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Mixed Convection Flow of Powell–Eyring Nanofluid near a Stagnation Point along a Vertical Stretching Sheet

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Abstract: A stagnation-point flow of a Powell–Eyring nanofluid along a vertical stretching surface is examined. The buoyancy force effect due to mixed convection is taken into consideration along with the Brownian motion and thermophoresis effect. The flow is investigated under active and passive controls of nanoparticles at the surface. The associating partial differential equations are converted into a set of nonlinear, ordinary differential equations using similarity conversions. Then, the equations are reduced to first-order differential equations before further being solved using the shooting method and *bvp4c* function in MATLAB. All results are presented in graphical and tabular forms. The buoyancy parameter causes the skin friction coefficient to increase in opposing flows but to decrease in assisting flows. In the absence of buoyancy force, there is no difference in the magnitude of the skin friction coefficient between active and passive controls of the nanoparticles. Stagnation has a bigger influence under passive control in enhancing the heat transfer rate as compared to when the fluid is under active control. Assisting flows have better heat and mass transfer rates with a lower magnitude of skin friction coefficient as compared to opposing flows. In this case, the nanofluid parameters, the Brownian motion, and thermophoresis altogether reduce the overall heat transfer rates of the non-Newtonian nanofluid.

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1. Introduction

Mixed convection flow occurs when effects from forced convection and natural convection mechanisms contribute significantly to heat transfer. According to Chen et al. [1], the buoyancy force that arises from the temperature difference will instigate a longitudinal pressure gradient. This phenomena will affect the flow area and the heat transfer rate from the surface. A stagnation flow in pure forced convection usually represents the rather symmetric flow in the vicinity of a stagnation point line. However, a mixed convection stagnation flow will no longer be symmetrical towards the stagnation line [2]. Earlier on, Merkin [3] discussed dual-solution phenomena that occur in mixed convection of a porous medium. By considering a simple time-dependent problem, they showed that the upper branch of solutions is stable whilst the lower branch is unstable. Ishak et al. [4] examined the mixed convection flow of an incompressible viscous fluid towards a stretching vertical surface near a stagnation-point. Their study shows that a dual solution exists for the opposing flow whilst a unique solution is available for the assisting flow. Later, they investigated the effects that stagnation mixed convection flow had when a constant magnetic field is applied normal to the vertical plate [5]. With applied suction and injection on the surface, their results show that the dual solution actually exists for both assisting and opposing flows. Suction as well as electromagnetic field increase the range of buoyancy for which the solution may exist. Bognar [6] decided to present a theoretical analysis on similarity

solutions for an opposing flow in a power-law fluid over a flat surface. They gave an analytic estimation for the critical value that depends on the power-law exponent and proved the existence of the analytic solution. Hayat et al. [7] applied the homotopy analysis method (HAM) in their study on the influence of thermal radiation on the MHD stagnation point flow with mixed convection. Both thermal radiation and magnetic parameter are found to improve the heat transfer rate of the fluid in assisting flows as well as in opposing flows. Aman et al. [8] considered a boundary slip in the mixed convection stagnation-point flow on a vertical surface. In their study, velocity slip helps to improve the heat transfer rate while the thermal slip worsens it. Abbas et al. [9] discussed the stagnation point flow of a Maxwell fluid on a stretching vertical surface with mixed convection. They found that increasing the fluid relaxation time represented by the Deborah number will increase the heat transfer rate in opposing flow but will decrease it in assisting flow. Hayat et al. [10] studied the stagnation-point flow of Casson fluid with mixed convection under convective boundary conditions. Their result shows that the Biot number has a qualitatively similar effect towards velocity and temperature profiles. Tian et al. [11] carried out a numerical inspection on MHD convective stagnation point flow of a non-Newtonian nanofluid over a stretching surface using collocation spectral method (CSM). They found that heat transfer in the boundary layer is heavily influenced by concentration of the saturated nanoparticles.

Heat transfer has wide applications in numerous devices and systems in automotive, materials processing, power station engineering, and thermal management, to name a few. The heat transfer fluid plays a major role in the performance of these devices and systems. Since Choi [12] introduced nanofluid to the world 20 years ago; it has gained lots of interest among researchers as a new heat transfer fluid due to its ability to improve a fluid's thermal conductivity. Nowadays, there is a lot of literature on the mixed convection stagnation point flow of nanofluids. Some of them include work from Makinde et al. [13] that studied the influences buoyancy force and magnetic field had on stagnation point flow towards a convectively heated surface. Here, the buoyancy force helps to improve the heat transfer rate of the fluid. Pal and Mandal [14] researched the effects of thermal radiation, heat generation, and viscous dissipation past a mixed convection flow over a stretching/shrinking surface that carries a nanofluid. Hsiao [15] investigated the stagnation point flow of a nanofluid with electrical magnetohydrodynamic (EMHD) and slip boundary effects on a stretching surface. Abbasi et al. [16] analyzed the mixed convection flow of a Jeffrey nanofluid while taking into consideration the effects of thermal radiation and double stratifications. Their result shows that thermal radiation increases the fluid temperature, but thermal stratification lowers the temperature. Othman et al. [17] did a study that covers a stagnation point of a mixed convection flow saturated with a controlled nanoparticle concentration over an impermeable, vertically stretched/shrunked surface. Results show that increasing values of the mixed convection parameter will raise the values of the skin friction coefficient as well as both heat and mass transfer rates near the wall surface. Zaib et al. [18] did an analysis focusing on the dual-nature solution for the entropy generation along a Riga surface. In their study, the dual solutions are obtained only in opposing flow. Alizadeh et al. [19] examined the impact of radiation and electromagnetic field in a mixed convection stagnation-point flow surrounding a cylinder. It is concluded that the radiative system produces a higher Nusselt number and less thermal irreversibilities. Additionally, changes in Biot number allow the modification of the predicted value of Nusselt numbers. Jamaludin et al. [20] investigated another mixed convection problem near a stagnation-point flow over a vertically stretched/shrunked plate with suction, thermal radiation, and heat source/sink. Their results confirmed that dual solutions appear within a particular range of the mixed convection parameters whereby the boundary layer separation takes place in the opposing flow region. They have also performed an analysis to verify the stability of the upper and lower branches' solutions. Recently, solar radiation effect with internal energy of thermal diffusion and thermal conductivity in MHD stagnation-point flow of Maxwell fluid were considered by Khan et al. [21] who concluded that radiation increases the rate of heat transfer.

Among the abundance of non-Newtonian fluids available, the Powell–Eyring fluid holds some advantages over the others, especially in chemical engineering. This particular fluid model is derived from the kinetic theory of liquids. Furthermore, the model can properly exhibit Newtonian behavior at low and high shear rates. Hayat et al. [22] did a study on MHD nonlinear stretching flow of a Powell–Eyring nanofluid where a zero nanoparticle mass flux condition was applied at the sheet. Their results show that a higher magnetic parameter value lowers the rate of the heat transfer at the surface, but it increases the skin friction coefficient. It was also found that Brownian motion and thermophoresis parameters reacted differently on the concentration field. Meanwhile, Hussain et al. [23] considered the radiative peristaltic flow of Powell–Eyring nanofluid in an asymmetric channel for their study. They found that the nonlinear radiation in the flow field increases the pressure gradient magnitude and reduces fluid temperature. Ibrahim and Godisa [24] presented finite element solutions of a boundary-layer flow of Powell–Eyring nanofluid over a nonlinear stretching sheet in the presence of heat generation/absorption and chemical reaction. Ogunsye et al. [25] used the nanofluid viscosity and thermal conductivity models derived from experimental data to investigate the flow and heat transfer of a Powell–Eyring nanofluid. The computational results show that heat transfer rate increases with increment of temperature ratio parameter and thermal radiation. Khan et al. [26] explored the flow of MHD mixed convection of Eyring–Powell nanofluid over a tilted plane with an exponentially varying viscosity. Ogunseye and Sibanda [27] studied the entropy generation in a Powell–Eyring Al_2O_3 –water nanofluid flow passing through a vertical channel subjected to a convective cooling. The result shows that the concentration of nanoparticles as well as the Brinkman number play important roles in reducing the entropy generation rate in that channel. In recent works, Alsaedi et al. [28] studied the influence of Bejan number and entropy rate on Eyring–Powell nanofluid flow over a stretchable surface, while Waqas et al. [29] presented a numerical analysis of a nonlinear radiating Eyring–Powell nanofluid flow while taking into consideration the effects of magnetic dipole energy as well as activation energy. Aziz et al. [30] did an entropy analysis for a Powell–Eyring hybrid nanofluid flow in which the model includes the effects of linear thermal radiation and viscous dissipation. One of their main results uncovered the superiority of the hybrid Powell–Eyring nanofluid as a thermal conductor as compared to the conventional nanofluids. It is also shown that the highest heat flux is produced using the smallest shape factor parameter.

In this paper, the analysis will be focused more on the comparative study of the Powell–Eyring nanofluid flow under active and passive controls of nanoparticles. The influence of other parameters towards the flow are also looked at in detail. The governing equations will be simplified using similarity transformation before being solved using a shooting technique and *bvp4c* function in MATLAB. All results are presented in graphical and tabular forms.

2. Problem Formulation

Take into account a steady two-dimensional mixed convection stagnation point flow of an incompressible Powell–Eyring nanofluid towards a vertical stretching surface. The Cartesian coordinate system is chosen in such a way that the x -axis is along the vertical linearly stretching surface at $y = 0$, and the fluid occupies the area $y > 0$ (Figure 1). Near the surface, velocity is assumed to be $u_w(x) = cx$ with temperature $T = T_w(x)$. Depending on the fluid environment setting, the fluid concentration is assumed to be constant $C = C_w$ under active control surroundings, but it will depend on the temperature gradient when put under passive control of nanoparticles. Velocity of the free stream flow is defined as $u_e(x) = ax$ with ambient temperature and concentration of T_∞ and C_∞ , respectively.

Figure 1. Geometry of the flow in Cartesian coordinates.

2.1. The Continuity and Momentum Equations

The governing equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \left(\nu + \frac{1}{\rho \beta c_*} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1}{2\rho \beta c_*^3} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + g[\beta_T(T - T_\infty) + \beta_C(C - C_\infty)] \quad (2)$$

where u and v are the velocity components along x - and y -axes, respectively. The term $u_e \frac{du_e}{dx}$ represents the stagnation flow as it signifies the fluid flow normally (at 90 degrees) towards the vertical plate surface at free stream velocity [15]. In the momentum equation for Powell–Eyring fluid [31], $\nu = \mu/\rho$ is the kinematic viscosity, ρ is the fluid density, and β and c_* are the fluid parameters. The mixed convection flow produces some buoyancy forces that change the flow field and heat and mass transfer rates from the surface. This is described in the last term of the momentum equation at which g is the acceleration due to gravity, β_T is the thermal expansion coefficient, β_C is the nanoparticle volumetric coefficient, and T and C are the fluid temperature and concentration. The boundary conditions are

$$\begin{aligned} u &= u_w(x) = cx, \quad v = 0 & \text{at } y = 0 \\ u &= u_e(x, t) = ax & \text{as } y \rightarrow \infty \end{aligned} \quad (3)$$

Here, a and c are positive constants. Similarity transformations are used by applying the following non-dimensional variables:

$$\begin{aligned}
\eta &= \sqrt{\frac{c}{v}}y, & \psi &= \sqrt{cv}xf(\eta), & \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \\
\phi(\eta) &= \frac{C - C_\infty}{C_\infty}, & & \text{for passive control of } \phi, \\
\phi(\eta) &= \frac{C - C_\infty}{C_w - C_\infty}, & & \text{for active control of } \phi.
\end{aligned} \tag{4}$$

where η is the similarity variable, and ψ is the stream function that satisfies (1) and

$$u = \frac{\partial \psi}{\partial y} = cf'(\eta) \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} = -\sqrt{cv}f(\eta). \tag{5}$$

By substituting (4) and (5) into the governing Equations (1) and (2), we have

$$(1 + \epsilon)f''' - (f')^2 + ff''' + r^2 - \epsilon\delta(f'')^2f''' + \gamma[\theta + N\phi] \tag{6}$$

where primes denote the differentiation with respect to η . Here, $r = a/c$ is the stagnation parameter, $\epsilon = \frac{1}{\mu\beta c_*}$ and $\delta = \frac{c^3x^2}{2c_*^2v}$ are the fluid parameters, $\gamma = \frac{g\beta_T(T-T_\infty)x^3/v^2}{(u_w(x)x/v)^2}$ is the buoyancy parameter, and $N = \frac{\beta_C(C-C_\infty)}{\beta_T(T-T_\infty)}$ is the buoyancy force ratio. The boundary condition (3) then becomes

$$\begin{aligned}
f(0) &= 0, & f'(0) &= 1, & \text{at } \eta &= 0 \\
f'(\infty) &= r, & & \text{as } \eta \rightarrow \infty
\end{aligned} \tag{7}$$

2.2. The Energy and Mass Transport Equations

The nanofluid energy and mass transport equations according to [32] are given by

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2} + \tau D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \tau \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \tag{8}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}. \tag{9}$$

where T and C are the fluid temperature and concentration, κ is the thermal diffusivity, τ is the effective heat capacity ratio of the nanoparticle and the fluid, D_B is the Brownian diffusion coefficient, and D_T is the thermophoretic diffusion coefficient. The appropriate boundary conditions are given as

$$\begin{aligned}
\text{At } y = 0 : \\
&T = T_w(x), \\
&\begin{cases} D_B \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial y} = 0, & \text{for passive control of } \phi \\ C = C_w(x), & \text{for active control of } \phi \end{cases} \\
\text{As } y \rightarrow \infty : \\
&T = T_\infty, \quad C = C_\infty
\end{aligned} \tag{10}$$

In a steady boundary layer flow, the temperature near the wall will be equal to the wall temperature, T_w , and as the flow moves further away from the wall, it will be equal to the ambient temperature, T_∞ . Under passive control of nanoparticles, it is assumed that the nanoparticle volume fraction is controlled passively on the surface via a temperature gradient. This is contrary to when the fluid is under active control of nanoparticles, where it is assumed to be constant at the surface. However, the nanoparticle volume concentration

will reach a uniform value C_∞ under both conditions as the flow moves further away. Applying the similarity transformation, the resulting equations will be

$$\theta'' + \text{Pr} [f\theta' + Nb\theta'\phi' + Nt(\theta')^2] = 0, \quad (11)$$

$$\phi'' + \text{LePr}f\phi' + \frac{Nt}{Nb}\theta'' = 0, \quad (12)$$

with $\text{Pr} = \nu/\kappa$ as the Prandtl number, Nb is the Brownian motion parameter where $Nb = \frac{\tau D_B C_\infty}{\nu}$ (for passive control of ϕ) and $Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu}$ (for active control of ϕ), $Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}$ is the thermophoresis parameter, and $\text{Le} = \kappa/D_B$ is the Lewis number. The boundary conditions (10) will then become

At $\eta = 0$:

$$\begin{aligned} \theta(0) &= 1, \\ \begin{cases} Nb\phi'(0) + Nt\theta'(0) = 0, \\ \phi(0) = 1, \end{cases} &\quad \begin{array}{l} \text{for passive control of } \phi, \\ \text{for active control of } \phi, \end{array} \end{aligned} \quad (13)$$

As $\eta \rightarrow \infty$:

$$\theta(\infty) = 0, \quad \phi(\infty) = 0,$$

2.3. Skin Friction Coefficient, Heat, and Mass Transfer Rate

The physical quantities of practical interest are the local skin friction coefficient Cf_x , the heat transfer rate or the local Nusselt number Nu_x , and the mass transfer rate also known as local Sherwood number Sh_x , which are defined as

$$Cf_x = \frac{\tau_w}{\rho u_w^2(x)}, \quad Nu_x = \frac{q_w x}{\alpha(T_w - T_\infty)}, \quad Sh_x = -\frac{q_m x}{D_B(C_w - C_\infty)}. \quad (14)$$

(for active control of ϕ)

Here, the surface shear stress τ_w [26], surface heat flux q_w , and the surface mass flux q_m are given as follows

$$\begin{aligned} \tau_w &= \left[\left(\nu + \frac{1}{\rho\beta c_*} \right) \frac{\partial u}{\partial y} - \frac{1}{6\beta c_*^3} \left(\frac{\partial u}{\partial y} \right)^3 \right]_{y=0}, \\ q_w &= -\alpha \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad q_m = -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}. \end{aligned} \quad (15)$$

Equation (14) can then be simplified into the dimensionless form below:

$$Cf_x Re_x^{1/2} = (1 + \epsilon) f''(0) - \frac{\epsilon \delta}{3} (f''(0))^3, \quad (16)$$

$$Nu_x Re_x^{-1/2} = -\theta'(0), \quad (17)$$

$$Sh_x Re_x^{-1/2} = -\phi'(0). \quad \text{(for active control of } \phi) \quad (18)$$

where $Re_x = U_w x / \nu$ stands for local Reynolds number. The local Sherwood number for passive control will follow the local Nusselt number accordingly with the boundary condition (10).

$$Sh_x Re_x^{-1/2} = \frac{Nt}{Nb} \theta'(0), \quad (19)$$

3. Method of Solution

The *bvp4c* function in MATLAB was utilized in solving the problem. First, the governing nonlinear ordinary differential equations (ODEs) (Equations (6), (11) and (12))

with the boundary conditions (Equations (7) and (13)) are rewritten as a system of seven first-order ODEs:

$$\begin{aligned}
 \chi_1' &= \chi_2, \\
 \chi_2' &= \chi_3, \\
 \chi_3' &= \frac{(\chi_2)^2 - \chi_1\chi_3 - r^2 - \gamma[x_4 + N\chi_6]}{1 + \epsilon - \epsilon\delta(\chi_3)^2}, \\
 \chi_4' &= \chi_5, \\
 \chi_5' &= -\text{Pr}(\chi_1\chi_5 + Nb\chi_5\chi_7 + Nt(\chi_5)^2), \\
 \chi_6' &= \chi_7, \\
 \chi_7' &= -Le\text{Pr}\chi_1\chi_7 - \frac{Nt}{Nb}\chi_5',
 \end{aligned} \tag{20}$$

with the boundary conditions

$$\begin{aligned}
 \chi_1(0) &= 0, & \chi_2(0) &= 1, & \chi_4(0) &= 1 \\
 \begin{cases} Nb\chi_7(0) + Nt\chi_5(0) = 0, \\ \chi_6(0) = 1, \end{cases} & & & & \begin{cases} \text{for passive control of } \phi. \\ \text{for active control of } \phi \end{cases} \\
 \chi_2(\infty) &= r, & \chi_4(\infty) &= 0, & \chi_6(\infty) &= 0
 \end{aligned} \tag{21}$$

where

$$\begin{aligned}
 \chi_1 &= f, & \chi_2 &= \chi_1' = f', & \chi_3 &= \chi_2' = f'', \\
 \chi_4 &= \theta, & \chi_5 &= \chi_4' = \theta', & & \\
 \chi_6 &= \phi, & \chi_7 &= \chi_6' = \phi'.
 \end{aligned} \tag{22}$$

For calculation purposes, $\eta \rightarrow \infty$ in this problem is set to $\eta_\infty = 10$, and the numerical solutions are evaluated at 100 equally spaced points with suitable initial guesses. The relative tolerance is set to 1×10^{-12} . To ensure the accuracy of our calculation and method of choice, results are compared with published results of others. Table 1 lists the compared values of $-\theta'(0)$ for assorted values of the thermophoresis parameter Nt . From the table, it is evident that the results are in good agreement with each other, thus validating the accuracy of the current results.

Table 1. Comparison table for values of Nusselt number, Nu for variation of thermophoresis parameter Nt , when $r = \epsilon = \delta = \gamma = N = 0$, $Pr = 3.97$, $Nb = 0.1$, and $Le = 10/Pr$.

Nt	Ishfaq et al. [33]	Current Study
	<i>RK4</i>	<i>bvp4c</i>
0.1	1.2764	1.27637
0.2	1.1836	1.18362
0.3	1.0957	1.09571
0.4	1.0131	1.01309
0.5	0.9361	0.93607

4. Discussion

This section discusses the impact that parameters $r, \epsilon, \delta, \gamma, N, Nt, Nb, Le$, and Pr have on the fluid flow characteristics, specifically the velocity, temperature, and concentration. Values of a particular parameter are varied, and the results are presented in graphical form in Figures 2–10. The physical properties of interest, which are the skin friction coefficient $f''(0)$, the heat transfer rate $-\theta'(0)$ or Nusselt number Nu , and the mass transfer rate

$-\phi'(0)$ or Sherwood number Sh , are calculated numerically and recorded in tabular form (Tables 2 and 3). The parameter values used throughout this section are $r = 0.2, \epsilon = 0.3, \delta = 0.1, \gamma = 1, N = Nt = Nb = 0.5, Le = 1$, and $Pr = 5$, unless stated otherwise.

Table 2. Values of $f''(0)$, $-\theta'(0)$, and $-\phi'(0)$ for parameters $r, \epsilon, \delta, \gamma$ and N under active and passive controls.

r	ϵ	δ	γ	N	$f''(0)$		$-\theta'(0)$		$-\phi'(0)$	
					Active	Passive	Active	Passive	Active	Passive
0.2	0.3	0.1	1	0.5	−0.3071	−0.5445	0.1620	1.2594	1.9210	−1.2594
				0.5	−0.1121	−0.3418	0.1628	1.3079	1.9985	−1.3079
				1.0	0.4303	0.2151	0.1673	1.4104	2.1538	−1.4104
				1.2	0.7128	0.5023	0.1698	1.4540	2.2185	−1.4540
0.2	0				−0.3009	−0.5904	0.1623	1.2478	1.9108	−1.2478
	0.1				−0.3041	−0.5736	0.1622	1.2520	1.9144	−1.2520
	0.3				−0.3071	−0.5445	0.1620	1.2594	1.9210	−1.2594
	0.5				−0.3074	−0.5198	0.1618	1.2656	1.9269	−1.2656
	0.7				−0.3062	−0.4985	0.1617	1.2710	1.9321	−1.2710
	0.3	0			−0.3073	−0.5438	0.1620	1.2595	1.9211	−1.2595
		0.1			−0.3071	−0.5445	0.1620	1.2594	1.9210	−1.2594
		0.3			−0.3068	−0.5457	0.1620	1.2591	1.9209	−1.2591
		0.5			−0.3065	−0.5469	0.1620	1.2589	1.9208	−1.2589
		0.7			−0.3063	−0.5482	0.1620	1.2586	1.9207	−1.2586
		0.1	−1		−1.4386	−1.1039	0.1479	1.1653	1.5966	−1.1653
			−0.5		−1.0936	−0.9507	0.1524	1.1939	1.7275	−1.1939
			0		−0.8081	−0.8081	0.1561	1.2184	1.8077	−1.2184
			0.5		−0.5493	−0.6733	0.1592	1.2399	1.8695	−1.2399
			1		−0.3071	−0.5445	0.1620	1.2594	1.9210	−1.2594
			1	0.0	−0.4274	−0.5383	0.1608	1.2560	1.8975	−1.2560
				0.5	−0.3071	−0.5445	0.1620	1.2594	1.9210	−1.2594
				1	−0.1899	−0.5513	0.1631	1.2625	1.9429	−1.2625
				2	0.0371	−0.5666	0.1652	1.2684	1.9829	−1.2684

Table 3. Values of $f''(0)$, $-\theta'(0)$, and $-\phi'(0)$ for parameters Nt, Nb, Le , and Pr under active and passive controls.

Nt	Nb	Le	Pr	$f''(0)$		$-\theta'(0)$		$-\phi'(0)$	
				Active	Passive	Active	Passive	Active	Passive
0.1	0.5	1	5	−0.3700	−0.5772	0.3120	1.5658	1.7157	−0.3132
0.3				−0.3364	−0.5615	0.2212	1.4090	1.8154	−0.8454
0.5				−0.3071	−0.5445	0.1620	1.2594	1.9210	−1.2594
0.7				−0.2815	−0.5263	0.1225	1.1202	2.0193	−1.5683
0.5	0.1			−0.2593	−0.5750	0.5876	1.2712	1.4680	−6.3558
	0.3			−0.3156	−0.5489	0.3172	1.2615	1.9149	−2.1025
	0.5			−0.3071	−0.5445	0.1620	1.2594	1.9210	−1.2594
	0.7			−0.2931	−0.5426	0.0785	1.2584	1.8860	−0.8989
	0.5	1		−0.3071	−0.5445	0.1620	1.2594	1.9210	−1.2594
		2		−0.3449	−0.5255	0.1135	1.1013	2.7166	−1.1013
		3		−0.3653	−0.5162	0.0963	1.0159	3.3040	−1.0159
		4		−0.3788	−0.5107	0.0873	0.9614	3.7925	−0.9614
		1	1	−0.1451	−0.4212	0.4777	0.6640	0.5283	−0.6640
			3	−0.2693	−0.5142	0.3335	1.0731	1.3429	−1.0731
			5	−0.3071	−0.5445	0.1620	1.2594	1.9210	−1.2594
			7	−0.3258	−0.5567	0.0697	1.3396	2.3250	−1.3396

4.1. Dimensionless Velocity, Temperature, and Concentration Fields

Figures 2 and 3 display the fluid profiles when stagnation parameter r and fluid parameter ϵ are varied. Both parameters seem to bring out the same outcome from the fluid flow. The fluid velocity increases when r and ϵ increase. The fluid velocity under active control is higher than the fluid velocity under passive control, but both eventually converge to a uniform velocity as the fluid moves away from the surface. As the velocity increases, both fluid temperature and concentration decrease. Heat is being dispersed quicker to the surroundings, and particles are being spread out farther with the fluid flow. In Figure 4, the effect of another fluid parameter δ is shown. Compared to fluid parameter ϵ , fluid parameter δ does not seem to make significant impact towards the fluid profiles. Minimal changes can be seen in velocity where the velocity seems to be decreasing with increasing values of δ , while changes in temperature and concentration are not visible.

$$\epsilon = 0.3, \delta = 0.1, \gamma = \text{Le} = 1, N = \text{Nb} = \text{Nt} = 0.5, \text{Pr} = 5$$

Figure 2. Impact of stagnation parameter r on (a) velocity, (b) temperature, and (c) nanoparticle volume fraction profiles.

$$r = 0.2, \delta = 0.1, \gamma = \text{Le} = 1, N = \text{Nb} = \text{Nt} = 0.5, \text{Pr} = 5$$

Figure 3. Impact of fluid parameter ϵ on (a) velocity, (b) temperature, and (c) nanoparticle volume fraction profiles.

Figure 4. Impact of fluid parameter δ on (a) velocity, (b) temperature, and (c) nanoparticle volume fraction profiles.

Influences of the mixed convection parameter γ and buoyancy ratio parameter N are presented in Figures 5 and 6. The outcome is similar to those of varied r and ϵ . Fluid velocity increases, while temperature and concentration decrease when the value of parameters γ and N increase. Here, $\gamma > 0$ represents assisting flow, $\gamma < 0$ represents opposing flow, and $\gamma = 0$ represents the flow when no buoyancy force is present. In assisting flow, fluid velocity is higher in active control than velocity in passive control. The opposite is true for the opposing flow. Relatively, the mixed convection parameter or the buoyancy parameter γ leaves a bigger impact towards the flow profile than N .

Figure 5. Impact of mixed convection parameter γ on (a) velocity, (b) temperature, and (c) nanoparticle volume fraction profiles.

Figure 6. Impact of buoyancy ratio parameter N on (a) velocity, (b) temperature, and (c) nanoparticle volume fraction profiles.

The effects of nanoparticles can be seen through thermophoresis parameter Nt and Brownian motion parameter Nb in Figures 7 and 8. The parameter Nt has increasing effects towards all three fluid profiles. Thermophoresis represents the movement of particles due to the temperature gradient. Increasing values of Nt mean increasing movement due to increasing temperature gradient. Concentration will also increase following the movement of the particles. On the other hand, Brownian motion represents the motion of particles due to their collision with each other. Increasing values of Nb will cause the fluid velocity and concentration to decrease, but as the collision produces heat, the fluid temperature will increase. However, note that temperature under passive control remains constant and is not affected by the changes in Nb .

Figure 7. Impact of thermophoresis parameter Nt on (a) velocity, (b) temperature, and (c) nanoparticle volume fraction profiles.

Figure 8. Impact of Brownian motion parameter Nb on (a) velocity, (b) temperature, and (c) nanoparticle volume fraction profiles.

Last but not least are the influences of Lewis numbers Le and Pr towards the fluid profiles as shown in Figures 9 and 10, respectively. Interesting results can be seen in the temperature profile between active and passive controls when Le is varied, where the trends turn out to be opposite to each other. Increasing Le will cause the temperature to rise under passive control but to decline under active control. Other than that, the fluid has a decreasing trend when Le and Pr increase.

Figure 9. Impact of Lewis number Le on (a) velocity, (b) temperature, and (c) nanoparticle volume fraction profiles.

Figure 10. Impact of Prandtl number Pr on (a) velocity, (b) temperature, and (c) nanoparticle volume fraction profiles.

4.2. Skin Friction Coefficient, Nusselt Number, and Sherwood Number

Tables 2 and 3 list the values of $f''(0)$, $-\theta'(0)$, and $-\phi'(0)$ under both active and passive controls. Magnitude of the skin friction coefficient shows similar trends under active control as well as passive control when γ , Nt , and Pr are varied. It increases with increasing γ in opposing flows but decreases with increasing γ in assisting flows. When there is no buoyancy force, the magnitude of $f''(0)$ will remain the same between active and passive controls. Other parameters will result in opposite trends of $f''(0)$ between active and passive controls. Parameter r , γ , N , and Pr will help in enhancing the heat transfer rate of the fluid, while parameters Nt , Nb , and Le do the opposite. Varying the fluid parameter ϵ decreases the heat transfer under active control but increases it under passive control. Meanwhile, fluid parameter δ has a very minimal influence on heat and mass transfer of the fluid, as the magnitude barely changes when δ increases and heat transfer remains stagnant under active control. Increasing values of parameters r , ϵ , γ , N , Nt , and Pr will contribute to improving the mass transfer rate of the fluid. Interestingly, parameter Nb shows a more dynamic trend under active control for skin friction coefficient and mass transfer rate. The result is highlighted in Figure 11. With small values of Nb , the trend of $f''(0)$ seems to be increasing, but at a certain point it starts to decline. On the other hand, $-\phi'(0)$ increases until Nb reaches a large enough number and starts to decrease.

Figure 11. Impact of Brownian motion parameter Nb with varied thermophoresis parameter Nt on (a) skin friction coefficient $f''(0)$ and (b) mass transfer rate $-\theta(0)$.

5. Conclusions

The model of a steady two-dimensional stagnation point flow of a Powell–Eyring nanofluid with mixed convection is examined. The main observations of the study are listed below:

- Stagnation parameter r has a bigger impact on the heat transfer rate under passive control of nanoparticles.
- Both fluid parameters ϵ and δ have minimal impacts on the fluid flow.
- Assisting flow has higher rates of heat and mass transfer as compared to the opposing flow.
- Both Brownian parameter, Nb , and thermophoresis parameter, Nt , lower the heat transfer rate of the fluid.
- The heat transfer rate is higher under passive control of nanoparticles than the heat transfer rate in active control of nanoparticles.

Active and passive controls of nanoparticles are practical areas that should not be simply avoided while stagnation points occur everywhere in many applications and instrumental designs. From the observations above, engineers, technologists, and manufacturers who use or are exposed to nanoparticles or nanofluids in their products or processes, especially involving heat exchanger systems, should take note of the importance of controlling the nanoparticles, as they can heavily affect the distribution of heat and mass fluxes that may cause defects to their instruments or final outputs.

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Nomenclature

Roman Letter

a	coefficient for stagnation velocity
c	coefficient for free stream velocity
c_*	fluid parameter
C	nanoparticle volume fraction (mol/m^3)
C_w	nanoparticle volume fraction near the wall surface for active control (mol/m^3)
C_∞	nanoparticle volume fraction outside the boundary layer region (mol/m^3)
C_{f_x}	local skin friction
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
f	space function
g	acceleration due to gravity (m/s^2)
Le	Lewis number
N	buoyancy force ratio
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number
Pr	Prandtl number
Re_x	Reynolds number
Sh_x	local Sherwood number
T	fluid temperature (K)
T_w	temperature near the wall surface (K)
T_∞	temperature outside the boundary layer region (K)
q_m	surface mass flux ($\text{kg}/\text{s m}^2$)
q_w	surface heat flux (W/m^2)
r	stagnation parameter
u	velocity component in x-direction (m/s)
u_e	free stream velocity (m/s)
u_w	velocity near the wall surface (m/s)
v	velocity component in y-direction (m/s)

Greek Letter

α	thermal conductivity (W/mK)
β	fluid parameter
β_T	thermal expansion coefficient
β_C	nanoparticle volumetric coefficient
χ	variable
δ	fluid parameter
ϵ	fluid parameter
γ	buoyancy parameter
κ	thermal diffusivity (m^2/s)
μ	dynamic viscosity (kg/ms)
ν	kinematic viscosity (m^2/s)
η	dimensionless distance
ψ	stream function
ρ	fluid density (kg/m^3)
τ	effective heat capacity ratio
τ_w	wall shear stress (Pa)
θ	temperature function
ϕ	nanoparticle volume fraction function

Superscript

$'$	differentiation with respect to η
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Subscript

w	surface conditions
∞	free stream conditions

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