

Article

Examining the Effects of Gradual Catastrophes on Capital Modelling and the Solvency of Insurers: The Case of COVID-19

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Abstract: This paper models the gradual elements of catastrophic events on non-life insurance capital with a particular focus on the impact of pandemics, such as COVID-19. A combination of actuarial and epidemiological models are handled by the Markovian probabilistic approach, with Feynman's path calculation and Dirac notations, in order to observe how a pandemic risk may affect an insurer via reduced business. We also examine how the effects of a pandemic can be taken into account both during and at the end of the process. Examples are also provided showing the potential effects of a pandemic on different types of insurance product.

Keywords: risk theory; catastrophe risk modelling; COVID-19; dependence; pandemic

1. Introduction

Insurance companies typically have the benefit of historical data to aid their forecasting. However, past data is not necessarily an accurate indication as to what will happen in the future and unexpected events are bound to occur. Moreover, the complex relationships between different parameters are not exactly known, and so one must attempt to predict the unpredictable in advance. This requires a comprehensive analysis in which dependence plays a crucial role.

Catastrophe modelling presents additional challenges due to the low probability of occurrence and hence infrequent data points. The data available for traditional catastrophes such as tornadoes is sparse and for new man-made catastrophes, such as terrorism, can be even more difficult to obtain. Thus, the margin of error in catastrophe modelling is typically higher than classical risk models, which affects the reliability and accuracy analyses. Another challenge in catastrophe risk analysis is that the catastrophe characteristics per se may have changed from past occurrences. For example, hurricanes may now be more clustered due to climate change (Papachristou 2020), and the COVID-19 pandemic has higher morbidity in relation to the elderly, which contrasts with previous pandemics (Cortis 2020).

Catastrophe insurance has attracted significant scholarly attention and has been extensively investigated. Typical catastrophe insurance investigations have historically focused on on pricing (Chang et al. 1996; Christensen and Schmidli 2000), capital allocation (Zanjani 2002), risk awareness (Lin 2020), risk classification (Porrini 2016), and pricing of reinsurance contracts (Dassios and Jang 2003). However, recent experiences show that there is a need for greater exploration of capital modelling in relation to pandemics in general insurance models.

Dependences among different risk factors are taken into consideration in insurance modelling. Examples of such research results are as follows:

- ruin models with dependence between the interclaim arrivals and the claim amounts in [Albrecher and Boxma \(2004\)](#); [Albrecher et al. \(2011\)](#),
- risk processes with dependence on reinsurance retention levels in [Nie et al. \(2010\)](#),
- risk processes with reserve-dependent premiums in [Albrecher et al. \(2013\)](#); [Ganesh et al. \(2007\)](#),
- dependence modelling in multivariate risk analysis in insurance data via copula functions in [Genest et al. \(2011\)](#), and
- dependence in dynamic claim frequency credibility models in [Purcaru and Denuit \(2003\)](#).

In addition to these, different dependency types such as between excessively large claims and fraud rates, and claim frequency and a policyholder's characteristics (such as age, gender, traffic record) can also be highlighted in terms of actuarial research interests.

The COVID-19 pandemic and the associated measures are having many different, and often interacting, impacts across various insurance lines. Some insurance products are experiencing higher profits in the short term. For example, as a result of lockdown measures across the world, fewer cars and miles are being driven. Hence the frequency and severity of car insurance claims is significantly lower despite a premium having been paid for a yearly cover. However, there are also many other risks such as reputational risks associated with insurers, and they are at risk of writing lower amounts of business in future. Furthermore, agile technology-focused InsurTech firms may take advantage of the evolving landscape to win market share and possibly offer improved products with better market fit ([Cortis et al. 2019](#)). An example would be the use of telematics devices for pay-as-you-go cover where essentially a driver would be paying "per-mile" for only the amount of driving done rather a fixed annual premium based on proxy factors ([Azzopardi and Cortis 2013](#)).

The main scope of this paper is to look at how the classical surplus process is modified by catastrophe risk and to then examine the dependence between the behaviour of the regular claim, premium rate, and occurrence of catastrophe risk in the modified surplus process. On the other hand, regular reported claims and collected premiums will be exposed to change by catastrophic risk that will impact the number of policyholders and the level of actuarial risk. Secondly, how the effects of the gradual catastrophe are taken into consideration during and at the end of the process are observed. Lastly, another aim of this paper is to provide more insight for further discussions.

In that respect, our paper contributes to the existing literature by evaluating the risk of ruin for an insurer following a decrease in business written. This could be due to higher mortality, new more efficient incumbents or lower client demand. This change leads to a longer term effect than the initial shock of a very large catastrophe claim. In order to consider the longer-term effects of decreased business, our work is original in considering transitions for both customers and non-customers.

Classical Surplus Process and Ruin Probability

The classical surplus process of an insurance company is defined by

$$R(t) = R(0) + ct - S(t)$$

where $R(t)$ is the capital at time t , $R(0) = u$ is initial capital, c is premium income per unit time and $S(t) = \sum_{i=1}^{N(t)} X_i$, $t \geq 0$ is a compound Poisson process representing the aggregate claim loss in the interval $(0, t]$. $\{X_i\}$ is a sequence of positive integer-valued i.i.d. random variables representing the successive regular claim amounts. $N(t)$ is a Poisson process of rate λ giving the total number of claims in the interval $(0, t]$. Let us consider a simple insurance random walk. When operational costs are not taken into account, changes in the capital of the insurance company in the interval $(t, t + \varepsilon]$ depend on

c and $S(\varepsilon)$, $\varepsilon > 0$. For the fixed premium rate, the probability of being $S(\varepsilon)$ will give us the probability of changes in the capital of the insurance company. Simply, for the compound Poisson process,

$$P(S(\varepsilon) = w) = \sum_{k=0}^{\infty} \frac{e^{-\lambda\varepsilon}(\lambda\varepsilon)^k}{k!} P(X_1 + X_2 + \dots + X_k = w).$$

For the integer-valued claims and $u + c\varepsilon > S(\varepsilon)$, the non-ruin probability with initial capital u can be expressed by

$$P_u(T > t) = \sum_{S(\varepsilon)=0}^{\lfloor u+c\varepsilon \rfloor} P(u \rightarrow u + c\varepsilon - S(\varepsilon)) P_{u+c\varepsilon-S(\varepsilon)}(T > t - \varepsilon) \quad (1)$$

where $T = \min\{x | R(x) \leq 0\}$ and $\lfloor u + c\varepsilon \rfloor = \max\{y | y \in \mathbb{Z} \text{ and } y < u + c\varepsilon\}$.

For the integer-valued initial capitals, a matrix form of Equation (1) can be written as

$$\begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} & \cdots & a_{0,n-1} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,n-1} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,n-1} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} & \cdots & a_{3,n-1} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{u,0} & a_{u,1} & a_{u,2} & a_{u,3} & \cdots & a_{u,n-1} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{n-1,0} & a_{n-1,1} & a_{n-1,2} & a_{n-1,3} & \cdots & a_{n-1,n-1} \end{pmatrix} \begin{pmatrix} P_0(T > t - \varepsilon) \\ P_1(T > t - \varepsilon) \\ P_2(T > t - \varepsilon) \\ P_3(T > t - \varepsilon) \\ \vdots \\ P_u(T > t - \varepsilon) \\ \vdots \\ P_{n-1}(T > t - \varepsilon) \end{pmatrix} = \begin{pmatrix} P_0(T > t) \\ P_1(T > t) \\ P_2(T > t) \\ P_3(T > t) \\ \vdots \\ P_u(T > t) \\ \vdots \\ P_{n-1}(T > t) \end{pmatrix} \quad (2)$$

$A \qquad f(t - \varepsilon) \qquad f(t)$

where the first matrix (A) is the transition matrix. Note that we consider 0 is the absorption (ruin) state in the transition matrix. Therefore, $P_0(T > t) = P_0(T > t - \varepsilon) = 0$. This means that ruin will occur when the capital of the insurance company will be zero or a negative value. This is different from the classical surplus process where 0 is not a ruin level.

$a_{i,j}$ (or $A_{i,j}$), the elements of our $n \times n$ dimensional transition matrix A , are defined by

$$a_{i,j} = \begin{cases} 1 & \text{for } i = j = 0; \\ 0 & \text{for } i = 0, j \neq 0; \\ 1 - \sum_{k=1}^{n-1} a_{i,k} & \text{for } j = 0, i \neq 0; \\ P(R(t + \varepsilon) = j | R(t) = i) & \text{for the other cases} \end{cases}.$$

In this circumstance, Equation (2) can be written in the following form:

$$A(x)f(t - x) = f(t)$$

where f is the column vector function representing non-ruin probabilities. Similarly, using the Chapman–Kolmogorov equation, we have

$$A(x + y)f(t) = A(x)A(y)f(t) = f(t + x + y).$$

In continuous time, the transition matrix can be found via the generator matrix.

$$A(0) = \lim_{t \rightarrow 0} A(t) = I, \quad A'(0) = \lim_{\varepsilon \rightarrow 0} \frac{A(\varepsilon) - I}{\varepsilon} = Q$$

where Q is called the generator of Markov process

$$Q = \begin{pmatrix} q_{0,0} & q_{0,1} & q_{0,2} & q_{0,3} & q_{0,4} & q_{0,5} & \cdots & q_{0,n-1} \\ q_{1,0} & q_{1,1} & q_{1,2} & q_{1,3} & q_{1,4} & q_{1,5} & \cdots & q_{1,n-1} \\ q_{2,0} & q_{2,1} & q_{2,2} & q_{2,3} & q_{2,4} & q_{2,5} & \cdots & q_{2,n-1} \\ q_{3,0} & q_{3,1} & q_{3,2} & q_{3,3} & q_{3,4} & q_{3,5} & \cdots & q_{3,n-1} \\ q_{4,0} & q_{4,1} & q_{4,2} & q_{4,3} & q_{4,4} & q_{4,5} & \cdots & q_{4,n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ q_{n-1,0} & q_{n-1,1} & q_{n-1,2} & q_{n-1,3} & q_{n-1,4} & q_{n-1,5} & \cdots & q_{n-1,n-1} \end{pmatrix}. \quad (3)$$

$n \times n$ dimensional generator matrix Q has negative values on the diagonal, which are equal to minus the sum of the other elements (off-diagonal) in each row (Löeffler and Posch 2011).

$$q_{i,i} = - \sum_{j=0, j \neq i}^{n-1} q_{i,j} \quad \text{and} \quad q_{i,j} = \lim_{\varepsilon \rightarrow 0} \frac{A_{i,j}(\varepsilon)}{\varepsilon} \geq 0.$$

For small enough ε and error term $o(\varepsilon)$,

$$A_{i,j} = q_{i,j}\varepsilon + o(\varepsilon) \quad \text{for} \quad i \neq j.$$

The matrix A is differentiable for all $t \geq 0$ with $A'(t) = A(t)Q = QA(t)$ and $A''(t) = Q^2A(t)$. A can be computed by

$$A(t + \Delta t) = A(t) + A(t)Q \Delta t + \frac{A(t)Q^2(\Delta t)^2}{2!} + o((\Delta t)^3) \quad \text{or} \quad A(t) = e^{tQ} = e^{-tH} \quad (4)$$

where Q is the generator operator of the Markovian process and H is a Hamiltonian operator. Discretisation of the semigroup is used in Section 3.

2. Catastrophe Risk

Catastrophe risk modelling is a research area that determines and analyses the effects of events with a very low probability of occurrence but with a high severity of losses, should they occur. Such events are often associated with both natural disasters and man-made occurrences (see Table 1).

Table 1. Examples of sudden, gradual, natural and man-made catastrophes.

	Sudden Catastrophes	Gradual Catastrophes
Natural	Earthquake, flood hurricane, tsunami, volcanic eruption	Epidemic/pandemic outbreak
Man-made	Terrorist attack	War, financial and political crises

In this paper, we consider catastrophes as singular events. However, the financial (and non-financial) impact of their occurrence may be sudden or gradual. For example, the damage of a flood can be considered to be measurable straight away over the period just after the flood. After the claims are settled, it can be treated as business as usual, unless there are any chronic elements such as climate or infrastructural changes that are increasing the frequency and/or severity of a flood. On the other hand, the gradual effects of a catastrophe typically occur over a longer period. In addition to this, if there is a certain length of time between the beginning and the end of a disaster like the COVID-19 outbreak, we consider it a gradual catastrophe.

Gradual catastrophes have very different features compared to sudden catastrophes in terms of behaviour of the claim arrival process, and uncertainty about the tragedy's severity and duration.

In essence, modelling of sudden catastrophic events in most types of non-life insurance can be based on the following elements in general.

- (i) A catastrophic event increases both the number of claims $N(t)$ and individual severity loss X suddenly.
- (ii) There is a dramatic fall in the number of insured people after the catastrophic event. When gradual catastrophes are involved, this shows the differences. In the next section, we discuss how a pandemic can be considered as a catastrophe with gradual features for many categories of non-life insurance and how to factor in the uncertainty in risk analysis.

3. Capital Modelling under Pandemic Risk in General Insurance

The COVID-19 pandemic will result in immediate large losses for insurers that offer cover for pandemic-related claims such as travel insurance, credit and surety-ship policies, and event cancellation (Wells 2020). Moreover, there is political pressure and there are class lawsuits for business interruption cover to pay out for loss of business despite this explicitly excluding pandemics (Sams 2020). The effects of a pandemic for these classes of insurers, as well as life insurers, are sudden.

The COVID-19 pandemic has resulted in motor insurers having a significantly lower frequency of claims due to lock-down measures and smart-working schemes, with some motor insurers returning premiums to customers (Barlyn 2020). Yet these insurers may now face other challenges.

Pandemics bring with them potential financial consequences. Even if a worldwide pandemic is brought under control within a short time period, insurance companies may face economic losses because of national and even global recessions, a higher number of lost workers due to death, or loss of market share due to new agile approaches adopted by nimble companies such as InsurTechs. While the impact of a pandemic on life and health insurance companies has been researched in several previous studies (see Toole et al. (2010); Weisbart (2006)), similar research on non-life insurance companies has not attracted the same level of interest, in part because such companies are less clearly affected by possible pandemic outbreaks (European Actuarial Consultative Group 2006).

The combination of actuarial and epidemiological models (see Feng and Garrido (2011)) is necessary, since the mixed model approach depends on several parameters like virus infectiousness, the effects of human behaviour, medical trials, and clinical data (Bagus 2008). For the sake of simplicity, only the spread and death rate of a virus (infection fatality rate) will be taken into account in our analysis.

3.1. Modification of Actuarial Process by the Pandemic Risk

Let t_b be the beginning time of the outbreak in the origin, t_s be a starting time when the first patient presents in the observed country, and t_e be the time when the pandemic disappears. In this case, $t_e > t_s > t_b$. We assume the non-life insurance company in the observed country will begin to be affected at time t_s , so the insurance company only has a time between t_s and t_b in order to take precautions. Precautions and some risk mitigation techniques against the pandemic outbreak are beyond this paper's scope.

Now, let us have a look at how to modify our model in light of a pandemic risk. During the pandemic, we assume that existing healthy individuals may potentially cease being customers or become sick. The same is reflected by healthy non-customers who may transition to become customers or sick. Sickness is divided into two stages: early onset and late stage. Those in the latter stage are not customers and run the risk of dying due to the pandemic. Hence, there is a Markov chain on a finite number of states.

$$S = \{\text{healthy customer, sick customer, healthy non-customer, sick non-customer in early stage, sick non-customer in late stage, death}\}$$

with transition probabilities $p(x, y)_{x, y \in S}$ as $\sum_{y \in S} p(x, y) = 1$ can be defined as sequence of $X_0, X_1, X_2 \dots$ while it satisfies the property $P(X_{t+1} = s_{t+1} | X_t = s_t, \dots, X_0 = s_0) = P(X_{t+1} = s_{t+1} | X_t = s_t)$ where $s_i, i = 0, 1, \dots$ are states in S .

Let us consider the following example. When a pandemic occurs:

- A healthy person may become sick with rate β .
- A sick person in the early stage may move to the late stage (intensive unit) with probability δ .
- A sick customer in the late stage may die with probability α .
- A healthy customer may stop being a customer with rate η .
- A non-customer may become a customer with probability γ .

The states during the pandemic period are displayed in Figure 1, and the transition probabilities are listed in Table 2.

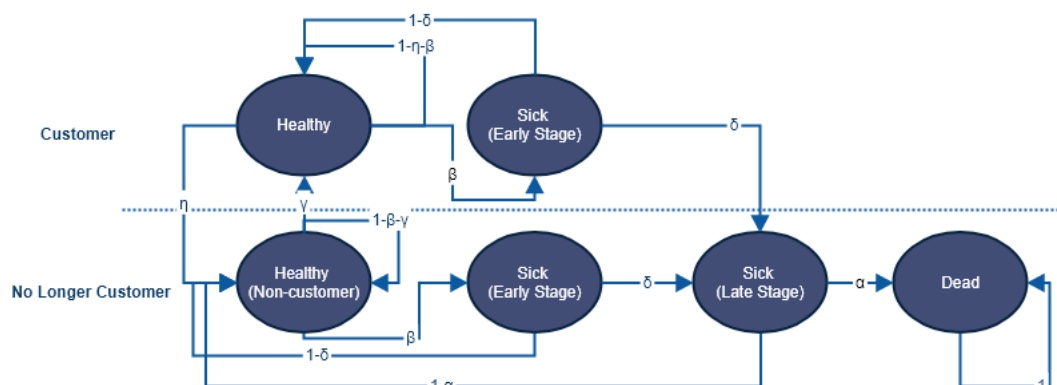


Figure 1. States in pandemic outbreak.

Table 2. Transition probabilities among the states.

	Death	Sick (Late Stage) Non-Customer	Sick (Early Stage) Non-Customer	Healthy Non-Customer	Sick (Early Stage) Customer	Healthy Customer
Death	1					
Sick (late stage) non-customer	α			$1 - \alpha$		
Sick (early stage) non-customer		δ		$1 - \delta$		
Healthy non-customer			β	$1 - \beta - \gamma$		γ
Sick (early stage) customer		δ				$1 - \delta$
Healthy customer				η	β	$1 - \beta - \eta$

In Table 2, α is the fatality rate of the patient in the late stage (in intensive care), δ is rate of change in the patient's condition from early stage to late stage, β is the spread rate of the pandemic disease,

and γ is the probability of buying a new insurance. The transition matrix of the epidemic process is thus represented as follows:

$$\bar{E} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \alpha & 0 & 0 & 1-\alpha & 0 & 0 \\ 0 & \delta & 0 & 1-\delta & 0 & 0 \\ 0 & 0 & \beta & 1-\beta-\gamma & 0 & \gamma \\ 0 & \delta & 0 & 0 & 0 & 1-\delta \\ 0 & 0 & 0 & \eta & \beta & 1-\beta-\eta \end{pmatrix}. \quad (5)$$

Note that normal deaths are not considered in Table 2 and Matrix (5). The rationale is that our goal is to determine the effects of the extraordinary situation, then to modify the classical surplus process including normal mortality by the unusual pandemic case.

Under a pandemic outbreak, in contrast to life and health insurance models, non-life insurance models are typically not exposed to dramatic changes in terms of claim severity. However, in motor insurance, a reduction in claim severity is expected due to a decrease in the density of vehicles and people on the road, which reduces the probability of multiple-vehicle collision. The main effects of pandemics in non-life insurance are a decrease in the value of assets, operational disruption, and a decrease in the number of policyholders. Hence the income and total aggregate claim amount will drop in connection with the loss rate of customers. Here, we assume the pandemic only affects the insurance company at time scale (t_s, t_e) .

Let x_i be the reserve of the insurance company at time t_i , $i = 0, 1, 2, \dots$. In the continuous space, the probability of the reserve of the insurance company at time t is modelled by Feynman path calculation and Dirac notations without the pandemic risk:

$$P(R(t) = x_n | R(0) = u) = (1 + o) \int_0^\infty \int_0^\infty \dots \int_0^\infty dx_1 dx_2 \dots dx_{n-1} \langle x_0 | A(t_1) | x_1 \rangle \langle x_1 | A(t_2 - t_1) | x_2 \rangle \dots \langle x_{n-1} | A(t - t_{n-1}) | x_n \rangle.$$

where o is error margin. Bra-ket notations $\langle x |$ and $|x\rangle = \langle x |^T$ are row and column vectors that represent reserve states in quantum modelling. Similarly, the probability for discrete reserves of insurance company at time t is given by:

$$P(R(t) = x_n | R(0) = u) = (1 + o) \sum_{x_1} \langle u | A(t_1) | x_1 \rangle \sum_{x_2} \langle x_1 | A(t_2 - t_1) | x_2 \rangle \sum_{x_3} \langle x_2 | A(t_3 - t_2) | x_3 \rangle \dots \sum_{x_{n-1}} \langle x_{n-2} | A(t_{n-1} - t_{n-2}) | x_{n-1} \rangle \langle x_{n-1} | A(t - t_{n-1}) | x_n \rangle. \quad (6)$$

where A is an operator defining $A(t_n - t_{n-1}) = e^{-(t_n - t_{n-1})H}$ with Hamiltonian operator H . The Hamiltonian operator is usually equal to minus the generator operator of the Markovian process mentioned in (4).

$$H = -Q.$$

In the previous papers; Lefèvre et al. (2018), Tamturk and Utev (2018, 2019) showed how to compute propagators according to different Hamiltonian operators and different claim distributions, and showed how to compute the expected reserve of an insurance company and its ruin probability. A propagator with a completeness equation is defined by

$$\begin{aligned} \langle x_i | e^{-\Delta t H} | x_{i+1} \rangle &= \int_0^{2\pi} \frac{dp}{2\pi} \langle x_i | e^{-\Delta t H} | p \rangle \langle p | x_{i+1} \rangle \\ &= \frac{1}{2\pi} \int_0^{2\pi} (e^{ix_i p} e^{-ix_{i+1} p}) e^{-\Delta t K_p} dp \end{aligned} \quad (7)$$

where K_p and $|p\rangle$ are the eigenvalue and eigenvector of the Hamiltonian operator, and i is a complex unit. Moreover, A can be defined via a transition matrix as $A(t_n - t_{n-1}) = A[\frac{t_n - t_{n-1}}{\Delta t}]$.

3.2. Taking the Pandemic Effects into Account at the End of the Process

The financial burden of a pandemic can be reflected in the computation either at the end of the pandemic process or during the process step by step. Now, our focus is on applying the effects of the pandemic at the end of the process. Let us give a probability amplitude without a pandemic.

$$\langle x_0 | A(t) | x_n \rangle = \langle x_0 | A(t_1) A(t_2 - t_1) \dots A(t - t_{n-1}) | x_n \rangle.$$

With pandemic risk,

$$\langle x_0 | A(t) | x_n \rangle = \langle x_0 | A(t_s) A(t_e - t_s) W(t_e - t_s) A(t - t_e) | x_n \rangle$$

where operator W is used for modifying the process by impact of the pandemic outbreak. The probability of the reserve of the insurance company at time t under the pandemic outbreak is defined by

$$P(R(t) = x_n | R(0) = u) = (1 + o) \sum_{x_1} \langle u | A | x_1 \rangle \sum_{x_2} \langle x_1 | A | x_2 \rangle \dots \sum_{x_s} \langle x_{s-1} | A | x_s \rangle \sum_{x_e} \langle x_s | A(t_e - t_s) W | x_e \rangle \sum_{x_{e+1}} \langle x_e | A | x_{e+1} \rangle \dots \sum_{x_{n-1}} \langle x_{n-2} | A | x_{n-1} \rangle \langle x_{n-1} | A | x_n \rangle. \quad (8)$$

In Equation (8), W can be defined for $x \in \mathbb{R}$ and a reserve function f by

$$f(x)W = \begin{cases} 0, & \text{if } x < \lfloor r(c - X) \rfloor \\ x - \lfloor r(c - X) \rfloor, & \text{if } x \geq \lfloor r(c - X) \rfloor \end{cases}.$$

The matrix form of W for $\Delta t = 1$ is

$$W = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & \dots & \dots & n-1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ \vdots \\ \lfloor r(c - X) \rfloor \\ \lfloor r(c - X) \rfloor + 1 \\ \lfloor r(c - X) \rfloor + 2 \\ \vdots \\ n-1 \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & \dots & 0 \\ 1 & 0 & 0 & 0 & \dots & \dots & 0 \\ 1 & 0 & 0 & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & \dots & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 & \dots & 0 \end{pmatrix} \end{matrix} \quad (9)$$

where r is the sum of the probabilities representing the rate of lost customers.

$$r = \left(\bar{E}^{\frac{t_e - t_s}{\Delta t}} \right)_{5,1} + \left(\bar{E}^{\frac{t_e - t_s}{\Delta t}} \right)_{5,2} + \left(\bar{E}^{\frac{t_e - t_s}{\Delta t}} \right)_{5,3}.$$

In motor insurance, changes in claim size throughout the pandemic should be taken into consideration, so X in matrix W in Equation (9) needs to be substituted with $X(100 - v)/100$, where v represents the drop rate of the claim size because of a lower probability of extremely large claims (such as a multiple car pileup) connected with the density of vehicles and people on the road during the pandemic. As a matter of course, v depends on precautions taken by a government like quarantine and restraints, so from the beginning of the pandemic to the end, the drop rate may change with respect to the political roadmap. To illustrate this point, new research about the impact of the lockdown on COVID-19 case deaths and rates of spread have begun to be published (see [Pachetti et al. 2020](#)).

For the dynamic t_s and t_e ,

$$P(R(t) = x_n | R(0) = u) = \sum_{i=2}^t \sum_{j=1}^i \left(A^{\frac{t_e}{\Delta t}} W A^{\frac{t-t_e}{\Delta t}} \right)_{u, x_n} P(t_e = i, t_s = j).$$

Changing the time between t_s and t_e will cause a change in W by producing a different r . Note that A and W are not commutative, so we have

$$A^{\frac{t_e}{\Delta t}} W A^{\frac{t-t_e}{\Delta t}} \neq A^{\frac{t}{\Delta t}} W.$$

In our model, the spread rate β and death rate p are taken as constant. In real epidemic models, they are unstable, so they may be changed by precautions or negative factors over time. In such a scenario, the rates would be dynamic in nature and a non-time homogeneous process would need to be considered.

In this circumstance, we need to substitute $\bar{E}(t_e - t_s) = \bar{E}^{\frac{t_e - t_s}{\Delta t}}$ with $\bar{E}(t_e - t_s) = \prod_{i=1}^{\frac{t_e - t_s}{\Delta t}} \bar{E}_i = \bar{E}_1 \bar{E}_2 \dots \bar{E}_{\frac{t_e - t_s}{\Delta t}}$ where elements of $\bar{E}_i$ consist of the variants α_i, β_i and γ_i .

$$\alpha_i = \int_{t_s + (i-1)\Delta t}^{t_s + i\Delta t} \alpha(t) dt, \quad \beta = \int_{t_s + (i-1)\Delta t}^{t_s + i\Delta t} \beta(t) dt \quad \text{and} \quad \gamma = \int_{t_s + (i-1)\Delta t}^{t_s + i\Delta t} \gamma(t) dt.$$

By means of this,

$$r = \left(\prod_{i=1}^{\frac{t_e - t_s}{\Delta t}} \bar{E}_i \right)_{5,1} + \left(\prod_{i=1}^{\frac{t_e - t_s}{\Delta t}} \bar{E}_i \right)_{5,2} + \left(\prod_{i=1}^{\frac{t_e - t_s}{\Delta t}} \bar{E}_i \right)_{5,3}.$$

The expected reserve of the insurance company in discrete reserve and time spaces is computed by

$$E[R(t)] = \sum_{x_n=0}^{n-1} P(R(t) = x_n | R(0) = u) x_n = \sum_{x_n}^{n-1} \sum_{i=2}^t \sum_{j=1}^i \left(A^{\frac{t_e}{\Delta t}} W A^{\frac{t-t_e}{\Delta t}} \right)_{u, x_n} P(t_e = i, t_s = j) x_n \quad (10)$$

where we note that W will be produced differently for each i and j .

3.3. Taking the Pandemic Effects into Account during the Process

In the previous subsection, the effects of the pandemic on insurance companies W was added to the computation at time t_e (end of the outbreak process). This may cause a large error margin for a long pandemic duration. To minimise the error in numerical computations, the effect of the pandemic must be applied throughout the entire process. Hence, the formula for the expected capital in Equation (10) needs to be modified by applying the shift operator $\frac{t_e - t_s}{\Delta t}$ times.

$$E[R(t)] = \sum_{x_n=0}^{n-1} P(R(t) = x_n | R(0) = u) x_n = \sum_{x_n}^{n-1} \sum_{i=2}^t \sum_{j=1}^i \left(A^{\frac{t_s}{\Delta t}} (\bar{A} \bar{W})^{\frac{t_e - t_s}{\Delta t}} A^{\frac{t-t_e}{\Delta t}} \right)_{u, x_n} P(t_e = i, t_s = j) x_n \quad (11)$$

where $\bar{W}$ represents average shifts due to the pandemic in unit time Δt while W is the changing amount in the entire pandemic process. By taking the shift amount as $\lfloor \frac{r(c-X)}{\frac{t_e - t_s}{\Delta t}} \rfloor$ instead of $\lfloor r(c-X) \rfloor$ in matrix W , $\bar{W}$ is obtained. The relationship between the two can be expressed by

$$W = \bar{W}^{\frac{t_e - t_s}{\Delta t}}.$$

If the expected capital is required without the use of matrices, taking $A = e^{-\Delta t H}$ as mentioned before, then

$$\langle x_i | A | x_{i+1} \rangle = \int_0^{2\pi} \frac{dp}{2\pi} \langle x_i | e^{-\Delta t H} | p \rangle \langle p | x_{i+1} \rangle = \frac{1}{2\pi} \int_0^{2\pi} (e^{ix_i p} e^{-ix_{i+1} p}) e^{-\Delta t K_p} dp \quad (12)$$

where $K_p = -cip + \lambda - \sum_{j=1}^{\infty} \lambda_j e^{-jip}$ for exponentially distributed claims from $H|p \rangle = K_p|p \rangle$ (for proof, see [Tamturk and Utev 2018](#)) in which $\lambda = \sum_{j=1}^{\infty} \lambda_j$ is the sum of the claim frequencies for $X = j$. When the effect of the pandemic is applied via shift operator $\bar{W}$,

$$\langle x_i | A \bar{W} | x_{i+1} \rangle = \int_0^{2\pi} \frac{dp}{2\pi} \langle x_i | e^{-\Delta t \bar{H}} | p \rangle \langle p | x_{i+1} \rangle = \frac{1}{2\pi} \int_0^{2\pi} (e^{ix_i p} e^{-ix_{i+1} p}) e^{-\Delta t \bar{K}_p} dp \quad (13)$$

where $\bar{H} = H_1 + H_2$ represents the sum of the Hamiltonian operators belonging to A and $\bar{W}$, respectively, and $\bar{K}_p = -\bar{r}cip + \bar{r}\lambda - \sum_{j=1}^{\infty} \bar{r}\lambda_j e^{-jip}$. Here, $\bar{r}$ is the average rate for policyholders during the pandemic outbreak,

$$\bar{r} = 1 - r.$$

If Equations (12) and (13) are substituted into Equation (8), the probability distribution of future capital, and therefore the expected capital, can easily be computed.

To minimise computational errors, a more realistic case where pandemic effects follow an incremental trend up to a point, and then begin to decrease, can be taken into account. However, more sensitive numerical methods are beyond the scope of this paper.

4. Discussion

The different behaviours of sudden and gradual catastrophe risks on non-life insurance modelling have been highlighted in this paper. In addition, we have shown that they affect the surplus process in several ways. In Table 3, we highlight the main effects of earthquake and pandemic catastrophes on household insurance.

Table 3. Summary of effects of catastrophe risks on household insurers.

	Sudden Catastrophe (Example: Earthquake)	Gradual Catastrophe (Example: Pandemic)
Overall effect:	sudden	gradual
Number of renewing or new policyholders:	possibly uptake due to awareness	slow-and-gradual drop
Average claim severity:	increase, possibly exacerbated by secondary effects such as demand surge	lower as individuals spend more time at home and can limit damage made by accidents
Claim frequency:	significant spike over a short period	lower for some types of claims such as theft, higher in others such as electrical damage due to over-use

Table 4 summarises the effects of a pandemic on motor and travel insurers in terms of overall effect, number of policyholders, average claim severity, and claim frequency.

Table 4. Summary of effects of pandemics on motor and travel insurers.

	Gradual catastrophe (Example: Pandemic) on Motor Insurance	Gradual Catastrophe (Example: Pandemic) on Travel Insurance
Overall effect:	gradual	sudden & gradual
Number of renewing or new policyholders:	gradual drop	sharp drop
Average claim severity:	lower as lock-down and less economic activity leads less exposure to large claims. Attritional claims possibly higher due to increased labour costs.	increase due to overseas travel cancellation
Claim frequency:	lower as per above	increase dramatically

Our aim is to generate more scholarly attention in relation to the effects of pandemics on non-life insurance companies by putting forward a simple approach demonstrating how to modify the normal insurance risk process by a pandemic risk. The model presented here has a number of limitations. For example, we do not include investment income on the assets held. However, it can be argued that many non-life insurers writing short-tail business tend to have short-duration assets, which in turn have low investment income, or even negative investment income in some cases.

Another limitation is that the requirements for insurers to be able to operate tend to be based on a risk measure well above the surplus being zero. For example, the Solvency II framework within the European Union requires an insurer to hold enough reserves to withstand a 1-in-200 year shock (Cortis 2019).

A vital consideration is that a comprehensive approach is needed to ensure precise and deep analysis. For example, fatality rate and spread rate are taken as the same for all policyholders in this paper, whereas in reality this should vary by the age, gender, health situation, and geography of policyholders. As such, a holistic analysis of policyholder data in interdisciplinary areas plays a significant role.

With regard to the reliability of actuarial data, it should be highlighted that actuarial analyses based on recent historical data generally will not work well during the pandemic period. The main reason for this is that almost all parameters in the predictive analytics are exposed to changes because of the lockdown, financial unrest, and varying disease and mortality rates. Therefore, instead of recent historical data, data and experiences for similar periods in the distant past, for example during the Spanish flu of 1918, Asian flu of 1957, and Hong Kong flu of 1968 outbreaks (European Actuarial Consultative Group 2006), might be needed for more precise predictions in actuarial computations. We leave this as another future subject of study.

In addition, how quickly a pandemic can be contained must be observed. To take precautions against pandemic risk, the effects of reinsurance contracts from different countries as a risk mitigation method should be analysed.

According to a survey for which most respondents worked in insurance roles, published by the British Government Actuary's Department (Government Actuary's Department 2020), increases in the expected cost and fraud rate related to the impact of COVID-19 on the overall economy are highlighted. Public liability, employer's liability, business interruption, property, and motor insurance types are listed from the greatest concern to the least. Even though the ongoing impact of the pandemic in the insurance industry is recognised, more research will be needed for the post-pandemic period.

A pandemic outbreak's impact on the financial stability of insurers depends on many factors such as medical capacity, population density, education, and awareness levels. Therefore, we highlight the necessity of interdisciplinary connections in agile working environments and call for more collaboration in this field.

5. Conclusions

In this paper we have presented a theoretical model that considers the gradual long-term effects of a catastrophe such as a pandemic. We modified the classical surplus model by applying a Markov chain with six states, which considers both customers and non-customers that may be healthy, at early and late stages of the sickness, or possibly deceased. This novel approach provides another facet for modelling the probability of ruin for an insurer when exposed to catastrophes, and different examples of the effects of different causes and insurance products were provided.

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