and Rao (1989) $D\beta^{HR}$, orthogonalize downside skewness for Harlow and Rao (1989) $OD\gamma^{HR}$, R^2 and Adj. R^2 , along with HH (high, high), HL (high, low), LH (low, high), and LL (Low, low) portfolios. Panel D explains the results of the CAPM with downside beta for Estrada (2002) $D\beta^E$, orthogonalize downside skewness for Estrada (2002) $OD\gamma^E$, R^2 and Adj. R^2 , along with HH (high, high), HL (high, low), LH (low, high), and LL (Low, low) portfolios.

If we look at the overall results, the values for R^2 and Adj. R^2 have increased, showing that combining the beta and downside beta provides a better measure of risk. In panel A, the null hypothesis is accepted for all except HL for orthogonalize downside skewness, where the null hypothesis is rejected at a 5% significant level. In panel B, the null hypothesis is rejected for HL and LL at a 5% significant level for downside beta for Hogan and Warren (1974). In panel C, the null hypothesis is accepted by all, so Harlow and Rao's (1989) model holds. In panel D, the null hypothesis is rejected for downside beta at the HL and LH for a 10% significant level, but, as 5% is considered as a benchmark, the model holds. For orthogonalize, the downside skewness null hypothesis is rejected for HL at a 5% significant level, so the model does not hold. There is a positive relationship between the risk and return in all the models and increased values of R^2 and Adj. R^2 , depicting the efficiency of combining downside beta and orthogonalize downside skewness.

Table 4 is also a double factor model; we are employing both beta and skewness as a measure for risk. As both beta and skewness are highly correlated, we orthogonalize them, represented by O. In Table 4, we orthogonalize beta instead of skewness; panel A illustrates the results of the CAPM with orthogonalize $O\beta$, downside skewness γ , R^2 , and Adj. R^2 , along with HH (high, high), HL (high, low), LH (low, high), and LL (Low, low) portfolios. Panel B depicts the results of the CAPM with orthogonalize downside beta $OD\beta^{HW}$ for Hogan and Warren (1974), downside skewness for Hogan and Warren (1974) $D\gamma^{HW}$, R^2 and Adj. R^2 , along with HH (high, high), HL (high, low), LH (low, high), and LL (Low, low) portfolios. Panel C shows the results of the CAPM with orthogonalize downside beta for Harlow and Rao (1989) $OD\beta^{HR}$, downside skewness for Harlow and Rao (1989) $D\gamma^{HR}$, R^2 and Adj. R^2 , along with HH (high, high), HL (high, low), LH (low, high), and LL (Low, low) portfolios. Panel D explains the results of the CAPM with orthogonalize downside beta for Estrada (2002) $OD\beta^E$, downside skewness for Estrada (2002) $D\gamma^E$, R^2 and Adj. R^2 , along with HH (high, high), HL (high, low), LH (low, high), and LL (Low, low) portfolios.

The overall values for R^2 and Adj. R^2 have increased, showing a positive outcome. In panel A, the null hypothesis is rejected for the HH and LL portfolios at a 1% significant level for orthogonalize $O\beta$, and downside skewness, the null hypothesis is rejected for the HH and LH at a 5% significant level and for LH at a 10% significant level; this model does not hold. In Hogan and Warren's (1974) model, the null hypothesis is rejected for orthogonalize beta $O\beta$ at a 10% significant level for the HH portfolios and a 5% significant level for the LL portfolios. The HH rejects the null hypothesis at a 5% significant level for downside skewness. Panel C illustrates that the null hypothesis is accepted and the high value results for R^2 and Adj. R^2 represent that Harlow and Rao's (1989) model is the most efficient one. In panel D, the null hypothesis is rejected for the HL at a 10% significant level and 5% significant level for orthogonalized downside beta and downside skewness, respectively.

These results state that the model with two risk proxies is the better choice among the given models, i.e., Harlow and Rao (1989) downside beta should be extended to include downside gamma. This model has better results in terms of the R^2 , risk–return relationship, and significance of factors. The importance of downside skewness follows with the results

of Galagedera and Brooks (2007); Ang et al. (2006); Harvey and Siddique (2000). These studies advocate for the use of the third moment in asset pricing models; however, using it with the second moment is the question that is answered in this study.

4. Conclusions

The study examines the capital asset pricing model developed by Sharpe (1964) and Lintner (1965) as the standard model in the asset pricing theory, which defines the first two moments as the target variable. The results show that the Sharpe–Lintner CAPM is inadequate for the equity market of Pakistan by explaining the economically and statistically significant role of the market risk for the determination of the expected return. In this paper, a detailed analysis identified a single risk factor instead of more risk factors. The returns on an asset in the equity market of Pakistan deviate from normality by indicating that investors are concerned about the higher moments return distribution. Two methods were carried out: first, the standard model was extended by taking the higher moments, and, second, investors put more weight on losses and less weight on gains; the lower moments are incorporated with the upside risk framework.

This research is to demonstrate the benefits of conditional non-linear pricing behavior, and, from the evidence, the results show that higher order pricing factors are associated with co-skewness and co-kurtosis. The results concluded that the investor is rewarded for the co-skewness risk for the higher moment model. However, the results provide marginal support for the reward of co-kurtosis risk. It is concluded that the improved form of the Sharpe–Lintner CAPM used by Kraus and Litzenberger (1976) is successful to some extent with data in the Karachi Stock Exchange. Finally, the empirical study states that the investigated conditional higher moments are used in explaining the cross-section of asset returns. The main aim of this research was to analyze which model performs better as the study estimates a multifactor asset pricing model with a downside risk based and higher moment based asset pricing model. It was thus demonstrated that the downside higher moments beat the downside risk. Under alternative assumptions, it was found out that the portfolio composition differs. The results indicate that the asset pricing relationship varies through time, and the conditional co-skewness is an important determinant for it. In general, the cross-sectional analysis discloses that, when the CAPM beta, downside beta, or downside gamma is included in the pricing model where the risk premium is associated with it, this is always positive, and the explanatory power varies in favor of downside gamma. When the downside gamma and downside beta are included together, the downside gamma appears to dominate the explanatory variable. Overall, from the statistical results, it was concluded that the downside gamma may be a more appropriate explanatory variable of asset price variation than the downside beta in emerging market data.

The study helps an investor in investment decisions about pricing an asset as well as forecasting returns. Subsequently, the investor can mitigate the risk to considerable limits using the Harlow and Rao (1989) extended two-factor model. Furthermore, a suitable asset pricing model helps in stabilizing the market by decreasing its volatility. Lastly, the proposed model in the study is conservative as it incorporates two risk measures at the same time. If the investor still wants to penalize the outcome, then the fourth moment, i.e., downside kurtosis, can be used. The decision, in this case, will be the most conservative for the investor. The emerging markets that are comparable to the market of Pakistan reveal different risk–return relationships, and the studies have found the existence of highly autocorrelated returns, volatile prices, and supernormal returns. This study will help portfolio managers investing in emerging markets and stocks to quantify the expected return and risk. This study also includes a downside risk perspective for risk averse investors who always avoid risk. There are some recommendations for a future study, such as the sample being divided into sub-samples from 2000–2002, 2003–2005, 2006–2008, 2009–2012, and 2013–2018. Furthermore, a comparative analysis is suggested with other developed countries to obtain better and more comprehensive results.

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Notes

- ¹ CAPM is independently developed by (Sharpe 1964; Lintner 1965; Mossin 1966; Treynor 1962). Treynor is considered by some as the pioneer for CAPM, but his first paper of 1962 “Toward a Theory of Market Value of Risky Assets” was not published until 1999. William Sharpe was eventually a winner of the 1990 Nobel Memorial Prize in Economic Sciences.
- ² Initial tests favour the risk–return relationship as outlined by CAPM (Beaver et al. 1970; Hamada 1972). However, (Black et al. 1972; Blume and Friend 1973; Fama and MacBeth 1973) provide weak empirical evidence on this relationship. Later, Post and van Vliet (2004) and (Ang et al. 2006; Tahir et al. 2013) also report similar results.
- ³ The first three are presented by (Abbas et al. 2011).
- ⁴ However, (Shanken 1982, 1987; Kandel and Stambaugh 1987) argue that, even though the stock market is not true market portfolio, it must nevertheless be a highly correlated proxy for the true market.
- ⁵ See (Basu 1977, 1983; Banz 1981; Stattman 1980; Reinganum 1981; Rosenberg et al. 1985; De Bondt and Thaler 1987; Fama and French 1992; Carhart 1997) for details.
- ⁶ However, (Gibbons 1982) rejects zero-beta CAPM.
- ⁷ For details, see (Kahneman and Tversky 1979; Gul 1991; Estrada 2007; Post and Levy 2005; Ayub et al. 2015).
- ⁸ For details on skewness, see (Adcock and Shutes 1998; Leland 1999; Harvey and Siddique 2000; Chen et al. 2001). For kurtosis, see (Fang and Lai 1997; Dittmar 2002; Chang et al. 2006).
- ⁹ (Rubinstein 1973) is the first to propose an assessment of financial asset price with more than two moments. His proposition is estimated by (Kraus and Litzenberger 1976; Ang and Chua 1979).
- ¹⁰ See for details (Sihem and Slaheddine 2014; Harvey and Siddique 2000).
- ¹¹ See for details (Elton et al. 2013).
- ¹² Following (Ang et al. 2006; Ayub et al. 2015).

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