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Multiple Slip Impact on the Darcy–Forchheimer Hybrid Nano Fluid Flow Due to Quadratic Convection Past an Inclined Plane

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Abstract: Nowadays, the problem of solar thermal absorption plays a vital role in energy storage in power plants, but within this phenomenon solar systems have a big challenge in storing and regulating energies at extreme temperatures. The solar energy absorber based on hybrid nanofluids tends to store thermal energy, and the hybrid nanofluids involve the stable scattering of different nano dimension particles in the conventional solvent at a suitable proportion to gain the desired thermophysical constraints. The authors focus on the behavior of the inclined plate absorber panel as the basic solution of water is replaced by a hybrid nanofluid, including Cu (Copper) and Al_2O_3 (Aluminum Oxide), and water is utilized as a base surfactant in the current investigation. The inclined panel is integrated into a porous surface with the presence of solar radiations, Joule heating, and heat absorption. The fundamental equations of the flow and energy model are addressed with the similarity transformations. The homotopy analysis method (HAM) via Mathematica software is used to explore the solution to this problem. Furthermore, the important physical characteristics of the rate of heat transfer, omission and absorption of solar radiation, and its impact on the solar plant are observed.

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Keywords: inclined plate; Darcy–Forchheimer flow; nanomaterials (Cu , Al_2O_3); heat source; HAM

1. Introduction

The importance of energy is inevitable in our lives. Researchers and engineers are focusing to introduce advanced and affordable techniques to fulfill the requirements. Solar energy resources are one of the important and cheap resources used within the energy sector. Flat panels and thermal transition solvents are used to convert sunlight into electricity. Sunlight is captured by the instruments via an absorbing panel, which transfers heat to the absorption liquid (mainly water, a water mixture, and Ethylene Glycol). Its negative aspect is the poor specific features of such ordinary solvents, which lead to bad energy properties. Mostly in the situation of fossil energy, the transition mechanism constrained their output. One of several activities that have sparked attention in recent times to improve the thermal effectiveness of such an innovation is converting standard working fluids into nanofluids. Nanofluids are sustainable emulsions of solid materials varying in size from 1 nm to 100 nm [1]. Nanofluid is also widely used for heat exchangers [2], storing solar energy [3,4], freezing mechanisms [5]. Suresh et al. [6] performed a study on synthesized hybrid nanofluids ($Al_2O_3 - Cu$ /water). The flow of nanofluids was investigated by Mebarek-Oudina [7] using a variety of basic fluids. Through the applications of the nanofluid, Li et al. [8] scrutinized the motion of nanofluids inside a porous conduit by applying the induced external power of the Buongiorno model. Momin [9] investigated the coupled convection in an inverted cylinder for laminar flow utilizing water- Al_2O_3 and a hybrid nanofluid. The findings of Zaim [10], Sheikholeslami [11], and Gul [12] are a few examples of scientific literature in the thermal field and energy systems that have used a theoretical and mathematical framework to handle heat transport and nanofluids.

Hybrid nanofluids are heat transport solutions that include two forms of nanomaterials immersed in a traditional fluid (water) and are utilized in a variety of heat transportation technologies, comprising electrochemical and nanosensors, manufacturing, and monitoring. Hayat and Nadeem [13] analyzed the performance of an extending surface on the energy conversion of a hybrid nanofluid. Investigators claimed that by introducing 5% of nano components, the Nusselt parameter had been significantly enhanced. Chamkha [14] modeled and computed the modeled movement of a $Cu - Al_2O_3 / H_2O$ hybrid nanofluid into a trapezoidal vessel as a function of time. Aziz [15] and Lund [16] planned the circulation of hybrid nanofluid across a rotating plate. The Al_2O_3 nanofluids flow over a horizontal cylinder with the combination of entropy generation is investigated by Mwesigye and Huan [17]. Lund [18] explored the role of viscous dissipation on a hybrid nanofluid ($Cu - Al_2O_3 / H_2O$) flow over a stretched surface. Sohail [19] also investigated the nanofluids' flow over a stretching surface. These materials are widely used in different industrial and engineering applications. Recently, Aladdin et al. [20] used the flow of $Cu - Al_2O_3 / H_2O$ nanofluids in the permeable medium with suction/injection applications. Kapen et al. [21], Ilyas et al. [22], and Shah [23] used $Cu - Al_2O_3 / H_2O$ nanofluids for a variety of applications.

The porous space applications may be considered in diverse areas such as environmental science, construction management, nuclear engineering, solar thermal science, and bioinformatics. Geothermal energy, chemical catalyst connections, circulation of blood in the lungs or the arteries, spongy heated tubes, and subterranean electrical wires are just a few of the operations that need fluid movement in porous space. For understanding the flow that occupies porous space, Darcy's rule is widely used. In terms of high-speed and turbulent repercussions in a porous medium, Darcy's idea is still incorrect. To modify the influence of inertia on relative permeability, Forchheimer [24] established a second-order polynomial for describing momentum. It is known as the Forchheimer factor, according to Muskat [25]. Numerous scholars studied the flow through porous media employing Darcy–Forchheimer principles in different geometries. Many of them will be addressed herein. Saif [26] proposed the motion of nanofluids via porous materials. Rasool [27] established the Darcy–Forchheimer nanofluid flow produced by a stretched surface. Sadiq [28] analyzed the Darcy–Forchheimer liquid motion through a spinning disc. Sheikholeslami [29] reported non-Darcy liquid within the transparent cylinder. Hayat [30,31] evaluated how the Darcy–Forchheimer model and *Electromagnetic-Hydrodynamic* (EMHD) affected the flow of a viscous fluid. The authors further investigated how the second law of thermodynamics can be used to analyze entropy generation. Kumar [32] employed thermal radiation to estimate the statistical solution of CNT nanofluid motion in the shape of converging and diverging tubes.

Thermal/solar radiation refers to the mechanism of energy transport processes that take the form of electromagnetic fields. The larger temperature variation between the two surfaces causes this process to develop. However, several technological activities took place to gain the desired amount of energy through advanced techniques. The relevance of that kind of radiation may be observed in heat transfer, particularly in aerospace architecture, solar plants, nuclear power plants, material sciences, furnace design, and glasses manufacturing [33,34]. The implementation of thermal radiation for heat transfer is illustrated in [35] with similar importance. Bilal [36] examined the effect of electromagnetic-hydrodynamic (EMHD) waves on a micro-polar fluid flow over a stretchable surface. Different scholars have examined a range of aspects for such a topic (see [37,38]).

The current study examines the effects of heat source, Darcy–Forchheimer model, radiation, and (Cu , and Al_2O_3) nanomaterials on the heat exchange rate and movement of H_2O as a common solvent, as well as its applications in enhancing the efficiency of inclined plate solar panels. The governing expression of heat is determined when viscous dissipation, Joule heating, and thermal radiation are taken into account. The HAM technique was utilized to evaluate the hybrid and nanofluid flows situation, which was based on differential equations. The thermal and velocity profiles are used to describe the assessment

of several significant factors. The suggested quantitative outcomes of the present study are equated to prior published results for validation purposes.

2. Mathematical Modeling

The two-dimensional, time-dependent flow of water-based hybrid nanofluids (a composition of Al_2O_3 and Cu) has been considered on an inclined plate. The inclined plate makes an angle θ with the vertical axis. The inclined plate in the shape of a solar panel is drawn in Figure 1. The Darcy–Forchheimer porous space is used in the mathematical model. The sheet surface is stretched with velocity $U_w = \frac{bx}{(1-\alpha t)}$, $a > 0$ along the x-axis as shown in the geometry.

Figure 1. Solar collector schematic model.

The stable dispersion of the solid materials (Cu and Al_2O_3) and base fluid (H_2O) is considered with slip physical conditions. In addition, energy formulation takes into consideration Joule heating, thermal radiation, and viscous dissipation. Moreover, T_w is the temperature of the wall and T_∞ the free stream. The basic governing equations for the Darcy–Forchheimer flow are written as.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v_{hnf} \left(\frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{\rho_{hnf}} F_0 u^2 \pm g \left[\beta_{hnf} (T - T_\infty) + \beta_{hnf}^2 (T - T_\infty)^2 \right] \cos \theta - \frac{v_{hnf}}{K} u, \quad (2)$$

$$\left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{k_{hnf}}{(\rho C p)_{hnf}} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{hnf}}{(\rho C p)_{hnf}} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{16}{3(\rho C p)_{hnf}} \left(\frac{\sigma^* T_\infty^3}{k^*} \frac{\partial^2 T}{\partial y^2} \right) + \frac{1}{(\rho C p)_{hnf}} Q_0 (T - T_\infty), \quad (3)$$

The permissible boundary conditions are

$$\begin{aligned} u &= \mu_{hnf} \left(\frac{\partial u}{\partial y} \right) + u_w, \quad v = 0, \quad -k_0 \left(\frac{\partial T}{\partial y} \right) = h(T - T_w), \quad \text{at } y = 0, \\ u &= 0, \quad v = 0, \quad T \rightarrow T_\infty, \quad \text{at } y \rightarrow \infty. \end{aligned} \quad (4)$$

Equation (1) concerns continuity, Equation (2) is about momentum, and Equation (3) stands for energy. The convection term is considered quadratic (nonlinear). The positive g

stand for the gravity force along the downward direction, whereas the negative g shows the contrasting force occurs due to opposite stretching against the gravity. Here, u, v are the velocity components along the x and y -directions, respectively. K is the porous surface parameter, $F_0 = \frac{C_b}{K^2}$ is the non-uniform Inertia coefficient, C_b is the positive constant, and Q_0 is the rate of heat generation/absorption. β_{hnf} is the thermal expansion coefficient of the hybrid nanofluids; ν_{hnf} is the kinematic viscosity of the hybrid nanofluids. Here, the nanomaterial volume fraction Cu is symbolized by ϕ_{Cu} , whereas $\phi_{Al_2O_3}$ indicated the Al_2O_3 nanoparticle.

The subscript hnf denotes the hybrid nanofluid, nf denotes nanofluid, and f denotes base fluid in Table 1. The thermophysical numerical values of the Al_2O_3 , Cu , and water are displayed in Table 2 from the existing literature.

Table 1. Mathematical Models of thermophysical properties.

Nanofluid	Hybrid Nanofluid
$\rho_{nf} = (1 - \phi_{Cu})\rho_f + \phi_{Cu}\rho_{Cu}$	$\rho_{hnf} = \left\{ (1 - \phi_{Al_2O_3})[1 - \phi_{Cu}]\rho_{nf} + \phi_{Cu}\rho_{Cu} \right\} + \phi_{Al_2O_3}\rho_{Al_2O_3}$
$(\rho\beta)_{nf} = (1 - \phi_{Cu})(\rho\beta)_f + \phi_{Cu}(\rho\beta)_{Cu}$	$(\rho\beta)_{hnf} = \left\{ (1 - \phi_{Al_2O_3})[1 - \phi_{Cu}](\rho\beta)_{nf} + \phi_{Cu}(\rho\beta)_{Cu} \right\} + \phi_{Al_2O_3}(\rho\beta)_{Al_2O_3}$
$(\rho C_p)_{nf} = (1 - \phi_{Cu})(\rho C_p)_f + \phi_{Cu}(\rho C_p)_{Cu}$	$\rho_{hnf} = \left\{ (1 - \phi_{Al_2O_3})[1 - \phi_{Cu}](\rho C_p)_{nf} + (1 - \phi_{Cu})(\rho C_p)_{nf} + \phi_{Cu}(\rho C_p)_{nf} + \phi_{Cu}(\rho C_p)_{Cu} \right\} + \phi_{Al_2O_3}(\rho C_p)_{Al_2O_3}$
$\mu_{nf} = \frac{\mu_f}{(1 - \phi_1)^{2.5}}$	$\mu_{nf} = \frac{\mu_{nf}}{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}}$
$\frac{k_{nf}}{k_f} = \frac{k_{Cu} + (n-1)k_f - (n-1)\phi_{Cu}(k_f - k_s)}{k_{Cu} + (n-1)k_f + \phi_{Cu}(k_f - k_s)}$	$\frac{k_{hnf}}{k_{nf}} = \frac{k_{Al_2O_3} + (n-1)k_{nf} - (n-1)\phi_{Al_2O_3}(k_{nf} - k_{Al_2O_3})}{k_{Al_2O_3} + (n-1)k_{nf} + \phi_{Al_2O_3}(k_{nf} - k_{Al_2O_3})}$ where $\frac{k_{nf}}{k_f} = \frac{k_{Cu} + (n-1)k_f - (n-1)\phi_{Cu}(k_f - k_{Cu})}{k_{Cu} + (n-1)k_f + \phi_{Cu}(k_f - k_{Cu})}$

Table 2. Nanocomposites and base fluid thermo-physical characteristics.

Property	H_2O	Al_2O_3	Cu
ρ ($Kg\ m^{-3}$)	997.1	3970	8933
c_p ($Kg^{-1}\ K^{-1}$)	4179	765	385
k ($Wm^{-1}\ K^{-1}$)	0.6071	40	400
$\beta \times 10^{-5}$ (K^{-1})	21	1.67	0.85

Introduction of the relevant dimensionless parameters.

$$u = f'(\eta) \frac{bx}{(1 - \alpha t)}, \quad v = -f(\eta) \sqrt{\frac{bv_f}{(1 - \alpha t)}}, \quad (T_w - T_\infty)\Theta(\eta) = T - T_\infty, \quad \eta = y \sqrt{\frac{b}{v_f(1 - \alpha t)}}, \quad (5)$$

$$f''' + \frac{\rho_{hnf}}{\rho_f} \frac{\mu_f}{\mu_{hnf}} \left[f f'' - (1 + Fr)(f')^2 - S \left(f' + \frac{\eta}{2} f'' \right) \right] \pm \frac{\mu_f}{\mu_{hnf}} \left[Gr\Theta + (Gr^*\Theta)^2 \right] \cos\theta - Kr f' = 0, \quad (6)$$

$$\left(\frac{k_{hnf}}{k_f} + Rd \right) \Theta'' + Pr \frac{(\rho C_p)_{hnf}}{(\rho C_p)_f} \left(f \Theta' - S \left(\Theta + \frac{\eta}{2} \Theta' \right) \right) + \frac{\mu_{hnf}}{\mu_f} Ec Pr (f'')^2 + Q Pr \Theta = 0, \quad (7)$$

$$\begin{aligned} f(0) &= 0, & f'(0) &= 1 + \gamma \left(\frac{\mu_{hnf}}{\mu_f} \right) f''(0), & \Theta'(0) &= -Bi(1 - \Theta(0)), \\ F(\infty) &= 0, & \Theta(\infty) &= 0. \end{aligned} \quad (8)$$

$$Fr = \frac{C_b}{K^{\frac{1}{2}}} x, Gr = \frac{g\beta_f(T_w - T_\infty)}{bu_w}, Gr^* = \frac{g\beta_f^2(T_w - T_\infty)^2}{bu_w}, \gamma = \sqrt{\frac{b}{v_f(1-\alpha t)}}, Rd = \frac{16}{3} \frac{\sigma^* T_\infty^3}{k^* k},$$

$$Bi = \frac{h_f}{k_0} \sqrt{\frac{v_f(1-\alpha t)}{b}}, Kr = \frac{v_f}{bk}, Ec = \frac{u_w^2}{c_p(T_w - T_\infty)}, Pr = \frac{v_f(\rho c_p)}{k_f}, Q = \frac{Q_0}{b(\rho c_p)_f}, S = \frac{\alpha}{b}. \quad (9)$$

Dimensionless expressions can be formed using the parameters mentioned above in Equation (5).

Interrelated constraints on the boundary are:

Where the above-resulting expressions represent the Darcy–Forchheimer parameter, the Grashof numbers in terms of linear and nonlinear convection, the slip parameter, the radiation variable, the Biot number, the Porosity parameter, the Eckert, Prandtl numbers, heat generation parameter, and unsteadiness parameter.

Additionally, the skin friction coefficients (C_{fx}), as well as the Nusselt number (Nu_x), are of relevance from a physical perspective:

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho_{hmf} (u_w)^2}, \quad Nu = \frac{x q_w}{k_{hmf} (T_w - T_\infty)}. \quad (10)$$

In Equation (10), τ_w signify the shear stress, where q_w denote the heat flux close to the sheet surface. Employing Equation (5), the above Equation (10) becomes:

$$C_f Re^{0.5} = \frac{2}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}} \cdot f''(0), \quad Nu Re^{-0.5} = -\left(\frac{k_{hmf}}{k_f} + Rd\right) \Theta'(0). \quad (11)$$

Method of Solution

In this section, the homotopy analysis method (HAM) is applied to solve Equations (6) and (7) with the help of the physical conditions given in Equation (8). This method was first introduced by Liao [39] for the solution of nonlinear problems. The recent advancement has been made by the authors [40–42] to make this method more effective. Mathematica software is utilized for this purpose. The following description provides a basic explanation of the model equation using the HAM method.

$$\widehat{F}(\eta) = -\left[\frac{\gamma}{(1+\gamma)}\right] \frac{\mu_{hmf}}{\mu_f} e^{-\eta}, \quad \widehat{\Theta}(\eta) = \frac{Bi}{(1+Bi)} e^{-\eta}, \quad (12)$$

$$L_{\widehat{F}}(\widehat{F}) = \widehat{f}''', \quad L_{\widehat{\Theta}}(\widehat{\Theta}) = \widehat{\Theta}'''. \quad (13)$$

Linear operators $L_{\widehat{F}}$, and $L_{\widehat{\Theta}}$ are signified as

$$L_{\widehat{F}}(c_1 + c_2\eta + c_3\eta^2) = 0, \quad L_{\widehat{\Theta}}(c_4 + c_5\eta) = 0. \quad (14)$$

We recognize the non-linear variables that are commonly referred to as $N_{\widehat{F}}$, and $N_{\widehat{\Theta}}$ in the scheme:

$$N_{\widehat{F}} \left[\widehat{F}(\eta; \zeta) \right] = \widehat{F}_{\eta\eta\eta} + \frac{\rho_{hmf}}{\rho_f} \frac{\mu_f}{\mu_{hmf}} \left[\widehat{F} \widehat{F}_{\eta\eta} - (1 + Fr) \widehat{F}_{\eta\eta}^2 - S \left(\widehat{F}_{\eta} + \frac{\eta}{2} \widehat{F}_{\eta\eta} \right) \right]$$

$$\pm \frac{\mu_f}{\mu_{hmf}} \left[Gr \widehat{\Theta} + \left(Gr \widehat{\Theta} \right)^2 \right] \cos \theta - K \widehat{F}_{\eta}, \quad (15)$$

$$N_{\widehat{\Theta}} \left[\widehat{F}(\eta; \zeta), \widehat{\Theta}(\eta; \zeta) \right] = \left(\frac{k_{hmf}}{k_f} + Rd \right) \widehat{\Theta}_{\eta\eta} + Pr \frac{(\rho c_p)_{hmf}}{(\rho c_p)_f} \left[\widehat{F} \widehat{\Theta}_{\eta} - S \left(\widehat{\Theta} + \frac{\eta}{2} \widehat{\Theta}_{\eta} \right) \right]$$

$$+ \frac{\mu_{hmf}}{\mu_f} Ec Pr \widehat{F}_{\eta\eta}^2 + Q Pr \widehat{\Theta}, \quad (16)$$

By utilizing the above expression zero-order set of the problem is:

$$(1 - \zeta) L_{\widehat{F}} \left[\widehat{F}(\eta; \zeta) - \widehat{F}_0(\eta) \right] = p \hbar_{\widehat{F}} N_{\widehat{F}} \left[\widehat{F}(\eta; \zeta) \right], \quad (17)$$

$$(1 - \zeta) L_{\widehat{\Theta}} \left[\widehat{\Theta}(\eta; \zeta) - \widehat{\Theta}_0(\eta) \right] = p \hbar_{\widehat{\Theta}} N_{\widehat{\Theta}} \left[\widehat{\Theta}(\eta; \zeta) \right], \quad (18)$$

Whereas BCs are:

$$\begin{aligned} \widehat{F}(\eta; \zeta) \Big|_{\eta=0} &= 0, & \frac{\partial \widehat{F}(\eta; \zeta)}{\partial \eta} \Big|_{\eta=0} &= 1 + \frac{\mu_{hmf}}{\mu_f} \gamma f''(0), \\ \widehat{F}(\eta; \zeta) \Big|_{\eta=\infty} &\rightarrow 0, & \frac{\partial \widehat{\Theta}(\eta; \zeta)}{\partial \eta} \Big|_{\eta=0} &= -B_i(1 - \Theta(0)), \\ \widehat{\Theta}(\eta; \zeta) \Big|_{\eta=\infty} &\rightarrow 0. \end{aligned} \quad (19)$$

where the embedding constraint is $\zeta \in [0, 1]$, to manipulate for congruence of solutions assumed $\hbar_{\widehat{F}}$, $\hbar_{\widehat{\Theta}}$ and $\hbar_{\widehat{\Phi}}$. At the value of $\zeta = 0$ and $\zeta = 1$, we have:

$$\widehat{F}(\eta; 1) = \widehat{F}(\eta), \quad \widehat{\Theta}(\eta; 1) = \widehat{\Theta}(\eta). \quad (20)$$

Considering Taylor's series, the expansion of $\widehat{F}(\eta; \zeta)$ and $\widehat{\Theta}(\eta; \zeta)$ at $\zeta = 0$ is

$$\begin{aligned} \widehat{F}(\eta; \zeta) &= \widehat{F}_0(\eta) + \sum_{n=1}^{\infty} \widehat{F}_n(\eta) \zeta^n, \\ \widehat{\Theta}(\eta; \zeta) &= \widehat{\Theta}_0(\eta) + \sum_{n=1}^{\infty} \widehat{\Theta}_n(\eta) \zeta^n. \end{aligned} \quad (21)$$

$$\widehat{F}_n(\eta) = \frac{1}{n!} \cdot \frac{\partial(\eta; \zeta)}{\partial \eta} \Big|_{p=0}, \quad \widehat{\Theta}_n(\eta) = \frac{1}{n!} \cdot \frac{\partial \widehat{\Theta}(\eta; \zeta)}{\partial \eta} \Big|_{p=0}. \quad (22)$$

While boundary conditions are:

$$\begin{aligned} f(0) &= 0, f'(0) = 1 + \frac{\mu_{hmf}}{\mu_f} \gamma f''(0), \\ F(\infty) &= 0, \Theta'(0) = -B_i(1 - \Theta(0)), \\ \Theta(\infty) &= 0. \end{aligned} \quad (23)$$

Now

$$\begin{aligned} \Re_{\widehat{F}}(\eta) &= \widehat{F}_{n-1}''' + \frac{\rho_{hmf}}{\rho_f} \frac{\mu_f}{\mu_{hmf}} \left[\sum_{j=0}^{w-1} \widehat{F}_{w-1-j} \widehat{F}_j'' - (1 + Fr) \widehat{F}_{n-1}'^2 - S \left(\widehat{F}_{n-1} + \frac{\eta}{2} \widehat{F}_{n-1} \right) \right] \\ &\pm \frac{\mu_f}{\mu_{hmf}} \left[Gr \widehat{\Theta}_{n-1} - \left(Gr \widehat{\Theta}_{n-1} \right)^2 \right] \cos \theta - K F_{n-1}' = 0, \end{aligned} \quad (24)$$

$$\begin{aligned} \Re_{\widehat{\Theta}}(\eta) &= \left(\frac{k_{hmf}}{k_f} + 1 \right) \widehat{\Theta}_{n-1}'' + \Pr \frac{(\rho C p)_{hmf}}{(\rho C p)_f} \left(\sum_{j=0}^{w-1} \widehat{F}_{w-1-j} \widehat{\Theta}_j' - S \left(\widehat{\Theta}_j + \frac{\eta}{2} \widehat{\Theta}_j' \right) \right) \\ &+ \frac{\mu_{hmf}}{\mu_f} Ec \Pr \widehat{F}_{n-1}'^2 + Q \Pr \widehat{\Theta}_{n-1} = 0, \end{aligned} \quad (25)$$

Whereas

$$\chi_n = \begin{cases} 0, & \text{if } \zeta \leq 1 \\ 1, & \text{if } \zeta > 1. \end{cases} \quad (26)$$

3. Results and Discussion

The influence of the various embedded parameters over the velocity and temperature profiles are investigated and displayed. These parameters include the inertia coefficient Fr , the Grashof numbers Gr, Gr^* , the Radiation factor Rd , the Prandtl number Pr , the Porosity factor Kr , the Eckert number Ec using the $Cu + Al_2O_3/H_2O$ hybrid nanofluid, and Cu/H_2O nanofluid. The geometry of the problem is shown in Figure 1. The total square residual sum of the obtained and displayed in Figure 2. The iterations are obtained up to the 30th order approximation. This shows that with the increasing number of iterations, the convergence rate increases. Figures 3–6 depict the performance of $F'(\eta)$ varied values of the developing parameters.

Figure 2. Total square residual sum of the problem.

Figure 3. (a–d) Impact of γ, S, Kr, Fr on $F'(\eta)$ for $Cu + Al_2O_3/H_2O$ versus Cu/H_2O .

Figure 4. (a,b) Impact of $Gr, \phi_{Cu}, \phi_{Al_2O_3}$ on $F'(\eta)$ for $Cu + Al_2O_3/H_2O$ versus Cu/H_2O .

Figure 5. (a–d) Influence of $B_i, S, \phi_{Cu}, \phi_{Al_2O_3}, Q$ on $\Theta(\eta)$ for $Cu + Al_2O_3/H_2O$ versus Cu/H_2O .

Figure 6. Parameter range of the common unsteady parameter S .

The influence of the slide factor γ on $F'(\eta)$ velocity distribution is illustrated in Figure 3a for the hybrid nano liquid ($Cu + Al_2O_3/H_2O$) and the nano liquid (Cu/H_2O). The figures indicate that the velocity profile $F'(\eta)$ declines by improving the sliding factor, but on the other hand, the temperature profiles show the opposite tendency. It is because a

reduction in surface friction between the stretched sheet and liquid occurs as the sliding factor increases. The thickness of the fluid layer also decreases as the slide parameter increases. However, an increase in the sliding parameter generates the frictional force, which allows more liquid to slide past the sheet and the deceleration of the flow, and the temperature field is increased due to the action of force. Moreover, the maximum velocity is observed for hybrid nano liquid ($Cu + Al_2O_3/H_2O$) compared with the nano liquid (Cu/H_2O).

The consequence of the unsteadiness factor S on the $F'(\eta)$ velocity profile for hybrid nanofluids ($Cu + Al_2O_3/H_2O$) and nanofluids (Cu/H_2O) is shown in Figure 3b. The velocity profiles are demonstrated to be reduced whenever the unsteadiness factor is increased. Because the width of the momentum boundary is reduced as the unsteadiness parameter is increased, the velocity along with the sheet drops. It is observed that $F'(\eta)$ is higher for hybrid nanofluid ($Cu + Al_2O_3/H_2O$) than nanofluid (Cu/H_2O). Figure 3c depicts the effect of the porosity parameter Kr . The impact of the Kr on $F'(\eta)$ for hybrid nanofluids ($Cu + Al_2O_3/H_2O$) and nanofluids (Cu/H_2O) is investigated. An increase in the permeability parameter reduces the fluid motion and this happens due to the resistance force generation due to the porous medium. Similarly, in evaluation, the $F'(\eta)$ velocity $Cu + Al_2O_3/H_2O$ is shown to be greater than that of Cu/H_2O .

In Figure 3d, a velocity profile for hybrid nanofluids ($Cu + Al_2O_3/H_2O$) and nanofluids (Cu/H_2O) is compared for various values of Fr (inertia coefficient parameter). The velocity profile shows a decreasing influence as the amount of the object increases in the graph Fr (Inertia coefficient factor). Inertia is a mass's physical ability to resist transition, allowing it to move more powerfully; as a result, it reduces fluid motion. It is also found that the velocity of fluid $F'(\eta)$ of Cu/H_2O nanofluids is lower than that of $Cu + Al_2O_3/H_2O$. The Grashof numbers (Gr) influence on the velocity profile consisting of hybrid nanofluids ($Cu + Al_2O_3/H_2O$) and nanofluids (Cu/H_2O) are displayed in Figure 4a. This graph shows that when the value rises, the fluid velocity rises as well. This is because Gr (the thermal Grashof number) includes both thermal and hydrodynamic buoyancy forces and arises on the boundary layer as a result of temperature changes. In addition, the inset plot reveals that Cu/H_2O the nanofluid velocity field $F'(\eta)$ is often lower than the ($Cu + Al_2O_3/H_2O$) hybrid nanofluid. The flow field of both hybrid nanofluid ($Cu + Al_2O_3/H_2O$) and (Cu/H_2O) nanofluids is influenced by nanoparticle concentration ϕ_{Cu} , $\phi_{Al_2O_3}$, as seen in Figure 4b. As the volume fraction increases, the proportion of nanoparticles increases, and as a result, the velocity and flow boundary layer reduces. Nanofluid (Cu/H_2O) flow slower than the ($Cu + Al_2O_3/H_2O$) hybrid nanofluid for volume fraction increment, it is because Cu/H_2O is denser than $Cu + Al_2O_3/H_2O$. The Biot number B_i is a dimensionless measure of the relative transit of external and interior resistances. The hot fluids heat the lower surface of an extending surface as the temperature of the sheet rises, resulting in convective heat transfer. As a result, the thermal boundary layer is enhanced, as displayed in Figure 5a for hybrid nanofluid ($Cu + Al_2O_3/H_2O$) and nanofluid (Cu/H_2O), respectively. Because the coefficient of heat transfer is in direct relation to B_i , the non-dimensional heat transfer rate for the hybrid nanofluid increases dramatically when compared with nanofluid.

Figure 5b illustrates that the $\Theta(\eta)$ temperature distribution for hybrid nano liquid ($Cu + Al_2O_3/H_2O$) is stronger compared with common nano liquid (Cu/H_2O). In this figure, the temperature profile increases with the positive incrementation in the value of S .

As shown in Figure 5c, the $\Theta(\eta)$ temperature profile of nanoscale volume fractions is influenced by a variety of factors ϕ_{Cu} , $\phi_{Al_2O_3}$. With increasing particle concentrations ϕ_{Cu} , $\phi_{Al_2O_3}$, this figure shows a significant rise in the temperature profile $\Theta(\eta)$. The cause for this is that adding varying volume fractions of nanoparticles ϕ_{Cu} , $\phi_{Al_2O_3}$ changes the heat features of the hosting fluid, which raises the temperature $\Theta(\eta)$. The hybrid nanofluid ($Cu + Al_2O_3/H_2O$) demonstrates its domination over nanofluid (Cu/H_2O) in this graph.

Figure 5d depicts the outlines of the $Cu + Al_2O_3/H_2O$ (hybrid nanofluid) and Cu/H_2O (nanofluid) versus thermal fields $\Theta(\eta)$ as a function of various values of Q . The $\Theta(\eta)$ is enhanced when the value of Q (heat source/sink) increases, as shown in Figure 5d. On

In Table 5, the enhancement in heat transfer rate for the hybrid nanofluid ($Cu + Al_2O_3/H_2O$) and nanofluids (Cu/H_2O) have been calculated in percentage and the effect of $\phi_{Cu}, \phi_{Al_2O_3}$ against $Nu_x Re_x^{-0.5}$. Figure 7 explains the % wise statistical data for each one parameter versus heat transfer rate, whereas Figure 8 is picketed for $\phi_{Cu}, \phi_{Al_2O_3}$ versus $Nu_x Re_x^{-0.5}$. In light of the obtained results, it has been observed that hybrid nanofluid is more efficient for augmentation of heat transfer rate in comparison with nanofluid.

Table 6 compares the current outcomes to those of Wang (Golra and Sidawi) [43,44] in order to explain the best agreement. It is observed that there is a clear consensus amongst the current findings and those in Refs. [43,44].

Table 5. For each nanoparticle, the heat transmission has been determined in percent $Pr = 6.2, S = 0.1, Ec = 0.3$, by applying the percentage % formula
 Percentage Increase = $\frac{\text{With Nano-particle}}{\text{Without Nano-particle}} \times 100 = \text{Result}$,
 Result – 100 = Percentage enhancement.

$\phi_{Cu}, \phi_{Al_2O_3}$	$\Theta'(0)$ Nanofluid <i>Cu</i>	$\Theta'(0)$ Hybrid Nanofluid <i>Cu+Al₂O₃</i>
0.0	2.06469 (0% Increase)	2.06469 (0% Increase)
0.01	2.09138 (1.29% Increase)	2.09857 (1.64% Increase)
0.02	2.11871 (2.62% Increase)	2.13360 (3.34% Increase)
0.03	2.14671 (3.97% Increase)	2.16984 (5.09% Increase)

Figure 7. Percentage versus $\phi_{Cu}, \phi_{Al_2O_3}$.

Figure 8. Versus $\phi_{Cu}, \phi_{Al_2O_3}$.Table 6. Comparison with [43,44] taking $Nu_x Re_x^{-0.5}$.

Pr	Ref. [43]	Ref. [44]	Present
6.2	1.54276	1.54288	1.54289
6.4	1.52165	1.52176	1.52178
6.6	1.50043	1.50054	1.50057

4. Conclusions

Physical characteristics of the hybrid nanofluid flow over an inclined surface are investigated in this study. H_2O (Water) is utilized as a base liquid in a hybrid nanofluid containing nanoparticles. The current study has looked at the Darcy–Forchheimer model, solar radiation, heat source, and viscous dissipation. The fundamental physical characteristics of the heat transfer rate and solar radiation influences are determined. The uses of these characteristics are related to the same solar panel design. The main outputs are obtained as:

- The velocity of the Cu/H_2O and $Cu + Al_3O_3/H_2O$ nanofluids decrease with increasing the slip parameter γ .
- The temperature profile $\Theta(\eta)$ was assessed using a higher number of B_i and Q variables.
- The heat transfer rate accelerates as the scale of the thermal radiation parameter increases, and as a result, the Nusselt number rises.
- The heat transfer rate of hybrid nanofluid ($Cu + Al_3O_3/H_2O$) seems to be higher than that of nanofluid (Cu/H_2O).
- Although nanofluids are more sticky than ordinary fluids, their boiling point is greater than that of conventional base liquids. It could help to increase the heat transfer capacity of the solar panel.

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