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Correction

Correction: Gušić, D. Prime Geodesic Theorems for Compact Locally Symmetric Spaces of Real Rank One. *Mathematics* 2020, *8*, 1762

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The author wishes to make the following correction to the paper [1].

It was found that there was a typo in the abstract section of [1]. Namely, it should be $O\left(x^{2\rho-\frac{\rho}{n}}(\log x)^{-1}\right)$ in place of $O\left(x^{2\rho-\frac{\rho}{\eta}}(\log x)^{-1}\right)$. This change has no material impact on the conclusions of the paper.

In [1], we derived a number of results on prime geodesic theorems for compact, even-dimensional, locally symmetric Riemannian manifolds of strictly negative sectional curvature. For the sake of readers and the overall completeness of the research, we would also like to complement [1] with analogous results in odd dimensions. Thus, we shall briefly prove that the results obtained in [1] remain valid if the dimension of the underlying locally symmetric space is assumed to be odd.

Let Y be a compact, n-dimensional (n odd), locally symmetric Riemannian manifold of strictly negative sectional curvature. The rest of the notation used below will be fully adopted from [1].

The following results hold true (see, [1] (Theorems 1–3) for the counterparts in the even-dimensional case).

check for updates

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Theorem 1. *Let Y be as above. Then*

$$\psi_0(x) = \sum_{p=0}^{n-1}(-1)^{p+1}\sum_{(\tau,\lambda)\in I_p}\sum_{\substack{\alpha\in S^{\mathbb{R}}_{p,\tau,\lambda}\\ 2\rho-\frac{\rho}{n}<\alpha\leq 2\rho}}\alpha^{-1}x^{\alpha} + O\left(x^{2\rho-\frac{\rho}{n}}\right),$$

where $S^{\mathbb{R}}_{p,\tau,\lambda}$ denotes the set of real singularities of $Z_S(s+\rho-\lambda,\tau)$.

Proof. We adjust the proof of Theorem 1 in [1] (pp. 6–9).

The singularities of $Z_S(s+\rho-\lambda,\tau)$ are given by Theorem 3.15 in [2] (pp. 113–115).

Since n is odd, there are only spectral singularities, so the part of the proof related to topological singularities is missing now. The actual proof (in odd dimensions) is therefore much simpler.

On page 7 in [1], Equation (9) becomes

$$\begin{aligned}\psi_{2n}(x) = {} & \sum_{j=0}^{2n}\alpha_{2n-j}x^{2n-j}\log x + \sum_{j=0}^{2n}\beta_{2n-j}x^{2n-j} + \\ & \sum_{p=0}^{n-1}(-1)^{p+1}\sum_{(\tau,\lambda)\in I_p}\sum_{\alpha\in S^{\mathbb{R}}_{p,\tau,\lambda}}\alpha^{-1}(\alpha+1)^{-1}...(\alpha+2n)^{-1}x^{\alpha+2n} + \\ & \sum_{p=0}^{n-1}(-1)^{p+1}\sum_{(\tau,\lambda)\in I_p}\sum_{\alpha\in S^{-\rho+\lambda}_{p,\tau,\lambda}}\alpha^{-1}(\alpha+1)^{-1}...(\alpha+2n)^{-1}x^{\alpha+2n},\end{aligned} \tag{1}$$

where $0, -1, ..., -2n \notin S^{\mathbb{R}}_{p,\tau,\lambda}$, $S^{-\rho+\lambda}_{p,\tau,\lambda}$ is the set of non-real singularities of $Z_S(s+\rho-\lambda,\tau)$, and α_j, β_j, $j \in \{0, 1, ..., 2n\}$ are some explicitly computable constants.

The first inequality in [1] (p. 9) is now read as

$$\psi_0(x) \leq \sum_{p=0}^{n-1} (-1)^{p+1} \sum_{(\tau,\lambda)\in I_p} \sum_{\substack{\alpha \in S^{\mathbb{R}}_{p,\tau,\lambda} \\ 0<\alpha\leq 2\rho}} \alpha^{-1} x^{\alpha} + O\left(x^{2\rho-1} d\right) + O\left(x^{\rho} K^{n-1}\right) + O\left(d^{-2n} x^{\rho+2n} K^{-n-1}\right) + O(\log x).$$

The assertion follows by taking $d = x^{1-\frac{\rho}{n}}$, $K = x^{\frac{\rho}{n}}$. □

Theorem 2. *(Prime Geodesic Theorem) Let Y be as above. Then*

$$\pi_{\Gamma}(x) = \sum_{p=0}^{n-1} (-1)^{p+1} \sum_{(\tau,\lambda)\in I_p} \sum_{\substack{\alpha \in S^{\mathbb{R}}_{p,\tau,\lambda} \\ 2\rho-\frac{\rho}{n}<\alpha\leq 2\rho}} \mathrm{li}(x^{\alpha}) + O\left(x^{2\rho-\frac{\rho}{n}} (\log x)^{-1}\right),$$

as $x \to \infty$, where $\pi_{\Gamma}(x)$ is the function counting prime geodesics on Y of length not larger than $\log x$.

Proof. Follows immediately from Theorem 1. □

Theorem 3. *(Gallagherian Prime Geodesic Theorem) Let Y be as above and $\varepsilon > 0$. There exists a set E of finite logarithmic measure such that*

$$\pi_{\Gamma}(x) = \sum_{p=0}^{n-1} (-1)^{p+1} \sum_{(\tau,\lambda)\in I_p} \sum_{\substack{\alpha \in S^{\mathbb{R}}_{p,\tau,\lambda} \\ 2\rho-\rho\frac{4n+1}{4n^2+1}<\alpha\leq 2\rho}} \mathrm{li}(x^{\alpha}) + O\left(x^{2\rho-\rho\frac{4n+1}{4n^2+1}} (\log x)^{\frac{n-1}{4n^2+1}-1} (\log\log x)^{\frac{n-1}{4n^2+1}+\varepsilon}\right),$$

as $x \to \infty$, $x \notin E$.

Proof. As a starting point, we take the explicit formula for $\psi_{2n}(x)$ given by Equation (1).

Bearing in mind the fact that topological singularities are missing, and proceeding in the same way as in [1] (pp. 10–12), we conclude that for $x \notin E$ (Cf. [1] (p. 12, relation (21)))

$$\psi_0(x) \leq \sum_{p=0}^{n-1} (-1)^{p+1} \sum_{(\tau,\lambda)\in I_p} \sum_{\substack{\alpha \in S^{\mathbb{R}}_{p,\tau,\lambda} \\ 0<\alpha\leq 2\rho}} \alpha^{-1} x^{\alpha} + O\left(x^{2\rho-1} d\right) + O\left(x^{\rho} Y^{n-1}\right) + O\left(\frac{x^{\alpha} (\log x)^{\beta} (\log\log x)^{\beta+\varepsilon}}{d^{2n}}\right) + O\left(d^{-2n} \frac{x^{\rho+2n}}{W^{n+1}}\right) + O(\log x). \tag{2}$$

The assertion of the theorem now follows by putting $d = x^{\frac{\alpha-2\rho+1}{2n+1}} (\log x)^{\frac{\beta}{2n+1}} (\log\log x)^{\frac{\beta}{2n+1}}$, $Y \sim x^{\frac{2\rho+4n-2\alpha}{2n+3}} (\log x)^{\frac{1-2\beta}{2n+3}} (\log\log x)^{\frac{1-2\beta}{2n+3}}$ into (2), $\alpha = \frac{8n^3+2n+\rho-6n\rho}{4n^2+1}$, $\beta = \frac{2n^2-n-1}{4n^2+1}$, and finalizing the argument in a standard way. □

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