



Article

# Diagnosis and Prediction of IIGPS' Countries Bubble Crashes during BREXIT

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Abstract: We herein employ an alternative approach to model the financial bubbles prior to crashes and fit a log-periodic power law (LPPL) to IIGPS countries (Italy, Ireland, Greece, Portugal, and Spain) during Brexit. These countries represent the five financially troubled economies of the Eurozone that have suffered the most during the Brexit referendum. It was found that all 77 crashes across the five IIGPS nations from 19 January 2015 until 17 February 2020 strictly followed a log-periodic power law or other LPPL signature. They all had a speculative bubble phase (following the power law growth) that was then followed by a sudden crash immediately after reaching a critical point. Furthermore, their pattern coefficients were similar as well. This study would surely assist policymakers around the Eurozone to predict future crashes with the help of these parameters.

Keywords: financial bubbles; market crashes; log-periodic power law; IIGPS' countries



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### 1. Introduction

Financial crashes have weakened the economy of various countries. A financial bubble is often outlined as a positive acceleration of asset prices that does not reflect an increase in their true value. With the trend of speculation amongst market players constantly on the rise, stock markets have become circumvented. Conceiving and predicting financial bubbles is of paramount importance in the stock markets in order to stabilize the economy.

Numerous controversies are based upon understanding the concepts behind the formation of bubbles. Studies have shown that major crashes also occur due to the consistent collective approach that several investors follow, thus further exemplifying an intermittent positive movement [1]. The issue of the financial crisis in the late 2000s was a matter of great concern that prompted both researchers and experts to conduct research for which the LPPL model was implemented. In terms of effective bubble detection, the log-periodic power law model was developed [2–7]. The aim of the log-periodic power law model is to investigate whether the LPPL patterns in the development of credit default swaps (CDS) can be conducive to default classification. Hence, this entire approach facilitates a computable analysis behind bank runs. Traces of LPPL patterns were observed in CDS spread variations during global economic collapses [8]. Researchers point to the fact that major crashes in stock markets are parallel to critical points studied in logarithmic science with log-periodic correction to scaling. They analyze the presence of log-frequency shift over time in the log-periodic oscillations prior to a crash by performing tests on two of the largest crashes, viz, October 1929 and October 1987 [9].

Over the past decade, the LPPL model has been widely used for detecting bubbles and crashes in various markets [10–20]. Hence, a model was created that would enable a reduction in demand with an increasing number of obstacles, causing stock prices to fall as a result of the power law. The model was applied to the Japanese Nikkei stock index from 1990 and the gold future prices after 1980, both after their peak levels, and it was subjected

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to various parametric and nonparametric analyses [21]. One of the studies revealed the significance of the log-periodic power law in the collapse of the Mont Blanc glacier in Italy, in which case the incident was predicted in advance, enabling officials to take the appropriate precautions. At the same time, the retrospective analysis of this incident was further used to establish a potential early warning system [22]. While acknowledging the challenges of fluctuations in the housing prices of Wuhan, China, especially so in the real estate market and the economy, the LPPL strategy was applied with the implementation of a multi-population genetic algorithm.

It was observed that this LPPL model conquers the challenges related to multivariate and univariate techniques. It will assist governments in formulating better policies for the real estate market and at the same time protect the interests of purchasers [23]. Empirical studies have revealed the fact that through the reformulated version of the LPPL by Filimonov and Sornette [24], it is possible to predict the embedded risk of future enigmatic events in the Indian stock market [25]. Researchers suggested that certain parameters such as time asymmetry, robust alliance between market players, definite rationality, and a probabilistic elucidation are essential in building a model of stock price variations. In addition, previous models based on log-periodic behavior were compared prior to the crash and accordingly, it was discovered that the model that adhered to the above specifications resulted in greater authenticity [26].

In this study, we adopt the LPPL methodology to detect the positive and negative bubbles in the IIGPS countries (Italy, Ireland, Greece, Portugal, and Spain) using the daily data of five stock indices (Irish Stock Exchange, Italian FTSE MIB Index, Athens Stock Exchange, Portuguese Stock Index, and Madrid Stock Exchange). IIGPS countries are usually referred to as PIIGS or GIPSI [27–32]. Since the above acronyms are derogatory, we herein use IIGPS as a new acronym for the five financially troubled economies of the Eurozone and cover the time period from 19 January 2015 to 17 February 2020, when the Brexit referendum occurred, in order to identify LPPL traces. This study is the first of its kind that identifies the existence of bubbles in the stock markets of the IIGPS countries (Italy, Ireland, Greece, Portugal, and Spain) with the advanced bubble detection methodology of the LPPLS confidence indicator. Thus motivated by previous research on the characteristics of bubbles, we aim to examine the characteristics of the recent bubbles that occurred from 2016–2020 (Brexit impact) in the IIGPS countries (Italy, Ireland, Greece, Portugal, and Spain), which, as mentioned above, suffered the most during the Brexit referendum.

The research questions that this paper attempts to answer are the following: (a) Is there a common pattern for all the IIGPS countries during Brexit? (b) Are Italy, Ireland, Greece, Portugal, and Spain, as financially troubled economies of the Eurozone, interconnected with each other (from the perspectives of the respective stock markets during the Brexit)?

The contribution of our research to the existing literature is threefold. Firstly, this is the first attempt to search for a common thread across the financial crashes of the IIGPS countries during Brexit using Filimonov and Sornette's [24] modified LPPL. Secondly, we differentiate from previous studies and test the robustness of the LPPL following the reformulated version of the LPPL calibrations proposed by Filimonov and Sornette [24]. Thirdly, the IIGPS or PIIGS countries are relatively vulnerable to financial crashes when compared to their Western EU counterparts, especially during structural events such as Brexit. If the LPPL model fits the past crashes of these countries without any glitches, that ensures future crash predictions for those countries by regulators well in advance (especially in the advent of a possible structural event as big as Brexit). The LPPL model fitted pretty well in our study and paves the way for future crash predictions well in advance for the IIGPS countries, before any large structural events. Market stability instruments such as circuit filters can be used to control volatility (downwards) well in advance.

The rest of the paper is organized as follows: Sections 2 and 3 describe the data and present the results. Section 4 concludes the paper.

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## 2. Data and Methodology

The study analyzed the daily closing prices of five stock indices (the Irish Stock Exchange, the Italian FTSE MIB Index, the Athens Stock Exchange, the Portuguese Stock Index, and the Madrid Stock Exchange). The time period was selected from the data that was collected for the period from 19 January 2015 to 17 February 2020 in order to identify LPPL traces. Each index had 1240 observations; thus, the total numbers of observations was 6200.

The study complied with the LPPL approach proposed by Filimonov and Sornette to support the identification of LPPL traces [24]). We began with the standard Johansen-Ledoit-Sornette (JLS) log-periodic power law algorithm:

$$y_t = A + B(t_c - t)^{\beta} + C(t_c - t)^{\beta} \cos(\omega \log(t_c - t)) + \Phi$$
 (1)

where  $t_c$  denotes the most plausible time of the market crash,  $\beta$  signifies the exponential growth,  $y_t$  is the price index at time t ( $y_t > 0$ ),  $\omega$  regulates the magnitude of the oscillations, and t is any time in the bubble preceding ( $t < t_c$ ). A, B, C, and  $\Phi$  are merely units with minor details. A signifies the expected value of  $y_t$  when the end of the bubble is reached at  $t_c$  (A > 0). B denotes the fall in  $y_t$  over the time period before the crash if C is near zero (B < 0). C is the proportional magnitude of the oscillations around the exponential growth (|C| < 1).

Fitting the LPPL algorithm into financial data, the bubble behavior is apprehended by log-periodic oscillations and proceeds with the stock index at the critical time of the crash  $(t_c)$ . Rapid acceleration in asset or equity prices is the prime indication followed by periodic fluctuations at a low magnitude, when t comes nearer to critical time  $(t_c)$ . Fundamentally,  $t = t_c$  is considered to be the most plausible time of the crash. However, the JLS algorithm required a reconstruction so as to avoid technical hitches. Filimonov and Sornette [24] have since amended the classical model into an advanced one:

$$y_t = A + B(t_c - t)^{\beta} + C_1(t_c - t)^{\beta} \cos(\omega \log(t_c - t)) + C_2(t_c - t)^{\beta} \sin(\omega \log(t_c - t))$$
 (2)

where  $C_1 = CCos\emptyset$  and  $C_2 = CSin\emptyset$ .

The advanced version of the LPPL algorithm has four linear variables (A, B,  $C_1$ ,  $C_2$ ) and three non-linear variables ( $t_c$ ,  $\omega$ ,  $\beta$ ). The four linear parameters (A, B,  $C_1$ ,  $C_2$ ) are based on the "standard slaving" principle. The subordination procedure is used to propagate non-linear parameters ( $\omega$ ,  $\beta$ ). This amendment enables the examination of the bubble formation followed by the anticipation of future crashes [24].

This latest version of the LPPL is used on a daily basis with various cryptocurrencies and commodities by Sornette at the Financial Crisis Observatory, ETH Zurich (https://er.ethz.ch/financial-crisis-observatory.html accessed on 28 January 2021). The original JLS model (2001) proposed three linear variables (A, B, C) and four non-linear variables ( $t_c$ ,  $\omega$ ,  $\beta$ ,  $\emptyset$ ). This was difficult to calibrate. Moreover, it was also not robust from the error (RMSE) perspective.

In conclusion, the advantages of the methodology used are that firstly, the LPPL has no competitor. In fact, Sornette runs a financial crisis observatory. However, they started with the JLS model [5]) back in 2001, which had three linear (A, B, C) and four non-linear parameters ( $\emptyset$ ,  $t_c$ ,  $\omega$ ,  $\beta$ ), which was difficult to calibrate and was not reliable in empirical data crunching. Filimonov–Sornette [24] (the model we used) has four linear (A, B,  $C_1$ ,  $C_2$ ) and three non-linear parameters (tc,  $\omega$ ,  $\beta$ ), making it a robust calibration.

# 3. Empirical Results

The LPPL framework is subject to certain restrictions. For instance, the drawdown threshold or all the stock indices were taken as  $\geq$ 7%. Primarily, drawdown is defined as an accretive fall from one local maximum value to the next proximal minimum value. Conjointly, Python codes were operated on the Anaconda 3 platform in order to generate empirical results. It is a surprising fact that Greece alone exhibited 22 events with cogent LPPL signatures. The prominent underlying facts, which validate the low economic growth

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of Greece, include the turmoil of the Great Recession, which led to budget deficits and a higher unemployment rate.

Table 1 represents the constraints on the LPPL variables associated with its literature. Tables 2–6 represent the coefficients of the LPPL framework providing the drawdown >7% in Ireland, Italy, Greece, Portugal, and Spain, respectively. The LPPL signatures were observed on 77 occasions.

**Table 1.** Stylized facts (http://finance.martinsewell.com/stylized-facts/ accessed on 28 January 2021) of LPPL.

Parameter	Constraint	Literature
A	(>0)	Kuropka and Korzeniowski [33]
В	(<0)	Lin Ren, and Sornette [34]
$C_1$	(Cos function)	Filimonov and Sornette [24]
$C_2$	(Sine function)	Filimonov and Sornette [24]
$t_c$	( <i>t</i> to ∞)	Kuropka and Korzeniowski [33]
β	(0.1 to 0.9)	Lin et al. [34]
ω	(4.8 to 13)	Johansen [35]

Note: The table above presents the constraints on the LPPL parameters of Equation (2) used in our empirical analysis. These stylized facts were found consistent in most previous literature.

**Table 2.** Coefficients of the LPPL framework providing a drawdown of >7% in Ireland.

Critical Time	$t_c$	β	ω	$\boldsymbol{A}$	В	$C_1$	$C_2$	DD (%)	RMSE
04 August 2015	23.68	0.31	8.14	8.8	0	0	0	-11.6%	0.0049
09 September 2015	12.17	0.16	9.78	8.77	0	0	0	-7.2%	0.0086
29 December 2015	8.95	0.47	9.47	8.82	0	0	0	-10%	0.0022
01 February 2016	10.7	0.31	7	8.98	0	0	0	-11.2%	0.0034
02 September 2016	25.42	0.12	7.51	8.74	0	0	0	-7.7%	0.0043
22 June 2016	33.39	0.37	8.53	8.42	0.28	0	0	-15.7%	0.0079
30 May 2016	18.36	0.53	9.45	8.77	0	0	0	-9.2%	0.0064
05 May 2017	31.5	0.4	12.9	8.43	0.46	0	0	-8.8%	0.0051
23 January 2018	9.05	0.85	7.32	8.9	0	0	0	-11%	0.0003
21 May 2018	36.56	0.26	7.6	8.94	0	0	0	-7.8%	0.0052
28 August 2018	11.31	0.36	8.73	8.84	0	0	0	-12%	0.0012
07 November 2018	9.86	0.58	10.82	8.75	0	0	0	-14%	0.002
04 July 2019	9.45	0.56	6.71	8.83	0	0	0	-9.8%	0.0006

Note: The table above presents the estimates of all the parameters of Equation (2) in the text. DD, or drawdown, is the gap between the local minima to the next local maxima and it is  $\geq$ 7%. RMSE is the root mean square error. All estimates were generated by the authors using Python language in the Anaconda program with a project Jupyter notebook (please see Figure A1 in the Appendix A).

**Table 3.** Coefficients of the LPPL framework providing a drawdown of >7% in Italy.

Critical Time	$t_c$	β	ω	A	В	<i>C</i> <sub>1</sub>	C <sub>2</sub>	DD (%)	RMSE
26 June 2015	7.34	0.51	10.5	10.08	0	0	0	-12%	0.0029
07 August 2015	26.52	0.16	10.27	10.07	0	0	0	-13.7%	0.0072
27 November 2015	68.89	0.18	11.62	10.01	-7.39	5.64	-2.16	-30%	0.0134
17 March 2016	28.25	0.6	12.66	9.88	0	2.59	3.28	-9%	0.0102
28 April 2016	16.64	0.2	12.53	9.84	0	1.7	1.11	-8%	0.0085
07 June 2016	10.78	0.38	11.5	9.73	0.04	0.01	0	-9%	0.0113
23 June 2016	7.75	0.18	6.15	9.95	0	0	0	-16%	0.006
29 January 2018	23.1	0.52	6.74	-111.23	4.91	4.4	2.09	-8.3%	0.0037
02 May 2018	26.03	0.43	9.91	10.22	0	0	0.04	-12%	0.0045
31 July 2018	28.47	0.42	9.75	10	0	0	0	-8%	0.0061
25 September 2018	17.32	0.72	12.29	9.98	0	0	0	-14%	0.0033
03 December 2018	9.95	0.51	9.34	9.88	0	0	0	-8%	0.0053
16 April 2019	90.99	0.44	9.33	10.01	0	0	6.72	-9.6%	0.0075
15 July 2019	31.93	0.11	11.37	10.09	0	0	0	-9.7%	0.007

Note: The table above presents the estimates of all the parameters of Equation (2) in the text. DD, or drawdown, is the gap between the local minima to the next local maxima and it is  $\geq$ 7%. RMSE is the root mean square error. All estimates were generated by the authors using Python language in the Anaconda program with a project Jupyter notebook (please see Figure A2 in the Appendix A).

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<b>Table 4.</b> Coefficients of the LPPL framework	providing a drawdown of >7% in Greece.
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Critical Time	$t_c$	β	ω	A	В	C <sub>1</sub>	C <sub>2</sub>	DD (%)	RMSE
24 February 2015	17.37	0.21	8.16	7.92	0	0	0.004	-23.8%	0.028
24 March 2015	43.94	0.24	8.5	8.66	0	0.07	0	-12.3%	0.035
28 May 2015	30.1	0.88	10.86	7.82	-2.6	1.47	3.4	-18.3%	0.014
26 June 2015	8.65	0.63	11.94	7.79	-2.72	-8.8	-7.95	-21.6%	0.007
11 August 2015	16.74	0.19	6.69	7.46	0	-5.26	-2.14	-22.6%	0.023
18 September 2015	23.43	0.39	9.6	7.62	-3.62	-7.08	-9.83	-9.2%	0.014
27 Öctober 2015	16.31	0.73	11.32	7.69	0	0	0	-24.8%	0.009
31 December 2015	14.01	0.47	7.5	7.49	0	0	2.19	-22%	0.01
01 February 2016	9.48	0.32	7.93	7.34	0	0	0	-26.3%	0.005
23 May 2016	74.24	0.45	10.38	7.51	0	0.001	0.003	-25.3%	0.029
25 January 2017	58.34	0.43	6.03	7.46	-3.09	-2.1	-2.26	-7.3%	0.016
16 August 2017	11.25	0.77	6.45	7.7	0	0	0	-12.8%	0.003
01 November 2017	4.96	0.34	8.73	7.62	0	0	0.004	-9%	0
24 January 2018	45.09	0.33	7.66	6.8	1.25	0.025	0.029	-7.7%	0.008
27 April 2018	24.03	0.55	8.74	7.77	0	0	0	-11%	0.009
18 July 2018	13.51	0.59	6.05	7.63	0	0.001	0	-9%	0.004
29 August 2018	10.55	0.86	6.94	7.58	0	0	0	-8.5%	0.002
26 September 2018	9.45	0.81	6.73	7.48	0.03	0	0	-11%	0.005
13 November 2018	23.82	0.65	8.32	7.46	0	0	0	-8%	0.009
03 December 2018	9.74	0.33	12.9	7.45	0	0.002	0	-9.5%	0.002
02 May 2019	24.96	0.65	9.51	7.61	0	4.34	9.78	-8%	0.004
31 July 2019	19.03	0.74	12.47	7.74	0	-7.22	0	-11.6%	0.005

Note: The table above presents the estimates of all the parameters of Equation (2) in the text. DD, or drawdown, is the gap between the local minima to the next local maxima and it is  $\geq$ 7%. RMSE is the root mean square error. All estimates were generated by the authors using Python language in the Anaconda program with a project Jupyter notebook (please see Figure A3 in the Appendix A).

Table 5. Coefficients of the LPPL framework providing a drawdown of >7% in Portugal.

Critical Time	$t_c$	β	ω	A	В	<i>C</i> <sub>1</sub>	$C_2$	DD (%)	RMSE
10 April 2015	63.55	0.54	10.95	9.22	0	0	0.001	-16.8%	0.009
16 Jûly 2015	7.37	0.56	10.19	8.75	-2.55	2.65	-2.2	-15.4%	0.005
04 November 2015	31.05	0.5	9.14	8.6	-6.08	-4.29	-8.27	-10%	0.01
24 December 2015	8.14	0.77	8.7	8.58	0	0	0.003	-14.7%	0.006
01 February 2016	7.74	0.22	12.67	8.54	-4.21	-1.91	5.58	-12.2%	0.003
22 March 2016	28.12	0.77	7.26	8.57	0	0	0	-8.3%	0.007
02 May 2016	16.78	0.75	7.58	8.53	-1.54	-1.28	-5.83	-16.2%	0.004
11 August 2016	39.34	0.18	8.89	8.48	0	0.003	0	-7.2%	0.01
20 October 2016	25.6	0.68	11.05	8.41	0.05	0	0	-7.7%	0.006
23 January 2018	18.22	0.29	12.02	8.78	-5.34	-2.74	-4.49	-8.5%	0.002
22 May 2018	13.35	0.2	8.72	8.66	0	0	-6.76	-7.2%	0.002
26 September 2018	16.69	0.27	8.14	8.59	0	0	0	-7.3%	0.002
03 December 2018	7.36	0.35	8.26	8.46	92.02	-8.11	-31.68	-8%	0
16 July 2019	32.23	0.59	7.23	8.57	0	0	8.75	-10.3%	0.005

Note: The table above presents the estimates of all the parameters of Equation (2) in the text. DD, or drawdown, is the gap between the local minima to the next local maxima and it is  $\geq$ 7%. RMSE is the root mean square error. All estimates were generated by the authors using Python language in the Anaconda program with a project Jupyter notebook (please see Figure A4 in the Appendix A).

**Table 6.** Coefficients of the LPPL framework providing a drawdown of >7% in Spain.

Critical Time	$t_c$	β	ω	A	В	C <sub>1</sub>	C <sub>2</sub>	DD (%)	RMSE
26 June 2015	9.51	0.22	10.26	9.34	0	0	0.001	-9%	0.003
20 July 2015	11.3	0.56	12.23	9.35	0	-2.96	0	-15.5%	0.001
17 September 2015	9.89	0.12	10.9	9.26	0	0	0.011	-8%	0.005
30 November 2015	13.91	0.84	12.97	9.25	-4.3	-3.33	-2.08	-9.8%	0.004
29 December 2015	12.03	0.1	8.07	9.15	0.002	0	0	-14.3%	0.008
01 February 2016	9.19	0.57	12.08	9.09	0	-3.9	0	-11.8%	0.003
11 March 2016	12.55	0.44	11.55	9.09	-1.12	-1.16	-1.64	-8.7%	0.002
25 May 2016	12.33	0.54	12.07	9.06	0.017	0.012	0.003	-11%	0.006
23 June 2016	6.7	0.51	9.48	10.74	-3.83	-1.15	-2.52	-14%	0
23 January 2018	16.17	0.5	12.85	9.26	-3.6	-3.84	1.24	-9%	0.003
11 May 2018	33.9	0.12	7.24	9.23	0	0	0	-7.8%	0.004
21 September 2018	12.68	0.23	10.04	9.25	0	0	0	-9.5%	0.001
30 November 2018	8.21	0.41	12.34	9.11	-7.6	-2.44	-4.73	-7.8%	0.004
16 July 2019	30.07	0.46	12.74	9.14	0	0	0	-9%	0.005

Note: The table above presents the estimates of all the parameters of Equation (2) in the text. DD, or drawdown, is the gap between the local minima to the next local maxima and it is  $\geq$ 7%. RMSE is the root mean square error. All estimates were generated by the authors using Python language in the Anaconda program with a project Jupyter notebook (please see Figure A5 in the Appendix A).

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In addition, the three stylized facts that emerged through the past 77crashes were:

- (1)  $\beta = 0.52 \pm 0.38$
- (2)  $\omega = 9.29 \pm 3.39$
- (3) Drawdown (%) as 7%

These stylized facts were consistent for the entire IIGPS economic group and for the specific period of study.

It is worth mentioning here that the LPPL is a generalized method as it was conceived by Sornette in the late 1990s. It indicates a common behavior of a group of interconnected underlying time series. Here, in this study, the power law coefficient  $\beta$  value range mentioned above signifies the extent of power law growth of a speculative bubble in these stock exchanges during Brexit. Since it is on the higher side (refer to Table 1), the pattern of the speculative bubble in the IIGPS countries would be steep in nature. Angular frequency, or  $\omega$ , clocked closer to the highest range (refer to Table 1), signifying an enormous increase in volatility during the bubble build-up phase in all of the IIGPS countries' stock exchanges during Brexit. The most important of is the "drawdown," or the bubble buildup phase measurement just before the crash. Our study found that all of the IIGPS countries' stock markets under a specified time period witnessed a total of 77 crashes. The drawdown before all those crashes was the same.

Furthermore, although the LPPL looks non-stationary, it is a stationary process due to the increments of the process. Hence, persistence (though a weaker one) would manifest. During the bubble build-up phase and even during the crash phase it will exhibit persistency due to the increments of its process.

The robustness of the LPPL model was supported through root mean square errors (RMSE) in all 77 cases. The accuracy was indicated by extremely lower values, which was exhibited in all the cases.

The predicted crashes were validated by analyzing the actual events that occurred in the Eurozone countries. Here is a brief synopsis of the events that triggered a crash during different critical points. Table 7 represents the crashes that had a drawdown point exceeding –10%, termed "large crashes" (double-digit drawdowns).

<b>Table 7.</b> Linking international	events with the	double-digit	drawdown(s).

Critical Time	Drawdown %	Events
24 February 2015	-23.8%	Decline in oil prices.
•		Impending uncertainties towards Greece as the Greek government is seeking approval for reform
		proposals, resulting in skittish behavior by investors.
24 March 2015	-12.3%	Stressful sentiments arise in markets with Greece's bank closure, capital controls, and a limit on bank
		withdrawals. Concerns grow for other European economies that might suffer a similar debt crisis.
10 April 2015	-16.8%	Greece makes a EUR 450 million debt repayment to the International Monetary Fund (IMF). Christin
-		Lagarde, head of the IMF, signals the risks of low growth and high unemployment.  The United States (US) urges Greece to terminate brinkmanship with its creditors. There are chances of
28 May 2015	-18.3%	Greece being pushed out of the Eurozone. Greece has yet to make a payment of EUR 300m on 5 June.
		Greece and its lenders need to agree on reforms pertaining to the Greek pension policy, Value Added Tax
26 June 2015	-21.6%	(VAT), and corporate taxes. Turbulence in Greek banks is growing as the public rushes to withdraw money,
•		which leads to Greek bank deposits falling to their lowest level.
16 July 2015	-15.4%	Borrowing costs of the Eurozone's weaker economies, viz, Italy, Spain, and Portugal, have escalated due to fear
16 July 2015		of a Grexit from Europe. A Grexit would turn the euro into a currency peg, causing distress among investors.
20 July 2015	-15.5%	Greek banks face complete nationalization.
04 August 2015	-11.6%	In Athens, major banking stocks are tyrannized. The Greek bank index has lost 29.46%. However,
O		Ireland's manufacturing sector is steered but factory gate prices tumble.  Greeks campaign for a no vote, which leads to debt relief from creditors and keeps Greece in the Eurozone.
07 August 2015	-13.7%	Greece makes a 20-day delayed payment to the IMF.
11 August 2015	-22.6%	Greece agrees to a third bailout package.
27 October 2015	-24.8%	Decline in the producers price index (PPI), growing concerns for the health of the US economy.
04 November 2015	-10%	Decline in the producers price index (PPI), growing concerns for the health of the US economy.  The market value of the Hellenic Financial Stability Fund (HFSF) holding shares in Greek banks is worth
04 November 2013	-10/0	less than EUR 3 billion.
27 November 2015	-30%	The European Central Bank releases its M3 money Supply. Loans to the private sector in the Eurozone
		jump to 1.2% in October after a 1.1% gain in September.
24 December 2015 29 December 2015	$-14.7\% \ -14.3\%$	The euro strengthens; growing tensions in Spain follow due to the country's indecisive election results.
29 December 2015 31 December 2015	-14.3% -22%	Grexit speculations arise; high unemployment, Ukraine conflicts, incursion of migrants in the Eurozone.  European shares are dragged down by weak commodity prices.
31 December 2013	ZZ /0	European shares are dragged down by weak continiounty prices.

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Table 7. Cont.

Critical Time	Drawdown %	Events
01 February 2016	-26.3%	Rising energy prices lead to inflation, which rises to 1.8%.
		ECB to further lower the bond-buying program. The EU agrees to delay Brexit until 31 January.
02 May 2016	-16.2%	Rising concerns for a Brexit.
23 May 2016	-25.3%	Greek bonds have hit their highest levels, indicating increasing confidence of investors. Greece approves the latest austerity measures.
25 May 2016	-11%	The Eurozone finally hails a breakthrough with Greece—a green light to receive more than EUR 10 billion in bailout funds is given.
22 June 2016	-15.7%	Prior speculations lead to public outrage. Predictions are made that if Britain votes to leave the European Union, then the GBP could fall against the USD.
23 June 2016	-16%	Drop in European stocks as the UK votes to exit the EU.
16 August 2017	-12.8%	The euro is strengthened against the USD, indicating economic revival signs. This leads to further predictions that the ECB could begin to cut back the money printing program that it has been running to
	440/	restructure the economic fallout.
23 January 2018	-11%	The Irish republican terror group that maimed a Catholic police officer in Northern Ireland states a ceasefire.
27 April 2018	-11%	The National Bank of Greece declares the protraction of the sale process for 75% of its subsidiary  Ethniki Insurance.
02 May 2018	-12%	Tensions in Italy arise due to fresh elections amid political turmoil in Spain.
28 August 2018	-12%	NAFTA is formulated between the USA, Mexico, and Canada.
25 September 2018	-14%	The EU clashes as budget proposals are extended. Italian Investors experience more than a 4% loss.
26 September 2018	-11%	Interest rate hike of 25 bps, with speculations from Fed's two-day monetary policy meeting. Brent oil hits a four-year high.
07 November 2018	-14%	Investors put aside their concerns over global trade tensions. Instead, they are bothered by the interest rate hike. They eagerly await the results of the US midterm elections.
16 July 2019 31 July 2019	$-10.3\% \\ -11.6\%$	Investors are skeptical regarding the prospects of Brexit, resulting in a drop in the GBP against the USD. Fed announces reduction in key borrowing costs by 25 bps to a new target band between 2% and 2.25%.

Note: This table shows certain events that are identified behind some of the double-digit crashes in the IIGPS countries' stock markets, consisting of a clear LPPL signature.

#### 4. Conclusions

We presented a model for market bubbles or crashes, termed the "log-periodic power law," or LPPL signature, in order to describe and diagnose situations when excessive expectations of future market price increases cause prices in the IIGPS countries (Italy, Ireland, Greece, Portugal and Spain) to be temporarily elevated and vice versa. We covered the time period from 19 January 2015 to 17 February 2020, a time which the Brexit referendum occurred, in order to identify LPPL traces.

The LPPL model for pre-crash bubbles on stock markets has important consequences. Our analysis led us to the following conclusions: We found a profound LPPL signature with common pattern parameters for all the IIGPS countries across all 77 bubble-crash cases. The speculative bubble followed by a crash remained a part of IIGPS during the Brexit period (extended).

LPPL-based models are apparently non-stationary and more of mean-shifting process. However, in detailed observation they are mean-reverting due to their increments in the process. It is not a strong stationarity, though. However, this stationarity further indicates persistence. Interestingly, a milder policy shock stays for longer in cases that border non-stationarity, according to a Portuguese research group [36]). We can draw a parallel and suggest transitory policy shocks for the IIGPS countries' stock markets based on our findings, which were similar.

We believe that these findings are useful to policymakers as a vision for appropriate policymaking, for entrepreneurs in financial companies of all sizes to develop their competitive strategies, and for investors to adjust their investment strategies.

Another economic implication is that most stock markets deviate from the efficient market theory (EMH) in practice. Hence, an alternate prediction mechanism would assist policymakers to control unnecessary volatility and safeguard investor wealth.

This study contributes to the existing literature as it is the first attempt to search for a common thread across the financial crashes of the IIGPS countries during Brexit using the Filimonov–Sornette modified LPPL. In addition, we differentiated from previous studies and tested the robustness of the LPPL following the reformulated version of the LPPL calibrations proposed by Filimonov and Sornette [24].

Our findings are in line with previous studies [8,18–20].

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The IIGPS or PIIGS countries are relatively vulnerable to financial crashes compared to their Western EU counterparts, especially during structural events such as Brexit. If the LPPL model fits their past crashes without any glitches, it ensures future crash prediction for those countries by regulators well in advance (especially in the advent of a possible structural event as big as Brexit). The LPPL model fitted pretty well in our study and paves the way for future crash predictions for the IIGPS countries well in advance, before any large structural events. Market stability instruments such as circuit filters can be used to control volatility (downwards) well in advance.

The limitations of the study concerning the limitations of the research methodology (LPPL) are twofold. Firstly, it tries to generalize, which may not work sometimes, and secondly, as drawdowns are case specific and empirically found, this cannot be generalized for a very long period.

Transitory policies are required for all the IIGPS countries. The debt-led growth policy probably needs to be revised or reconsidered. Future crashes can be forecasted well in advance (for the IIGPS countries) using the specific stylized facts found during our study. Thus, volatility curbing measures could prevent further crashes and safeguard the public wealth invested in the respective stock markets.

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#### **Abbreviations**

The following abbreviations are used in this manuscript:

IIGPS Italy, Ireland, Greece, Portugal, and Spain

LPPL Log-periodic power law

BREXIT British exit

CDS Credit default swaps EMH Efficient market theory

Grexit Greek exit

IMF International Monetary Fund

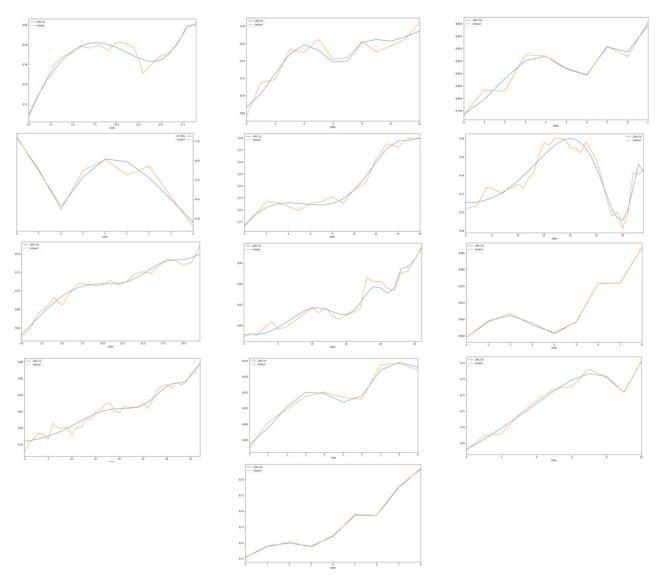
US United States
VAT Value added tax
PPI Producers price index

NAFTA North American Free Trade Agreement

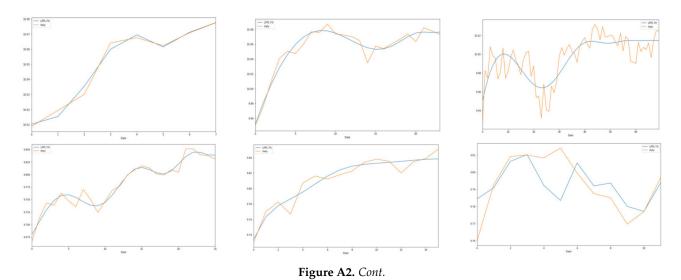
RMSE Root mean square errors
GBP British pound sterling rates
USD United States dollar
JLS Johansen-Ledoit-Sornette

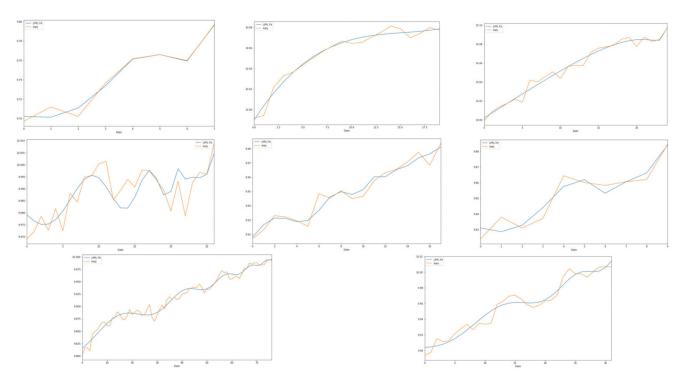
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# Appendix A. Proof of LPPL Signatures

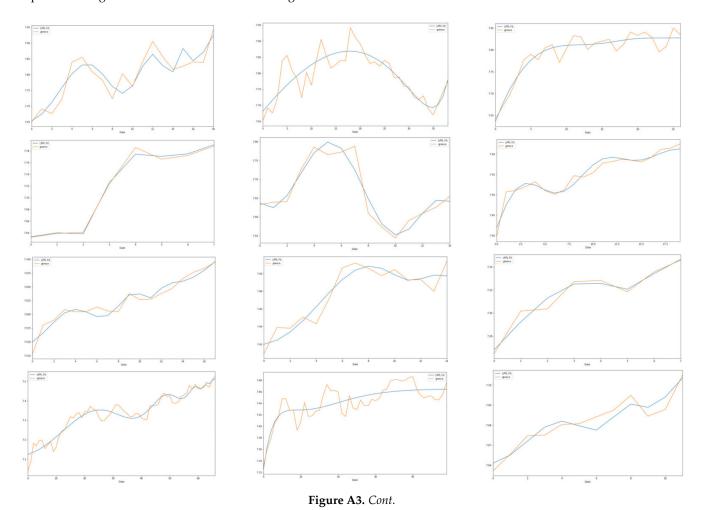


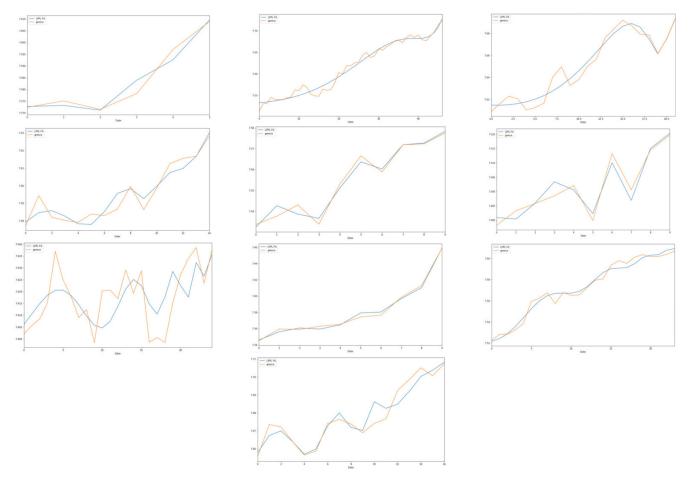
**Figure A1.** LPPL Signatures in the Irish Stock Exchange (ISEQ) 15 January 2015 to 14 January 2020. Log-periodic power law signatures in the Irish Stock Exchange.



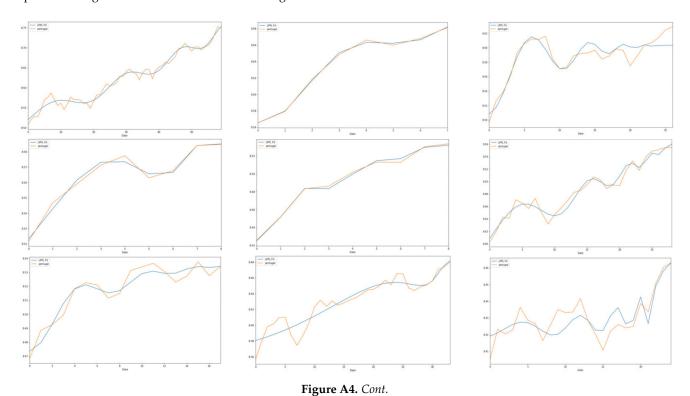


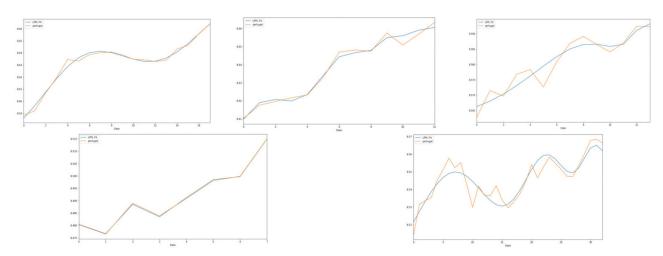
**Figure A2.** LPPL Signatures in the Italian Stock Exchange (FTSE MIB) 15 January 2015 to 30 December 2019. Log-periodic power law signatures in the Italian Stock Exchange.



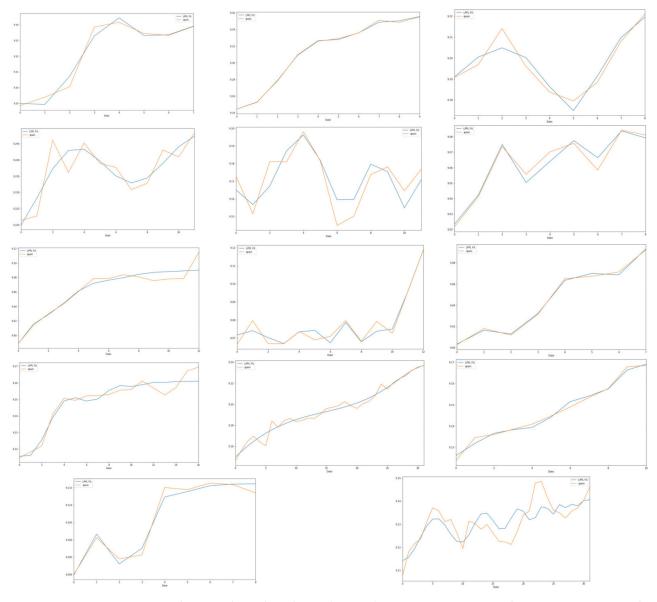


**Figure A3.** LPPL Signatures in the Greek Stock Exchange (FTSE/ATHEX) 19 January 2015 to 17 February 2020. Log-periodic power law signatures in the Greek Stock Exchange.





**Figure A4.** LPPL Signatures in the Portuguese Stock Exchange (PSI 20) 15 January 2015 to 14 January 2020. Log-periodic power law signatures in the Portuguese Stock Exchange.



**Figure A5.** LPPL Signatures in the Spanish Stock Exchange (IBEX 35) 19 January 2015 to 17 February 2020. Log-periodic power law signatures in the Spanish Stock Exchange.

#### References

1. Zhou, W.; Sornette, D. 2000–2003 real estate bubble in the UK but not in the USA. *Phys. A Stat. Mech. Its Appl.* **2003**, 329, 249–263. [CrossRef]

- 2. Johansen, A.; Sornette, D.; Ledoit, O. Predicting financial crashes using discrete scale in variance. J. Risk 1999, 1, 5–32. [CrossRef]
- 3. Drozdz, S.; Ruf, F.; Speth, J.; Wójcik, M. Imprints of log-periodic self-similarity in the stock market. *Eur. Phys. J. B* **1999**, *10*, 589–593. [CrossRef]
- 4. Sornette, D.; Johansen, A.; Bouchaud, J.P. Stock market crashes, precursors and replicas. J. Phys. 1996, 6, 167–175. [CrossRef]
- 5. Johansen, A.; Ledoit, O.; Sornette, D. Crashes as critical points, International. J. Theor. Appl. Financ. 2000, 3, 219–255. [CrossRef]
- 6. Feigenbaum, J.A.; Freund, P.G. Discrete scale invariance in stock markets before crashes Internat. *J. Int. J. Mod. Phys. B* **1996**, 10, 3737–3745. [CrossRef]
- 7. Sornette, D.; Johansen, A. Significance of log-periodic precursors to financial crashes. Quant. Financ. 2001, 1, 452–471. [CrossRef]
- 8. Wosnitza, J.H.; Denz, C. Liquidity crisis detection: An application of log-periodic power law structures to default prediction. *Phys. A Stat. Mech. Its Appl.* **2013**, 392, 3666–3681. [CrossRef]
- 9. Sornette, D.; Johansen, A. Large financial crashes. Phys. A Stat. Mech. Its Appl. 1997, 245, 411–422. [CrossRef]
- 10. Zhou, W.; Huang, Y.; Chen, J. The bubble and anti-bubble risk resistance analysis on the metal futures in China. *Phys. A Stat. Mech. Its Appl.* **2018**, 503, 947–957. [CrossRef]
- 11. Gerlach, J.C.; Demos, G.; Sornette, D. Dissection of Bitcoin's multiscale bubble history from 2012 to 2018. *R. Soc. Open Sci.* **2019**, *6*, 180643. [CrossRef]
- 12. Zhou, W.; Sornette, D. Renormalization group analysis of the 2000-2002 anti-bubble in the US S&P500 index: Explanation of the hierarchy of 5 crashes and prediction. *Phys. A Stat. Mech. Its Appl.* **2003**, 330, 584–604.
- 13. Wątorek, M.; Drozdz, S.; Oświęcimka, P. World financial 2014-2016 market bubbles: Oil negative-US dollar positive. *Acta Phys. Pol. A* 2016, 129, 932–936. [CrossRef]
- 14. Li, C. Log-periodic view on critical dates of the Chinese stock market bubbles. *Phys. A Stat. Mech. Its Appl.* **2017**, *465*, 305–311. [CrossRef]
- 15. Sornette, D.; Demos, G.; Zhang, Q.; Cauwels, P.; Filimonov, V.; Zhang, Q. Real-time prediction and post-mortem analysis of the Shanghai 2015 stock market bubble and crash. *J. Invest. Strateg.* **2015**, *4*, 77–95. [CrossRef]
- 16. Drozdz, S.; Gębarowski, R.; Minati, L.; Oświęcimka, P.; Wątorek, M. Bitcoin market route to maturity? Evidence from return fluctuations, temporal correlations and multiscaling effects. *Chaos* **2018**, *28*, 071101. [CrossRef]
- 17. Zhou, W.; Sornette, D. Analysis of the real estate market in Las Vegas: Bubble, seasonal patterns, and prediction of the CSW indices. *Phys. A Stat. Mech. Its Appl.* **2008**, *387*, 243–260. [CrossRef]
- 18. Shu, M.; Zhu, W. Detection of Chinese stock market bubbles with LPPLS confidence indicator. *Phys. A Stat. Mech. Its Appl.* **2020**, 557, 124892. [CrossRef]
- 19. Yao, C.Z.; Li, H.Y. A study on the bursting point of Bitcoin based on the BSADF and LPPLS methods. *North Am. J. Econ. Financ.* **2021**, *55*, 101280. [CrossRef]
- 20. Brée, D.; Joseph, N.L. Testing for financial crashes using the Log Periodic Power Law model. *Int. Rev. Financ. Anal.* **2013**, 30, 287–297. [CrossRef]
- 21. Sornette, D.; Johansen, A. Financial "Anti-Bubbles": Log-Periodicity in Gold and Nikkei Collapses. *Int. J. Mod. Phys.* **1999**, *10*, 563–575. [CrossRef]
- 22. Faillettaz, J.; Funk, M.; Vagliasindi, M. Time forecast of a break-off event from a hanging glacier. *Cryosphere* **2016**, *10*, 1191–1200. [CrossRef]
- 23. Dong, S.; Wang, Y.; Gu, Y.; Shao, S.; Liu, H.; Wu, S.; Li, M. Predicting the turning points of housing prices by combining the financial model with genetic algorithm. *PLoS ONE* **2020**. [CrossRef] [PubMed]
- 24. Filimonov, V.; Sornette, D. A stable and robust calibration scheme of the log-periodic power law model. *Phys. A Stat. Mech. Its Appl.* **2013**, 392, 3698–3707. [CrossRef]
- 25. Ghosh, B.; Kenourgios, D.; Francis, A.; Bhattacharyya, S. How well the log periodic power law works in an emerging stock market? *Appl. Econ. Lett.* **2020**, 1–7. [CrossRef]
- 26. Johansen, A.; Sornette, D. Modeling the stock market prior to large crashes. *Eur. Phys. J. B Condens. Matter Complex Syst.* **1999**, *9*, 167–174. [CrossRef]
- 27. Christopoulos, A.; Papathanasiou, S.; Kalantonis, P.; Chouliaras, A. An Investigation of Cointegration and Casualty Relationships between the PIIGS' Stock Markets. *Eur. Res. Stud.* **2014**, *17*, 109–123. [CrossRef]
- 28. Mobarek, A.; Muradoglu, G.; Mollah, S.; Hou, A.J. Determinants of time varying co-movements among international equity markets during crisis and non-crisis periods. *J. Financ. Stab.* **2016**, *24*, 1–11. [CrossRef]
- 29. Moagăr-Poladian, S.; Clichici, D.; Stanciu, C.V. The Comovement of Exchange Rates and Stock Markets in Central and Eastern Europe. *Sustainability* **2019**, *11*, 3985. [CrossRef]
- 30. Niţoi, M.; Pochea, M.M. Time-varying dependence in European equity markets: A contagion and investor sentiment driven analysis. *Econ. Model.* **2020**, *86*, 133–147. [CrossRef]
- 31. Koutsokostas, D.; Papathanasiou, S. Mutual funds in Greece: Case study of domestic equity mutual funds during financial crisis. *Managerial Financ.* **2017**, *43*, 812–827. [CrossRef]

Mathematics 2021, 9, 1003 14 of 14

32. Koutsokostas, D.; Papathanasiou, S.; Balios, D. Adjusting for risk factors in mutual fund performance and performance persistence. *J. Risk Financ.* **2019**, 20, 352–369. [CrossRef]

- 33. Kuropka, I.; Korzeniowski, P. Forecasting the Critical Points of Stock Markets' Indices Using Log-Periodic Power Law. *Ekonometria* **2013**, *1*, 100–110.
- 34. Lin, L.; Ren, R.E.; Sornette, D. The Volatility-confined LPPL Model: A Consistent Model of "Explosive" Financial Bubbles with Mean-reverting Residuals. *Int. Rev. Financ. Anal.* **2014**, 33, 210–225. [CrossRef]
- 35. Johansen, A. Characterization of Large Price Variations in financial Markets. *Phys. A Stat. Mech. Its Appl.* **2003**, 324, 157–166. [CrossRef]
- 36. Belbute, J.M.; Pereira, A.M. Do Global CO2 Emissions from Fuel Consumption Exhibit Long Memory? A Fractional Integration Analysis. *Appl. Econ.* **2017**, *49*, 4055–4070. [CrossRef]